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ROBUST CONSTRAINED MODEL PREDICTIVE CONTROL
FOR PIECEWISE AFFINE SYSTEMS
USING SATURATED LINEAR FEEDBACK CONTROLLER

Mr. Santo Wijaya

A Thesis Submitted in Partial Fulfillment of the Requirements
for the Degree of Master of Engineering Program in Electrical Engineering
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ซานโต วิจายา: การควบคุมเชิงทำนายแบบจำลองด้วยเงื่อนไขคงทนสำหรับระบบสัมพรรคเป็นช่วง ด้วยตัวควบคุมป้อนกลับเชิงเส้นแบบอิ่มตัว (ROBUST CONSTRAINED MODEL PREDICTIVE CONTROL FOR PIECEWISE AFFINE SYSTEMS USING SATURATED LINEAR FEEDBACK CONTROLLER), อ. ที่ปรึกษา: ผศ. ดร.มานพ วงศ์สายสุวรรณ, 69 หน้า

ระบบสัมพรรคเป็นช่วงจัดอยู่ในหมวดหมู่ของระบบลูกผสมที่กำลังเป็นที่สนใจ ระบบสัมพรรคเป็นช่วงสามารถพิจารณาเป็นกลุ่มของแบบจำลองตามปกติสำหรับระบบไม่เชิงเส้น เนื่องจากเราสามารถประมาณความไม่เป็นเชิงเส้นประเภทหนึ่งได้ด้วยฟังก์ชันเชิงเส้นหรือฟังก์ชันสัมพรรค การประมาณในลักษณะนี้กำลังถูกวิจัยกันอย่างกว้างขวาง วิทยานิพนธ์ฉบับนี้นำเสนอการประยุกต์ใช้กรอบการออกแบบตัวควบคุมเชิงทำนายแบบจำลองด้วยเงื่อนไขคงทนสำหรับระบบเวลาเต็มหน่วยซึ่งมีความไม่แน่นอนเชิงพารามิเตอร์ กฎการควบคุมใช้ฟังก์ชันเลียปูนอฟที่แปรตามพารามิเตอร์ซึ่งตรงกับจุดยอดของรูปหลายด้านที่บรรยายความไม่แน่นอนของระบบสัมพรรคเป็นช่วง เพื่อให้ระบบมีสมรรถนะคงทน พิจารณากฎการควบคุมป้อนกลับแบบเชิงเส้นที่มีความอิ่มตัวเพื่อให้ได้ระบบป้อนกลับสัมพรรคเป็นช่วงที่ดัดแปลงง่ายและมีเสถียรภาพคงทน การออกแบบแบ่งออกเป็นสองส่วน ส่วนแรก พิจารณาเฉพาะกฎการควบคุมคงทนสำหรับระบบสัมพรรคเป็นช่วงเปลี่ยนแปลงตามเวลาที่มีความไม่แน่นอนและไม่มีเวลาประวิง ส่วนที่สอง เน้นการออกแบบตัวควบคุมคงทนสำหรับระบบสัมพรรคเป็นช่วงเปลี่ยนแปลงตามเวลาที่มีความไม่แน่นอนและมีเวลาประวิง นอกจากนั้น เรายังแยกพิจารณาระบบอีกสองกลุ่มย่อย คือ ระบบระบบสัมพรรคเป็นช่วงที่มีเวลาประวิงคงที่ และระบบที่มีเวลาประวิงเปลี่ยนแปลงตามเวลา เราออกแบบโดยเปลี่ยนปัญหาให้เป็นปัญหาการหาค่าเหมาะสมที่สุดโดยใช้อสมการเมทริกซ์เชิงเส้นและแก้ปัญหาแบบออนไลน์เพื่อประกันเสถียรภาพคงทนของระบบวงรอบปิด หากปัญหาการหาค่าเหมาะสมที่สุดนี้มีคำตอบ ก็จะสามารถประกันเสถียรภาพเชิงเส้นกำกับของระบบได้ ผลการจำลองเชิงตัวเลขแสดงให้เห็นว่ากรอบการออกแบบที่นำเสนอนี้เหมาะสมและมีประสิทธิภาพดี

ภาควิชา ......วิศวกรรมไฟฟ้า...... **ลายมือชื่อนิสิต** ......SANTO......
สาขาวิชา ......วิศวกรรมไฟฟ้า...... **ลายมือชื่อ อ.ที่ปรึกษาวิทยานิพนธ์หลัก** ....
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KEYWORDS: ROBUST CONSTRAINED MODEL PREDICTIVE CONTROL / UNCERTAIN PIECEWISE AFFINE SYSTEM / PARAMETER-DEPENDENT LYAPUNOV FUNCTION / LINEAR MATRIX INEQUALITIES

SANTO WIJAYA: ROBUST CONSTRAINED MODEL PREDICTIVE CONTROL FOR PIECEWISE AFFINE SYSTEMS USING SATURATED LINEAR FEEDBACK CONTROLLER, THESIS ADVISOR: ASST. PROF. MANOP WONGSAISUWAN, Ph.D., 69 pp.

Piece-Wise Affine (PWA) systems belong to a promising class of hybrid systems. PWA systems can be considered as a natural model class for nonlinear systems since they can be used to approximate a range of nonlinearities by a set of affine/ linear function. This class of representation is still an active research area. This thesis presents the application of the Robust Constrained Model Predictive Control (RCMPC) design framework for discrete-time PWA systems under parametric uncertainties. In order to guarantee robust performance, the control law applies a parameter-dependent Lyapunov function which corresponds to the vertices of the polytopic uncertainties of the PWA systems. We consider saturated linear feedback control law in deriving tractable and robustly stable closed-loop PWA systems. The design approach is divided into two parts. The first part focuses on the design of a robust control law for uncertain time-varying PWA systems with delay-free. The second part emphasizes on the design of a robust control law for uncertain time-varying PWA systems with time-delays. Moreover, we consider two sub-parts of the systems, which are PWA systems with time-invariant delays and PWA systems with time-varying delays. The design formulations are then cast as a Linear Matrix Inequalities (LMIs) optimization problem and the algorithms are solved on-line to guarantee the robust stability of the closed-loop systems. Once a feasible solution is found, the system is guaranteed to be asymptotically stable for time $k \geq 0$. The numerical results demonstrate that the framework adopted here is suitable and effective.

Department Electrical Engineering
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Student's signature SANTO W.
Advisor's signature

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List of Notations

Symbols

$\prec, \preceq, \succ, \succeq$	Componentwise inequalities (for vectors)
max, min	Componentwise maximum or minimum (for vectors)
R	Input weighting matrix of the cost function
Q	State weighting matrix of the cost function
Ω_i	Uncertainties set of the corresponding systems mode
F	State feedback gain matrix
P	Lyapunov matrix
J_∞	Worst-case performance index
γ	Upper bound of J_∞
$(.)^T$	Transpose of (.)
$*$	Symmetric matrix structure in LMIs
$x(k+j\|k)$	State at time $k+j$, predicted based on the measurement at time k
$y(k+j\|k)$	Plant output at time $k+j$, predicted based on the measurement at time k
$u(k+j\|k)$	Control input at time $k+j$, computed at time k
$\bar{x}$	$= [\ x\ \ 1\ ]^T$,
$\vec{u}(k)$	$= \left[u(k)^T, u(k+1)^T, u(k+2)^T, \dots\right]^T$, $\vec{u}$ is infinite dimensional.
$w(k) \in \mathbb{R}^{(\tau_p+1)n}$	$= \left[x(k)^T, x(k-\tau_1)^T, x(k-\tau_2)^T, \dots, x(k-\tau_p)^T\right]^T$,
$v(k) \in \mathbb{R}^{(p+2)n}$	$= \left[x(k)^T, x(k-1)^T, \dots, x(k-\tau_1)^T, \dots, \dots, x(k-\tau_{p-1}+1)^T, \dots, x(k-\tau_p)^T\right]^T$.
$\mathfrak{L}_d$	$\triangleq \{\underline{\tau}_d, \underline{\tau}_d+1, \dots, \overline{\tau}_d\}$,
$\mathfrak{m}(k)$	$\triangleq [\bar{x}(k)^T, \bar{x}(k-1)^T, \bar{x}(k-2)^T, \dots, \bar{x}(k-\overline{\tau}_p)^T]^T$,
$\breve{\mathfrak{m}}(k)$	$\triangleq [\bar{x}(k)^T, \mathfrak{m}_1(k)^T, \dots, \mathfrak{m}_p(k)^T]^T$,
$\mathfrak{m}_d(k)$	$= [\bar{x}(k-\underline{\tau}_d)^T, \dots, \bar{x}(k-\overline{\tau}_d)^T]^T$,

Acronyms

Co	Convex hull
MPC	Model Predictive Control
RCMPC	Robust Control Model Predictive Control
PWA	Piece-Wise Affine
LMIs	Linear Matrix Inequalities
LPV	Linear Parameter-Varying
TID	Time-Invariant Delay
TVD	Time-Varying Delay

CHAPTER I

INTRODUCTION

1.1 Research Motivation

At the beginning, Model Predictive Control (MPC) is developed to meet the specialized control needs of power plants and petroleum refineries, but it can now be found in a wide variety of application areas and has been applied successfully in the process industries (see [1, 2] and the references therein).

Due to its advantages over other control methods, and the fact that many new results of MPC technology breakthrough have been proposed by researchers in the recent years (see [3]– [4]), we can think of that MPC is still being further developed as an interesting topic. Most of the recent proposed methods are based on Robust Constrained MPC (RCMPC), and the optimization problem is cast as LMIs. This method is able to handle the drawbacks occurred in the existing MPC-based control techniques. One of these is its inability to explicitly incorporate plant model uncertainties. Since the design algorithm is based on the prior knowledge of the model, it is obvious that the obtained benefits will be affected by the plant-model mismatch. The other drawback is the computational complexities of the optimization problems that need to be solved through conventional linear or quadratic programming [5]. We are interested in doing the research on the RCMPC using LMIs because of the above reasons. With this method we are able to incorporate input and output constraints, and a description of the plant uncertainties, and guarantee certain robustness properties.

The theory of the MPC for linear systems is quite mature, but its extension to hybrid systems is still an active research area. Recent research has been focused on developing stabilizing controllers for hybrid systems and in particular for PWA systems (see [6–12] and the references therein). PWA systems belong to the promising class of representation of nonlinear systems by approximating the nonlinearity with linear or affine functions. PWA systems can be considered as a natural model class for nonlinear systems since they have been used to represent a range of nonlinearities such as dead zones, saturations, and hysteresis with arbitrary accuracy [9]. In practice, uncertainties and robustness are central themes in the modeling and anaylsis of PWA systems [6]. The research is focusing on the implementation of RCMPC controller synthesis to the PWA systems.

1.2 Literature Review

MPC is an effective multivariable constrained control algorithm in which dynamic optimization problem is solved on-line. At each sampling time, MPC uses an explicit process model to compute process inputs so as to optimize future plant behaviour over a time interval known as the prediction horizon [13]. The optimization yields an optimal control sequence and the first control in this sequence is applied to the plant. At the next sampling time, the optimization problem is reformulated and solved with new measurements obtained from the system [5, 14]. However, the receding horizon strategy does not inherently imply the closed-loop stability and optimality. In these areas, the subject has developed to a stage where it has achieved sufficient maturity to warrant the active interest in nonlinear control. The stability method that we employ in the research is the direct approach using Lyapunov quadratic function. Model uncertainties are also taken into the consideration to maintain the robustness of the MPC algorithm [3, 15].

For an uncertain system, an infinite horizon performance function can also be used to guarantee the robust stability of MPC algorithm. In RCMPC algorithm, a min-max optimization problem needs to be solved online because the forecast of the system behaviour is not exclusive. Hence, the design problem of RCMPC can be reduced to making min-max optimization problem with infinite horizon performance function tractable [3]. Kothare et al. [5] proposed an RCMPC based on LMI in which a linear state feedback control law is used to transformed the optimization problem with infinite variables into one with finite variables and maintain the state vector inside invariant feasible sets. This method provides the basic of the design of RCMPC algorithm. Furthermore, the state feedback RCMPC has been studied extensively in [15–18]. A thorough overview of robustness in MPC is given in [19].

However, the performance of the systems is highly limited in the presence of actuator saturation. There are many proposed methods on the stability of the systems in the presence of input saturation because it is a classical issue in the real systems. Li et al. [3] proposed an RCMPC formulation to tackle this issue by using saturated linear feedback controller, in which the saturation control can be described as a convex hull of a group of linear control. Besides actuator saturation, the other real-life problem concerns dealing with the time-delay. Time-delay systems were discussed extensively in [20]. An overview of some control approaches and open problems in time-delay systems was presented. In [18], the authors proposed an observer-based robust control for uncertain linear systems with time-delay. An application of RCMPC for systems with state-delay can be found in [4, 21]. In [7], the RCMPC for systems with time-delay has been generalized to time-invariant state and input delay and then the authors extended it furthermore to time-varying state and input delay in [22].

PWA systems belong to a class of hybrid systems, for which the switching rule between different linear/affine dynamics is given by polyhedral partition of the state+input set. *Hybrid systems* are dynamical systems whose behaviour is determined by both continuous and discrete dynamics. Such systems are characterized by both variables or signals that take value from continuous sets, and variables that take values from discrete, typically finite, sets. These continuous or discrete-valued variables or signals may either depend on independent variables such as time, which also may be continuous or discrete, or be driven asynchronously by external or internal discrete events [11]. The modeling power of PWA systems has already been shown in several applications, such as switched power converters, optimal control of DC-DC converters and direct torque control of three-phase induction motors, application to automotive systems, and systems biology, to mention just a few (see [2, 12], and the references therein).

PWA systems have been studied extensively in [23]. A computational approach for stability analysis of PWA systems and an extension of some aspects in linear systems and quadratic criteria to PWA systems and piecewise quadratic Lyapunov criteria. An approach to optimal control of PWA systems using feedback laws derived from the solution of Hamilton-Jacobi-Bellman equation was presented. In [24] the authors have proven the equivalence of PWA systems and other class of hybrid systems such as Linear Complementarity (LC) systems, and Extended Linear Complementarity (ELC) systems, Max-Min-Plus-Scaling (MMPS) systems, and Mixed Logic Dynamical (MLD) systems. Each modeling framework has its advantages and the equivalence of the hybrid dynamical systems allows one to easily transfer the theoretical properties and tools from one class to another. In [12], a study of stability and robustness of PWA systems using MPC approach was proposed. In the area of PWA with time-delay systems, the stability analysis of PWA with time-delay systems using piecewise quadratic Lyapunov function was proposed in [25]. XiangYong Mu et al. [26] introduced state feedback control strategy based on ellipsoid for Piece-Wise Affine system with time-delay.

Despite that PWA systems received a lot of attention in the last decades, unfortunately, an application of RCMPC for PWA systems pays to a lesser extent. One of the first results in extending the RCMPC algorithm for PWA systems is obtained in [9]. Multiple model MPC technique involving a sequence of local state feedback matrices, and utilizing a single quadratic Lyapunov function was presented in the paper. In [6] the design of linear state-feedback control law, and multiple quadratic Lyapunov functions was extended from the previous result, the uncertain PWA systems has the form in which the parameters of each submodel in each polyhedral partition of the state space has a different polytopic uncertainties description.

In the thesis, we apply the result in [3] to the uncertain PWA systems. The saturated control law yields a less conservative result then the algorithm proposed in [6]. We are also extending the results in [7, 22] to the uncertain PWA systems with time-delays. In particular, we will show via numerical examples that the developed algorithms are suitable and effective.

1.3 Thesis Objective

The main objective of this research is to apply the Robust Constrained MPC design framework to discrete-time PWA systems under parametric uncertainties. In order to guarantee robust performance, the control law applies a parameter-dependent Lyapunov function which corresponds to the vertices of the polytopic uncertainties of the PWA systems. We consider saturated linear feedback control law in deriving tractable and robustly stable closed-loop PWA systems.

The design approach is divided into two parts. The first part focuses on the design of a robust control law for uncertain time-varying PWA systems with delay-free. The second part emphasizes on the design of a robust control law for uncertain time-varying PWA systems with time-delays. Moreover, we consider two sub-parts of PWA systems with time-delays, which are PWA systems with time-invariant delays and PWA systems with time-varying delays. The design formulations are then cast as a Linear Matrix Inequalities optimization problem and solved on-line to guarantee the robust stability of the closed-loop systems.

1.4 Scope of Thesis

1. To develop an extension of RCMPC for PWA systems with delay-free using saturated linear feedback controller.
2. To develop RCMPC for PWA systems with time-invariant delays using saturated linear feedback controller.
3. To extend further RCMPC for PWA systems with time-varying delay using saturated linear feedback controller.

1.5 Methodology

1. Literature review on RCMPC for PWA systems with delay-free using linear feedback controller
2. Derivation of an extension of RCMPC for PWA systems with delay-free using saturated linear feedback controller.
3. Literature review on RCMPC for LPV systems with time-invariant and time-varying delays systems.
4. Derivation of a new extension of RCMPC for PWA systems with time-invariant and time-varying delays.
5. Development of a computer program for implementing all of the design formulations.
6. Simulation under MATLAB environment and comparison of the results with existing control methods.

1.6 Contributions

1. An extended design formulation of RCMPC for PWA systems with delay-free using saturated linear feedback control law.
2. A new design formulation of RCMPC for PWA systems with time-invariant delays using saturated linear feedback controller.
3. An extension of design formulation of RCMPC for PWA systems with time-varying delays using saturated linear feedback controller.
4. A computational tool for RCMPC used in several applications.

1.7 Structure of Thesis

The organization of the thesis is as follows. In the next chapter, the mathematical preliminary is explained. Chapter 3 presents RCMPC for PWA systems with delay-free. Chapter 4 presents RCMPC for PWA systems with time-invariant delays. Chapter 5 presents RCMPC for PWA systems with time-varying delays. Chapter 6 presents numerical examples of the design formulations. In the last chapter, conclusions are given.

CHAPTER II

MATHEMATICAL PRELIMINARY

In this chapter, an overview of the fundamental theory used in designing RCMPC for PWA systems is given.

2.1 Piece-Wise Affine Systems

A PWA dynamical systems is a nonlinear systems described by

$$\begin{cases} x(k+1) &= f(x(k), u(k), k) \\ y(k) &= g(x(k), u(k), k) \end{cases} \tag{2.1}$$

whose right-hand side is a piecewise affine function of its arguments. It is assumed that a PWA systems and a corresponding partition of the state space with polytopic cells $\mathcal{X}_i,\ i \in \mathcal{I}$. We are concentrating on discrete-time models of PWA dynamical systems. The motivation stems from the need to analyze these systems and to solve optimization problems, for which the continuous time counterpart would not be easily computable. Fig. 2.1 illustrates the PWA system dynamics [12].

For systems (2.1), we adopt the following definition of trajectories or solutions presented in [23].

Definition 2.1 (Trajectory). *Let $x(k) \in \cup_{i \in I} \mathcal{X}_i$ be an absolutely continuous function. We say that $x(k)$ is a trajectory of the system (2.1) on $[k_0, k_f]$ if, for almost all $k \in [k_0, k_f]$, the equation $x(k+1) = f(x(k), u(k), k)$ holds for all i with $x(k) \in \mathcal{X}_i$.*

From an analysis point of view, however, the main obstacle will be the cases when no continuation of a trajectory in the sense of Definition 2.1 is possible. The following definition allow us to single out such situations. For sake of clarity, we will present the main results in this thesis for systems without attractive sliding modes.

Definition 2.2 (Attractive sliding mode). *The system (2.1) is said to have an attractive sliding mode at x_s if there exists a trajectory with final state x_s but no trajectory with initial state x_s.*

For the first part of the research, we consider the following time-varying discrete-time PWA systems with delay-free

$$\begin{cases} x(k+1) &= A_i(k)x(k) + a_i(k) + B_i(k)u(k) \\ y(k) &= C_i(k)x(k) \end{cases} \quad \text{for } x(k) \in \mathcal{X}_i. \tag{2.2}$$

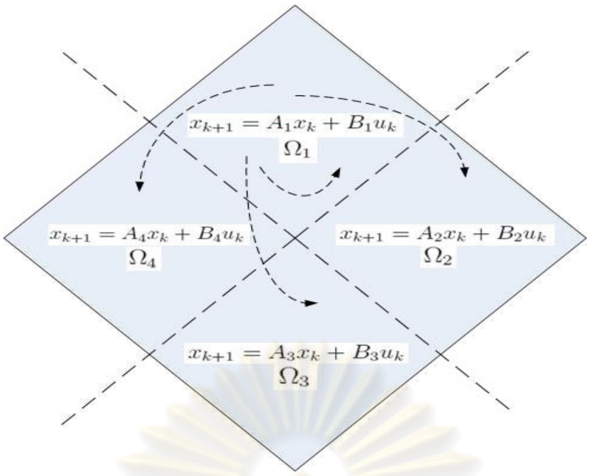

Figure 2.1: A PWA system: in each polyhedral region a different affine dynamics is active. The system dynamics changes when the state crosses the switching boundaries (denoted by dotted diagonal lines).

The next part, we consider the following time-varying discrete-time PWA systems with time-delay

$$\begin{cases} x(k+1) &= A_i(k)x(k) + \sum_{d=1}^{p} A_{di}(k)x(k-\tau_d(k)) \\ & \quad +a_i(k) + B_i(k)u(k) \\ y(k) &= C_i(k)x(k) \end{cases} \quad \text{for } x(k) \in \mathcal{X}_i. \tag{2.3}$$

We consider in particular the systems (2.3) in the form of two main models:

1. Systems (2.3.1) - PWA systems with time-invariant delay.
   Where $\tau_d,\ d \in \mathcal{D} \triangleq \{1, 2, \ldots, p\}$ is integer time-invariant state-delay satisfying $0 < \tau_1 < \tau_2 < \ldots < \tau_p$.
2. Systems (2.3.2) - PWA systems with time-varying delay.
   Where $\tau_d(k),\ d \in \mathcal{D}$ is integer time-varying state-delay satisfying $0 < \underline{\tau}_1 < \tau_1 < \overline{\tau}_1 < \underline{\tau}_2 < \tau_2 < \overline{\tau}_2 < \ldots < \underline{\tau}_p < \tau_p < \overline{\tau}_p$.

Here, $x \in \mathbb{R}^n$ is the discrete state vector, $u \in \mathbb{R}^m$ is the input vector, and $y \in \mathbb{R}^m$ is the output vector at the discrete-time instant $k \geq 0$. The matrices $A_i, A_{1i}, \ldots, A_{pi}, a_i, B_i, C_i$ are time-varying and of compatible dimensions, respectively.

The regions $\mathcal{X}_i \subseteq \mathbb{R}^n$ are assumed to be closed (possibly unbounded) n-dimensional convex polyhedra which we call *cells*. Following [11, 12, 23], each cell is constructed as the intersection of a finite number (p_i) of half spaces

$$\mathcal{X}_i = \left\{ x | H_i^T x - g_i \leq 0 \right\}, \tag{2.4}$$

where $H_i = [h_{i1}, h_{i2}, \ldots, h_{ip_i}]$, $g_i = [g_{i1}, g_{i2}, \ldots, g_{ip_i}]^T$. Moreover, the set of cell indices is denoted $\mathcal{I}$ and the union of all cells, $\mathcal{X} = \cup_{i \in \mathcal{I}} \mathcal{X}_i$, will be referred to as *partition*. We also assume that the cells have disjoint interior so that any two cells may only share common boundary, $\mathcal{X}_i \cap \mathcal{X}_j = \emptyset, \forall i \neq j$.

Many results in this thesis are concerned with the analysis of equilibria. Unless stated otherwise, we will assume that the equilibrium point of interest is located at $x = 0$. It is then convenient to let $\mathcal{I}_0 \subseteq \mathcal{I}$ be the set of indices for cells that contain the origin and $\mathcal{I}_1 \subseteq \mathcal{I}$ be the set of indices for cells that do not contain the origin. It is assumed that $a_i = 0$ for $i \in \mathcal{I}_0$.

2.2 A Matrix Parameterization

Matrix parameterization will be useful in deriving the RCMPC of PWA sytems formulation subject to systems condition $x \in \mathcal{X}_i, i \in \mathcal{I}_1$. For convenient treatment of affine terms, we define

$$\bar{x}(k) = \begin{bmatrix} x(k) \\ 1 \end{bmatrix}$$

Throughout this thesis, a bar over a signal vector denotes the *augmentation* of the vector with the unit element 1. Somewhat informally, a bar over a matrix indicates that it has been modified to be compatible with the augmented signal vector. Hence, the PWA systems (2.2, 2.3.1, 2.3.2) can be parameterized directly to introduce compact notation of the model,

$$\left.\begin{aligned} x(k+1) &\triangleq A_i(k)x(k) + \sum_{d=1}^{p} A_{di}(k)x(k-\tau_d(k)) + B_i(k)u(k), \\ y(k) &\triangleq C_i(k)x(k), \end{aligned}\right\} \text{for } x(k) \in \mathcal{X}_i, i \in \mathcal{I}_0 \tag{2.5}$$

$$\left.\begin{aligned} \bar{x}(k+1) &\triangleq \bar{A}_i(k)\bar{x}(k) + \sum_{d=1}^{p} \bar{A}_{di}(k)\bar{x}(k-\tau_d(k)) + \bar{B}_i(k)u(k), \\ y(k) &\triangleq \bar{C}_i(k)\bar{x}(k), \end{aligned}\right\} \text{for } x(k) \in \mathcal{X}_i, i \in \mathcal{I}_1 \tag{2.6}$$

with

$$\bar{A}_i(k) = \begin{bmatrix} A_i(k) & a_i(k) \\ 0_{1\times n} & 0 \end{bmatrix}, \bar{A}_{di}(k) = \begin{bmatrix} A_{di}(k) & 0 \\ 0_{1\times n} & 0 \end{bmatrix},$$

$$\bar{B}_i(k) = \begin{bmatrix} B_i(k) \\ 0 \end{bmatrix}, \bar{C}_i(k) = \begin{bmatrix} C_i(k) & 0 \end{bmatrix},$$

where $d = 0$ represents systems (2.2), and $d > 0$ represents systems (2.3.1, 2.3.2).

