ผลของรูปแบบของท่อสัมผัสแบบอากาศยกชนิดไหลวนภายนอกต่ออุทกพลศาสตร์และการถ่ายเทมวลสารระหว่างวัฏภาคก๊าซและของเหลว

นายยุทธนา รุจิรัตนากุล

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิศวกรรมศาสตรมหาบัณฑิต
สาขาวิชาวิศวกรรมเคมี ภาควิชาวิศวกรรมเคมี
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ปีการศึกษา 2550

EFFECTS OF CONFIGURATIONS OF EXTERNAL LOOP AIRLIFT CONTACTOR ON HYDRODYNAMICS AND GAS-LIQUID MASS TRANSFER

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A Thesis Submitted in Partial Fulfillment of the Requirements
for the Degree of Master of Engineering Program in Chemical Engineering

Department of Chemical Engineering
Faculty of Engineering
Chulalongkorn University

Academic Year 2007

Thesis Title	EFFECTS OF CONFIGURATIONS OF EXTERNAL LOOP AIRLIFT CONTACTOR ON HYDRODYNAMICS AND GAS-LIQUID MASS TRANSFER
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ยุทธนา รุจิรัตนากุล : ผลของรูปแบบของท่อสัมผัสแบบอากาศยกชนิดไหลวนภายนอกต่ออุทกพลศาสตร์และการถ่ายเทมวลสารระหว่างวัฏภาคก๊าซและของเหลว (EFFECTS OF CONFIGURATIONS OF EXTERNAL LOOP AIRLIFT CONTACTOR ON HYDRODYNAMICS AND GAS-LIQUID MASS TRANSFER.) อ. ที่ปรึกษา : รศ. ดร. ประเสริฐ ภวสันต์, 93 หน้า.

วัตถุประสงค์ของงานวิจัยนี้คือการศึกษาประสิทธิภาพของถังสัมผัสแบบอากาศยกชนิดไหลวนภายนอกต่อค่าทางอุทกพลศาสตร์และการถ่ายเทมวลสาร โดยการเปลี่ยนความยาวของท่อที่ต่อระหว่างส่วนให้อากาศและส่วนที่ไม่ให้อากาศ เปลี่ยนความสูงของส่วนให้อากาศและส่วนที่ไม่ให้อากาศ และทำการเปลี่ยนแปรงรูปร่างของถังสัมผัสแบบอากาศยก ซึ่งกำหนดอัตราส่วนระหว่างพื้นที่ในการไหลลงและพื้นที่ในการไหลขึ้นของของไหลไว้คงที่ที่ 0.269 จากผลการศึกษาแสดงให้เห็นว่าการเปลี่ยนแปลงรูปร่างของถังสัมผัสอากาศชนิดไหลวนภายนอกนี้มีผลต่อปริมาณก๊าซในระบบ ความเร็วของของเหลว และค่าการถ่ายเทมวลสาร โดยผลของการเพิ่มความยาวของท่อที่ต่อระหว่างส่วนให้อากาศและส่วนที่ไม่ให้อากาศ และผลจากการเพิ่มความสูงของส่วนให้อากาศและไม่ให้อากาศมีผลในทิศทางเดียวกันคือ เมื่อเพิ่มความยาวของท่อที่ต่อระหว่างส่วนให้อากาศและส่วนที่ไม่ให้อากาศจาก 20 ถึง 40 เซนติเมตร และเพิ่มความสูงของส่วนให้อากาศและไม่ให้อากาศจาก 100 ถึง 140 เซนติเมตร พบว่าการเคลื่อนที่ของของเหลวเร็วขึ้น 11.3 และ 7.2 เปอร์เซ็นต์ ทำให้ปริมาณก๊าซในระบบลดลง 8.6 และ 4.3 เปอร์เซ็นต์ และค่าสัมประสิทธิ์การถ่ายเทมวลสารเชิงปริมาตรลดลง 8.2 และ 5.6 เปอร์เซ็นต์ ตามลำดับ และเมื่อทำการเปลี่ยนแปลงรูปร่างที่แตกต่างกันพบว่า รูปร่างของถังสัมผัสแบบอากาศยกแบบที่มีส่วนที่ต่อระหว่างส่วนให้อากาศและส่วนที่ไม่ให้อากาศเป็นลักษณะเฉียงลงและรูปร่างของถังสัมผัสแบบอากาศยกแบบปิดในส่วนของการแยกก๊าซในส่วนที่ไม่ให้อากาศ จากผลการทดลองพบว่า รูปแบบทั้งสองแบบนี้แสดงผลของความเร็วของของเหลวที่ลดลง ปริมาณก๊าซในระบบเพิ่มขึ้น และค่าสัมประสิทธิ์การถ่ายเทมวลสารเชิงปริมาตรเพิ่มขึ้นเมื่อเทียบกับรูปแบบของถังสัมผัสแบบอากาศยกในรูปแบบปรกติ