2.3 Models for Uncertain Systems

Two paradigms of uncertain systems that are commonly encountered in robust control, namely, a *polytopic* uncertain model, and a *norm-bound* uncertain model. These paradigms arise from two different modeling and identification procedures. We emphasizes a *polytopic* uncertain model, and considering the case when the system matrices, generally, $[A_i|A_{di}|B_i] \in \Omega_i, \forall d \in \mathcal{D} \triangleq \{1, 2, \ldots, p\}, \forall i \in \mathcal{I}$, for each cell can be written as a convex combination of matrices $\Omega_i^1, \ldots, \Omega_i^L$. In other words, we assume that for every k there exist scalars $\lambda_l(k) \geq 0$ with $\sum_l \lambda_l(k) = 1$ such that $\Omega_i(k)$ can be written as

$$\Omega_i(k) = \sum_{l=1}^{L} \lambda_l(k)\Omega_i^l, \quad \forall i \in \mathcal{I}. \tag{2.7}$$

We will then consider the family of models obtained by considering all admissible $\lambda_l(k)$. For notational convenience, we will for each cell $\mathcal{X}_i$ associate an index set $L(i)$ that specifies the matrices that are used in the inclusion. We will then rewrite (2.7) as

$$\Omega_i(k) \in \text{Co}\left\{\Omega_i^l\right\} \triangleq \text{Co}\left\{\left[A_i^1|A_{1i}^1|\dots|A_{pi}^1|B_i^1\right],\dots,\left[A_i^L|A_{1i}^L|\dots|A_{pi}^L|B_i^L\right]\right\}, \tag{2.8}$$

and $[A_{il}|A_{dil}|B_{il}] \triangleq [A_i^l|A_{di}^l|B_i^l], \forall d \in \mathcal{D}, \forall l \in \mathcal{L} \triangleq \{1,2,\dots,L\}$ represents a vertices of the convex hull for each of the polyhedral regions partition $i \in \mathcal{I}$. For $d = 0$, the model represents uncertain time-varying PWA systems (2.2), and for $d > 0$, the model represents uncertain time-varying PWA systems with time-delay (2.3.1, 2.3.2).

Polytopic uncertain models can be developed as follows. Suppose that for the (possibly nonlinear) system under consideration, we have input/output data set at different operating points, or at different times. From each data set, we develop a number of linear models for the active regions partition $\Omega_i,\ i \in \mathcal{I}$ (for simplicity, we assume that the various linear models involve the same state vector). Then it is reasonable to assume that analysis and design methods for the polytopic system (2.2), (2.3.1), and (2.3.2) with vertices given by the linear model are applied to the real system.

2.4 Saturated Linear Feedback Control Law

A linear state-feedback controller $u(k+j|k) = Fx(k+j|k)$ has been used in [5, 10, 17] for LPV systems, and it has been extended in [6] for uncertain PWA systems. This control structure makes the optimization problem tractable. In the paper, we propose the following saturated linear feedback controller

$$u(k+j|k) = \begin{cases} \sigma(F(k)x(k+j|k)), & x(k+j|k) \in \mathcal{X}_i, i \in \mathcal{I}_0, \\ \sigma(\bar{F}(k)\bar{x}(k+j|k)), & x(k+j|k) \in \mathcal{X}_i, i \in \mathcal{I}_1, \end{cases} \tag{2.9}$$

where $j \geq 0,\ \ F(k) \in \mathbb{R}^{m\times n}, \bar{F}(k) \in \mathbb{R}^{m\times(n+1)}$, and $\sigma(.)$ is a saturated function with the saturation levels given by a vector $\bar{u} \in \mathbb{R}^m$. In particular, for $r = 1,2,\dots,m$,

$$\sigma(u) = \left[\sigma(u_1)^T,\dots,\sigma(u_m)^T\right]^T, \text{ and } \sigma(u_r) = \begin{cases} u_{r,\max}, & u_r \geq \ u_{r,\max} \\ u_r, & u_r \in \ [-u_{r,\max}, u_{r,\max}] \\ -u_{r,\max}, & u_r \leq \ -u_{r,\max} \end{cases}$$

In order to apply (2.9) to the robust constrained MPC algorithm, the saturation function needs to be described as a linear polytope [27, 28] . Let f_r be the r-th row of the matrix F. We define the symmetric polyhedron

$$\mathcal{L}(F) = \{x \in \mathbb{R}^n : |f_r x| \leq u_{r,\max}, r = 1,2,\dots,m\}.$$

If the control u does not saturate for all $r = 1,2,\dots,m$, that is $x \in \mathcal{L}(F)$, then the saturated linear feedback controller (2.9) is in the form of general linear state feedback control law

$$u(k+j|k) = \begin{cases} F(k)x(k+j|k), & x(k+j|k) \in \mathcal{X}_i, i \in \mathcal{I}_0, \\ \bar{F}(k)\bar{x}(k+j|k), & x(k+j|k) \in \mathcal{X}_i, i \in \mathcal{I}_1. \end{cases} \tag{2.10}$$

Let Φ be the set of $m \times m$ diagonal matrices whose diagonal elements are either 0 or 1. There are 2^m elements in Φ. Suppose that each element of Φ is labeled as $D_q, q = 1,2,\dots,2^m$, i.e.

$\Phi = \{D_q, q = 1, 2, \ldots, 2^m\}$. If D_q belongs to Φ, then denote $D_q^- = I - D_q$. Obviously, D_q^- is also an element of Φ. For example, if $m = 2$ then

$$\Phi = \{D_1, D_2, D_3, D_4\},$$
$$\Phi = \left\{ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}.$$

Lemma 2.1 (Saturated linear feedback control law). *Let vectors $u,\ v \in \mathbb{R}^m$. Suppose that for $x \in \mathbb{R}^n$, if $x \in \mathcal{L}(H)$, then*

$$\sigma(u) \in \mathrm{Co}\left\{ D_q u + D_q^- v : q \in \mathcal{Q} \triangleq \{1, 2, \ldots, 2^m\} \right\}.$$

This means that for two given feedback matrices $F, H \in \mathbb{R}^{m\times n}$, and suppose that

$$|h_r x| \leq u_{r,\max}, \quad \forall r = 1, 2, \ldots, m, \tag{2.11}$$

in terms of Lemma 2.1, we have

$$\sigma(Fx) \in \mathrm{Co}\left\{ D_q F x + D_q^- H x : q \in \mathcal{Q} \right\}.$$

Hence, we can obtain the polytope description of saturated function

$$\begin{cases} \sigma(Fx) = \psi x = \sum_{q=1}^{2^m} \lambda_q (D_q F + D_q^- H) x, & \sum_{q=1}^{2^m} \lambda_q = 1, \lambda_q \in [0,1], x \in \mathcal{X}_i, i \in \mathcal{I}_0, \\ \sigma(\bar{F}\bar{x}) = \bar{\psi}\bar{x} = \sum_{q=1}^{2^m} \lambda_q (D_q \bar{F} + D_q^- \bar{H}) \bar{x}, & \sum_{q=1}^{2^m} \lambda_q = 1, \lambda_q \in [0,1], x \in \mathcal{X}_i, i \in \mathcal{I}_1. \end{cases} \tag{2.12}$$

The benefit of this approach is that the input constraints are satisfied by saturation function naturally and no constraints are imposed on the controller gain F directly. So the controller will have higher gain than the RCMPC algorithm using linear feedback controller, which can utilize the control region more sufficient, and has a better control performance. However, the representation of the saturation function as linear polytope introduces conservatism in the formulation.

2.5 Model Predictive Control

MPC is an open-loop control design procedure where at each sampling time k, plant measurements are obtained and a model of the process is used to predict future outputs of the system subject to system dynamics, input, and output constraints. Fig. (2.2) depicts the basic idea behind MPC. At each discrete-time instant k, the measured variables and the process model (2.2, 2.3.1, 2.3.2) are used to predict the future behaviour of the controlled plant over a specified prediction horizon N_p. This is achieved by considering a future control scenario as the input sequence applied to the process model, which must be calculated such that certain desired constraints, and objectives are fulfilled. To do that, a cost function $J_p(k)$ is minimized subject to constraints, yielding an optimal sequence of controls over a specified control horizon N_u, and described as following

$$\min_{u(k+j|k),\ j=0,1,\ldots,(m-1)} J_p(k),$$

where p is output or prediction horizon, m is input or control horizon.

According to the receding horizon control strategy, only the first element of the computed optimal sequence of controls is then applied to the plant. To incorporate feedback, the optimal open-loop input is implemented only until the next sampling time. Using the system state at time $k+1$, the whole procedure (prediction and optimization) is repeated, moving the control and prediction horizon forward.

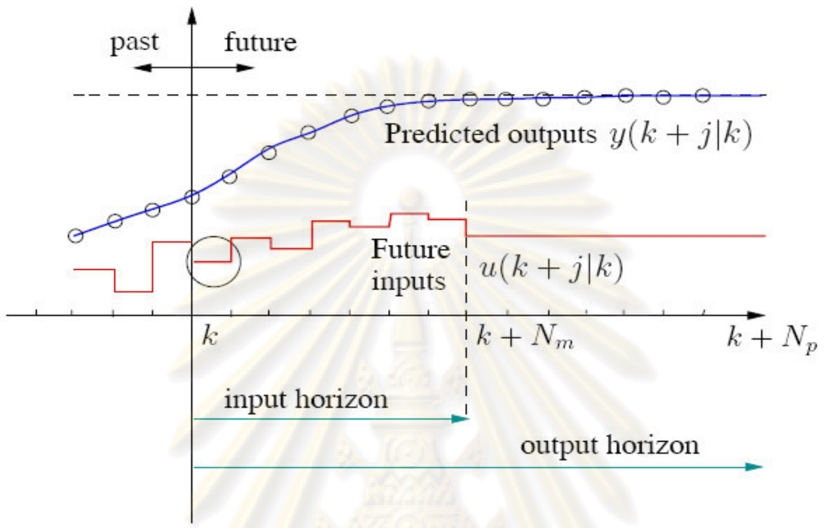

Figure 2.2: MPC scheme.

2.5.1 Objective Function

The thesis emphasizes the case of infinite control and prediction horizon (i.e. $N_u = N_p = \infty$). The control objective is to minimize an infinite horizon linear quadratic cost function

$$J_\infty(k) = \begin{cases} \sum_{j=0}^{\infty} x(k+j|k)^T Q x(k+j|k) + u(k+j|k)^T R u(k+j|k), & x(k) \in \mathcal{X}_i, i \in \mathcal{I}_0, \\ \sum_{j=0}^{\infty} \bar{x}(k+j|k)^T \bar{Q} \bar{x}(k+j|k) + u(k+j|k)^T R u(k+j|k), & x(k) \in \mathcal{X}_i, i \in \mathcal{I}_1, \end{cases} \tag{2.13}$$

where $Q, \bar{Q}$ and R are symmetric, positive-definite matrices denoting suitable weighting matrices.

It is well known that the infinite approach can guarantee nominal stability of the closed-loop system.

2.5.2 Constraints

In this work we consider the Euclidean norm constraints on the input $u(k+j|k)$ for the unsaturated linear feedback RCMPC, given respectively as

$$\|u(k+j|k)\|_2 \leq u_{\max}, \quad k \geq 0, j \geq 0. \tag{2.14}$$

Constraints on the input are typically *hard* constraints, since they represent limitations on the process equipment (such as valve saturation) and as such cannot be relaxed or *softened*.

2.6 Robust Model Predictive Control

A control system is robust if it is insensitive to differences between the actual system and the model of the system which was used to design the controller, and if the performance of the system meets the specified range. These differences are referred to as model/plant mismatch or simply model uncertainties. Any statement about *robustness* of a particular control algorithm must make reference to a specific uncertainties range as well as specific stability and performance criteria.

2.6.1 Robust Stability

Robust stability is the basic closed-loop requirement, i.e., stability in the presence of uncertainties. In MPC, various design procedures achieve robust stability in two different ways. The first approach is by indirectly specifying the performance objective and uncertainties description in such a way that the optimal control computations lead to robust stability. The second approach is by directly enforcing a type of a robust contraction constraint which guarantees that the state will shrink for all plants in the uncertainties set.

2.6.2 Robust Performance

In the main stream of robust control literature, *robust performance* is measured by determining the worst performance over the specified uncertainties range. In direct extension of this definition, it is natural to set up a new *robust* MPC objective where the control action is selected to minimize the worst-case value of the objective function. Many attempts have been made to synthesize such a robust MPC, but they all had more or less drawbacks in terms of addressing robust stability or on-line implementation. For more details on this topic, the reader is referred to [5, 19].

This thesis provides an extension methods of RCMPC to PWA systems (2.2, 2.3.1, 2.3.2). We use a formulation to calculate a state feedback control law and saturated state feedback control law that minimizes an upper bound on the robust performance and by using Lyapunov arguments, guarantees the robust stability. For fairly general uncertainties descriptions, the optimization problem can be expressed as a set of LMIs for which an efficient solution techniques exist.

2.7 Lyapunov Theory for Discrete-Time Systems

The Lyapunov stability theorem for discrete-time systems is reviewed. For discrete-time systems, we use the forward difference

$$\Delta V(x(k)) = V(x(k+1)) - V(x(k)).$$

The next Lemma establish the conditions necessary for a discrete-time PWA systems to be stable [29],

Lemma 2.2 (Stability of Discrete-Time Systems). *Consider the discrete-time PWA systems (2.2, 2.3.1, 2.3.2). Suppose there exists a scalar function $V(x)$ continuous in x such that*

- $V(x) > 0 \quad \forall x \neq 0$,
- $\Delta V(x) < 0 \quad \forall x \neq 0$,
- $V(0) = 0$,

locally in region $\mathcal{X}_i$ or at which partition that the state may be entered at the next sampling time $(k+1)$, $\mathcal{X}_j, \forall i \neq j$, then the equilibrium state $x = 0$ is asymptotically stable and $V(x)$ is a Lyapunov function

The general approaches for construction of Lyapunov function that has been suggested for PWA systems is known as quadratic stability. For PWA systems (2.2, 2.3.1, 2.3.2) with polytopic uncertainties (2.8), a natural stability approach is to check the existence of quadratic Lyapunov function that depends not only on the systems state but also on the uncertain parameter (λ). If we denote $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_L]^T$, we can state the following definition.

Definition 2.3 (Robust stability of PWA systems with delay-free). *The uncertain discrete time PWA systems (2.2) is said to be stable if there exists a Lyapunov function*

$$V(x(k), \lambda(k)) = \begin{cases} x(k)^T P_i(\lambda(k)) x(k), & x(k) \in \mathcal{X}_i, \forall i \in \mathcal{I}_0, \\ \bar{x}(k)^T \bar{P}_i(\lambda(k)) \bar{x}(k), & \bar{x}(k) \in \mathcal{X}_i, \forall i \in \mathcal{I}_1, \end{cases} \tag{2.15}$$

where $P_i(\lambda(k)), \bar{P}_i(\lambda(k))$ are symmetric, positive-definite matrices, such that $\Delta V(x(k), \lambda(k)) < 0$ for all non-zero $x(k) \in \mathbb{R}^n$ and admissible parameter $\lambda(k)$. Similarly, the uncertain discrete time PWA systems (2.2) is said to be robustly stabilizable if there exists a saturated state feedback control law (2.9) such that the resulting closed-loop systems is robustly stable for all admissible uncertain parameter $\lambda(k)$.

Definition 2.4 (Robust stability of PWA systems with time-invariant delay). *The uncertain discrete time PWA systems (2.3.1) is said to be stable if there exists a modified Lyapunov-Krasovskii function (for simplicity, we omit $\lambda(k)$ from the formulation)*

$$V(v(k)) = \begin{cases} x(k)^T P_i x(k) + \sum_{\hat{j}=1}^{\tau_1} x(k-\hat{j})^T P_{1i} x(k-\hat{j}) + \sum_{\hat{j}=\tau_1+1}^{\tau_2} x(k-\hat{j})^T P_{2i} x(k-\hat{j}) + \\ \quad \ldots + \sum_{\hat{j}=\tau_{p-1}+1}^{\tau_p} x(k-\hat{j})^T P_{pi} x(k-\hat{j}), \quad x(k) \in \mathcal{X}_i, \forall i \in \mathcal{I}_0, \\ \bar{x}(k)^T \bar{P}_i \bar{x}(k) + \sum_{\hat{j}=1}^{\tau_1} \bar{x}(k-\hat{j})^T \bar{P}_{1i} \bar{x}(k-\hat{j}) + \sum_{\hat{j}=\tau_1+1}^{\tau_2} \bar{x}(k-\hat{j})^T \bar{P}_{2i} \bar{x}(k-\hat{j}) + \\ \quad \ldots + \sum_{\hat{j}=\tau_{p-1}+1}^{\tau_p} \bar{x}(k-\hat{j})^T \bar{P}_{pi} \bar{x}(k-\hat{j}), \quad \bar{x}(k) \in \mathcal{X}_i, \forall i \in \mathcal{I}_1, \end{cases} \tag{2.16}$$

where $P_i, \bar{P}_i, P_{1i}, \bar{P}_{1i}, \ldots, P_{pi}, \bar{P}_{pi}$ *are symmetric, positive-definite matrices, such that* $\Delta V(v(k)) < 0$ *for all non-zero* $v(k) \in \mathbb{R}^{(p+2)n}$ *and admissible parameter* $\lambda(k)$*. Similarly, the uncertain discrete time PWA systems (2.3.1) is said to be robustly stabilizable if there exists a saturated state feedback control law (2.9) such that the resulting closed-loop systems is robustly stable for all admissible uncertain parameter* $\lambda(k)$*.*

Definition 2.5 (Robust stability of PWA systems with time-varying delay). *The uncertain discrete time PWA systems (2.3.2) is said to be stable if there exists a modified Lyapunov-Krasovskii function (for simplicity, we omit* $\lambda(k)$ *from the formulation)*

$$V(\mathfrak{m}(k)) = \begin{cases} x(k)^T P_i x(k) + \sum_{d=1}^{p} \left(\sum_{j=\mathcal{I}_d}^{\overline{\tau}_d} x(k-j)^T P_{ji} x(k-j) \right), x(k) \in \mathcal{X}_i, i \in \mathcal{I}_0. \\ \bar{x}(k)^T \bar{P}_i \bar{x}(k) + \sum_{d=1}^{p} \left(\sum_{j=\mathcal{I}_d}^{\overline{\tau}_d} \bar{x}(k-j)^T \bar{P}_{ji} \bar{x}(k-j) \right), \bar{x}(k) \in \mathcal{X}_i, i \in \mathcal{I}_1. \end{cases} \tag{2.17}$$

where $P_i, \bar{P}_i, P_{1i}, \bar{P}_{1i}, \ldots, P_{pi}, \bar{P}_{pi}$ *are symmetric, positive-definite matrices, such that* $\Delta V(\mathfrak{m}(k)) < 0$ *for all non-zero* $\mathfrak{m}(k)$ *and admissible parameter* $\lambda(k)$*. Similarly, the uncertain discrete time PWA systems (2.3.2) is said to be robustly stabilizable if there exists a state feedback control law* $u(k) = F(k)x(k)$ *or saturated state feedback control law* $u(k) = \sigma(F(k)x(k))$ *such that the resulting closed-loop systems is robustly stable for all admissible uncertain parameter* $\lambda(k)$*.*

In fact, there is no general and systematic way to formally determine $P_i(.)$ as a function of the uncertain parameter $\lambda(k)$. A traditional way of addressing this problem is to look for a multiple Lyapunov and modified Lyapunov-Krasovskii matrix $P_i(.) = P_i$ which renders condition (2.15) satisfied. Furthermore, a multiple matrix P_i that satisfies the condition given in Definition 2.3, 2.4, and 2.5 can be found by using efficient LMI tools. The quadratic stability, however, is somewhat conservative.

In the attempt to reduce the conservatism, the 'new' stability condition has been proposed in [16]. The benefit of this stability condition is that it consists in the introduction of an extra degrees of freedom which allows to get a control law without an explicit dependence on the Lyapunov function. Specifically, it is based on the sufficient LMI conditions. We derive extension of the proposed method in [16] for PWA systems (2.2, 2.3.1, 2.3.2).

Theorem 2.1 (New robust stability condition for PWA systems with delay-free). *The uncertain discrete-time PWA systems (2.2) is robustly stabilizable if there exist* L *symmetric matrices* Q_{ij} *with* $j = 1, 2, \ldots, L$*, for all* $i \in \mathcal{I}_0$ *and a pair of matrices* Y, G *satisfying the following LMIs.*

$$\begin{bmatrix} G + G^T - Q_{ij} & * \\ A_{ij}G + B_{ij}(D_q Y + D_q^- Z) & Q_{il} \end{bmatrix} > 0, \quad \forall j = 1, 2, \ldots, L, \ \forall l = 1, 2, \ldots, L, \ \forall i \in \mathcal{I}_0. \tag{2.18}$$

Furthermore, the state feedback matrix is given by

$$F = YG^{-1} \tag{2.19}$$

Proof. Assume there exists $H \in \mathbb{R}^{m\times n}$ satisfying

$$|h_r x(k+j|k)| \le u_{r,\max}, \;\; \forall j \ge 0, \; r = 1,2,\ldots,m,$$

From (2.18), G is of full rank and Q_{ij} is strictly positive definite. Therefore,

$$(G - Q_{ij})^T Q_{ij}^{-1}(G - Q_{ij}) \ge 0,$$

which is equivalent to

$$G^T Q_{ij}^{-1} G \ge G^T + G - Q_{ij}. \tag{2.20}$$

Then, defining $F \triangleq YG^{-1}, H \triangleq ZG^{-1}$, satisfying (2.18) leads to

$$\begin{bmatrix} G^T Q_{ij}^{-1} G & * \\ (A_{ij} + B_{ij}(D_q F + D_q^- H)G & Q_{il} \end{bmatrix} > 0,$$

which is equivalent to

$$\begin{bmatrix} G & * \\ 0 & Q_{il} \end{bmatrix}^T \begin{bmatrix} Q_{ij}^{-1} & * \\ Q_{il}^{-1}(A_{ij} + B_{ij}(D_q F + D_q^- H)) & Q_{il}^{-1} \end{bmatrix} \begin{bmatrix} G & * \\ 0 & Q_{il} \end{bmatrix} > 0.$$

Letting $P_{ij} = Q_{ij}^{-1}$ and $P_{il} = Q_{il}^{-1}$, one gets

$$\begin{bmatrix} P_{ij} & * \\ P_{il}(A_{ij} + B_{ij}(D_q F + D_q^- H)) & P_{il} \end{bmatrix} > 0, \quad \forall j,l = 1,2,\ldots,L, \; \forall i \in \mathcal{I}_0, \; \forall q \in \mathcal{Q}.$$

For each j, multiply the above corresponding $l = 1,2,\ldots,L$ inequalities by $\delta_l(k), \; \delta_l(k) \ge 0, \; \sum_{l=1}^{L} \delta_l(k) = 1$, and sum. Multiply the resulting $j = 1,2,\ldots,L$ inequalities by $\lambda_j(k)$ and sum. For each $q = 1,2,\ldots,2^m$, multiply the corresponding by $\lambda_q(k), \lambda_q \ge 0, \sum_{q=1}^{2^m} \lambda_q(k) = 1$ to get

$$\begin{bmatrix} P_i(k) & * \\ P_{i+}(k)(A_i + B_i\psi(k)) & P_{i+}(k) \end{bmatrix} > 0, \quad \forall i \in \mathcal{I}_0,$$

where $P_i(k) = \sum_{j=1}^{L} \lambda_j(k) P_{ij}$, $P_{i+}(k) = \sum_{l=1}^{L} \delta_l(k) P_{il}$.
Applying Schur complements [30], then it is equivalent to

$$P_i(k) - (A_i + B_i\psi(k))^T P_{i+}(A_i + B_i\psi(k)) > 0, \quad \forall i \in \mathcal{I}_0.$$

Choose the Lyapunov function of the form

$$V(x(k)) = x(k)^T P_i(k) x(k), \quad x(k) \in \mathcal{X}_i, \forall i \in \mathcal{I}_0.$$

Then, we can conclude immediately from the above inequality that

$$\begin{aligned} \Delta V(x(k)) &= V(x(k+1)) - V(x(k)) \\ &= x(k)^T \left[(A_i + B_i\psi(k))^T P_i(k+1)(A_i + B_i\psi(k)) - P_i(k) \right] x(k) \\ &= x(k)^T \left[(A_i + B_i\psi(k))^T P_{i+}(k)(A_i + B_i\psi(k)) - P_i(k) \right] x(k) < 0, \end{aligned}$$

for all non-zero $x \in \mathbb{R}^n$. It follows immediately from Definition 2.3 that the uncertain discrete-time PWA systems (2.2) is robustly stabilizable.
Notice that we can extend the theorem for the augmented condition where $x \in \mathcal{X}_i, i \in \mathcal{I}_1$ directly. Thus, the proof will be omitted. □

Theorem 2.2 (New robust stability condition for PWA systems with time-invariant delay). *The uncertain discrete-time PWA systems (2.3.1) is robustly stabilizable if there exist* L *symmetric matrices* Q_{ij} *with* $j = 1, 2, \ldots, L$*, for all* $i \in \mathcal{I}_0$ *and a pair of matrices* Y, G *satisfying the following LMIs.*

$$\begin{bmatrix} G + G^T - Q_{ij} & * & * & * \\ G & Q_{1il} & * & * \\ 0 & 0 & \widehat{M} & * \\ A_{ij}G + B_{ij}(D_qY + D_q^- Z) & 0 & \widehat{A} & Q_{il} \end{bmatrix} > 0, \quad \forall j, l = 1, 2, \ldots, L, \ \forall i \in \mathcal{I}_0, \tag{2.21}$$

where

$$\begin{aligned} \widehat{A} &= [A_{1il}, 0, A_{2il}, 0, \ldots, \ldots, A_{pil}], \\ \widehat{M} &= \begin{bmatrix} Q_{1il} & * & * & * & \ldots & * \\ Q_{1il} & Q_{2il} & * & * & \ldots & * \\ 0 & 0 & Q_{2il} & * & \ldots & * \\ 0 & 0 & Q_{2il} & Q_{3il} & \ldots & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & Q_{pil} \end{bmatrix}. \end{aligned}$$

Furthermore, the state feedback matrix is given by

$$F = YG^{-1} \tag{2.22}$$

Proof. The proof is a natural extension of that of the uncertain PWA systems (2.2) in Theorem 2.1. □

The above Theorems 2.1, 2.2 are based on Definitions 2.3, 2.4 to search for a state feedback law that robustly stabilizes the closed-loop system with a parameter-dependent Lyapunov matrix $P_i(\lambda(k)) = \sum_{j=1}^{L} \lambda_j(k) P_{ij}$ and $P_{ij} = Q_{ij}^{-1}$. It is interesting to note that, in contrast with the quadratic stability synthesis, the determination of the control (2.19) does not directly depend on the Lyapunov matrices P_i which are used to build the parameter-dependent Lyapunov matrix $P_i(\lambda(k))$.

2.8 Linear Matrix Inequalities

We give a brief introduction to LMI and some optimization problems based on LMIs. For more details, the interested reader is referred to Boyd et al. [30].

A Linear Matrix Inequalities or LMI is a matrix inequality of the form

$$F(x) \triangleq F_0 + \sum_{i=1}^{m} x_i F_i > 0, \tag{2.23}$$

where $x \in \mathbb{R}^m$ is the variable and the symmetric matrices $F_i = F_i^T \in \mathbb{R}^{n \times n}$, $i = 0, 1, \ldots, m$ are given. The inequality expressions in (2.23), $F(x) > 0$ means that $F(x)$ is positive definite.

We will also encounter a non-strict LMIs, which have the form

$$F(x) \geq 0 \tag{2.24}$$

The inequality expressions in (2.24) means that $F(x)$ is positive semi-definite.

Multiple LMIs $F_1(x) > 0, \ldots, F_n(x) > 0$ can be expressed as the single LMI

$$F(x) = \mathrm{diag}(F^{(1)}, \ldots, F^{(p)}(x)) > 0.$$

Therefore we will make no distinction between a set of LMIs and a single LMI, i.e., "the LMI $F_1(x) > 0, \ldots, F_n(x) > 0$" will mean "the LMI $\mathrm{diag}(F^{(1)}(x), \ldots, F^{(p)}(x)) > 0$".

The LMI (2.23) is a convex constraint on x, i.e., the set $\{x | F(x) > 0\}$ is convex. It can represent a wide variety of convex constraints on x. In particular, constraints that arise in control theory and have been used in the formulation in this thesis, such as Lyapunov function can all be cast in the form of an LMI. Therefore, LMI is an useful tool in this thesis, because the problem can be solved in an efficient and reliable way.

When the matrices F_i are diagonal, the LMI $F(x) > 0$ is just a set of linear inequalities. Nonlinear (convex) inequalities are converted to LMI form using Schur complements.

Theorem 2.3 (Schur complements). *Let $Q(x) = Q(x)^T$, $R(x) = R(x)^T$, and $S(x)$ depend affinely on x. Then the LMI*

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0$$

is equivalent to the matrix inequalities

$$R(x) > 0, \quad Q(x) - S(x)R(x)^{-1}S(x)^T > 0,$$

or equivalently

$$Q(x) > 0, \quad R(x) - S(x)Q(x)^{-1}S(x)^T > 0.$$

We often encounter problems in which the variables are matrices, for example, the constraint $P > 0$, where the entries of P are the optimization variables. In such cases we will not write out the LMI explicitly in the form $F(x) > 0$, but instead make clear which matrices are the variables.

The LMI-based problem of central importance to this thesis is that of minimizing a linear objective subject to LMI constraints:

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & F(x) > 0 \end{array} \tag{2.25}$$

Here, F is a symmetric matrix that depends affinely on the optimization variable x, and c is a real vector of appropriate size. The MATLAB *YALMIP toolbox* [31] has ready packages for solving the feasibility problem and solving the linear objective optimization problem subject to a set of LMIs (2.25).

The observation about LMI-based optimization that is *LMI problems are tractable*. LMI problems can be solved in polynomial time, which means that it has low computational complexity and numerical experience shows that these algoritms solve LMI problems with extreme efficiency. Therefore, it is well-suited for online implementation which is essential for MPC algorithm.

CHAPTER III

RCMPC FOR PWA SYSTEMS WITH DELAY-FREE

This chapter presents the first strategy to employ RCMPC to PWA systems with delay-free. Section 3.2 discusses about RCMPC using state feedback law. The algorithm is then extended using saturated state feedback law in section 3.3. The knowledge on deriving RCMPC in this chapter will be used to extend the strategy to PWA systems with time-delay in the next chapter.

Remark 3.1. *The formulation RCMPC for PWA systems (2.2, 2.3.1, 2.3.2) from this chapter and the next three chapters only concerns with the analysis of equilibria, where the affine terms* $a_i = 0$ *for* $x \in \mathcal{X}_i, i \in \mathcal{I}_0$. *The robust performance objective is aimed at designing a predictive controller that brings the systems to the origin* $(x = 0, u = 0)$ *and at each time k, minimizing the infinite horizon quadratic performance objective. Nevertheless, the formulation can be augmented directly using the matrix parameterization method as explained in the previous chapter for systems with conditions* $a_i \neq 0, x \in \mathcal{X}_i, i \in \mathcal{I}_1$.

3.1 Derivation of Upper Bound

The system is described by (2.2) with the associated uncertainties set (2.8). The system state $x(k)$ is assumed to be measurable. As mentioned in Section 2.6, the minimization of the nominal objective function (2.13) at each sampling time k is replaced by the minimization of a robust performance objective as follows

$$\min_{\vec{u}(k+j|k)} \max_{[A_i(k+j)|B_i(k+j)]\in\Omega_i, i\in\mathcal{I}_0, j\geq 0} J_\infty(k), \tag{3.1}$$

where $Q > 0$, $R > 0$ are given weighting matrices.

As proposed in [5], the min-max problem (3.1) is used to minimize the worst-case objective function by deriving an upper bound among all time-varying plants $[A_i(k)|B_i(k)] \in \Omega_i$. At sampling time k, we define a quadratic Lyapunov function (2.15). Suppose V satisfies the following inequality for all $x(k+j|k), u(k+j|k), j \geq 0$ satisfying (2.2), and for any $[A_i(k)|B_i(k)] \in \Omega_i, i \in \mathcal{I}_0, j \geq 0$

$$V(x(k+j+1|k)) - V(x(k+j|k)) \leq -\left[\|x(k+j|k)^T\|_Q^2 + \|u(k+j|k)^T\|_R^2\right]. \tag{3.2}$$

For the robust performance objective function to be finite, we let $x(\infty|k) = 0$ and hence, $V(x(\infty|k)) = 0$. Summing (3.2) from $j = 0$ to $j = \infty$, we obtain the following inequality

$$\max_{[A_i(k+j)|B_i(k+j)]\in\Omega_i, i\in\mathcal{I}_0} J_\infty(k) \leq V(x(k|k)) \leq \gamma, \tag{3.3}$$

where γ is an upper bound of the robust performance objective. Therefore, the robust constrained MPC algorithm (2.13) has been redefined to synthesize, at each time step k, a constant saturated state-feedback control law to minimize the following optimization problem

$$\begin{aligned} &\min_{\vec{u}(k+j|k)} \gamma, \\ \text{subject to } &(2.11), (3.2), (3.3). \end{aligned} \tag{3.4}$$

The goal of the robust MPC algorithm has, therefore, been redefined to synthesize, at each sampling time k, a (saturated) state feedback control law to minimize γ. As in standard MPC, only the first computed input $u(k|k)$ is implemented. At the next sampling time, the state $x(k+1)$ is measured and the optimization is repeated to recompute F. The following subsection gives conditions for the existence of the matrix P_i satisfying (2.13) and the corresponding state feedback matrix F.

3.2 State feedback RCMPC for PWA systems with delay-free

3.2.1 Control Algorithm

This part states the main results of state feedback RCMPC strategy for PWA systems with delay-free.

Theorem 3.1 (State feedback robust unconstrained MPC for PWA systems with delay-free). *Let $x(k|k) = x(k)$ be the state of the system (2.2) measured at sampling time k in partition $\mathcal{X}_i$. Suppose the switching sequence of the PWA system from one partition to another is known. Then the optimization problem (3.1) with a state-feedback control law $u(k+j|k) = Fx(k+j|k)$, $F \in \mathbb{R}^{m\times n}$, $j \geq 0$ can be solved by the following LMIs*

$$\min_{G,Y,Q_{il},Q_{\tilde{i}l}} \gamma, \tag{3.5}$$

subject to

$$\begin{bmatrix} 1 & * \\ x(k|k) & Q_{il} \end{bmatrix} \geq 0, \tag{3.6}$$

$$\begin{bmatrix} G + G^T - Q_{il} & * & * & * \\ A_{il}G + B_{il}Y & Q_{\tilde{i}l} & * & * \\ Q^{\frac{1}{2}}G & 0 & \gamma I & * \\ R^{\frac{1}{2}}Y & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \tag{3.7}$$

$\forall i \in \mathcal{I}_0$, $\forall l \in \mathcal{L}$, where $\forall \tilde{i}$ such that $(\tilde{i}, i) \in \mathcal{W}$, $\mathcal{W}$ is the set of all possible switching sequences defined as $\mathcal{W} \triangleq \mathcal{I}_0 \times \mathcal{I}_0$. On the other hand Q_{il} and $Q_{\tilde{i}l}$ are symmetric matrices, and $F = YG^{-1}$.

Proof. See [6]. □

Note that the variables in this problem should be strictly written as $Q_{il}(k), Q_{\tilde{i}l}(k), Y(k), F(k)$, etc. to emphasize that they are computed at time k. For notational convenience, we omit the time index here.

Theorem 3.1 formulates the robust unconstrained MPC problem, and derives an upper bound on the robust performance objective. We extend the formulation incorporating the input constraints as an LMI constraints in the robust MPC problem. As a first step, we need to establish the following lemma which will also be required to prove robust stability. The lemma is an extension to PWA systems in [5].

Lemma 3.1 (Invariant ellipsoid). *Consider the system (2.2) with the associated uncertainties set Ω_i described by (2.8). At sampling time k, suppose there exist $Q_i > 0$, γ, and $F = YG^{-1}$ such that (3.7) holds. Also suppose that $u(k+j|k) = Fx(k+j|k), j \geq 0$. Then if*

$$x(k|k)^T Q_i^{-1} x(k|k) \leq 1, \text{or } \; x(k|k)^T P_i x(k|k) \leq \gamma \; \text{ with } \; Q_i = \gamma P_i, \quad \forall i \in \mathcal{I}_0,$$

then

$$\max_{[A_i(k+j)|B_i(k+j)] \in \Omega_i, j \geq 1} x(k+j|k)^T Q_i^{-1} x(k+j|k) < 1, \forall i \in \mathcal{I}_0, \tag{3.8}$$

or equivalently

$$\max_{[A_i(k+j)|B_i(k+j)] \in \Omega_i, j \geq 1} x(k+j|k)^T P_i x(k+j|k) < \gamma, \forall i \in \mathcal{I}_0. \tag{3.9}$$

Thus, $\mathcal{E} = \{z | z^T Q_i^{-1} z \leq 1\} = \{z^T P_i z \leq \gamma\}$ is an invariant ellipsoid for the predicted states of the uncertain PWA systems with delay-free. Fig. 3.1 illustrates the graphical representation of the state-invariant ellipsoid [5].

Remark 3.2. *The maximization in (3.8) and (3.9) is over the set Ω_i of time-varying PWA models that can be used for prediction of the future states of the system. This maximization leads to the "worst-case" value of $x(k+j|k)^T Q_i^{-1} x(k+j|k)$ (equivalently, $x(k+j|k)^T P_i x(k+j|k)$) at every instant of time $k+j, j \geq 1$.*

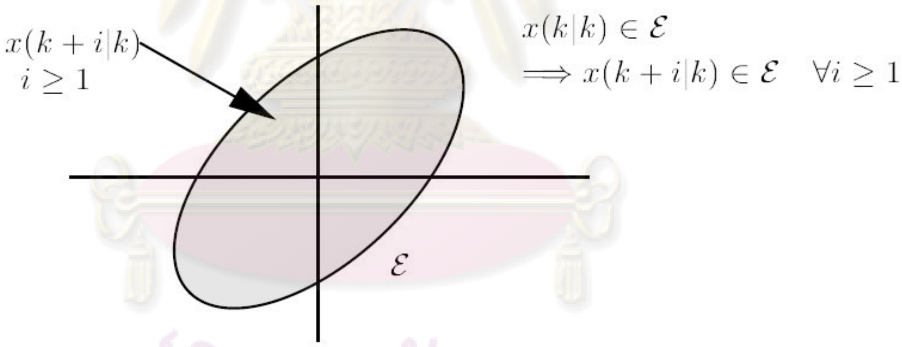

Figure 3.1: Graphical representation of the state-invariant ellipsoid $\mathcal{E}$ in 2-dimensions.

Proof. Because of the PWA systems with delay-free comprises of several linear systems, then the proof is a natural extension from [5], so it will be omitted here. □

In industry, many processes are subject to constraints on the control input. The explicit handling of constraints may allow the process to operate closer to constraints and optimal operating conditions. In LMI framework, input constraints is formulated as follows. For Euclidean norm constraint (2.14), at sampling time k the constraint is imposed on the present and the entire horizon of future manipulated variables, although only the first control move $u(k|k) = u(k)$ is implemented. Based on [5] and following [16, 30], we extend the formulation to PWA systems with delay-free as follows

$$\begin{bmatrix} u_{\max}^2 I & * \\ Y^T & G + G^T - Q_{il} \end{bmatrix} \geq 0, \;\; \forall i \in \mathcal{I}_0, \forall l \in \mathcal{L} \tag{3.10}$$

From (2.20), then (3.10) implies

$$\begin{bmatrix} u_{\max}^2 I & * \\ Y^T & G^T Q_{il}^{-1} G \end{bmatrix} \geq 0, \;\; \forall i \in \mathcal{I}_0, \forall l \in \mathcal{L}$$

Substituting $F = YG^{-1}$ into the inequality above multiplying the resulting inequality from the left-hand and right-hand sides by diag$[I, G]$, we have

$$\begin{bmatrix} u_{\max}^2 I & * \\ F^T & Q_{il}^{-1} \end{bmatrix} \geq 0, \;\; \forall i \in \mathcal{I}_0, \forall l \in \mathcal{L}$$

Substituting $Q_{il} = \gamma P_{il}^{-1}$ and multiplying the resulting inequalities by $\lambda_l(k+j)$ and summing up for $l = 1, 2, \ldots, L$, we obtain

$$\begin{bmatrix} u_{\max}^2 I & * \\ F^T & \frac{1}{\gamma} P_i(k+j) \end{bmatrix} \geq 0, \;\; \forall i \in \mathcal{I}_0$$

Applying Schur complement to the inequality above and multiplying the resulting inequality from the left-hand and right-hand sides by $x(k+j|k)$ and taking into account of (2.10), we obtain

$$\frac{1}{u_{\max}^2} u(k+j|k)^T u(k+j|k) \leq \frac{1}{\gamma} x(k+j|k)^T P_i x(k+j|k) \tag{3.11}$$

Since the inequality (3.2) implies that $V(x(k+j|k))$ strictly decreases as j goes to ∞ and $V(k|k) \leq \gamma$ from (3.3), we have

$$\frac{1}{\gamma} x(k+j|k)^T P_i x(k+j|k) \leq 1, \;\; \forall i \in \mathcal{I}_0, \forall j \geq 0 \tag{3.12}$$

Hence, from (3.11) and (3.12), we conclude that (3.10) holds.

Theorem 3.2 (State feedback RCMPC for PWA systems with delay-free). *Let $x(k) = x(k|k)$ be the state of the uncertain systems (2.2) measured at sampling time k. Suppose that the uncertainties set is defined by a polytope as in (2.8). Then the state-feedback matrix F in the control law $u(k+j|k) = Fx(k+j|k)$ for $k, j \geq 0$ that minimizes the upper bound γ on the robust performance objective function at sampling time k and satisfies a set of specified input constraints is given by*

$$F = YG^{-1}$$

where G is full-rank and Y is obtained from the solution of the following linear objective minimization problem

$$\begin{aligned} &\min_{G, Y, Q_{il}, Q_{\bar{i}l}} \gamma \\ &\textit{subject to} \;\; (3.6), (3.7), (3.10). \end{aligned} \tag{3.13}$$

3.2.2 Robust stability

In order to prove robust stability of the closed loop, we need to establish the following lemma.

Lemma 3.2 (Feasibility). *Any feasible solution of the optimization in Theorem 3.2 at time k is also feasible for all times $t > k$. Thus, if the optimization problem in Theorem 3.2 is feasible at time k, then it is feasible for all times $t > k$.*

Proof. The proof of this lemma is a natural extension of that proposed in [5] by Kothare et al.

□

Theorem 3.3 (Robust stability). *The feasible receding horizon state feedback control law obtained from Theorem 3.2 robustly asymptotically stabilizes the closed-loop PWA systems with delay-free (2.2).*

Proof. See [6]. □

3.3 Saturated state feedback RCMPC for PWA systems with delay-free

3.3.1 Control Algorithm

By applying the polytopic description of the saturated linear feedback controller (2.12), the optimization problem (3.4) can be reduced to an LMI optimization problem for uncertain system (2.2).

Theorem 3.4 (Saturated state feedback RCMPC for PWA systems with delay-free). *Let $x(k|k) = x(k)$ be the state of the system (2.2) measured at sampling time k in partition $\mathcal{X}_i, i \in \mathcal{I}_0$. Suppose the switching sequence of the PWA system from one partition to another is known. Then the optimization problem (3.4) with a saturated state-feedback control law $u(k+j|k) = \sigma(Fx(k+j|k))$ can be solved by the following LMIs*

$$\min_{G,Y,Z,Q_{il},Q_{\tilde{i}l}} \gamma \tag{3.14}$$

subject to

$$\begin{bmatrix} 1 & * \\ x(k|k) & Q_{il} \end{bmatrix} \geq 0, \tag{3.15}$$

$$\begin{bmatrix} G + G^T - Q_{il} & * & * & * \\ A_{il}G + B_{il}(D_qY + D_q^- Z) & Q_{\tilde{i}l} & * & * \\ Q^{\frac{1}{2}}G & 0 & \gamma I & * \\ R^{\frac{1}{2}}(D_qY + D_q^- Z) & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \tag{3.16}$$

$$\begin{bmatrix} X & * \\ Z^T & G + G^T - Q_{il} \end{bmatrix} \geq 0, \; X_{rr} \leq u_{r,\max}^2, r = 1, 2, \ldots, m, \tag{3.17}$$

$\forall i \in \mathcal{I}_0$, $\forall l \in \mathcal{L}$, $\forall q \in \mathcal{Q}$, where $\forall \tilde{i}$ such that $(\tilde{i}, i) \in \mathcal{W}$, $\mathcal{W}$ is the set of all possible switching sequences defined as $\mathcal{W} \triangleq \mathcal{I}_0 \times \mathcal{I}_0$. On the other hand Q_{il} and $Q_{\tilde{i}l}$ are symmetric matrices, and $F = YG^{-1}$.

Proof. We only need to consider the condition (2.11), because the saturated controller subject to input constraint (2.14). We use the similar techniques as in [3, 5] to transform the saturated condition into LMI representation. Assume there exists $H \in \mathbb{R}^{m \times n}$ satisfy

$$|h_r x(k+j|k)| \leq u_{r,\max}, \;\; \forall j \geq 0, \; r = 1, 2, \ldots, m. \tag{3.18}$$

If the conditions in (3.15), and (3.16) are feasible, we obtain invariant ellipsoid as the polyhedral cell bounding for predicted state $x(k+j|k)$. From (2.20), then (3.18) implies

$$\begin{bmatrix} X & * \\ Z^T & G^T Q_{il}^{-1} G \end{bmatrix} \geq 0, X_{rr} \leq u_{r,\max}^2, r = 1, 2, \ldots, m \;\; \forall i \in \mathcal{I}, \forall l \in \mathcal{L},$$

Substituting $H = ZG^{-1}$ into the inequality above multiplying the resulting inequality from the left-hand and right-hand sides by diag$[I, G^{-1}]$, and substituting $Q_{il} = \gamma P_{il}^{-1}$, we obtain

$$\begin{bmatrix} X & * \\ H^T & \frac{1}{\gamma} P_{il} \end{bmatrix} \geq 0, X_{rr} \leq u_{r,\max}^2, r = 1, 2, \ldots, m \;\; \forall i \in \mathcal{I}, \forall l \in \mathcal{L}.$$

Applying Schur complement to the inequality above and multiplying the resulting inequality from the left-hand and right-hand sides by $x(k+j|k)$, we obtain

$$\begin{aligned} \{Hx(k+j|k)\}^T X^{-1} \{Hx(k+j|k)\} &\leq \tfrac{1}{\gamma} x(k+j|k)^T P_{il} x(k+j|k), \\ X_{rr} &\leq u_{r,\max}^2, \; r = 1, 2, \ldots, m \end{aligned} \tag{3.19}$$

Condition in (3.19) is equivalent to

$$\frac{1}{u_{r,\max}^2} \{h_r x(k+j|k)\}^T \{h_r x(k+j|k)\} \leq \frac{1}{\gamma} x(k+j|k)^T P_{il} x(k+j|k), r = 1, 2, \ldots, m. \tag{3.20}$$

Since the inequality (3.2) implies that $V(x(k+j|k))$ strictly decreases as j goes to ∞ and $V(k|k) \leq \gamma$ from (3.3), we have

$$\frac{1}{\gamma} x(k+j|k)^T P_{il} x(k+j|k) \leq 1, \;\; \forall i \in \mathcal{I}, \forall l \in \mathcal{L}, \forall j \geq 0. \tag{3.21}$$

Hence, from (3.20) and (3.21), we conclude that (3.17) holds.