ในการทำวิจัยนี้ ยังได้ทำการออกแบบโมเดลทางคณิตศาสตร์เพื่อทำนายค่าการละลายของออกซิเจนในของเหลวของถังสัมผัสแบบอากาศยกชนิดไหลวนภายนอก โดยการจำลองถังสัมผัสแบบอากาศยกชนิดไหลวนภายนอกเป็นอนุกรมของถังปฏิกิริยา ซึ่งได้แก่ ส่วนที่ให้อากาศ ส่วนที่ไม่ให้อากาศ ส่วนของท่อที่ต่อระหว่างส่วนที่ให้อากาศและไม่ให้อากาศทั้งด้านบนและด้านล่างถูกจำลองให้เป็นโมเดลของการกระจายตัว และส่วนที่ทำการแยกอากาศทั้งสองด้านถูกจำลองให้เป็นโมเดลของถังกวน ผลจากการคำนวณที่ได้จากโมเดลแสดงให้เห็นว่า แม้ว่าถังสัมผัสแบบอากาศยกในงานวิจัยนี้จะมีหลายรูปแบบส่งผลให้มีค่าการละลายของออกซิเจนที่หลากหลาย แต่เมื่อทำการเปรียบเทียบระหว่างค่าการละลายของก๊าซออกซิเจนที่ได้จากการทดลองและจากผลการคำนวณของโมเดลนั้นพบว่ามีค่าใกล้เคียงกันในทุกรูปแบบของถังสัมผัสแบบอากาศยกที่ได้ทำการศึกษาในงานวิจัยนี้

ภาควิชา วิศวกรรมเคมี ลายมือชื่อนิสิต ยุทธนา รุจิรัตนากุล

สาขาวิชา วิศวกรรมเคมี ลายมือชื่ออาจารย์ที่ปรึกษา

ปีการศึกษา 2550

4870693221: MAJOR CHEMICAL ENGINEERING

KEY WORD: AIRLIFT CONTACTER / EXTERNAL LOOP HYDRODYNAMICS / MASS TRANSFER

YUTTANA RUJIRUTTANAKUL: EFFECTS OF CONFIGURATIONS OF EXTERNAL LOOP AIRLIFT CONTACTOR ON HYDRODYNAMICS AND GAS-LIQUID MASS TRANSFER. THESIS ADVISER: ASSOC.PROF. PRASERT PAVASANT, Ph.D., 93 pp.

Various configurations of external loop airlift contactor (ELALC) were examined for their hydrodynamic and mass transfer behavior using tap water. These behaviors was manipulated by changing the length of connection tubes (L_c), height of riser and downcmoer (L_h) and other configurations while keeping the ratio between downcomer and riser cross sectional area constant at 0.269. The results showed that the change of configuration could have effects on: gas holdup, liquid velocity, and gas-liquid mass transfer. Increasing L_c and L_h seemed to pose the same effect on the airlift performance. For instance, an increase in L_c from 20 to 40 cm, and L_h from 100 to 140 cm, could increase liquid velocity at 11.3 and 7.2%, decreased overall gas holdup at 8.6 and 3.3% and decreased the overall volumetric mass transfer coefficient at 8.2 and 5.6%, respectively. The effect of configurations on ELALC was investigated. ELALCs with inclined connection tubes and with close downcomer were compared with regular ELALC in terms of hydrodynamic and gas-liquid mass transfer behaviors. The two configurations presented large agglomerate of gas bubbles on the top connection tubes which resulted in more gas holdups and less downcomer liquid velocity when compared with regular ELALC.

The mathematical model was developed based on the mass conservation principals to predict the gas-liquid mass transfer in the ELALC. Basically ELALC was represented by a series of reactors, i.e. the riser, downcomer, top and bottom connection tubes were represented by the dispersion model, and the two gas separators represented by the stirred tank model. The simulation results were found to be reasonably accurate for all experimental conditions employed in this work.

Department....Chemical Engineering... Student's signature...ยุทธนา รุจิรุตนากุล...

Field of study..Chemical Engineering... Adviser's signature.......[signature].......

Academic year........2007........

ACKNOWLEDGEMENTS

This thesis will never have been completed without the help and supports of many people who are gratefully acknowledged here. Firstly, I would like to express my sincere gratitude to Associate Professor Prasert Pavasant, my advisor, for his suggestions, guidance, warm encouragement and generous supervision throughout my master program. I am also grateful to Associate Professor Dr. Tharathon Mongkhonsi, Assistant Professor Dr. Artiwan Shotipruk and Assistant Professor Dr. Navadol Laosiripojana for their helpful and many valuable comments.

Special gratitudes are for the Thailand Research Fund and the Graduated School Research Fund for finantially supporting my work here. I would like to gratefully thank the Department of Chemical Engineering that gave a chance for studying in the field of engineering. I cannot forget to express my thankfulness to my lovely friends, Numtip, Oam, Ni, Natchiko, Supersert’s group research: P’Beau P’Puud P’Boom, P’Poot, P’Ron, P’Nui, P’Mod, Phong, Tik, X, M, Mag, Kaey, Note, Toey, Big, Mauy, Bass, N’Gim.

Finally, I would like to express me sincere my sincere indebtedness to my family for their worth supports throughout my Master courses.