Define $F \triangleq YG^{-1}$. With the same procedure as the proof of Theorem 2.1, satisfying (3.16) leads to

$$\begin{bmatrix} G^T Q_{il}^{-1} G & * & * & * \\ (A_{il} + B_{il}(D_q F + D_q^- H))G & Q_{\tilde{i}l} & * & * \\ \gamma^{-\frac{1}{2}} Q^{\frac{1}{2}} G & 0 & I & * \\ \gamma^{-\frac{1}{2}} R^{\frac{1}{2}} (D_q F + D_q^- H) G & 0 & 0 & I \end{bmatrix} \geq 0,$$

which is equivalent to

$$\begin{bmatrix} \gamma Q_{il}^{-1} & * & * & * \\ \gamma Q_{\tilde{i}l}^{-1} (A_{il} + B_{il}(D_q F + D_q^- H)) & \gamma Q_{\tilde{i}l}^{-1} & * & * \\ Q^{\frac{1}{2}} & 0 & I & * \\ R^{\frac{1}{2}} (D_q F + D_q^- H) & 0 & 0 & I \end{bmatrix} \geq 0,$$

where left-hand and right-hand sides are multiplied by diag $\left[\gamma^{-\frac{1}{2}} G, \gamma^{-\frac{1}{2}} Q_{\tilde{i}l}, I, I\right]$.
Letting $P_{il} = \gamma Q_{il}^{-1}, P_{\tilde{i}l} = \gamma Q_{\tilde{i}l}^{-1}$, we gets

$$\begin{bmatrix} P_{il} & * & * & * \\ P_{\tilde{i}l}(A_{il} + B_{il}(D_q F + D_q^- H)) & P_{\tilde{i}l} & * & * \\ Q^{\frac{1}{2}} & 0 & I & * \\ R^{\frac{1}{2}} (D_q F + D_q^- H) & 0 & 0 & I \end{bmatrix} \geq 0, \quad \forall l \in \mathcal{L}, \forall i \in \mathcal{I}_0, \forall q \in \mathcal{Q}.$$

For each $q \in \mathcal{Q} = 1, 2, \ldots, 2^m$, multiply the corresponding inequalities by $\lambda_q(k+j)$, $\lambda_q \geq 0, \sum_{q=1}^{2^m} \lambda_q(k+j) = 1$ and sum up to get

$$\begin{bmatrix} P_{il} & * & * & * \\ P_{\tilde{i}l}(A_{il} + B_{il}\psi) & P_{\tilde{i}l} & * & * \\ Q^{\frac{1}{2}} & 0 & I & * \\ R^{\frac{1}{2}}\psi & 0 & 0 & I \end{bmatrix} \geq 0, \quad \forall l \in \mathcal{L}, \forall i \in \mathcal{I}_0. \tag{3.22}$$

We define the PWA system (2.2) with uncertainties described by (2.8) into L PWA subsystems in each of polyhedral partition $\mathcal{X}_i$

$$x(k+j+1|k) = A_{il}(k+j)x(k+j|k) + B_{il}(k+j)u(k+j|k), \quad l = 1, 2, \ldots, L, \forall i \in \mathcal{I}_0,$$

then the corresponding Lyapunov function V at time $k+j$ is defined as follow

$$V(x(k+j|k)) = x(k+j|k)^T P_{il} x(k+j|k), \quad j \geq 0, x \in \mathcal{X}_i, \forall i \in \mathcal{I}_0. \tag{3.23}$$

At sampling time $k+j+1$, assume the state $x(k+j+1|k)$ is implemented by the saturated state feedback (2.9) enter partition $\mathcal{X}_{\tilde{i}}$

$$V(x(k+j+1|k)) = x(k+j+1|k)^T P_{\tilde{i}l} x(k+j+1|k) = \Xi^T P_{\tilde{i}l} \Xi, \tag{3.24}$$

where $\Xi = A_{il}(k+j)x(k+j|k) + B_{il}(k+j)\sigma(Fx(k+j|k))$, for all $i \in \mathcal{I}_0, l \in \mathcal{L}$, and for all $\tilde{i}$ such that $(\tilde{i}, i) \in \mathcal{W}$, where $\mathcal{W}$ is the set of all possible switching sequences defined as $\mathcal{W} \triangleq \mathcal{I}_0 \times \mathcal{I}_0$. Then, we can see straightforward from inequalities (3.22), (3.23), and (3.24) that

$$\begin{aligned} \Delta V(x(k+j|k)) &= V(x(k+j+1|k)) - V(x(k+j|k)) \\ &= x(k+j|k)^T \left\{ [A_{il}(k+j) + B_{il}(k+j)\psi]^T P_{\tilde{i}l} [A_{il}(k+j) + B_{il}(k+j)\psi] \right. \\ &\quad \left. - P_{il} \right\} x(k+j|k) \\ &\leq -\left[x(k+j|k)^T Q x(k+j|k) + u(k+j|k)^T R u(k+j|k) \right], \end{aligned}$$

which meets the performance constraints (3.2). Hence, the inequality (3.16) holds.

Next, we show that the inequality (3.15) holds. By applying congruence transformation to the resulting inequality with diag$[1, Q_{il}^{-1}]$, we have

$$\begin{bmatrix} 1 & * \\ Q_{il}^{-1} x(k|k) & Q_{il}^{-1} \end{bmatrix} \geq 0.$$

Substituting $Q_{il} = \gamma P_{il}^{-1}$ and applying congruence transformation to the resulting inequality with diag$[1, \gamma P_{il}^{-1}]$, we obtain

$$\begin{bmatrix} 1 & * \\ x(k|k) & \gamma P_{il}^{-1} \end{bmatrix} \geq 0,$$

which, by the Schur complements, yields

$$x(k|k)^T P_{il} x(k|k) \leq \gamma.$$

Hence, we conclude that the first inequality (3.15) holds. □

3.3.2 Robust Stability

Standard linear feedback controller can be considered as a special case of saturated linear feedback controller when the input value within the prescribed bounds. Hence, Theorem 3.3 can be applied to saturated state feedback RCMPC for PWA systems with delay-free. The implementation of saturated linear feedback controller to the system has a similar robust stability condition as the standard linear feedback controller.

3.4 Augmented Formulation

Theorem 3.5 (Augmented saturated state feedback RCMPC for PWA systems with delay-free). *Let $\bar{x}(k|k) = \bar{x}(k)$ be the state of the system (2.2) measured at sampling time k in partition $\mathcal{X}_i, i \in \mathcal{I}_1$. Suppose the switching sequence of the PWA system from one partition to another is known. Then the optimization problem (3.4) with a saturated state-feedback control law $u(k+j|k) = \sigma(\bar{F}\bar{x}(k+j|k))$ can be solved by the following LMIs*

$$\min_{\bar{G},\bar{Y},\bar{Z},\bar{Q}_{il},\bar{Q}_{\tilde{i}l}} \gamma \tag{3.25}$$

subject to

$$\begin{bmatrix} 1 & * \\ \bar{x}(k|k) & \bar{Q}_{il} \end{bmatrix} \geq 0, \tag{3.26}$$

$$\begin{bmatrix} \bar{G} + \bar{G}^T - \bar{Q}_{il} & * & * & * \\ \bar{A}_{il}\bar{G} + \bar{B}_{il}(D_q\bar{Y} + D_q^-\bar{Z}) & \bar{Q}_{\tilde{i}l} & * & * \\ \bar{Q}^{\frac{1}{2}}\bar{G} & 0 & \gamma\bar{I} & * \\ R^{\frac{1}{2}}(D_q\bar{Y} + D_q^-\bar{Z}) & 0 & 0 & \gamma I \end{bmatrix} \geq 0, \tag{3.27}$$

$$\begin{bmatrix} X & * \\ \bar{Z}^T & \bar{G} + \bar{G}^T - \bar{Q}_{il} \end{bmatrix} \geq 0, \; X_{rr} \leq u_{r,\max}^2, r = 1, 2, \ldots, m, \tag{3.28}$$

$\forall i \in \mathcal{I}_1$, $\forall l \in \mathcal{L}$, $\forall q \in \mathcal{Q}$, where $\forall \tilde{i}$ such that $(\tilde{i}, i) \in \mathcal{W}$, $\mathcal{W}$ is the set of all possible switching sequences defined as $\mathcal{W} \triangleq \mathcal{I}_1 \times \mathcal{I}_1$, on the other hand $\bar{Q}_{il}$ and $\bar{Q}_{\tilde{i}l}$ are symmetric matrices respectively, and $\bar{F} = \bar{Y}\bar{G}^{-1}$.

Proof. The proof follows Theorem 3.4. □

Based on Theorem 3.4 and Theorem 3.5, we state the algorithm for the implementation of saturated state feedback RCMPC for PWA sytems. It is given as follows

Algorithm 3.1 (Saturated state feedback RCMPC for PWA systems with delay-free).

1. *Get the measured state $x(k), x \in \mathcal{X}_i, \forall i \in \mathcal{I}$.*

2. *For $x \in \mathcal{X}_i, i \in \mathcal{I}_0$,*

   - *Solve $\min_{G,Y,Z,Q_{il},Q_{\tilde{i}l}} \gamma$ s.t. (3.15),(3.16),(3.17) and compute $F(k)$.*
   - *Apply $u(k) = \sigma(F(k)x(k))$ to the process.*

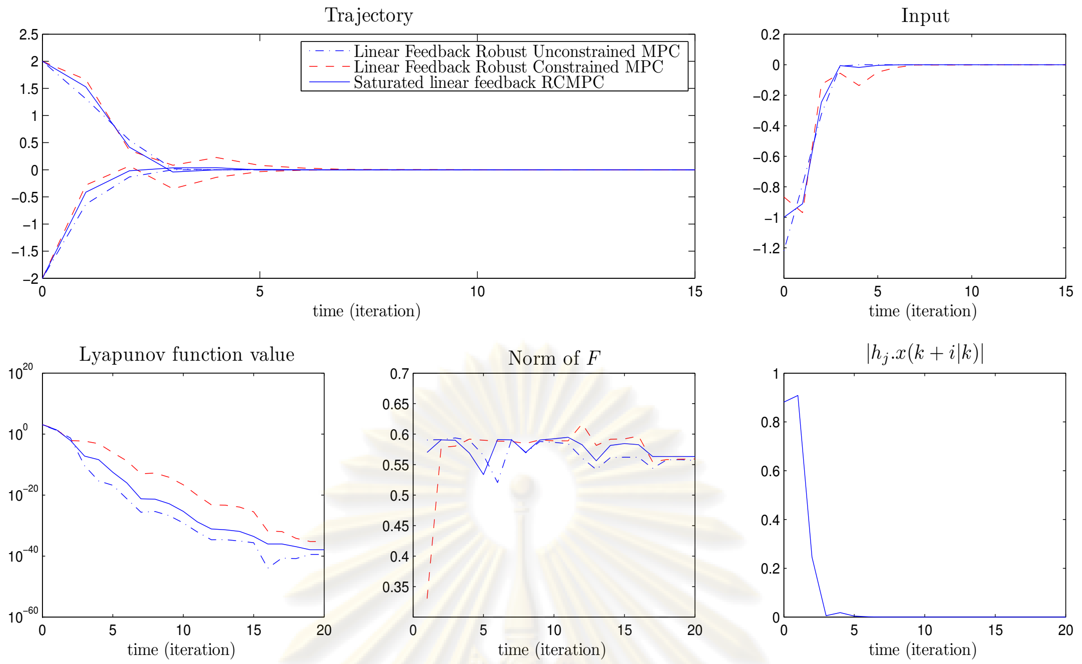

Figure 3.2: Comparison of the proposed method with the existing methods under input constraint $|u| \leq 1$.

3. *For* $x \in \mathcal{X}_i, i \in \mathcal{I}_1$,

   - *Solve* $\min_{\bar{G},\bar{Y},\bar{Z},\bar{Q}_{il},\bar{Q}_{\tilde{i}l}} \gamma$ *s.t. (3.26),(3.27),(3.28) and compute* $\bar{F}(k)$.
   - *Apply* $u(k) = \sigma(\bar{F}(k)\bar{x}(k))$ *to the process.*

4. *Set* $k := k + 1$ *and go to 1.*

3.5 Numerical Example

Consider the following uncertain PWA system with the partitioning

$$x(k+1) = 0.8 \begin{bmatrix} \cos(\alpha(k)) & -\sin(\alpha(k)) \\ \sin(\alpha(k)) & \cos(\alpha(k)+\beta(k)) \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$\alpha(k) = \begin{cases} \frac{\pi}{3} & [\,1, 0\,]x(k) \geq 0 \\ -\frac{\pi}{3} & [\,1, 0\,]x(k) < 0 \end{cases}$$

$$0 < \beta < 0.5$$

The uncertain PWA system has two modes, where

$$\mathcal{X}_1 = \{x | [1, 0]x \geq 0\}, A_1(k) = \begin{bmatrix} \frac{2}{5} & -\frac{2\sqrt{3}}{5} \\ \frac{2\sqrt{3}}{5} & \frac{2}{5} + \beta(k) \end{bmatrix}$$

$$\mathcal{X}_2 = \{x | [1, 0]x < 0\}, A_2(k) = \begin{bmatrix} \frac{2}{5} & \frac{2\sqrt{3}}{5} \\ -\frac{2\sqrt{3}}{5} & \frac{2}{5} + \beta(k) \end{bmatrix}$$

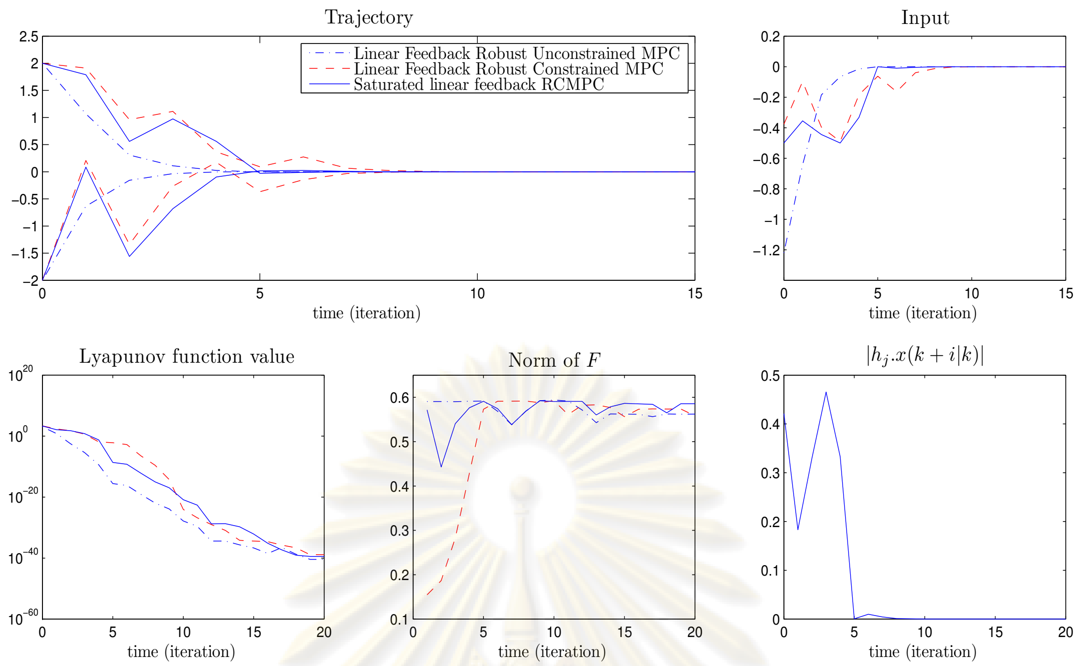

Figure 3.3: Comparison of the proposed method with the existing methods under input constraint $|u| \leq 0.5$.

The system with uncertainties $0 < \beta(k) < 0.5$, then we can conclude that $A_1(k) \in \Omega_1 = \text{Co}\{A_{11}, A_{12}\}$, and $A_2(k) \in \Omega_2 = \text{Co}\{A_{21}, A_{22}\}$

$$A_{11}(k) = \begin{bmatrix} \frac{2}{5} & -\frac{2\sqrt{3}}{5} \\ \frac{2\sqrt{3}}{5} & \frac{2}{5} \end{bmatrix}, \quad A_{12}(k) = \begin{bmatrix} \frac{2}{5} & -\frac{2\sqrt{3}}{5} \\ \frac{2\sqrt{3}}{5} & \frac{7}{10} \end{bmatrix},$$

$$A_{21}(k) = \begin{bmatrix} \frac{2}{5} & \frac{2\sqrt{3}}{5} \\ -\frac{2\sqrt{3}}{5} & \frac{2}{5} \end{bmatrix}, \quad A_{22}(k) = \begin{bmatrix} \frac{2}{5} & \frac{2\sqrt{3}}{5} \\ -\frac{2\sqrt{3}}{5} & \frac{7}{10} \end{bmatrix},$$

$$B_1 = B_2 = B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

At the time when the states enter partition $\mathcal{X}_1$, then the uncertainties $\Omega_1 = \text{Co}\{(A_{11}, B), (A_{12}, B)\}$, and when the states enter partition $\mathcal{X}_2$, then the uncertainties $\Omega_2 = \text{Co}\{(A_{21}, B), (A_{22}, B)\}$.

We consider two cases in the simulation, where the maximum input constraint:

$$u^1_{\max} = 1, \quad u^2_{\max} = 0.5$$

With tuning parameters $Q = I_2$, $R = 1$, and given initial condition $x(0) = [\,-2, 2\,]^T$, the LMI conditions in Theorem 3.4 can be solved. On a 2.2 GHz Intel Centrino Core 2 Duo Processor, with 2 GB RAM, the CPU time required to compute the online algorithm in Theorem 3.4 are 5.62 seconds for $u^1_{\max} = 1$, and 5.56 seconds for $u^2_{\max} = 0.5$.

Figures 3.2 and 3.3 shown the closed-loop response of the uncertain PWA system and the corresponding control signal for the two cases respectively. In particular, the simulation intended to compare the performance of saturated linear feedback RCMPC, and the linear feedback RCMPC algorithm. Notice that the simulation result of linear feedback robust unconstrained MPC also given here only as a reference of the closed-loop system performance without input constraint.

The saturated linear feedback RCMPC has better performance because it effectively utilizes the control region. The effectiveness of the proposed method can also be seen in Fig. 3.3 where $u_{\max} = 0.5$. The system states trajectory reaches the origin faster than the previous linear feedback RCMPC method, while the input constraint is tighter. Further analysis concludes that, for the two cases, the Lyapunov function value decreases along the sampling time which guarantees the robust stability of the closed-loop systems, and the assumption of polytopic description for the saturated linear feedback controller (5.13) is achieved.

CHAPTER IV

RCMPC FOR PWA SYSTEMS WITH TIME-INVARIANT DELAY

In practice, uncertainties and robustness are the main topic of research in the modeling and analysis of nonlinear systems. It is also well known, that time-delay cannot be avoided in industry everywhere. There has been extensive research addressed in this area (see [5,20,25,32], and the references therein). We extend the robust MPC strategy for uncertain PWA systems in [6] into uncertain PWA systems with time-invariant delay. In the design, we adopt multiple quadratic Lyapunov functions corresponding to different vertices of the uncertainties polytope in different partitions as the upper bound function.

4.1 Derivation of Upper Bound

The system is described by (2.3.1) with the associated uncertainties set (2.8). The system state $x(k)$ is assumed to be measurable. As mentioned in section 2.6, the minimization of the nominal objective function (2.13) at each sampling time k is replaced by the minimization of a robust performance objective as follows

$$\min_{\vec{u}(k+j|k)} \max_{[A_i(k+j)|A_{di}(k+j)|B_i(k+j)]\in\Omega_i, i\in\mathcal{I}_0, j\geq 0} J_\infty(k), \tag{4.1}$$

where $Q > 0$, $R > 0$ are given weighting matrices.

The min-max problem (4.1) is used to minimize the worst-case objective function by deriving an upper bound among all time-varying plants $[A_i(k)|A_{di}(k)|B_i(k)] \in \Omega_i, i \in \mathcal{I}_0$. Because the parameter of time-delay is been considered in the polytopic description, therefore, a modified PWA quadratic Lyapunov-Krasovskii function is used in the formulation, and defined as follow [5]:

$$\begin{aligned} V(v(k)) = \;& x(k)^T P_i x(k) + \sum_{\hat{j}=1}^{\tau_1} x(k-\hat{j})^T P_{1i} x(k-\hat{j}) + \sum_{\hat{j}=\tau_1+1}^{\tau_2} x(k-\hat{j})^T P_{2i} x(k-\hat{j}) + \dots \\ & + \sum_{\hat{j}=\tau_{p-1}+1}^{\tau_p} x(k-\hat{j})^T P_{pi} x(k-\hat{j}), \quad x \in \mathcal{X}_i, \; i \in \mathcal{I}_0. \end{aligned} \tag{4.2}$$

At sampling time k, suppose V satisfies the following inequality for all $x(k+j|k), u(k+j|k), j \geq 0$, satisfying (2.3.1), and for any $[A_i(k+j)|A_{di}(k+j)|B_i(k+j)] \in \Omega_i, i \in \mathcal{I}_0$,

$$V(v(k+j+1|k)) - V(v(k+j|k)) \leq -\left[\|x(k+j|k)\|_Q^2 + \|u(k+j|k)\|_R^2\right]. \tag{4.3}$$

For the robust performance objective function to be finite, we let $\lim_{j\to\infty} x(k+j|k) = 0$, $\lim_{j\to\infty} u(k+j|k) = 0$, and $\lim_{j\to\infty} V(v(k+j|k)) = 0$. Summing (4.3) from $j = 0$ to $j = \infty$, we obtain the following inequality

$$\max_{[A_i(k+j)|A_{di}(k+j)|B_i(k+j)]\in\Omega_i, i\in\mathcal{I}_0} J_\infty(k) \le V(v(k|k)) \le \gamma, \tag{4.4}$$

where γ is an upper bound of the robust performance objective. Therefore, the robust constrained MPC algorithm (2.13) has been redefined to synthesize, at each time step k, a constant saturated state-feedback control law to minimize the following optimization problem

$$\begin{aligned} &\min_{\vec{u}(k+j|k)} \gamma, \\ &\text{subject to } (2.11), (4.3), (4.4). \end{aligned} \tag{4.5}$$

4.2 Control Algorithm

By applying the polytopic description of the saturated linear feedback controller (2.12), the optimization problem (4.5) can be reduced to an LMI optimization problem for uncertain systems (2.3.1).

Remark 4.1. *As stated in the previous chapter that the standard linear feedback controller can be considered as a special case of saturated linear feedback controller. In this chapter, we only consider the derivation of saturated linear feedback controller as a more general formulation concept to PWA systems with TID.*

Theorem 4.1 (Saturated state feedback RCMPC for PWA systems with TID). *Let $x(k|k) = x(k)$ be the state of the systems (2.3.1) measured at sampling time k in partition $\mathcal{X}_i, i \in \mathcal{I}_0$. Suppose the switching sequence of the PWA system from one partition to another is known. Then the optimization problem (4.5) with a saturated state-feedback control law $u(k+j|k) = \sigma(F(k)x(k+j|k))$ can be solved by the following LMIs*

$$\min_{Y,Z,Q_{\tilde{i}l},Q_{il},Q_{dil}} \gamma \tag{4.6}$$

subject to

$$\begin{bmatrix} 1 & * \\ v(k|k) & \Gamma \end{bmatrix} \ge 0, \ \forall d \in \mathcal{D}, \tag{4.7}$$

$$\begin{bmatrix} G+G^T-Q_{il} & * & * & * & * & * \\ G & Q_{1il} & * & * & * & * \\ Q^{\frac{1}{2}}G & 0 & \gamma I & * & * & * \\ R^{\frac{1}{2}}(D_qY+D_q^-Z) & 0 & 0 & \gamma I & * & * \\ 0 & 0 & 0 & 0 & \widehat{M} & * \\ A_{il}G+B_{il}(D_qY+D_q^-Z) & 0 & 0 & 0 & \widehat{A} & Q_{\tilde{i}l} \end{bmatrix} \ge 0, \tag{4.8}$$

$\forall q \in \mathcal{Q}, \forall i \in \mathcal{I}_0, \forall l \in \mathcal{L}, \forall \tilde{i}$ *such that* $(\tilde{i}, i) \in \mathcal{W}$,

$$\begin{bmatrix} X & * \\ Z^T & G+G^T-Q_{il} \end{bmatrix} \ge 0, \ X_{rr} \le u_{r,\max}^2, r = 1, 2, \ldots, m, \ \forall i \in \mathcal{I}_0, \forall l \in \mathcal{L}, \tag{4.9}$$

where Q_{il}, Q_{dil}, *and* $Q_{\tilde{i}l}$ *are symmetric matrices, and* $F = YG^{-1}$, *with*

$$\begin{aligned}
\Gamma &= diag\left\{Q_{il}, Q_{1il}, ..., Q_{1il}, Q_{2il}, ..., Q_{2il}, \ldots, Q_{pil}\right\}, \\
\widehat{A} &= \left[A_{1il}, 0, A_{2il}, 0, \ldots, \ldots, A_{pil}\right], \\
\widehat{M} &= \begin{bmatrix} Q_{1il} & * & * & * & \ldots & * \\ Q_{1il} & Q_{2il} & * & * & \ldots & * \\ 0 & 0 & Q_{2il} & * & \ldots & * \\ 0 & 0 & Q_{2il} & Q_{3il} & \ldots & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & Q_{pil} \end{bmatrix}.
\end{aligned}$$

Proof. We define the PWA system (2.3.1) with uncertainties described by (2.8) into L PWA subsystems in each of polyhedral partition $\mathcal{X}_i$

$$x(k+1) = A_{il}(k)x(k) + \sum_{d=1}^{p} A_{dil}(k)x(k-\tau_d) + B_{il}(k)u(k), \quad \forall l \in \mathcal{L}.$$

At sampling time $k+j+1$, assume the state $x(k+j+1|k)$ is implemented by the saturated state feedback (2.9), there exists $H \in \mathbb{R}^{m \times n}$ satisfying

$$|h_r x(k+j|k)| \leq u_{r,\max}, \quad \forall j \geq 0, \;\; r = 1, 2, \ldots, m. \tag{4.10}$$

The condition (4.10) can be transformed into LMIs (4.9). The delay-term does not affect the LMIs of saturated constraint (4.9). Thus the proof will not be repeated here, due to the similarity condition as in Theorem 4.1. Then we obtain the state

$$\begin{aligned}
x(k+j+1|k) = [&A_{il}(k+j) + B_{il}(k+j)\psi, A_{1il}(k+j), \\
&\ldots, A_{pil}(k+j)]\, w(k+j|k), \quad \forall j > 0, \forall i \in \mathcal{I}_0, \forall l \in \mathcal{L},
\end{aligned} \tag{4.11}$$

where $\psi \triangleq \sum_{q=1}^{2^m} \lambda_q (D_q F + D_q^- H)$.

By applying (4.2) and assume that the state (4.11) enter partition $\mathcal{X}_{\tilde{i}}$, we obtain

$$\begin{aligned}
\Delta V(v(k|k)) &= V(v(k+j+1|k)) - V(v(k+j|k)) \\
&= \begin{bmatrix} x(k+j+1|k) \\ w(k+j|k) \end{bmatrix}^T \begin{bmatrix} P & 0 \\ 0 & \widehat{P} \end{bmatrix} \begin{bmatrix} x(k+j+1|k) \\ w(k+j+1|k) \end{bmatrix},
\end{aligned} \tag{4.12}$$

where $\widehat{P} = \text{diag}\ \{P_{1il} - P_{il}, P_{2il} - P_{1il}, \ldots, P_{pil}\}$, for all $\tilde{i}$ such that $(\tilde{i}, i) \in \mathcal{W}$, where $\mathcal{W}$ is the set of all possible switching sequences defined as $\mathcal{W} \triangleq \mathcal{I}_0 \times \mathcal{I}_0$.

By applying (4.11), and (4.12) into (4.3),

$$\begin{aligned}
V(v(k+j+1|k)) - V(v(k+j|k)) + \|x(k+j|k)\|_Q^2 + \|u(k+j|k)\|_R^2 &\leq 0, \\
w(k+j|k)^T \Delta(k+j) w(k+j|k) &\leq 0,
\end{aligned}$$

where

$$\Delta = \begin{bmatrix} \Pi & * & * & \dots & * \\ A_{1il}^T P_{\tilde{i}l}(A_{il} + B_{il}\psi) & \Pi_0 & * & \dots & * \\ A_{2il}^T P_{\tilde{i}l}(A_{il} + B_{il}\psi) & A_{2il}^T P_{\tilde{i}l} A_{1il} & \Pi_1 & \dots & * \\ \vdots & \vdots & \vdots & \ddots & * \\ A_{pil}^T P_{\tilde{i}l}(A_{il} + B_{il}\psi) & A_{pil}^T P_{\tilde{i}l} A_{1il} & A_{pil}^T P_{\tilde{i}l} A_{2il} & \dots & \Pi_p \end{bmatrix},$$

with (note that we omit the time $(k+j)$-term for space consideration)

$$\begin{aligned} \Pi &= -P_{il} + P_{1il} + (A_{il} + B_{il}\psi)^T P_{\tilde{i}l}(A_{il} + B_{il}\psi) + Q + \psi^T R \psi, \\ \Pi_0 &= -P_{1il} + P_{2il} + A_{1il}^T P_{\tilde{i}l} A_{1il}, \\ \Pi_1 &= -P_{2il} + P_{3il} + A_{2il}^T P_{\tilde{i}l} A_{2il}, \\ \Pi_p &= -P_{pil} + A_{pil}^T P_{\tilde{i}l} A_{pil}. \end{aligned}$$

This can be rewritten as

$$\begin{bmatrix} P_{il} - P_{1il} - Q - \psi^T R \psi & * \\ 0 & \bar{M} \end{bmatrix} - \begin{bmatrix} (A_{il} + B_{il}\psi)^T \\ \bar{A}^T \end{bmatrix} P_{\tilde{i}l} \begin{bmatrix} (A_{il} + B_{il}\psi) \\ \bar{A} \end{bmatrix}^T \geq 0,$$

where $\bar{M} = \text{diag}\ \{P_{1il} - P_{2il}, P_{2il} - P_{3il}, \dots, P_{pil}\}, \bar{A} = \left[A_{1il}^T, A_{2il}^T, \dots, A_{pil}^T\right]^T$.
Applying Schur complement, then Δ is equivalent to

$$\begin{bmatrix} P_{il} - P_{1il} - Q - \psi^T R \psi & * & * \\ 0 & \bar{M} & * \\ (A_{il} + B_{il}\psi) & \bar{A} & P_{\tilde{i}l}^{-1} \end{bmatrix} \geq 0. \tag{4.13}$$

Define

$$\begin{aligned} &P_{il} \triangleq \gamma Q_{il}^{-1} > 0, P_{\tilde{i}l} \triangleq \gamma Q_{\tilde{i}l}^{-1} > 0, \\ &P_{dil} \triangleq \gamma Q_{dil}^{-1} > 0, \forall d \in \mathcal{D}, \quad F \triangleq Y Q_{il}^{-1}, H \triangleq Z Q_{il}^{-1}. \end{aligned} \tag{4.14}$$

By multiplying left-hand and right-hand sides of Δ by diag $\left\{Q_{il}^{-1} G, I, I, \dots, I\right\}$, then applying several steps of Schur complement with the same procedure as the proof of Theorem 2.1, satisfying (4.8) leads to

$$\begin{bmatrix} G^T Q_{il}^{-1} G & * & * & * & * & * \\ G & Q_{1il} & * & * & * & * \\ \gamma^{-\frac{1}{2}} Q^{\frac{1}{2}} G & 0 & I & * & * & * \\ \gamma^{-\frac{1}{2}} R^{\frac{1}{2}} (D_q F + D_q^- H) G & 0 & 0 & I & * & * \\ 0 & 0 & 0 & 0 & \widehat{M} & * \\ (A_{il} + B_{il}(D_q F + D_q^- H)) G & 0 & 0 & 0 & \widehat{A} & Q_{\tilde{i}l} \end{bmatrix} \geq 0, \tag{4.15}$$

$$\forall q \in \mathcal{Q}, \forall i \in \mathcal{I}_0, \forall l \in \mathcal{L}, \forall \tilde{i} \text{ such that } (\tilde{i}, i) \in \mathcal{W}.$$

Thus the inequality (4.8) holds.

Next, we show that the inequality (4.7) holds. By applying congruence transformation to the resulting inequality with diag$[1, \Gamma^{-1}]$, we have

$$\begin{bmatrix} 1 & * \\ \Gamma^{-1} v(k|k) & \Gamma^{-1} \end{bmatrix} \geq 0.$$

Substituting $\widehat{\Gamma} = \left[\gamma P_{il}^{-1}, \gamma P_{1il}^{-1}, \ldots, \gamma P_{pil}^{-1}\right]$ and applying congruence transformation to the resulting inequality with $\text{diag}[1, \gamma\widehat{\Gamma}^{-1}]$, we obtain

$$\begin{bmatrix} 1 & * \\ v(k|k) & \gamma\widehat{\Gamma}^{-1} \end{bmatrix} \geq 0,$$

which, by the Schur complements, yields

$$v(k|k)^T \widehat{\Gamma} v(k|k) \leq \gamma. \tag{4.16}$$

Hence, we conclude that the first inequality (4.7) holds. □

4.3 Robust Stability

Lemma 4.1 (Feasibility for PWA systems with TID). *Any feasible solution of the optimization in Theorem 4.1 at time k is also feasible for all times $t > k$. Thus, if the optimization problem in Theorem 4.1 is feasible at time k, then it is feasible for all times $t > k$.*

Proof. The proof of this lemma is a natural extension from Theorem 3.2.
Assume that the optimization in Theorem 4.1 is feasible at time k. The LMI problem which depends explicitly on the measured state $v(k|k) = v(k)$ of the system is the following

$$\begin{bmatrix} 1 & * \\ v(k|k) & \Gamma \end{bmatrix} \geq 0, \quad \forall i \in \mathcal{I}_0.$$

Thus, to prove the lemma, we need only to prove that the LMI is feasible for all future measured states $v(k+j|k+j) = v(k+j), j \geq 1$. Now, feasibility of the problem at time k implies satisfaction of (4.8) and (4.9), which, using Lemma 3.1, in turn imply for the uncertainties set description that (3.8) is satisfied. Thus, for any Ω_i we must have

$$v(k+j|k)^T \Gamma^{-1} v(k+j|k) < 1, \forall i \in \mathcal{I}_0.$$

Since the state measured at time $k+1$, that is, $v(k+1|k+1) = v(k+1)$, equals $[A_{il}(k) + B_{il}(k)\psi|A_{1il}|\ldots|A_{dil}]w(k|k)$ for some $[A_{il}(k)|A_{dil}|B_{il}(k)] \in \Omega_i, \forall d \in \mathcal{D}$, it must also satisfy this inequality, i.e.,

$$v(k+1|k+1)^T \Gamma^{-1} v(k+1|k+1) < 1, \forall i \in \mathcal{I}_0.$$

Thus, the feasible solution of the optimization problem at time k is also feasible at time $k+1$. Hence, the optimization is feasible at time $k+1$. This argument can be continued for time $k+2, k+3, \ldots$ to complete the proof. □

Theorem 4.2 (Robust stability for PWA systems with TID). *The feasible receding horizon state feedback control law obtained from Theorem 4.1 robustly asymptotically stabilizes the closed-loop PWA systems (2.3.1).*

Proof. To prove the asymptotic stability, we will establish, according to Definition 2.4, that $V(v(k|k)) = v(k|k)^T\Gamma(k)v(k|k)$, where $\Gamma(k) > 0$ element-wise is obtained from the optimal solution at time k, is a strictly decreasing Lyapunov function for the closed-loop.

First, let assume that Theorem 4.1 is feasible at time $k = 0$. Lemma 4.1 then ensures feasibility of the problem at all times $k > 0$. Because of $\Gamma(k+1)$ is optimal whereas $\Gamma(k)$ is only feasible at time $k+1$, we must have

$$v(k+1|k+1)^T\Gamma(k+1)v(k+1|k+1) \leq v(k+1|k+1)^T\Gamma(k)v(k+1|k+1). \tag{4.17}$$

We know from Lemma 3.1 that if $u(k+j|k) = \sigma(F(k)x(k+j|k)), j \geq 0$ ($F(k)$ is obtained from the optimal solution at time k), then for any $[A_{il}(k)|A_{dil}(k)|B_{il}(k)] \in \Omega_i$, we must have

$$v(k+1|k)^T\Gamma(k)v(k+1|k) < v(k|k)^T\Gamma v(k|k), \quad v(k|k) \neq 0. \tag{4.18}$$

Since the measured state $v(k+1|k+1) = v(k+1)$ equals
$[A_{il}(k) + B_{il}(k)\psi|A_{1il}|\ldots|A_{dil}]w(k|k)$ for some $[A_{il}(k)|A_{dil}(k)|B_{il}(k)] \in \Omega_i$, it must also satisfy (4.18). Combining this with inequality (4.17) we conclude that

$$v(k+1|k+1)^T\Gamma(k+1)v(k+1|k+1) \leq v(k|k)^T\Gamma(k)v(k|k), \quad v(k|k) \neq 0.$$

Thus, $V(v(k|k)) = v(k|k)^T\Gamma(k)v(k|k)$ is strictly decreasing Lyapunov function for the closed-loop. We therefore conclude that $v(k) \to 0$ as $k \to \infty$. By checking Lyapunov stability conditions (4.8), we conclude that the controller guarantees asymptotic stability. ☐

4.4 Augmented Formulation

Theorem 4.3 (Augmented saturated state feedback RCMPC for PWA systems with TID). *Let $\bar{x}(k|k) = \bar{x}(k)$ be the state of the systems (2.3.1) measured at sampling time k in partition $\mathcal{X}_i, i \in \mathcal{I}_1$. Suppose the switching sequence of the PWA system from one partition to another is known. Then the optimization problem (4.5) with a saturated state-feedback control law $u(k+j|k) = \sigma(\bar{F}(k)\bar{x}(k+j|k))$ can be solved by the following LMIs*

$$\min_{\bar{Y},\bar{Z},\bar{Q}_{\tilde{i}l},\bar{Q}_{il},\bar{Q}_{dil}} \gamma \tag{4.19}$$

subject to

$$\begin{bmatrix} 1 & * \\ \bar{v}(k|k) & \Gamma \end{bmatrix} \geq 0, \quad \forall d \in \mathcal{D}, \tag{4.20}$$

$$\begin{bmatrix} \bar{G} + \bar{G}^T - \bar{Q}_{il} & * & * & * & * & * \\ \bar{G} & \bar{Q}_{1il} & * & * & * & * \\ \bar{Q}^{\frac{1}{2}}\bar{G} & 0 & \gamma\bar{I} & * & * & * \\ R^{\frac{1}{2}}(D_q\bar{Y} + D_q^-\bar{Z}) & 0 & 0 & \gamma I & * & * \\ 0 & 0 & 0 & 0 & \widehat{M} & * \\ \bar{A}_{il}\bar{G} + \bar{B}_{il}(D_q\bar{Y} + D_q^-\bar{Z}) & 0 & 0 & 0 & \widehat{A} & \bar{Q}_{\tilde{i}l} \end{bmatrix} \geq 0, \tag{4.21}$$

$\forall q \in \mathcal{Q}, \forall i \in \mathcal{I}_1, \forall l \in \mathcal{L}, \forall \tilde{i}$ *such that* $(\tilde{i}, i) \in \mathcal{W}$,

$$\begin{bmatrix} X & * \\ \bar{Z}^T & \bar{G}+\bar{G}^T-\bar{Q}_{il} \end{bmatrix} \geq 0,\ X_{rr} \leq u_{r,\max}^2, r=1,2,\ldots,m,\ \ \forall i \in \mathcal{I}_1, \forall l \in \mathcal{L}, \quad (4.22)$$

where $\bar{Q}_{il}$, $\bar{Q}_{dil}$, and $\bar{Q}_{\tilde{i}l}$ are symmetric matrices respectively, and $\bar{F} = \bar{Y}\bar{G}^{-1}$, with

$$\begin{aligned} \Gamma &= diag\left\{\bar{Q}_{il}, \bar{Q}_{1il}, ..., \bar{Q}_{1il}, \bar{Q}_{2il}, ..., \bar{Q}_{2il}, \ldots, \bar{Q}_{pil}\right\}, \\ \widehat{A} &= \left[\bar{A}_{1il}, 0, \bar{A}_{2il}, 0, \ldots, \ldots, \bar{A}_{pil}\right], \\ \widehat{M} &= \begin{bmatrix} \bar{Q}_{1il} & * & * & * & \ldots & * \\ \bar{Q}_{1il} & \bar{Q}_{2il} & * & * & \ldots & * \\ 0 & 0 & \bar{Q}_{2il} & * & \ldots & * \\ 0 & 0 & \bar{Q}_{2il} & \bar{Q}_{3il} & \ldots & * \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \ldots & \bar{Q}_{pil} \end{bmatrix}. \end{aligned}$$

Proof. The proof follows Theorem 4.1 □

4.5 Numerical Example

Consider the following uncertain PWA system with the partitioning

$$\begin{aligned} x(k+1) &= \theta \begin{bmatrix} \cos(\alpha(k)) & -\sin(\alpha(k)) \\ \sin(\alpha(k)) & \cos(\alpha(k)+\beta(k)) \end{bmatrix} x(k) \\ &+0.1 \begin{bmatrix} \cos(\alpha(k)) & -\sin(\alpha(k)) \\ \sin(\alpha(k)) & \cos(\alpha(k)+\beta(k)) \end{bmatrix} x(k-\tau_1) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k) \\ \alpha(k) &= \begin{cases} \frac{\pi}{3} & [\,1,0\,]x(k) \geq 0 \\ -\frac{\pi}{3} & [\,1,0\,]x(k) < 0 \end{cases} \\ \theta &> 0,\ 0 < \beta < 0.3,\ \tau_1 = 2 \end{aligned}$$

From the uncertain PWA system we can see that the system has two modes

$$\mathcal{X}_1 = \{x | [1,0]x \geq 0\}, \mathcal{X}_2 = \{x | [1,0]x < 0\},$$

We consider the system with uncertainties $0 < \beta(k) < 0.3$, the maximum input constraint $u_{\max} = 0.2$, and $\theta = 0.6$. At the time when the states enter partition $\mathcal{X}_1$, then the uncertainties $\Omega_1 = \mathrm{Co}\{(A_{11}, A_{111}, B), (A_{12}, A_{112}, B)\}$, and when the states enter partition $\mathcal{X}_2$, then the uncertainties $\Omega_2 = \mathrm{Co}\{(A_{21}, A_{121}, B), (A_{22}, A_{122}, B)\}$.

With tuning parameters $Q = I_2$, $R = 1$, and given initial conditions $x(-2) = x(-1) = x(0) = [\,-2, 2\,]^T$, the LMI conditions in Theorem 4.1 can be solved. On a 2.4 GHz Intel Centrino Core 2 Duo Processor, with 1 GB RAM, the CPU time required to compute the online algorithm in Theorem 4.1 is 12.75 s.

Figure 4.1 shows the closed-loop response of the uncertain PWA system and the corresponding control signal for the two cases of standard linear feedback controller and saturated linear feedback controller. The effectiveness of the proposed method can be seen since the saturated linear feedback controller is able to drive the system states trajectory to reach the origin faster, while keeping the

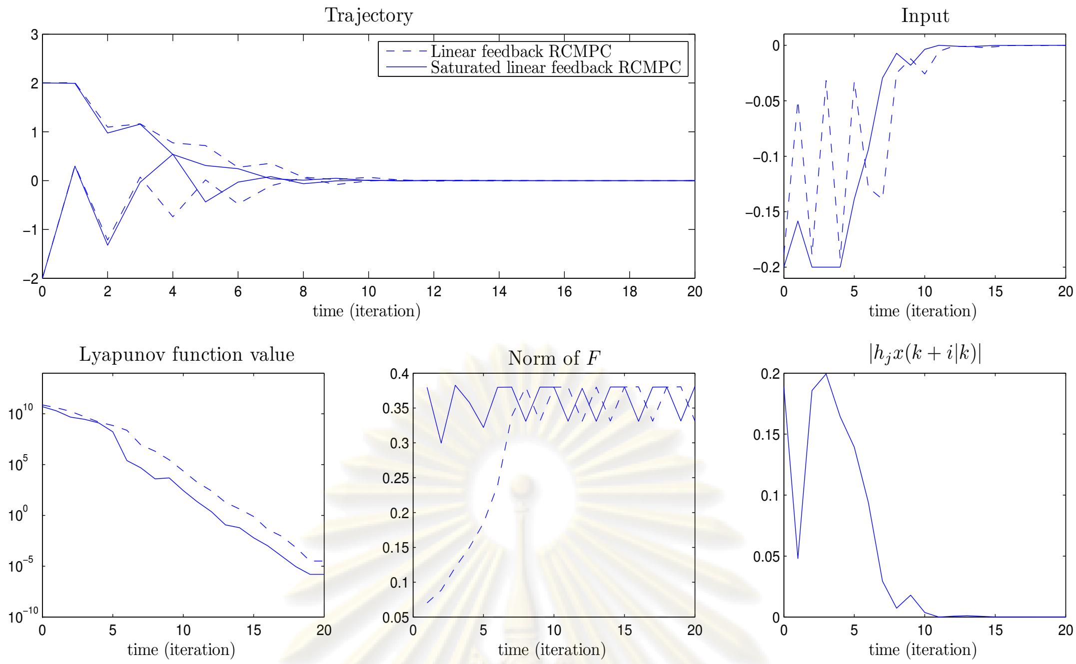

Figure 4.1: Comparison of the proposed method with input constraint $|u| \leq 0.2$.

input constraint within the prescribed bound. The Lyapunov function value decreases along the sampling time which guarantees the robust stability, and the assumption for the saturated linear feedback controller to be able to describe in polytopic description (5.13) is also achieved. Thus, the the objective in designing a control algorithm for robust constrained MPC for uncertain PWA systems with time-delay incorporating saturated linear feedback controller is achieved.

CHAPTER V

RCMPC FOR PWA SYSTEMS WITH TIME-VARYING DELAY

The uncertain and time-varying delays are addressed to a lesser extent. In [4], the authors present an algorithm considering only one state delay. This delay is assumed unknown, but with a known upper bound. In [22], the authors extended their idea to uncertain and time-varying delays systems by utilising the augmented state feedback to improve optimality and stability.

In this part, we generalized the results from the last chapter. We deal with systems that allow state delays to be uncertain and time-varying. This delay is assumed unknown, but with a known upper and lower bound.

5.1 Derivation of Upper Bound

The system is described by (2.3.2) with the associated uncertainties set (2.8). The system state $x(k)$ is assumed to be measurable. As mentioned in section 2.6, the minimization of the nominal objective function (2.13) that regulates the system to the origin $(x = 0, u = 0)$ at each sampling time k is replaced by the minimization of a robust performance objective

$$\min_{\bar{u}(k+j|k)} \max_{[\bar{A}_i(k+j), \bar{A}_{di}(k+j), \bar{B}_i(k+j)] \in \Omega_i, \tau_d \in \mathfrak{L}_d} J_\infty(k), \tag{5.1}$$

subject to (2.14), with

$$J_\infty \triangleq \sum_{j=0}^{\infty} \left[\|\bar{x}(k+j|k)\|_{\bar{Q}}^2 + \|u(k+j|k)\|_R^2 \right], x \in \mathcal{X}_i, i \in \mathcal{I}, \tag{5.2}$$

where $\bar{Q} = \bar{Q}^T > 0,\ R = R^T > 0$ are given weighting matrices. The optimization problem in this work has incorporated τ_d in the ”max” operator.

The min-max problem (5.1) is used to minimize the worst-case objective function by deriving an upper bound among all time-varying plants $\left[\bar{A}_i(k), \bar{A}_{di}(k), \bar{B}_i(k)\right] \in \Omega_i$, $\tau_d \in \mathfrak{L}_d$. Because the parameter of time-delay is considered in the polytopic description, therefore, a modified PWA quadratic Lyapunov-Krasovskii functional is used in the formulation, and defined as follow [22].

$$V(\mathfrak{m}(k)) = \bar{x}(k)^T \bar{P}_i \bar{x}(k) + \sum_{d=1}^{p} \left(\sum_{j=\underline{\tau}_d}^{\overline{\tau}_d} \bar{x}(k-j)^T \bar{P}_{ji} \bar{x}(k-j) \right), \bar{x} \in \mathcal{X}_i, i \in \mathcal{I}. \tag{5.3}$$

In particular, the expression for the candidate Lyapunov function in each region can be recast as [23]

$$\bar{x}(k)^T \bar{P}_i \bar{x}(k) = \begin{bmatrix} x(k) \\ 1 \end{bmatrix}^T \begin{bmatrix} P_i & * \\ q_i^T & r_i \end{bmatrix} \begin{bmatrix} x(k) \\ 1 \end{bmatrix}, \tag{5.4}$$

and it is also applied to the delayed candidate Lyapunov function $(\bar{P}_{ji})$, respectively.

At sampling time k, suppose V satisfies the following inequality for all $\bar{x}(k+j|k), u(k+j|k), j \geq 0$, satisfying (2.3.2), and for any $\left[\bar{A}_i(k+j), \bar{A}_{di}(k+j), \bar{B}_i(k+j)\right] \in \Omega_i$, $\tau_d \in \mathfrak{L}_d$

$$V(\mathfrak{m}(k+j+1|k)) - V(\mathfrak{m}(k+j|k)) \leq -\left[\|\bar{x}(k+j|k)\|_Q^2 + \|u(k+j|k)\|_R^2\right]. \tag{5.5}$$

For the robust performance objective function to be finite, we let $\lim_{j\to\infty}\bar{x}(k+j|k) = 0$, $\lim_{j\to\infty}u(k+j|k) = 0$, and $\lim_{j\to\infty}V(\mathfrak{m}(k+j|k)) = 0$. Summing (5.5) from $j = 0$ to $j = \infty$, we obtain the following inequality

$$\max_{[\bar{A}_i(k+j)|\bar{A}_{di}(k+j)|\bar{B}_i(k+j)]\in\Omega_i,\tau_d\in\mathfrak{L}_d} J_\infty(k) \leq V(\mathfrak{m}(k|k)) \leq \gamma, \tag{5.6}$$

where γ is an upper bound of the robust performance objective. Therefore, the robust constrained MPC algorithm (5.1) has been redefined to synthesize, at each time step k, a constant state-feedback control law to minimize the following optimization problem

$$\begin{gathered}\min_{\vec{u}(k+j)} \gamma, \\ \text{subject to } (2.11), (5.5), (5.6).\end{gathered} \tag{5.7}$$

5.2 Control Algorithm

By applying the polytopic description of the saturated linear feedback controller (2.12), the optimization problem (5.7) can be reduced to an LMI optimization problem for uncertain system (2.3.2).