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LIST OF ABBREVIAL & NOTATIONS

Abbreviations

ALC	Airlift contactor
ILALC	Internal loop airlift contactor
ELALC	External loop airlift contactor
BC	Bubble column

Notations

A	cross-sectional area [m^2]
a	specific gas-liquid interfacial area of bubble per volume of reactor [m^2/m^3]
C	oxygen concentration in the liquid [kg/m^3]
C^*	equilibrium oxygen concentration in the liquid [kg/m^3]
C_0	initial oxygen concentration in the liquid [kg/m^3]
C_D	drag coefficient [-]
d_b	bubble diameter [m]
D	column diameter [m]
D_G	gas phase dispersion coefficient [m^2/s]
D_L	liquid phase dispersion coefficient [m^2/s]
g	gravitational acceleration [m/s^2]
h	height [m]
h_D	dispersion height [m]
h_L	unaerated liquid height [m]
k_L	liquid phase mass transfer coefficient [m/s]
k_La	overall volumetric oxygen transfer coefficient [1/s]
K_La	overall volumetric oxygen transfer coefficient [1/s]
H	henry constant [g/g]
L	length [m]
L_c	length of connection tubes between riser and downcomer [m]
L_h	height of riser and downcomer [m]
N	molar flux [mol/s·m^2]
$\bar{O}_G$	dimensionless oxygen concentration in gas phase [-]
O_G	oxygen concentration in gas phase [kg/m^3]
$\bar{O}_L$	dimensionless oxygen concentration in liquid phase [-]

Notations

O_L	oxygen concentration in liquid phase [kg/m^3]
O_L^*	saturation concentration of transferring gas or solute in liquid [kg/m^3]
P	pressure [Pa]
Q_G	volumetric gas flowrate [m^3/s]
Q_L	volumetric liquid flowrate [m^3/s]
$\bar{t}$	dimensionless time [-]
t	time [s]
u_t	terminal velocity of bubbles [m/s]
u_L	superficial liquid velocity [m/s]
u_{sg}	superficial gas velocity [m/s]
v	velocity [m/s]
V	volume [m^3]
v_G	linear gas velocity [m/s]
v_L	linear liquid velocity [m/s]
V_L	liquid volume [m^3]
v_s	slip velocity [m/s]
V_t	gas volume at the top of the contactor [m^3]
x	liquid path length [m]
z	length [m]
Z	dimensionless length [-]

Greek symbols

Δ	difference [-]
ε	gas holdup [-]
ρ	density [kg/m^3]
τ	dimensionless time [-]

Subscript

balance	balance between riser and downcomer
ct	top connection tube
cb	bottom connection tube
D	dispersion
d	downcomer
dt	draft tube

Subscript

G	gas
h	height
in	input
L	liquid
o	overall
out	output
$out1$	output on gas-liquid separator1
$out2$	output on gas-liquid separator2
r	riser
$s1$	gas-liquid separator1
$s2$	gas-liquid separator2

CHAPTER I

INTRODUCTION

1.1 Motivations

Microorganisms have been extensively used in the manufacture of several products such as pharmaceuticals, enzymes, chemical substances, food, etc. and often they are employed in the treatment of wastewater. In aerobic processes, the availability of oxygen in the culture is essential for the growth of the microorganisms. Hence, numerous research works had been carried out to develop gas-liquid devices that can provide high oxygen transfer rate without attaining high operating cost or high shear rate. Airlift contactors are one of those recent developed gas-liquid contactors with several advantages over agitating devices, as they are easy to design, with no moving parts, low power requirement and low shear rate. In comparison with bubble columns, airlift contactors have high mixing performance due to the fluid circulation. This makes airlift contactors attractive for the culture of microbial systems.

Airlift contactors can be divided into two main types according to their physical appearance, i.e. internal loop airlift contactor (ILALC) and external loop airlift contactor (ELALC) as shown in Figure 1.1. The configuration of ELALC is generally more complicated than ILALC as it has two separated columns operating as riser and downcomer with connection tubes connecting the two columns together. This structure makes the design of ELALC more intricate than that of ILALC as there are more design parameters that could significantly affect the behavior of the system. In other words, ELALC can be designed to be operated with a complete gas disengagement leading to a maximal liquid circulation which could improve mixing and heat transfer (or vice versa). Examples of the various design configurations of the airlift are shown in Figure 1.2. Figure 1.2 (b) illustrates the configuration where the gas disengagement section in the downcomer is close which enhances the gas holdup in the system. Figure 1.2 (c) illustrates the inclined the connection tubes which could reduce gas holdup in downcomer. In addition, to minimize gas holdup in downcomer, the ELALC can be designed by enlarging the gas separator section as this drastically reduces the liquid velocity allowing a near complete separation of gas bubbles from liquid (Figure 1.2.(d))

There have been limited number of research articles concerning the design of ELALCs, and this would be described in detail in the next chapter. Most of the previous researches have focused their investigation on the effects of A_d/A_r, length of connection tubes and volume of airlift contactor and other types of ELALC such as inclined the connection tubes and closed downcomer gas liquid separator. As the detail of ELALC is quite diverged, it was difficult to have a comprehensive knowledge on the design of such system. Thus so far, the research at the Department of Chemical Engineering, Faculty of Engineering, Chulalongkorn University only focused on the internal loop type. The external loop airlift was considered significant for the future development and therefore this work intend to fill in the missing information regarding the design of ELALCs and would concentrate on characterizing the performance of external loop airlift contactors (ELALC) under the various design configurations. The performance of the system would be monitored from the hydrodynamic and mass transfer behavior of such system.

1.2 Objectives

1.2.1 To investigate the effect of the different configuration of external loop ALC on hydrodynamic and mass transfer behavior.

1.2.2 To establish empirical correlations to predict hydrodynamic behavior and mass transfer rate in external loop airlift contactors.

1.2.3 To establish mathematical model to predict hydrodynamic behavior and mass transfer rate in external loop airlift contactors.

1.3 Scopes of this work

1.3.1 The investigations were restricted to bench-scale external loop airlift contactors with dimensions as shown in Table 3.1.

1.3.2 The investigations were performed in an air-water system only.

1.3.3 In all investigations, the airlift contactor systems were subjected to the following assumptions:

- The system was isothermal which operates at room temperature, and the effect of the dynamics of the dissolved oxygen at electrode is negligible.
- The system was operated at atmospheric pressure.

1.3.4 The investigations of mass transfer characteristics were restricted to oxygen transfer only

1.3.6 The range of superficial gas velocity employed in this work was between 0.5-10 cm/s.

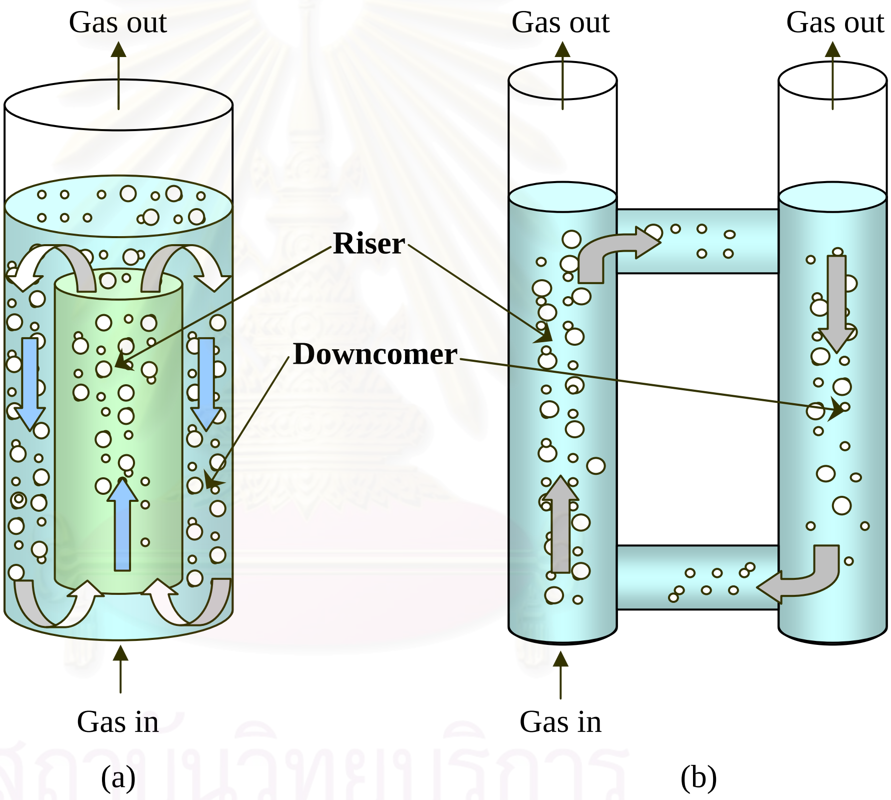

Figure 1.1 The type of airlift contactors: (a) internal loop (ALC), (b) external loop (ALC)