Theorem 5.1. *Let $\bar{x}(k|k) = \bar{x}(k)$ be the state of the system (2.3.2) measured at sampling time k in partition $\mathcal{X}_i, i \in \mathcal{I}$. Suppose the switching sequence of the PWA system from one partition to another is known. Then the optimization problem (3.4) with a saturated state-feedback control law $u(k+j|k) = \sigma(\bar{F}(k)\bar{x}(k+j|k))$ can be solved by the following LMIs*

$$\min_{\bar{Y},\bar{Z},\bar{Q}_{\tilde{i}l},\bar{Q}_{il},\bar{Q}_{dil}} \gamma, \tag{5.8}$$

subject to

$$\begin{bmatrix} 1 & * \\ \mathfrak{m}(k|k) & \Gamma \end{bmatrix} \geq 0, \quad \forall d \in \mathcal{D}, \forall l \in \mathcal{L}, \forall i \in \mathcal{I}, \tag{5.9}$$

$$\begin{bmatrix} \bar{G}+\bar{G}^T-\bar{Q}_{il} & * & * & * & * & * \\ \bar{G} & \bar{Q}_{\mathcal{I}_1 il} & * & * & * & * \\ \bar{Q}^{\frac{1}{2}}\bar{G} & 0 & \gamma\bar{I} & * & * & * \\ R^{\frac{1}{2}}\widehat{\psi} & 0 & 0 & \gamma\bar{I} & * & * \\ 0 & 0 & 0 & 0 & \widehat{M} & * \\ \bar{A}_{il}\bar{G}+\bar{B}_{il}\widehat{\psi} & 0 & 0 & 0 & \widehat{A} & \bar{Q}_{\tilde{i}l} \end{bmatrix} \geq 0, \tag{5.10}$$

$$\forall q \in \mathcal{Q}, \forall i \in \mathcal{I}, \forall l \in \mathcal{L}, \forall \tilde{i} \text{ such that } (\tilde{i}, i) \in \mathcal{W},$$

$$\begin{bmatrix} X & * \\ \bar{Z}^T & \bar{G}+\bar{G}^T-\bar{Q}_{il} \end{bmatrix} \geq 0,\ X_{rr} \leq u_{r,\max}^2, r = 1, 2, \ldots, m \ \ \forall l \in \mathcal{L}, \forall i \in \mathcal{I}, \tag{5.11}$$

where $\bar{Q}_{il}$, $\bar{Q}_{dil}$, *and* $\bar{Q}_{\tilde{i}l}$ *are symmetric matrices respectively, and* $\bar{F} = \bar{Y}\bar{G}^{-1}$, *with*

$$
\begin{aligned}
\widehat{\psi} \quad &= D_q\bar{Y} + D_q^{-}\bar{Z},\\
\Gamma \quad &= diag\left\{\bar{Q}_{il}, \bar{Q}_{\underline{\tau}_1 il}, \ldots, \bar{Q}_{\overline{\tau}_1 il}, \bar{Q}_{\underline{\tau}_2 il}, \ldots, \bar{Q}_{\overline{\tau}_2 il}, \ldots, \bar{Q}_{\underline{\tau}_p il}, \ldots, \bar{Q}_{\overline{\tau}_p il}\right\},\\
\widehat{A} \quad &= \left[\bar{A}^T_{1il,\underline{\mu}_1}, 0, \bar{A}^T_{1il,\underline{\mu}_1+1}, 0, \ldots, \bar{A}^T_{1il,\overline{\mu}_1}, \bar{A}^T_{2il,\underline{\mu}_2}, 0, \ldots, \bar{A}^T_{2il,\overline{\mu}_2}, \ldots, \bar{A}^T_{pil,\underline{\mu}_p}, 0, \ldots, \bar{A}^T_{pil,\overline{\mu}_p}\right]^T,\\
\bar{A}_{dil,\underline{\mu}_d} \quad &= \begin{cases} \bar{A}_{dil}, & \underline{\mu}_d = \underline{\tau}_d,\\ 0, & \underline{\mu}_d \neq \underline{\tau}_d,\end{cases}\\
\widehat{M} \quad &= diag\left[\check{M}_{P,1}, \check{M}_{P,2}, \ldots, \check{M}_{P,p}\right],\\
\check{M}_{P,d} \quad &= \begin{bmatrix} \bar{Q}_{(\underline{\tau}_d)il} & * & \cdots & *\\ \bar{Q}_{(\underline{\tau}_d)il} & \bar{Q}_{(\underline{\tau}_{d+1})il} & \cdots & *\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \bar{Q}_{(\overline{\tau}_d)il}\end{bmatrix}.
\end{aligned}
$$

Proof. We define the PWA system (2.3.2) with uncertainties described by (2.8) into L PWA subsystems in each of polyhedral partition $\mathcal{X}_i$, and assume the time-delay is uncertain and time-varying,

$$\bar{x}(k+1) = \bar{A}_{il}(k)\bar{x}(k) + \sum_{d=1}^{p}\bar{A}_{dil}(k)\bar{x}(k-\tau_d(k)) + \bar{B}_{il}(k)u(k), \quad \forall l \in \mathcal{L}. \tag{5.12}$$

At sampling time $k+j+1$, assume the state $\bar{x}(k+j+1|k)$ is implemented by the saturated state feedback (2.9), there exists $\bar{H} \in \mathbb{R}^{m\times(n+1)}$ satisfy

$$|\bar{h}_r\bar{x}(k+j|k)| \leq u_{r,\max}, \quad \forall j \geq 0, \;\; r = 1, 2, \ldots, m. \tag{5.13}$$

The condition (5.13) can be transformed into LMIs (5.11). The delay-term does not affect the LMIs of saturated constraint (5.11). Thus the proof will not be repeated here, due to the similarity condition as in Theorem 4.1.

Then we obtain the state, $\forall j \geq 0$,

$$\bar{x}(k+j+1|k) = \left[\bar{A}_{il}(k+j) + \bar{B}_{il}(k+j)\psi, \check{A}_{1il}(k+j), \ldots, \check{A}_{pil}(k+j)\right]\check{\mathfrak{m}}(k+j|k), \tag{5.14}$$

for all $i \in \mathcal{I}$, fora all $l \in \mathcal{L}, \tau_d \in \mathfrak{L}_d$, where

$$
\begin{aligned}
&\psi \triangleq \sum_{q=1}^{2^m}\lambda_q(D_q\bar{F} + D_q^{-}\bar{H}),\\
&\check{A}_{dil}(k+j) = [\bar{A}_{dil,\underline{\mu}_d}(k+j), \bar{A}_{dil,\underline{\mu}_d+1}(k+j), \ldots, \bar{A}_{dil,\overline{\mu}_d}(k+j)].
\end{aligned}
$$

By applying (5.3) and assume that the state (5.14) enter partition $\mathcal{X}_{\tilde{i}}$, we obtain

$$V(\mathfrak{m}(k+j+1|k)) - V(\mathfrak{m}(k+j|k)) = \begin{bmatrix} \bar{x}(k+j+1|k)\\ \check{\mathfrak{m}}(k+j|k)\end{bmatrix}^T \begin{bmatrix} \bar{P} & 0\\ 0 & \widehat{P}\end{bmatrix} \begin{bmatrix} \bar{x}(k+j+1|k)\\ \check{\mathfrak{m}}(k+j|k)\end{bmatrix}, \tag{5.15}$$

where

$$
\begin{aligned}
&\widehat{P} = \text{diag }\left\{\check{P}_{1il}, \check{P}_{2il}, \ldots, \check{P}_{pil}\right\},\\
&\check{P}_{dil} = \text{diag }\left\{\bar{P}_{(\underline{\tau}_d+1)il} - \bar{P}_{(\underline{\tau}_d)il}, \bar{P}_{(\underline{\tau}_d+2)il} - \bar{P}_{(\underline{\tau}_d+1)il}, \ldots, \bar{P}_{(\overline{\tau}_d)il}\right\},
\end{aligned}
$$

for all $\tilde{i}$ such that $(\tilde{i}, i) \in \mathcal{W}$, and $\mathcal{W}$ is the set of all possible switching sequences defined as $\mathcal{W} \triangleq \mathcal{I} \times \mathcal{I}$.

By applying (5.14), and (5.15) into (3.2), we obtain

$$\check{\mathfrak{m}}(k+j|k)^T \Delta(k+j) \check{\mathfrak{m}}(k+j|k) \leq 0, \tag{5.16}$$

where

$$\Delta = \begin{bmatrix} \Pi_0 & * & \dots & * \\ \Xi_{0,1} & \Pi_1 & \dots & * \\ \vdots & \vdots & \ddots & * \\ \Xi_{0,p} & \Xi_{1,p} & \dots & \Pi_p \end{bmatrix},$$

with (note that we omit the time $(k+j)$-term for space consideration)

$$\begin{aligned}
\Lambda_{il} &= \bar{A}_{il} + \bar{B}_{il}\psi, \\
\Pi_0 &= \bar{P}_{(\underline{\tau}_1)il} - \bar{P}_{il} + \Lambda_{il}^T \bar{P}_{\tilde{i}l} \Lambda_{il} + \bar{Q} + \psi^T R \psi, \\
\Pi_1 &= \text{diag}\left\{ \bar{P}_{(\underline{\tau}_1+1)il} - \bar{P}_{(\underline{\tau}_1)il} + \bar{A}_{1il,\underline{\tau}_1}^T \bar{P}_{\tilde{i}l} \bar{A}_{1il,\underline{\tau}_1}, \dots, -\bar{P}_{(\bar{\tau}_1)il} + \bar{A}_{1il,\bar{\tau}_1}^T \bar{P}_{\tilde{i}l} \bar{A}_{1il,\bar{\tau}_1} \right\}, \\
\Pi_p &= \text{diag}\left\{ \bar{P}_{(\underline{\tau}_p+1)il} - \bar{P}_{(\underline{\tau}_p)il} + \bar{A}_{pil,\underline{\tau}_p}^T \bar{P}_{\tilde{i}l} \bar{A}_{pil,\underline{\tau}_p}, \dots, -\bar{P}_{(\bar{\tau}_p)il} + \bar{A}_{pil,\bar{\tau}_p}^T \bar{P}_{\tilde{i}l} \bar{A}_{pil,\bar{\tau}_p} \right\}, \\
\Xi_{0,1} &= \left[\bar{A}_{1il,\underline{\mu}_1}^T \bar{P}_{\tilde{i}l} \Lambda_{il}, \dots, \bar{A}_{1il,\bar{\mu}_1}^T \bar{P}_{\tilde{i}l} \Lambda_{il} \right]^T \\
\Xi_{0,p} &= \left[\bar{A}_{pil,\underline{\mu}_p}^T \bar{P}_{\tilde{i}l} \Lambda_{il}, \dots, \bar{A}_{pil,\bar{\mu}_p}^T \bar{P}_{\tilde{i}l} \Lambda_{il} \right]^T
\end{aligned}$$

This can be rewritten as

$$\begin{bmatrix} \bar{P}_{il} - \bar{P}_{\underline{\tau}_1 il} - \bar{Q} - \psi^T R \psi & * \\ 0 & M \end{bmatrix} - \begin{bmatrix} \Lambda_{il}^T \\ N^T \end{bmatrix} \bar{P}_{\tilde{i}l} \begin{bmatrix} \Lambda_{il} \\ N \end{bmatrix}^T \geq 0, \tag{5.17}$$

where

$$\begin{aligned}
M &= \text{diag}\left\{ \bar{P}_{(\underline{\tau}_1+1)il} - \bar{P}_{(\underline{\tau}_1)il}, \dots, -\bar{P}_{(\bar{\tau}_1)il}, \dots, \bar{P}_{(\underline{\tau}_p+1)il} - \bar{P}_{(\underline{\tau}_p)il}, \dots, -\bar{P}_{(\bar{\tau}_p)il} \right\}, \\
N &= \left[\bar{A}_{1il,\underline{\mu}_1}^T, \dots, \bar{A}_{1il,\bar{\mu}_1}^T, \dots, \bar{A}_{pil,\underline{\mu}_p}^T, \dots, \bar{A}_{pil,\bar{\mu}_p}^T \right]^T.
\end{aligned} \tag{5.18}$$

Applying Schur complement, then Δ is equivalent to

$$\begin{bmatrix} \bar{P}_{il} - \bar{P}_{1il} - \bar{Q} - \psi^T R \psi & * & * \\ 0 & M & * \\ \Lambda_{il} & N & \bar{P}_{\tilde{i}l}^{-1} \end{bmatrix} \geq 0. \tag{5.19}$$

Define

$$\begin{aligned}
&\bar{P}_{il} \triangleq \gamma \bar{Q}_{il}^{-1} > 0, \bar{P}_{\tilde{i}l} \triangleq \gamma \bar{Q}_{\tilde{i}l}^{-1} > 0, \bar{P}_{dil} \triangleq \gamma \bar{Q}_{dil}^{-1} > 0, \forall d \in \mathcal{D}, \tau_d \in \mathfrak{L}_d \\
&\bar{F} \triangleq \bar{Y} \bar{Q}_{il}^{-1}, \bar{H} \triangleq \bar{Z} \bar{Q}_{il}^{-1}.
\end{aligned} \tag{5.20}$$

Using the new measure of robust stability in [16]

$$\begin{aligned}
&\left(\bar{G} - \bar{Q}_{il}\right)^T \bar{Q}_{il}^{-1} \left(\bar{G} - \bar{Q}_{il}\right) \geq 0, \quad \text{which is equivalent to} \\
&\bar{G}^T \bar{Q}_{il}^{-1} \bar{G} \geq \bar{G}^T + \bar{G} - \bar{Q}_{ij},
\end{aligned} \tag{5.21}$$

and by multiplying left-hand and right-hand sides of Δ by diag $\left\{\bar{Q}_{il}^{-1}\bar{G}, \bar{I}, \bar{I}, \ldots, \bar{I}\right\}$ and applying several steps of Schur complement, we can transform Δ into (5.10).

Suppose that (5.10) is satisfied, then the closed-loop system is asymptotically stable, and we obtain the inequality (3.3). By applying the definition (5.20) and Schur complement, (3.3) is equivalent to

$$\begin{bmatrix} 1 & * \\ \mathfrak{m}(k|k) & \Gamma \end{bmatrix} > 0, \tag{5.22}$$

□

5.3 Robust Stability

Lemma 5.1 (Feasibility for PWA systems with TVD). *Any feasible solution of the optimization in Theorem 5.1 at time k is also feasible for all times $t > k$. Thus, if the optimization problem in Theorem 5.1 is feasible at time k, then it is feasible for all times $t > k$.*

Proof. The proof of this lemma is a natural extension from Lemma 4.1. □

Theorem 5.2 (Robust stability for PWA systems with TVD). *The feasible receding horizon state feedback control law obtained from Theorem 5.1 robustly asymptotically stabilizes the closed-loop PWA systems (2.3.2).*

Proof. The proof of this theorem is a natural extension from Theorem 4.2. □

5.4 Numerical Example

Consider the following discrete-time PWA system with uncertain and time-varying delays

$$\begin{aligned}
&x(k+1) = \theta A_i(k)x(k) + 0.1A_{1i}(k)x(k-\tau_1(k)) + B_i(k)u(k), x \in \mathcal{X}_i, i \in \mathcal{I}_0 \\
&A_i(k) = A_{1i}(k) = \begin{bmatrix} \cos(\alpha(k)) & -\sin(\alpha(k)) \\ \sin(\alpha(k)) & \cos(\alpha(k)+\beta(k)) \end{bmatrix}, \ B_i(k) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \\
&\alpha(k) = \begin{cases} \frac{\pi}{3} & [\,1, 0\,]x(k) \geq \ 0 \\ -\frac{\pi}{3} & [\,1, 0\,]x(k) < \ 0 \end{cases} \\
&\theta = 0.75, \ 0 < \beta(k) < 0.5, \ 1 \leq \tau_1(k) \leq 3, \ u_{\text{max}} = 0.5.
\end{aligned} \tag{5.23}$$

From (5.23), we can see that the system has two modes

$$\begin{aligned}
\mathcal{X}_1 = \{x | [1 \ \ 0]x \geq 0\}, A_1(k) = A_{11}(k) = \begin{bmatrix} \frac{2}{5} & -\frac{2\sqrt{3}}{5} \\ \frac{2\sqrt{3}}{5} & \frac{2}{5} + \beta(k) \end{bmatrix}, \\
\mathcal{X}_2 = \{x | [1 \ \ 0]x < 0\}, A_2(k) = A_{12}(k) = \begin{bmatrix} \frac{2}{5} & \frac{2\sqrt{3}}{5} \\ -\frac{2\sqrt{3}}{5} & \frac{2}{5} + \beta(k) \end{bmatrix},
\end{aligned}$$

With the uncertainty parameter $0 < \beta(k) < 0.5$ and the time varying integer $1 \leq \tau_1(k) \leq 3$, we conclude that when

$$\begin{aligned}
x(k) \in \mathcal{X}_1, \Omega_1 = \text{Co}\left\{(A_1^1, A_{11,1}^1, A_{11,2}^1, A_{11,3}^1, B_1^1), (A_1^2, A_{11,1}^2, A_{11,2}^2, A_{11,3}^2, B_1^2)\right\}, \\
x(k) \in \mathcal{X}_2, \Omega_2 = \text{Co}\left\{(A_2^1, A_{12,1}^1, A_{12,2}^1, A_{12,3}^1, B_2^1), (A_2^2, A_{12,1}^2, A_{12,2}^2, A_{12,3}^2, B_2^2)\right\},
\end{aligned}$$

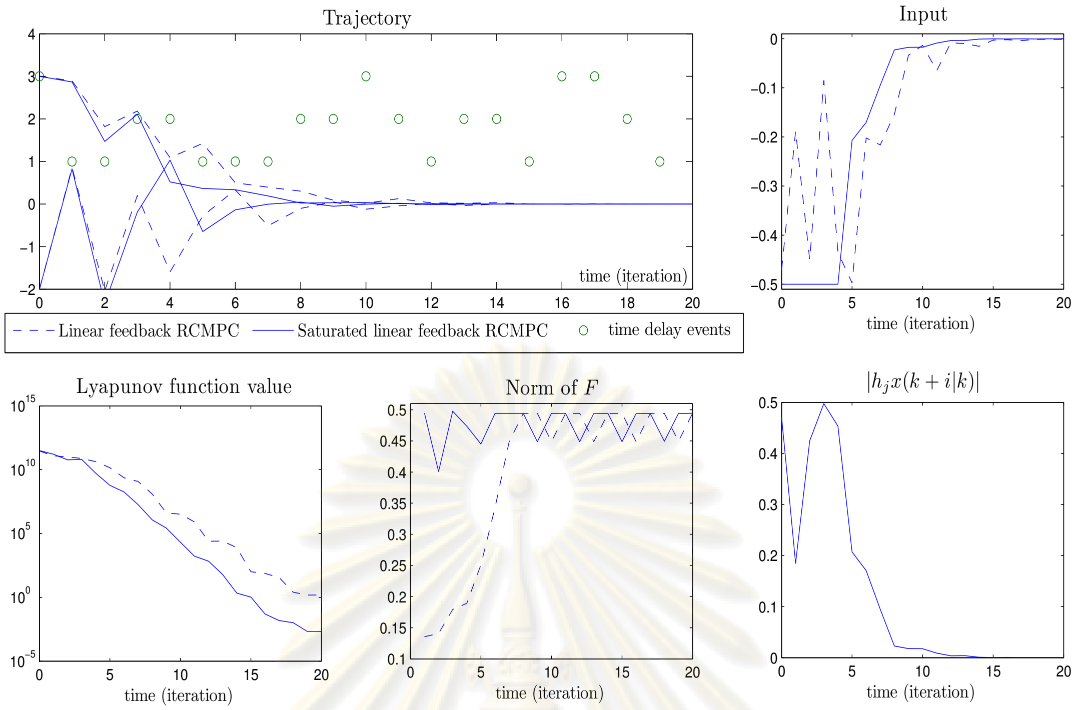

Figure 5.1: Saturated-RCMPC and non saturated-RCMPC simulation result.

with $A^1_{11,1} = A^1_{11}$, if $\tau_1(k) = 1$, or $A^1_{11} = 0$ otherwise, respectively.

The LMI conditions in Theorem 5.1 can be solved using the YALMIP toolbox for the tuning parameters $Q = I_2$, $R = 1$, $x(-3) = x(-2) = x(-1) = x(0) = [\,-2, 3\,]^T$. On a 2.4 GHz Intel Centrino Core 2 Duo Processor, with 1 GB RAM, the CPU time required to compute 20 iterations on the online algorithm in Theorem 5.1 is 37.25 s. Figure 5.1 shows the simulation results of the state responses, optimal input, Lyapunov function, norm of state feedback, and the saturated controller function requirement.

CHAPTER VI

APPLICATION EXAMPLES

We will show that the proposed algorithm is suitable and effective through several following examples. For these examples, the YALMIP toolbox with SEDUMI solver is used to compute the solution of the LMI problems.

6.1 Cruiser PWA Model

Consider the following cruiser PWA model taken from [Corona and Schutter, 2006], with a few modification to incorporate uncertainties and time-delay in the system

$$x(k+1) = A_i(k)x(k) + \theta A_{1i}(k)x(k-\tau_1(k)) + B_i(k)u(k), \text{ for } x_2(k) < (\geq)\, 0, i = 1(2) \in \mathcal{I}_0,$$

$$A_1 = A_{11} = \begin{bmatrix} 1 & 0.97+\beta(k) \\ 0 & 0.99+\beta(k) \end{bmatrix}, B_1 = \begin{bmatrix} 2.31 \\ 4.61 \end{bmatrix},$$
$$A_2 = A_{12} = \begin{bmatrix} 1 & 0.98+\beta(k) \\ 0 & 0.96+\beta(k)) \end{bmatrix}, B_2 = \begin{bmatrix} 2.28 \\ 4.54 \end{bmatrix},$$
$$\theta = 0.1,\ 0 < \beta(k) < 1,\ 1 \leq \tau_1(k) \leq 3,\ u_{\max} = 0.5,$$

where $x_1(k), x_2(k), u(k)$ are the cruise position, speed velocity, and throttle or brake. The system has been discretized with time sampling $T_s = 1s$. The objective is to bring the cruise to $x_1 = 0\,m$ from initial position.

We setup the initial condition $x(-3) = x(-2) = x(-1) = x(0) = [3, 0]^T$. The LMI conditions in Theorem 5.1 can be solved using the YALMIP toolbox for the tuning parameters $Q = I_2$, $R = 1$. On a 2.4 GHz Intel Centrino Core 2 Duo Processor, with 1 GB RAM, the CPU time required to compute 20 iterations on the online algorithm in Theorem 5.1 is 43.06 s. Figure 6.1 shows the simulation results of the state responses, optimal input, Lyapunov function, norm of state feedback, and the saturated controller function requirement respectively.

6.2 Autonomous Land Vehicle

This example is the autonomous land vehicle model adapted in [26]. Consider that the objective is to design a controller that force a cart on the $x-y$ plane to follow the straight line $y = 0$ with a constant velocity $u_0 = 1\mathrm{m/s}$. Assume that a controller has already been designed to maintain a constant forward velocity. The cart's path is then controlled by the torque T about the z axis according to the following discrete-time equations

$$x(k+1) = A_i x(k) + a_i(k) + B_i u(k)$$

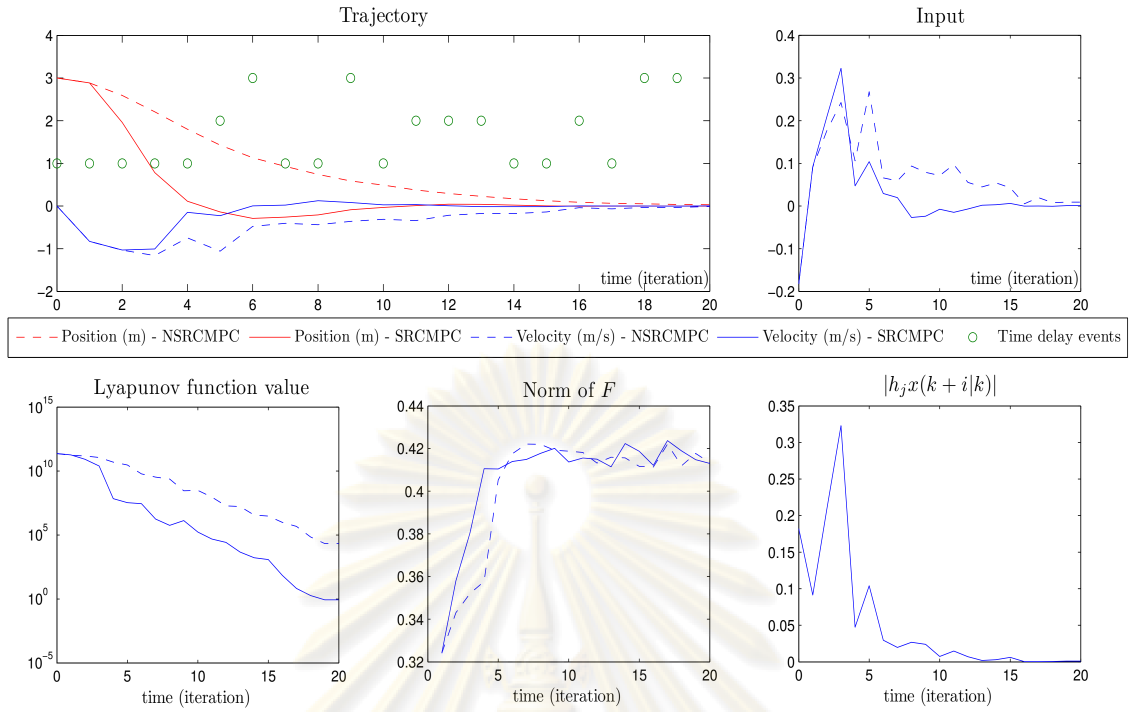

Figure 6.1: Saturated-RCMPC and non saturated-RCMPC simulation result for Cruiser PWA model.

where

$$A_1 = A_5 = \begin{bmatrix} 0 & 0.7 & 0 \\ 0 & -0.007 & 0 \\ 0.309 & 0 & \beta(k) \end{bmatrix}, A_2 = A_4 = \begin{bmatrix} 0 & 0.7 & 0 \\ 0 & -0.007 & 0 \\ 0.914 & 0 & \beta(k) \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0.7 & 0 \\ 0 & -0.007 & 0 \\ 1 & 0 & \beta(k) \end{bmatrix}, B_1 = B_2 = B_3 = B_4 = B_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$a_1 = -a_5 = \begin{bmatrix} 0 \\ 0 \\ -0.757 \end{bmatrix}, a_2 = -a_4 = \begin{bmatrix} 0 \\ 0 \\ -0.216 \end{bmatrix}, a_3 = 0.$$

The state of the system is $(x_1, x_2, x_3) = (\psi, \omega, y)$, where ψ is the heading angle with time derivative ω, and y is the cart's distance from the line $y = 0$. The input of the system $u = T$, where T is the control torque. We consider parametric uncertainties β in the range $0 < \beta(k) < 1$ and assumed to be arbitrarily time-varying in the indicated range of variation. Assume all the states are measurable and the trajectories can start from any possible initial angle in the range $\psi_0 \in [-\frac{3\pi}{5}, \frac{3\pi}{5}]$, and any initial distance from the line. The heading angle (ψ) is approximated by a piecewise affine function yielding a piecewise affine system with five regions as follows

$$\begin{aligned} \Omega_1 &= \left\{x | x_1 \in \left(-\tfrac{3\pi}{5}, -\tfrac{\pi}{5}\right)\right\}, \Omega_2 = \left\{x | x_1 \in \left(-\tfrac{\pi}{5}, -\tfrac{\pi}{15}\right)\right\}, \\ \Omega_3 &= \left\{x | x_1 \in \left(-\tfrac{\pi}{15}, \tfrac{\pi}{15}\right)\right\}, \\ \Omega_4 &= \left\{x | x_1 \in \left(\tfrac{\pi}{15}, \tfrac{\pi}{5}\right)\right\}, \Omega_5 = \left\{x | x_1 \in \left(\tfrac{\pi}{5}, \tfrac{3\pi}{5}\right)\right\}. \end{aligned}$$

We construct an initial state $x(0) = [3\pi/15, 0, 3]^T$. The system is subject to input constraint $\|u(k+j)|k)\|_2 \leq 1$N.m, $j \geq 0$. And with tuning parameters $Q = I_3$, $R = 1$, the LMI conditions in Theorem

3.4 and Theorem 3.5 can be solved. On a 2.4 GHz Intel Centrino Core 2 Duo Processor, with 1 GB RAM, the CPU time required to compute the online algorithm is 4.67 s. We obtained the following results as depicted in Fig. 6.2. Also included in the figure are the active regions and Lyapunov function value over time iterations, the norm of F as a function of time for the saturated state feedback RCMPC, and the norm condition for the saturated function can be applied with polytopic description.