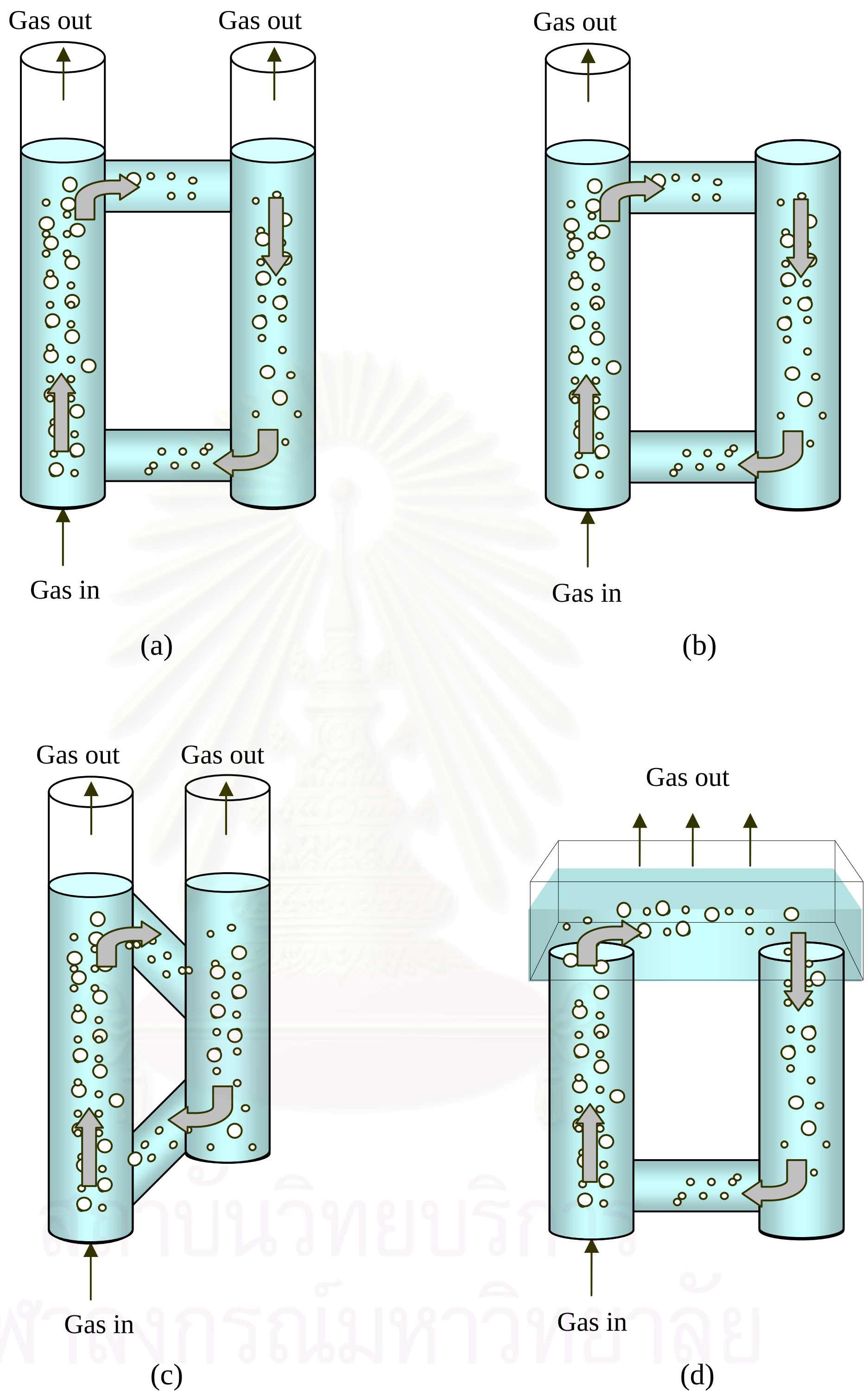

Figure 1.2 Various type of external loop airlift contractor: (a) regular configuration of ELALC (b) ELALC with closed downcomer gas liquid separator (c) ELALC with inclined the connection tubes (d) ELALC with liquid pool gas liquid separator.

CHAPTER II

BACKGROUNDS & LITERATURE REVIEW

2.1 Airlift contactors

Airlift contactors (ALCs) consist of a liquid pool divided into two distinct zones. The different gas holdups in the aerated and unaerated zones cause different bulk densities of the fluid in these regions resulting in fluid circulation in the system. The airlift contactors can be divided into two types and three different regions as delineated below.

2.1.1 Types and configurations of airlift contactors

In general, airlift contactors can be separated into two types as shown in Figure 1.1

(1) Internal loop airlift contactors (ILALC) which contains a vertical draft tube by which a loop channel for the fluid is formed in the airlift (Figure 1.1 (a)).

(2) External loop airlift contactors (ELALC) which consists of two vertical tubes (riser and downcomer) connected by horizontal conduits at the top and bottom sections (Figure 1.1 (b)).

2.1.2 Three main regions of airlift contactors (Schematic flow directions shown in Figure 2.1)

(1) Riser: Gas is added to the liquid in this section which makes this fluid lighter than that in downcomer. Fluid moves up to the top of riser by two mechanisms, i.e. gas lift momentum transfer and (ii) the buoyant force due to the different fluid density between riser and downcomer.

(2) Gas-liquid separator: Gas and liquid are separated in this section. The gas is disengaged from the airlift contactor resulting in a heavier liquid which moves down the downcomer.

(3) Downcomer: At the bottom of the airlift, the liquid moves from downcomer to riser together with the input gas. This completes the circulation of the liquid within the airlift system.

2.2 Gas-liquid hydrodynamics and mass transfer

Major parameters that control the behavior of airlift contactors are gas holdup, liquid velocity and mass transfer. Understanding the relationships between the various parameters is essential for a reliable description of the airlift systems. Details on hydrodynamic behavior and gas-liquid mass transfer of the airlift system are described below.

2.2.1 Hydrodynamics behavior

2.2.1.1 Gas holdup

The volume fraction of the gas-phase in the gas-liquid dispersion is known as the gas void fraction or the gas holdup. The overall gas holdup (ε) is the ratio between the volume of gas phase and the total volume of reactor which can be expressed as:

$$\varepsilon = \frac{V_G}{V_G + V_L} \tag{2.1}$$

where: V_G is the gas volume and V_L the liquid volume.

In airlift contactors, gas holdups are different in the various parts of the system. In general, gas holdups are described using three quantities, i.e. overall gas holdup, riser gas holdup and downcomer gas holdup. The three holdups can be correlated as follows:

$$\varepsilon = \frac{A_r \varepsilon_r + A_d \varepsilon_d}{A_r + A_d} \tag{2.2}$$

where: ε_r is the riser gas holdup, ε_d the downcomer gas holdup, A_r the riser cross sectional area and A_d the downcomer cross sectional area.

In fact, The overall gas holdup can be determined from the volume expansion method where

$$\varepsilon_o = 1 - \frac{h_L}{h_D} \tag{2.3}$$

when ε_o is the overall gas holdup, h_L the unaerated liquid height and h_D the dispersed liquid height.

Riser and downcomer gas holdups can be determined from the information on the pressure drop measured from the two side-ports of the column where

$$\varepsilon_d = 1 - \frac{\Delta Z_{manometer}}{\Delta H} \tag{2.4}$$

where ΔZ is the pressure difference of defined liquid level and ΔH is a distance of liquid level in the airlift column.

Table 2.1 summarizes literature that dealt with ELALC and ILALC in various conditions.

Effects of airlift configurations on gas holdups

In the study of the behavior of external loop airlift systems (35-64 L and A_d/A_r = 0.11-0.53), Choi (1993; 2001; 2002) demonstrated that the airlift with both open and close downcomers behaved similarly and an increase in A_d/A_r resulted in a decrease in riser and overall gas holdup. Larger scale external loop airlifts seemed to yield a higher riser gas holdup (Benyahia and Jones, 1998; Ghirardini et al., 1992)

One of the distinct parameters for external loop airlift system is the length of the connection conduit between riser and downcomer. Choi (1993) showed that this parameter could also pose some influence on the behavior of the system where an increase in this length (from 10 to 50 cm) reduced the riser gas holdup by 18% and the overall gas holdup by 30%. A much stronger effect was found for the downcomer gas holdup which was reduced by as much as 76% under the increase of conduit length.

The height of the external loop airlift was also found to have direct influence on riser gas holdup and an increase in the height caused the riser and overall gas holdups to decrease (Bentifraouine et al., 1997; Choi, 2002). Again, the effect of column height on downcomer gas holdup was the same as on other gas holdups but the magnitude was stronger.

A closed downcomer section for the external loop airlift system was found to affect the riser and downcomer gas holdups. Bentifraouine et al. (1997) and Choi (1993; 2001) reported that the riser gas holdup was high when the downcomer was closed, however, the increase in riser gas holdup with the close downcomer might depend on several factors. For instance, Bentifraouine et al. (1997) reported a 24% in riser gas holdup in the airlift with Ad/Ar = 0.225, whereas Choi (1993; 2001) reported about 14% in the system with Ad/Ar = 0.53. Choi (1993; 2001) also measured the downcomer gas holdup in the closed downcomer airlift which was reported to be twice as much as that in the system with open downcomer.

Comparison between external and internal loop airlift contactors and bubble columns

Generally gas holdups in riser of the airlift system are in the range from 0-20% depending on the setup and the aeration rate. Figure 2.2 demonstrates the level of riser gas holdups obtained from the various setups of airlift systems. However, some trends could be generalized from this survey. Firstly, the level of gas holdups in ELALC and ILALC were quite close particularly at low range of u_{sg}. Next, the internal loop system seemed to provide a larger riser gas holdup, especially at high u_{sg}. It is interesting to note that the bubble column often gave a higher gas holdup than in the airlift systems

2.2.1.2 Liquid velocity

Liquid velocity in the airlift system is caused by two main reasons. Firstly there is a momentum transfer from the gas input to the system, and this forces the liquid to move in the same direction with the gas bubbles. Secondly, the difference in apparent fluid densities between riser and downcomer also forces the fluid in the riser to move up and that in downcomer to move down. The replacement of liquid in riser from that in downcomer completes the circulation of liquid in the system.

Generally, liquid velocity is measured in terms of linear liquid velocity defined as:

$$v_L = \frac{x_L}{t} \tag{2.5}$$

where: v_L is liquid velocity, x_L the liquid path length and t is times for liquid complete movement.

u_L is called superficial liquid velocity which is calculated based on the empty column area. The actual liquid velocity is always higher than superficial velocity as the area of liquid movement is always intercepted by the presence of gas in the riser and downcomer. The superficial liquid velocity and actual liquid velocity can be related as:

$$v_{Lr} = \frac{u_{Lr}}{1-\varepsilon_r} \tag{2.6}$$

and

$$v_{Ld} = \frac{u_{Ld}}{1-\varepsilon_d} \tag{2.7}$$

where v_{Lr} and v_{Ld} are actual linear liquid velocities in riser and downcomer, respectively, u_{Lr} and u_{Ld} are superficial liquid velocities in riser and downcomer, respectively.

The relationship of superficial liquid velocity in riser and downcomer can be expressed using the mass conservation rule:

$$u_{Lr} A_r = u_{Ld} A_d \tag{2.8}$$

Effects of airlift configurations on liquid velocity

For external loop airlift contactors (ELALC), Choi (1996; 2001; 2002) demonstrated that an increase in A_d/A_r in the range of 0.11-0.53 led to an increase in riser liquid velocity, for all the ELALCs investigated which included both open and close downcomer sections. Kawase et al. (1994) studied ELALC (23 - 26L) with inclined connection tubes and A_d/A_r between 0.204 - 0.458 and reported that an increase in A_d/A_r, reduced the value of downcomer liquid velocity.

The volume of the airlift systems was also proven to have effects on liquid velocity. It was shown in the systems of 6.5 - 64 L and 2 - 800 L that a higher liquid volume resulted in a faster riser liquid velocity (Ghirardini et al., 1992; Lindert and Hochberk, 1992) Similarly, Bentifraouine et al. (1997) and Choi (2002) studied the effect of liquid height on the liquid velocity and concluded that an increase in the liquid height enhanced the riser liquid velocity. The extension of the length of the connection tubes could also increase riser liquid velocity but the effect of the connection tube length was not as strong as that of A_d/A_r (Choi, 1996).

The various configurations of ELALC such as the open and close downcomer sections could also affect the liquid velocity. The same conclusion could be drawn from the two different articles where the closed downcomer airlift caused the liquid to move slower than the open downcomer system (Bentifraouine et al., 1997; Choi, 1996; 2001)

Comparison between external and internal loop airlift contactors

The liquid velocity in internal loop was often reported to be higher than that in external loop airlift. Figure 2.3 illustrates this quite clearly and at the same aeration rate, the internal loop would result in a faster liquid velocity. Normally the liquid velocity in internal loop would take the value of about 20-30 cm/s, however, the external loop saw this value at as low as 5-20 cm/s.