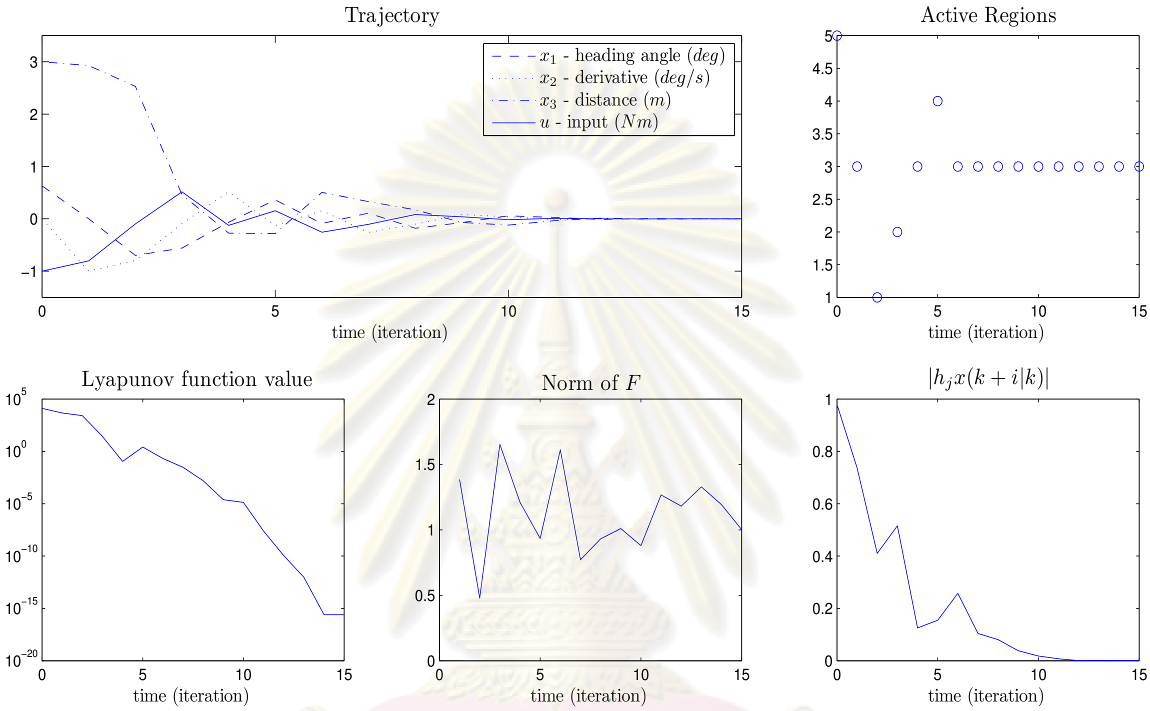

Figure 6.2: Saturated RCMPC algorithm applied to a Land Vehicle Model.

6.3 Inverted Cart-Pendulum

This example illustrates the stabilization at the inverted position (open-loop unstable equilibrium point) of a pendulum-cart system using the RCMPC for PWA system with time-varying delay formulation. Assume the pendulum can start anywhere within $\pm 30^o$ of vertical, the full nonlinear dynamics from [33] is chosen. The problem can be addressed by approximating the nonlinear dynamics with a PWA model and then a RCMPC controller can be designed to stabilize the inverted pendulum [33].

With $\bar{x}$ corresponding to the position of the cart and θ the angle of the pendulum ($\theta = \pm\pi$ at the vertical position), the state was chosen to be $x = [\, x_1 \;\; x_2 \;\; x_3 \;\; x_4 \,]^T = [\, \bar{x} \;\; \theta \;\; \dot{\bar{x}} \;\; \dot{\theta} \,]^T$. The nonlinear dynamics of the cart-pendulum are then

$$\dot{x}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ f_1(x(t)) \\ f_2(x(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{l}{ml\cos(x_2(t)) - l(M+m)} \\ \frac{1}{ml\cos(x_2(t)) - l(M+m)} \end{bmatrix} u(t), \tag{6.1}$$

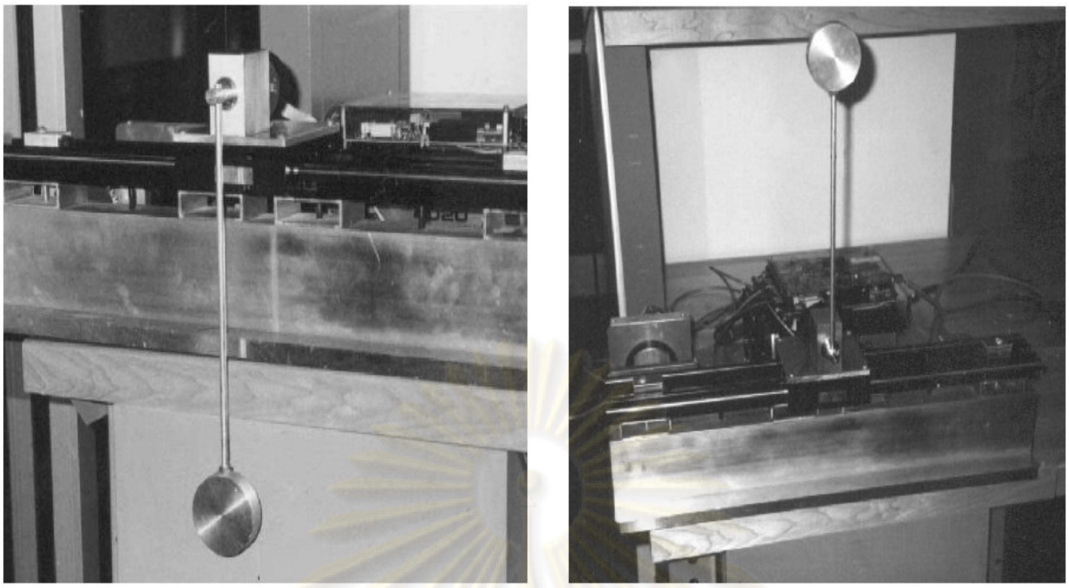

Figure 6.3: Photographs of the Pendulum Cart with pendulum down (left) and pendulum up (right).

$$f_1(x(t)) = \frac{1}{ml\cos(x_2(t)) - l(M+m)}\left(lcx_3(t) - b\cos(x_2(t))x_4(t) + mgl\sin(x_2(t))\cos(x_2(t))\right)$$
$$f_2(x(t)) = \frac{1}{ml\cos(x_2(t)) - l(M+m)}\left(-cx_3(t) + \tfrac{(M+m)b}{ml}x_4(t) - (M+m)g\sin(x_2(t))\right)$$

where M is the mass of the cart, m is the mass of the pendulum, l is the length of the pendulum, u is the force applied to the cart, g is the gravitational acceleration, b and c are the translational and rotational viscous damping coefficients. For this particular example, $M = 1.525$ kg, $m = 0.15$ kg, $l = 0.314$ m, $g = 9.8$ ms^{-2}, $b = c = 0.005$ Ns/m.

A PWA approximation of the nonlinear dynamics is obtained by linearizing (6.1) into three open-loop unstable equilibrium points of the pendulum angle by the x_2 values $\{\frac{20}{18}\pi, \pi, \frac{16}{18}\pi\}$. By change of coordinate $x_2' = x_2 - \pi$, we obtained a PWA model

$$\dot{x}(t) = A_i(t)x(t) + a_i(t) + B_i(t)u(t), \;\; x(t) \in \mathcal{X}_i, i \in \mathcal{I}, \tag{6.2}$$

with region partitions

$$\begin{cases} \Omega_i = \left\{x | x_2' \in \left(\frac{3}{18}\pi, \frac{1}{18}\pi\right), x \in \mathcal{X}_i\right\}, \;\; i = 1 \in \mathcal{I}_1, \\ \Omega_i = \left\{x | x_2' \in \left[\frac{1}{18}\pi, -\frac{1}{18}\pi\right], x \in \mathcal{X}_i\right\}, \;\; i = 2 \in \mathcal{I}_0, \\ \Omega_i = \left\{x | x_2' \in \left(-\frac{1}{18}\pi, -\frac{3}{18}\pi\right), x \in \mathcal{X}_i\right\}, \;\; i = 3 \in \mathcal{I}_1, \end{cases} \tag{6.3}$$

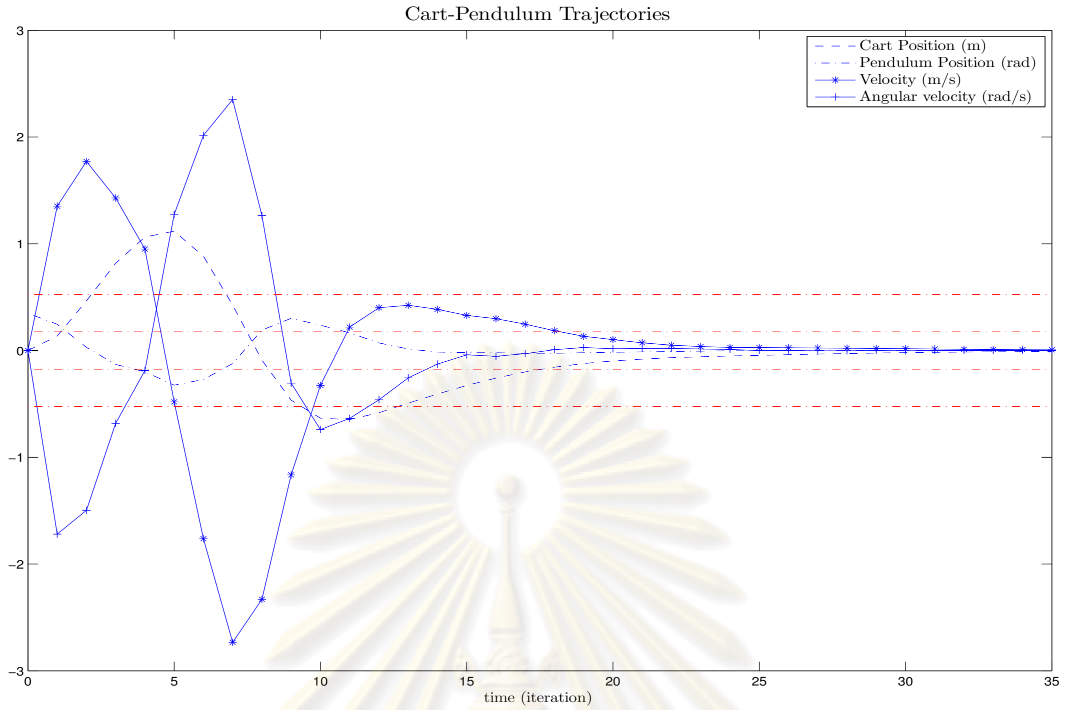

Figure 6.4: Saturated-RCMPC simulation result for Cart-Pendulum PWA model - systems trajectories.

and the model matrices, where $M_t = 0.9397ml - l(M+m)$,

$$A_1 = A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{1.0496gm^2l^2 - 0.9397mgl^2(M+m)}{M_t^2} & \frac{lc}{M_t} & -\frac{0.9397b}{M_t} \\ 0 & \frac{0.9397gl(M+m)^2 - mgl(M+m)}{M_t^2} & -\frac{c}{M_t} & \frac{(M+m)b}{mlM_t} \end{bmatrix}, \quad B_1 = B_3 = \begin{bmatrix} 0 \\ 0 \\ -\frac{l}{M_t} \\ -\frac{1}{M_t} \end{bmatrix},$$

$$a_1 = -a_3 = \begin{bmatrix} 0 \\ 0 \\ \frac{0.0066mgl^2(M+m) - 0.0644gm^2l^2}{M_t^2} \\ \frac{0.0277mgl(M+m) + 0.014gl(M+m)^2}{M_t^2} \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{mg}{M} & -\frac{c}{M} & \frac{b}{Ml} \\ 0 & \frac{g(M+m)}{Ml} & \frac{c}{Ml} & -\frac{(M+m)b}{Mml^2} \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ -\frac{1}{Ml} \end{bmatrix}, \quad a_2 = 0.$$

The PWA model (6.2) is then discretized with sampling time, $T_s = 0.2$ seconds. To apply the RCMPC for PWA system with time-varying delay algorithm, we assume that the pendulum position is perturbed by time delay $\tau_1(k)$, and there is uncertainty $\beta(k)$ in the cart velocity due to the physical condition, and/ or the approximation error. For this case, $1 \leq \tau_1(k) \leq 2$, and $0 < \beta(k) < 0.03$. To that end, we have the discretized PWA system with perturbed time-delay and uncertainty as follows

$$x(k+1) = \bar{A}_i x(k) + \bar{A}_{1i} x(k - \tau_1(k)) + \bar{a}_i + \bar{B}_i u(k), \quad x(k) \in \mathcal{X}_i, i \in \mathcal{I}, \qquad (6.4)$$

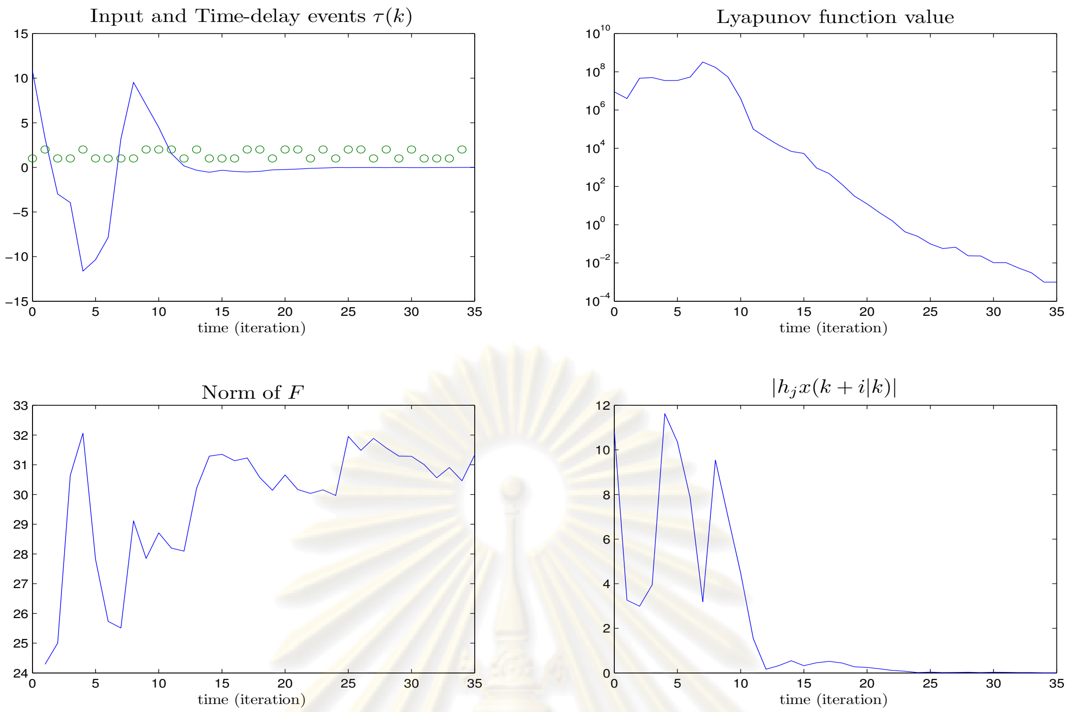

Figure 6.5: Saturated-RCMPC simulation result for Cart-Pendulum PWA model.

where

$$\bar{A}_1 = \bar{A}_3 = \begin{bmatrix} 1 & -0.0208\alpha & 0.2073+\beta(k) & -0.0012 \\ 0 & 1.7410\alpha & 0.0002+\beta(k) & 0.2480 \\ 0 & -0.2198\alpha & 0.9993+\beta(k) & -0.0183 \\ 0 & 7.8449\alpha & 0.0026+\beta(k) & 1.6495 \end{bmatrix}, \quad \bar{B}_1 = \bar{B}_3 = \begin{bmatrix} 0.0141 \\ -0.0486 \\ 0.1375 \\ -0.5147 \end{bmatrix},$$

$$\bar{a}_1 = -\bar{a}_3 = \begin{bmatrix} -0.0005 \\ 0.1521 \\ -0.0104 \\ 1.0115 \end{bmatrix}, \quad \bar{A}_{11} = \bar{A}_{13} = \begin{bmatrix} 0 & -0.0208(1-\alpha) & 0 & 0 \\ 0 & 1.7410(1-\alpha) & 0 & 0 \\ 0 & -0.2198(1-\alpha) & 0 & 0 \\ 0 & 7.8449(1-\alpha) & 0 & 0 \end{bmatrix}.$$

$$\bar{A}_2 = \begin{bmatrix} 1 & -0.0227\alpha & 0.2073+\beta(k) & -0.0013 \\ 0 & 1.8058\alpha & 0.0002+\beta(k) & 0.2520 \\ 0 & -0.2416\alpha & 0.9993+\beta(k) & -0.0200 \\ 0 & 8.5955\alpha & 0.0026+\beta(k) & 1.7123 \end{bmatrix}, \quad \bar{B}_2 = \begin{bmatrix} 0.0142 \\ -0.0493 \\ 0.1385 \\ -0.5261 \end{bmatrix},$$

$$\bar{a}_2 = 0, \quad \bar{A}_{12} = \begin{bmatrix} 0 & -0.0227(1-\alpha) & 0 & 0 \\ 0 & 1.8058(1-\alpha) & 0 & 0 \\ 0 & -0.2416(1-\alpha) & 0 & 0 \\ 0 & 8.5955(1-\alpha) & 0 & 0 \end{bmatrix}.$$

The constant α is the retarded coefficient [34], which satisfies $\alpha \in [0, 1]$. The limits correspond to the delay term, respectively. In this case, we assume $\alpha = 0.99$.

A RCMPC controller is then designed with the closed-loop equilibrium points of all polytopic regions are placed at $x_{cl} = [\, x_1 \;\; x_2' \;\; x_3 \;\; x_4 \,]^T = [\, 0 \;\; 0 \;\; 0 \;\; 0 \,]^T$. Assume the maximum input constraint $u_{\max} = 15$ Nm. We set an initial points $x(-2) = x(-1) = x(0) = \left[\, 0 \;\; \frac{2}{18}\pi \;\; 0 \;\; 0 \,\right]^T$. The LMI conditions in Theorem 5.1 can be solved using YALMIP toolbox for the tuning parameters

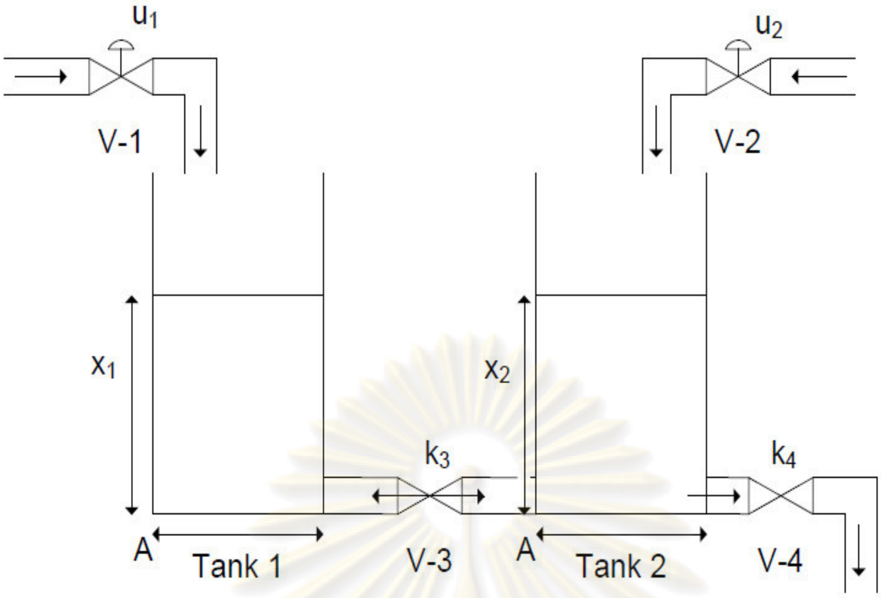

Figure 6.6: Two tanks level system.

$Q = I_4$, and $R = 1$, respectively. On a 2.4 GHz Intel Centrino Core 2 Duo Processor, with 1 GB RAM, the CPU time required to compute 35 iterations on the online algorithm is 57.44 seconds. Figures 6.4, and 6.5 show the simulation results of the state responses, optimal input, Lyapunov function, norm of state feedback, and the saturated controller function requirement, respectively.

6.4 Two-Tank Level System

Consider the two-tank level system as depicted in Fig.6.6. The control objective is to stabilize the level of the two tanks at 0.5 meter. Assume that the valve V-3 and V-4 are always open, and the two tanks have the same dimension. The problem can be addressed by approximating the nonlinear dynamics with a PWA model and then a RCMPC controller can be designed to stabilize the system.