2.2.2 Gas-liquid mass transfer

Gas-liquid mass transfer in airlift system is one of the most important characters for aerobic systems. The mass transfer between phases is often described by the Two-film model where both liquid and gas films on both sides of the bubble could exert some mass transfer resistance. The mechanisms of gas-liquid mass transfer can be generalized into four steps as follows:

1. the transport from inside the bubble to gas film
2. the transfer through the gas film to the gas-liquid interface
3. the transfer from the gas-liquid interface through the liquid film
4. the transport in the bulk liquid

The mass transfer resistance on the liquid side is much higher than the gas side, and therefore the overall mass transfer resistance is typically controlled solely by the resistance of the liquid film.

In cases where equilibrium can be explained by Henry's law, and the overall flux expressed as the overall concentration driving force in the liquid phase is:

$$N_{O_2} = k_L (C^*_{O_2} - C_{O_2}) \tag{2.9}$$

where N_{O_2} is the molar flux of oxygen, k_L is the overall mass transfer coefficient based on liquid phase, $C^*_{O_2}$ is dissolved oxygen at equilibrium and C_{O_2} is dissolved oxygen at any time.

Often the gas-liquid mass transfer rate and the flux are related by:

$$aN_{O_2} = \frac{dC_{O_2}}{dt} \tag{2.10}$$

where *a* is the specific mass transfer area which is equal to the total mass transfer area divided by the dispersed volume in the system. Therefore the rate of oxygen mass transfer can be written as:

$$\frac{dC_L}{dt} = k_L a (C^*_L - C_L) \tag{2.11}$$

where $k_L a$ is the overall volumetric mass transfer coefficient.

Effects of various configurations on k_La

The overall volumetric mass transfer coefficient or k_La in the external loop airlift system depends significantly on the design of the airlift system. Most reports stated that an increase in A_d/A_r decreased the level of k_La in the system (Choi, 1993; 2001 and Kawase et al, 1996). For example, the diameter of the column was also reported to have adverse effect on k_La where an airlift with larger diameter was found to give lower k_La than that with small diameter (both with the same A_d/A_r) (Benyahia and Jones, 1998). However, this might not be the case when the ratio between the column diameter and height was remained constant. For instance, (Lindert and Hochberk, 1992) reported that k_La in the small airlift system was almost the same as that in a larger column airlift when the column height and diameter were kept at the same ratio.

The length of the connection tube was also reported to have influence on k_La, but the effect was only marginal (Choi, 1993). The airlift with close downcomer seemed to be able to give positive effect on k_La. This effect could be as much as 30% increase as reported by Choi, 1993; 2001.

Comparison external loop and internal loop airlift contactors

The range of k_La obtained from external loop airlift systems covered the value from 0-0.06 1/s which was wider than that for internal loop of 0-0.04 1/s. Figure 2.4 shows that, for the same range of u_{sg}, k_La from both internal and external loop overlapped. However, as there were more designed variables for external loop airlift, the variation of k_La in the external loop system was much larger than that of the internal loop.

Table 2.1 Literature review

Author (year)	Reference No.	Details	volume (l)	*h* (m)	L_c (m)	A_d/A_r (-)	u_{sg} (cm/s)	v_{Lr} (cm/s)	v_{Ld} (cm/s)	riser gas holdup (-)	downcomer gas holdup (-)	overall gas holdup (-)	k_La (1/s)
Choi (1993; 1996)	1	regular ELALC	42	1.77	0.3	0.11	2 - 18	9 - 16	-	0.058 - 0.170	-	-	0.012 - 0.060
			48	1.77	0.3	0.28	2 - 18	17 - 27	-	0.050 - 0.150	-	-	0.010 - 0.055
	2		56	1.77	0.3	0.53	2 - 18	22 - 29	-	0.040 - 0.145	0.005 - 0.043	0.029 - 0.113	0.008 - 0.050
			52	1.77	0.1	0.53	2 - 18	18 - 23	-	0.050 - 0.165	0.007 - 0.092	0.036 - 0.142	0.011 - 0.063
			56	1.77	0.3	0.53	2 - 18	22 - 28	-	0.040 - 0.145	0.005 - 0.043	0.028 - 0.113	0.008 - 0.050
			60	1.77	0.5	0.53	2 - 18	20 - 32	-	0.035 - 0.135	0.004 - 0.022	0.025 - 0.099	0.007 - 0.045
	3	Bubble column	35	1.77	-	-	2 - 18	-	-	0.080 - 0.260	-	-	0.030 - 0.105
Choi (2002)		ELALC with all opened	58	1.86	0.3	0.11	2 - 18	8 - 17	-	0.042 - 0.166	-	-	-
		gas liquid separator	61	1.86	0.3	0.28	2 - 18	19 - 37	-	0.044 - 0.160	-	-	-
			64	1.86	0.3	0.53	2 - 18	25 - 54	-	0.044 - 0.157	-	-	-
	4		50	1.78	0.3	0.53	2 - 18	5 - 24	-	0.044 - 0.157	0.002 - 0.080	0.031 - 0.132	-
			57	1.82	0.3	0.53	2 - 18	23 - 48	-	0.033 - 0.123	0.003 - 0.032	0.023 - 0.094	-
	5		64	1.86	0.3	0.53	2 - 18	25 - 54	-	0.031 - 0.105	0.003 - 0.011	0.022 - 0.075	-
Choi (2001)	6	ELALC with closed	35	1.77	0.2	0.11	2 - 18	9 - 14	-	0.060 - 0.180	0.015 - 0.042	0.056 - 0.167	0.016 - 0.082
		downcomer gas liquid	38	1.77	0.2	0.28	2 - 18	18 - 16	-	0.045 - 0.170	0.007 - 0.066	0.037 - 0.149	0.013 - 0.078
	7	separator	42	1.77	0.2	0.53	2 - 18	23 - 16	-	0.040 - 0.165	0.003 - 0.096	0.028 - 0.143	0.011 - 0.074
Kawase et al.		ELALC with inclined the	23	1.37	-	0.204	0.4 - 4.5	-	24 - 37	-	-	-	0.010 - 0.045
(1994; 1996)	8	connection tubes	26	1.37	-	0.458	0.4 - 4.5	-	22 - 34	-	-	-	0.008 - 0.035
	9	bubble column	16	1.37	-	-	0.4 - 4.5	-	-	-	-	-	0.012 - 0.047
Gavrilescu		regular ELALC, lab scale	1.2	1.16	-	0.111	1.6 - 17.5	5 - 9	46 - 86	0.08 - 0.25	-	-	0.008 - 0.054
(1995; 1996)			1.4	1.16	-	0.360	1.6 - 17.5	12 - 20	32 - 57	0.07 - 0.23	-	-	0.006 - 0.046
	10		1.9	1.16	-	1.000	1.6 - 17.5	24 - 44	23 - 42	0.05 - 0.18	-	-	0.005 - 0.031
		pilot scale	157	4.70	-	0.040	1.0 - 12.0	3 - 6	62 - 121	0.03 - 0.14	-	-	0.016 - 0.086
	11		170	4.70	-	0.122	1.0 - 12.0	5 - 10	42 - 88	0.05 - 0.16	-	-	0.008 - 0.071
Al-Masry and Abasaeed (1998)	12	regular ELALC	600	4.00	-	0.25	2 - 13	20 - 37	-	0.04 - 0.130	-	-	0.012 - 0.046