In general, the nonlinear dynamics of the two tanks system depends on the level of each tank

$$\left.\begin{aligned} \dot{x}_1 &= \tfrac{1}{A}\left(q_1u_1 - k_3\sqrt{x_1 - x_2}\right) \\ \dot{x}_2 &= \tfrac{1}{A}\left(q_2u_2 + k_3\sqrt{x_1 - x_2} - k_4\sqrt{x_2}\right) \end{aligned}\right\} x_1 > x_2 \tag{6.5}$$

$$\left.\begin{aligned} \dot{x}_1 &= \tfrac{1}{A}\left(q_1u_1\right) \\ \dot{x}_2 &= \tfrac{1}{A}\left(q_2u_2 - k_4\sqrt{x_2}\right) \end{aligned}\right\} x_1 = x_2 \tag{6.6}$$

$$\left.\begin{aligned} \dot{x}_1 &= \tfrac{1}{A}\left(q_1u_1 + k_3\sqrt{x_2 - x_1}\right) \\ \dot{x}_2 &= \tfrac{1}{A}\left(q_2u_2 - k_3\sqrt{x_2 - x_1} - k_4\sqrt{x_2}\right) \end{aligned}\right\} x_1 < x_2 \tag{6.7}$$

where q_1, q_2 are the control valve liquid flow rate, u_1, u_2 are the percent opening of the control valve, k_3, k_4 are the valve coefficient. For this particular system, $A = 0.0346\ \text{m}^2$, $q_1 = q_2 = 2.8510\ \text{m}^3/\text{s}$, $k_3 = k_4 = 3.4892\ \text{m}^{2.5}/\text{s}$. A PWA approximation of the nonlinear dynamics is obtained by linearizing (6.5), (6.6), (6.7) into three open-loop unstable equlibrium points of the two tanks level where each of the x_1 and x_2 in $\{0.3\ \text{m}, 0.5\ \text{m}, 0.7\ \text{m}\}$. We obtained 9 set of PWA model because of the level in the two tanks can vary each other in the three operating regions. By change of coordinate, $x_1' = x_1 - 0.5$, $x_2' = x_2 - 0.5$, $u_1' = u_1$, and $u_2' = u_2 - \frac{k_4}{2q_2\sqrt{0.5}}$, then

$$\dot{x}(t) = A_i(t)x(t) + a_i(t) + B_i u(t), \tag{6.8}$$

where $u(t) = [\, u_1'(t),\ u_2'(t)\,]^T,\ x(t) = [\, x_1'(t),\ x_2'(t)\,]^T \in \mathcal{X}_i,\ i \in \mathcal{I}$, and the region partitions

$$\begin{cases} \Omega_i = \{x | x_1' \in (-0.1, 0.1) \text{ and } x_2' \in (-0.1, 0.1), x \in \mathcal{X}_i\},\ i = 1 \in \mathcal{I}_0, \\ \Omega_i = \{x | x_1' \in (0.3, 0.1] \text{ and } x_2' \in (0.3, 0.1], x \in \mathcal{X}_i\},\ i = 2 \in \mathcal{I}_1, \\ \Omega_i = \{x | x_1' \in (-0.3, -0.1] \text{ and } x_2' \in (-0.3, -0.1], x \in \mathcal{X}_i\},\ i = 3 \in \mathcal{I}_1, \\ \Omega_i = \{x | x_1' \in (0.3, 0.1] \text{ and } x_2' \in (-0.1, 0.1), x \in \mathcal{X}_i\},\ i = 4 \in \mathcal{I}_1, \\ \Omega_i = \{x | x_1' \in (-0.3, -0.1] \text{ and } x_2' \in (-0.1, 0.1), x \in \mathcal{X}_i\},\ i = 5 \in \mathcal{I}_1, \\ \Omega_i = \{x | x_1' \in (-0.1, 0.1) \text{ and } x_2' \in (0.3, 0.1], x \in \mathcal{X}_i\},\ i = 6 \in \mathcal{I}_1, \\ \Omega_i = \{x | x_1' \in (-0.1, 0.1) \text{ and } x_2' \in (-0.3, -0.1], x \in \mathcal{X}_i\},\ i = 7 \in \mathcal{I}_1, \\ \Omega_i = \{x | x_1' \in (0.3, 0.1] \text{ and } x_2' \in (-0.3, -0.1], x \in \mathcal{X}_i\},\ i = 8 \in \mathcal{I}_1, \\ \Omega_i = \{x | x_1' \in (-0.3, -0.1] \text{ and } x_2' \in (0.3, 0.1], x \in \mathcal{X}_i\},\ i = 9 \in \mathcal{I}_1, \end{cases} \tag{6.9}$$

The model matrices for each of the operating regions

$$A_1 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{k_4}{2A\sqrt{0.5}} \end{bmatrix},\ a_1 = 0,\ A_2 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{k_4}{2A\sqrt{0.7}} \end{bmatrix},\ a_2 = \begin{bmatrix} 0 \\ \frac{k_4}{2A}\left(\frac{1}{\sqrt{0.5}} - \frac{1.2}{\sqrt{0.7}}\right) \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{k_4}{2A\sqrt{0.3}} \end{bmatrix},\ a_3 = \begin{bmatrix} 0 \\ \frac{k_4}{2A}\left(\frac{1}{\sqrt{0.5}} - \frac{0.8}{\sqrt{0.3}}\right) \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -\frac{k_3}{2A\sqrt{0.2}} & \frac{k_3}{2A\sqrt{0.2}} \\ \frac{k_3}{2A\sqrt{0.2}} & -\left(\frac{k_3}{2A\sqrt{0.2}} + \frac{k_4}{2A\sqrt{0.5}}\right) \end{bmatrix},\ a_4 = \begin{bmatrix} -\frac{k_3}{2A}\sqrt{0.2} \\ \frac{k_3}{2A}\sqrt{0.2} \end{bmatrix},\ A_5 = A_4,\ a_5 = -a_4,$$

$$A_6 = \begin{bmatrix} -\frac{k_3}{2A\sqrt{0.2}} & \frac{k_3}{2A\sqrt{0.2}} \\ \frac{k_3}{2A\sqrt{0.2}} & -\left(\frac{k_3}{2A\sqrt{0.2}} + \frac{k_4}{2A\sqrt{0.7}}\right) \end{bmatrix},\ a_6 = \begin{bmatrix} \frac{k_3}{2A}\sqrt{0.2} \\ -\frac{k_3}{2A}\sqrt{0.2} + \frac{k_4}{2A}\left(\frac{1}{\sqrt{0.5}} - \frac{1.2}{\sqrt{0.7}}\right) \end{bmatrix},$$

$$A_7 = \begin{bmatrix} -\frac{k_3}{2A\sqrt{0.2}} & \frac{k_3}{2A\sqrt{0.2}} \\ \frac{k_3}{2A\sqrt{0.2}} & -\left(\frac{k_3}{2A\sqrt{0.2}} + \frac{k_4}{2A\sqrt{0.3}}\right) \end{bmatrix},\ a_7 = \begin{bmatrix} -\frac{k_3}{2A}\sqrt{0.2} \\ \frac{k_3}{2A}\sqrt{0.2} + \frac{k_4}{2A}\left(\frac{1}{\sqrt{0.5}} - \frac{0.8}{\sqrt{0.3}}\right) \end{bmatrix},$$

$$A_8 = \begin{bmatrix} -\frac{k_3}{2A\sqrt{0.4}} & \frac{k_3}{2A\sqrt{0.4}} \\ \frac{k_3}{2A\sqrt{0.4}} & -\left(\frac{k_3}{2A\sqrt{0.4}} + \frac{k_4}{2A\sqrt{0.3}}\right) \end{bmatrix},\ a_8 = \begin{bmatrix} -\frac{k_3}{2A}\sqrt{0.4} \\ \frac{k_3}{2A}\sqrt{0.4} + \frac{k_4}{2A}\left(\frac{1}{\sqrt{0.5}} - \frac{0.8}{\sqrt{0.3}}\right) \end{bmatrix},$$

$$A_9 = \begin{bmatrix} -\frac{k_3}{2A\sqrt{0.4}} & \frac{k_3}{2A\sqrt{0.4}} \\ \frac{k_3}{2A\sqrt{0.4}} & -\left(\frac{k_3}{2A\sqrt{0.4}} + \frac{k_4}{2A\sqrt{0.7}}\right) \end{bmatrix},\ a_9 = \begin{bmatrix} \frac{k_3}{2A}\sqrt{0.4} \\ -\frac{k_3}{2A}\sqrt{0.4} + \frac{k_4}{2A}\left(\frac{1}{\sqrt{0.5}} - \frac{1.2}{\sqrt{0.7}}\right) \end{bmatrix},$$

$$B_1 = B_2 = B_3 = B_4 = B_5 = B_6 = B_7 = B_8 = B_9 = \begin{bmatrix} \frac{q_1}{A} & 0 \\ 0 & \frac{q_2}{A} \end{bmatrix},$$

The PWA model (6.8) is then discretized with sampling time, $T_s = 7$ seconds. To apply the RCMPC for PWA system with time-varying delay algorithm, we assume that the level measurements in both of the tanks are perturbed by time delay $\tau_1(k)$, and there is uncertainty $\beta(k)$ due to the physical condition, and/ or the approximation error. For this case, $1 \leq \tau_1(k) \leq 2$, and $0 < \beta(k) < 1.2$. To that end, we have the discretized PWA system with perturbed time-varying delay and uncertainty as

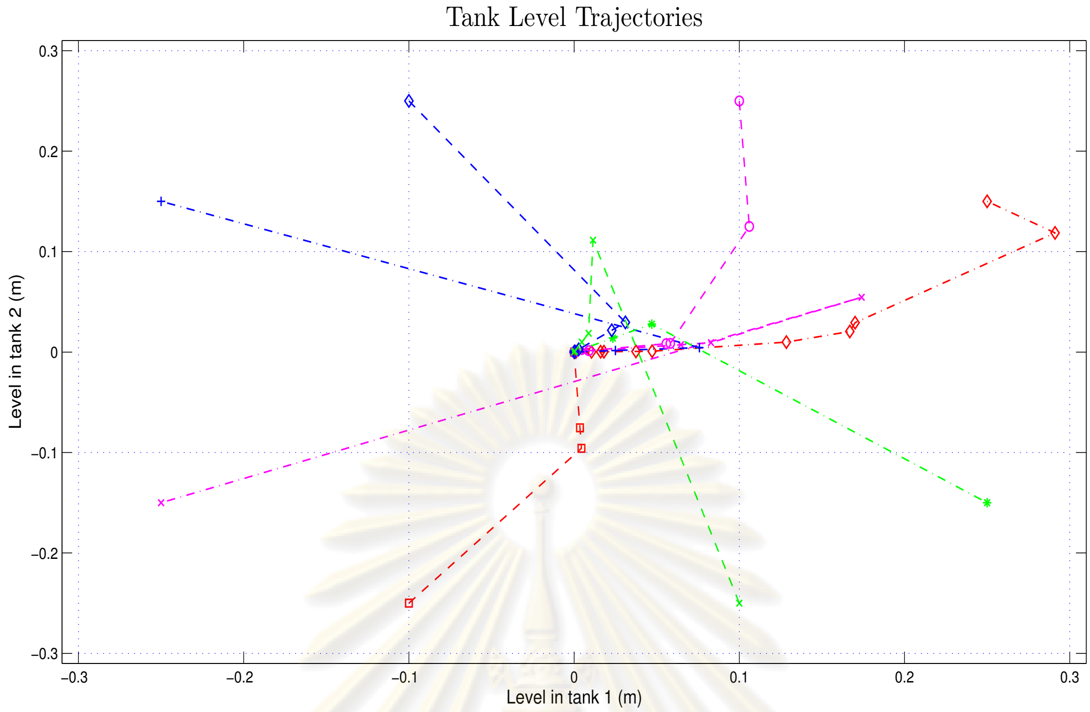

Figure 6.7: Two tanks level system trajectories.

follows

$$x(k+1) = \bar{A}_i x(k) + \bar{A}_{1i} x(k-\tau_1(k)) + \bar{a}_i + \bar{B}_i u(k), \quad x(k) \in \mathcal{X}_i, i \in \mathcal{I}, \tag{6.10}$$

where

$$\bar{A}_i = \begin{bmatrix} v_{11}\alpha + \beta(k) & v_{12}\alpha \\ v_{21}\alpha & v_{22}\alpha + \beta(k) \end{bmatrix}, \; \bar{A}_{1i} = \begin{bmatrix} v_{11}(1-\alpha) & v_{12}(1-\alpha) \\ v_{21}(1-\alpha) & v_{22}(1-\alpha) \end{bmatrix}, i = \{1, 2, \ldots, 9\},$$

The values of $v_{11}, v_{12}, v_{21}, v_{22}$ depend on the $\bar{A}$ matrices in each of the operating regions. The constant α is the retarded coefficient [34], which satisfies $\alpha \in [0, 1]$. The limits correspond to the delay term, respectively. In this case, we assume $\alpha = 0.8$.

A RCMPC controller is then designed with the closed-loop equilibrium points of all polytopic regions are placed at $x_{cl} = [\, x_1' \;\; x_2' \,]^T = [\, 0 \;\; 0 \,]^T$. Assume the maximum input constraint $u_{\max} = 1$. We set an initial points $x(-2) = x(-1) = x(0)$ into several initial liquid levels. The LMI conditions in Theorem 5.1 can be solved using YALMIP toolbox for the tuning parameters $Q = I_2$, and $R = I_2$, respectively. On a 2.4 GHz Intel Centrino Core 2 Duo Processor, with 1 GB RAM, the CPU time required to compute 10 iterations on the online algorithm is 64.5 seconds totally for all of the initial points. Figure 6.7 shows the trajectories of the level in the two tanks for several initial liquid level, respectively.

Remark 6.1. *The RCMPC with saturated controller is feasible to be applied online in the two-tank system example due to the slow-dynamics of the tank system. On the other hand, it should be noted that the online optimization problems lead to a computational burden due to the size of LMIs. Thus, this formulation is not feasible to be applied online for the fast-dynamics systems, as in examples 6.1, 6.2, and 6.3.*

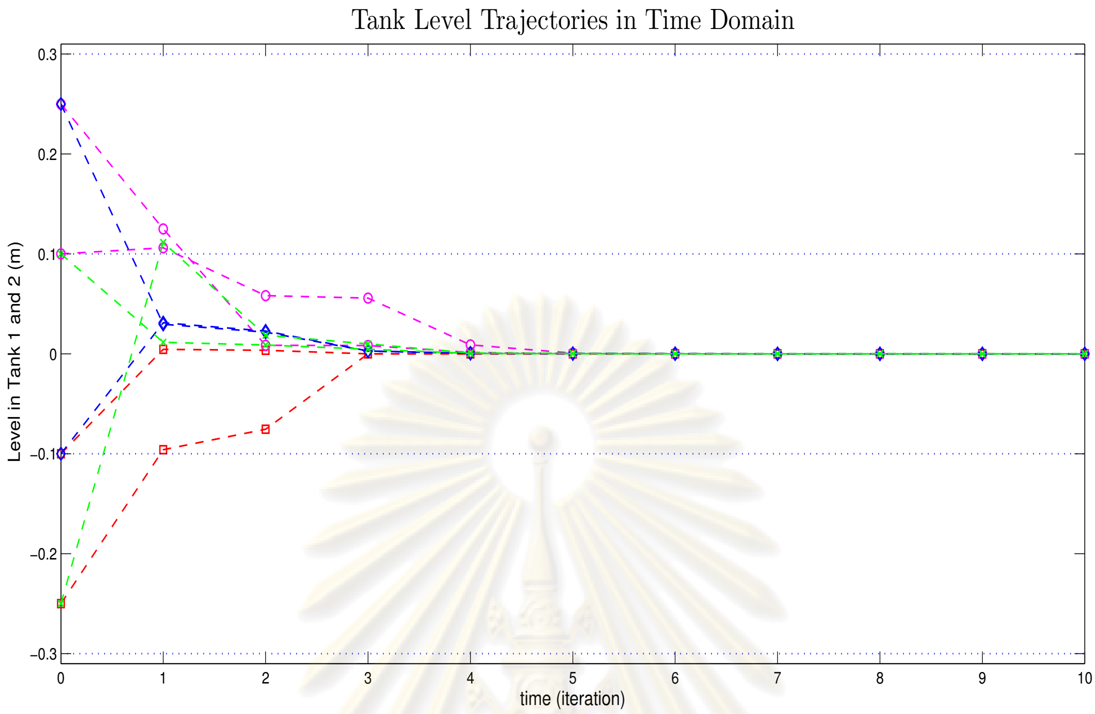

Figure 6.8: Two tanks level system system trajectories in time domain-1.

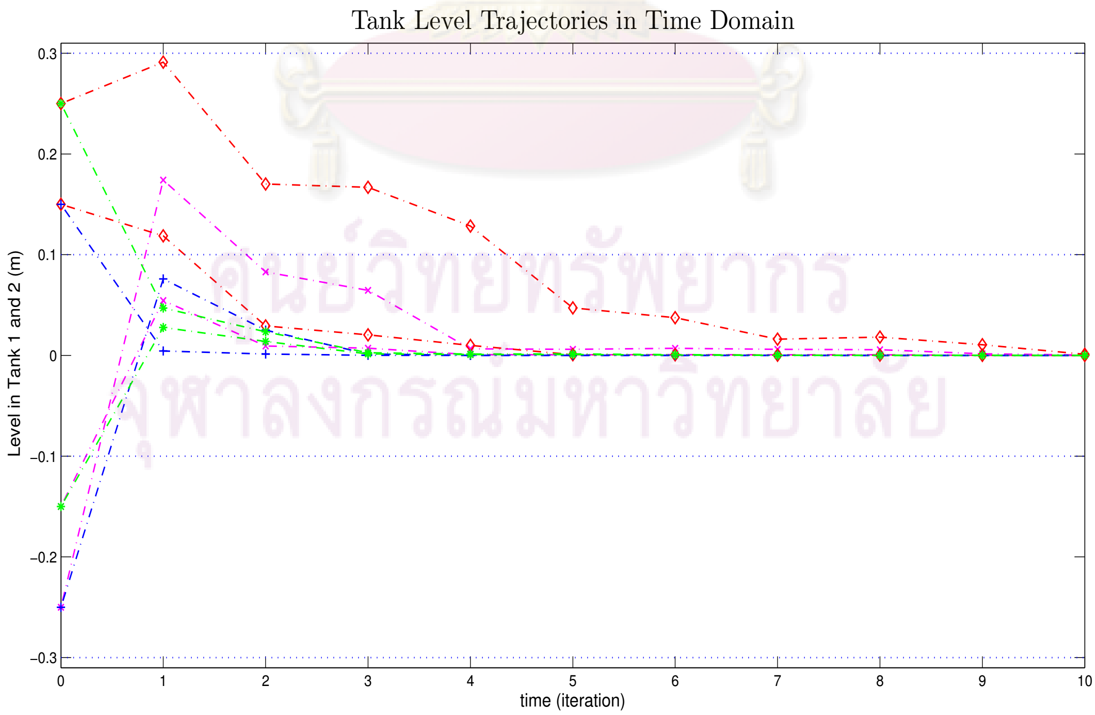

Figure 6.9: Two tanks level system trajectories in time domain-2.

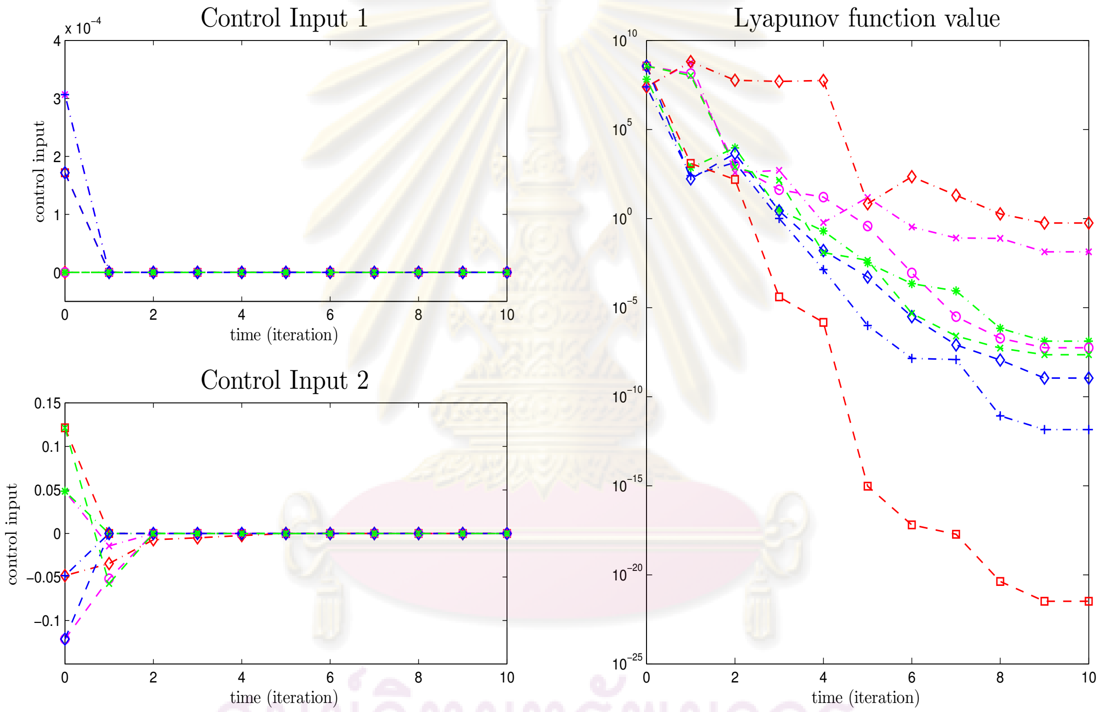

Figure 6.10: Two tanks level system system control valve input and Lyapunov function value.

CHAPTER VII

CONCLUSIONS

7.1 Summary

This thesis has proposed an extension of Robust Constrained Model Predictive Control framework to stabilize Piece-Wise Affine systems using saturated linear feedback controller. The online optimization problem of RCMPC can be transformed into an LMI optimization, which can be handled by currently available software with extreme efficiency. By properly constructing a Lyapunov function and/or a Lyapunov-Krasovskii function, LMIs conditions for closed-loop stability and constraints satisfaction have been obtained. Then, the effectiveness of the proposed method has been demonstrated through some application examples. To summarize the thesis, we highlight main topics in the following.

Chapter 1 briefly introduces the motivation behind the research. Next, the literature review is given to cover an overview of RCMPC framework as well as the development of an emerging and promising PWA systems. Afterward, we present the thesis scope and research contributions.

In Chapter 2, a basic knowledge with some important concepts and tools to be used throughout the thesis are presented. The description of PWA systems and the systems matrix parameterization are introduced in Sections 2.1 and 2.2. The mathematical representations of the uncertainties are discussed in Section 2.3. Section 2.4 presents the definition of saturated linear feedback control law. The remaining sections include a MPC and RCMPC framework, the Lyapunov theory in discrete-time systems, and LMIs optimization tool.

Chapter 3 presents the detail steps in the extensions formulation of a RCMPC framework to stabilize PWA system with delay-free time. An upper bound of the worst-case performance is derived, then the optimization problems is reformulated into LMIs optimization approach. A normal and an augmented approach of saturated state feedback strategies are presented. Robust stability of formulation is also taken into consideration, while the numerical example shows the effectiveness of the formulation.

Chapters 4 and 5 propose other two RCMPC framework to stabilize the PWA systems with time- delay. In Chapter 4, the framework is used to stabilize the PWA systems with time-invariant delay. A more general approach is considered in Chapter 5, where the framework is used to stabilize the PWA systems with time-varying delay satisfying upper and lower bound of certain delay value. An upper bound of the worst-case performance is derived, then the optimization problems is reformulated into LMIs optimization approach. A normal and an augmented approach of saturated state feedback strategies are presented. Robust stability of formulation is also taken into consideration, while the numerical example shows the effectiveness of the formulation.

Finally, four different application examples illustrated the RCMPC formulation in Chapter 6.

Moreover, some conclusions are made in chapter 7, together with some extensions and potential improvements of the proposed technique.

7.2 Possible Extensions

1. Reference trajectory tracking

   The research in this thesis is mainly to the infinite horizon regulator with zero target. In optimal tracking problems, the systems output are required to track a reference trajectory $y(k) = C_r x_r(k)$, where the reference state x_r is computed from the equation

   $$x_r(k+1) = A_r x_r(k), \quad x_r(0) = x_{r0}.$$

   The choice of performance objective for the robust trajectory tracking objective is

   $$J_\infty \triangleq \sum_{j=0}^{\infty} \left[\|Cx(k+j|k) - C_r x_r(k+j)\|_Q^2 + \|u(k+j|k)\|_R^2\right], x \in \mathcal{X}_i, i \in \mathcal{I},$$

   The plant dynamics can be augmented by the reference trajectory dynamics to reduce the robust trajectory tracking problem.

2. Systems with input delay

   In practice, the delay can occur not only in the state of the systems, but also in the systems input. We deal with systems that allow input delay and state delays to be uncertain and time-varying. At sampling time $k \geq \tau$, we would like to design a saturated state feedback control law

   $$u(k+j-\tau(k)|k) = \sigma(Fx(k+j-\tau(k)|k)), \quad j \geq 0,$$

   to minimize the following infinite horizon robust performance objective

   $$J_\infty \triangleq \sum_{j=0}^{\infty} \left[\|x(k+j|k)\|_Q^2 + \|u(k+j-\tau(k)|k)\|_R^2\right], x \in \mathcal{X}_i, i \in \mathcal{I},$$

3. Norm-bound uncertain PWA model

   This paradigm consists of a PWA model with uncertainties or perturbations appearing in the feedback loop.

   $$\left.\begin{array}{rcl} x(k+1) &=& A_i x(k) + B_i u(k) + B_{pi} p(k), \\ y(k) &=& C_i x(k), \\ q(k) &=& C_{qi} x(k) + D_{qui} u(k), \\ p(k) &=& (\Delta q)(k). \end{array}\right\} x(k) \in \mathcal{X}_i, i \in \mathcal{I}.$$

   The operator Δ is a block-diagonal, with $\Delta_i : \mathbb{R}^{n_i} \to \mathbb{R}^{n_i}$. Each Δ_i is assumed to be either a *repeated scalar* block or a *full* block, and models a number of factors, such as nonliniearities, dynamics or parameters, that are unknown, unmodeled, or neglected. A number of control systems with uncertainties can be handled using this framework.

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Biography

Born in Palembang, Indonesia, in 1981, Santo Wijaya obtained his Bachelor degree in physics engineering from Institute Technology of Bandung, Indonesia, in 2004. He was granted AUN/SEED-Net Scholarship (www.seed-net.org) to pursue his Master's degree in electrical engineering at Chulalongkorn University, Thailand, since 2008. He studied and did his research in Control Systems Research Laboratory, Department of Electrical Engineering, Faculty of Engineering, Chulalongkorn University, Thailand.

Throughout the graduate studies, Santo's research was under the supervision of Assistant Professor Manop Wongsaisuwan. His field of interest includes robust control, model predictive control, linear matrix inequalities, computer aided design of control systems, and its applications in control.

List of Publications

1. S. Wijaya, and M. Wongsaisuwan. Robust MPC for PWA With Time-Delay Systems Using Saturated Linear Feedback Controller. in *Proc. of 2nd ICCAE conference*. (2010): 57–61.

2. S. Wijaya, and M. Wongsaisuwan. Robust Constrained MPC for PWA Systems Using Saturated Linear Feedback Controller, in *Proc. of ECTI-CON conference*. (2010): accepted for publication.

3. S. Wijaya, and M. Wongsaisuwan. Robust constrained MPC for PWA with time-varying delay systems using saturated linear feedback controller. *IEEE Trans. on Automatic Control*. (2010): paper submitted.