Table 2.1 (cont.)

Author (year)	Reference No.	Details	volume (l)	h (m)	L_c (m)	A_d/A_r (-)	u_{sg} (cm/s)	v_{lr} (cm/s)	v_{ld} (cm/s)	riser gas holdup (-)	downcomer gas holdup (-)	overall gas holdup (-)	$k_l a$ (1/s)
Abasaeed (1998)			1100	6.20	-	1.00	1.7 - 13	62 - 76	-	0.02 - 0.095	-	-	0.004 - 0.016
Benyahia and Jones	13	ELALC with all opened	13.4	1.73	0.5	1	0.8 - 8.5	-	-	0.038-0.185	-	-	0.003-0.024
(1998)	14	gas liquid separator	107.8	1.73	0.5	1	0.2 - 2.8	-	-	0.004-0.095	-	-	0.002-0.011
Bentifraouine et al.	15	regular ELALC	42	1.20	0.36	0.225	0.3 - 6.3	3.5 - 12.7	-	0.005 - 0.105	-	-	-
(1997)	16	ELALC with closed	42	1.20	0.36	0.225	0.3 - 6.3	3.5 - 7.5	-	0.005 - 0.130	-	-	-
		downcomer gas liquid separator											
		ELALC with all opened	42	1.00	0.36	0.225	0.3 - 6.3	4.5 - 15	-	0.006 - 0.106	-	-	-
		gas liquid separator	69	1.30	0.36	0.225	0.3 - 6.3	5.0 - 19	-	0.006 - 0.100	-	-	-
	17		92	1.60	0.36	0.225	0.3 - 6.3	5.5 - 21	-	0.006 - 0.096	-	-	-
Guo et al. (1997)		regular ELALC	16.5	1.70	0.33	1	0.3 - 4.7	-	10 - 23	-	0.01-0.065		0.001 - 0.0055
Onken and Weiland (1980)	18	regular ELALC	83	8.50	-	0.248	0.8 - 9.5	11 - 40	-	0.007-0.070	-	-	-
Ghirardini et al.		regular ELALC	6.5	2.00	-	1	24 - 80	65 - 80	-	0.035 - 0.075	-	-	-
(1992)			12	2.00	-	1	20 - 85	60 - 140	-	0.040 - 0.115	-	-	-
			64	2.00	-	1	20 - 120	45 - 190	-	0.045 - 0.190	-	-	-
Lindert and		regular ELALC	2	0.70	-	0.212	1 - 7	6.0 - 8.0	-	-	-	-	0.007 - 0.032
Hochberk (1992)	19		80	2.90	-	0.078	1 - 7	4.5 - 10.0	-	0.025 - 0.120	-	-	0.007 - 0.030
	20		800	7.90	-	0.111	1 - 7	11.0 - 22.5	-	0.035 - 0.100	-	-	0.007 - 0.034
	21	internal loop airlift, annulus sparged	70	2.10	-	0.067	1 - 7	5.5 - 13.0	-	-	-	-	0.007 - 0.037
Korpijarvi et al.	22	regular ELALC	30	2.50	-	0.609	1 - 6	13.5 - 16.9	20.6 - 31.0	0.025 - 0.090	-	-	-

Table 2.1 (cont.)

Author (year)	Reference No.	Details	volume (l)	h (m)	L_c (m)	A_d/A_r (-)	u_{sg} (cm/s)	v_{lr} (cm/s)	v_{ld} (cm/s)	riser gas holdup (-)	downcomer gas holdup (-)	overall gas holdup (-)	k_la (1/s)
Korpijarvi et al.	23	ILALC	15	1.25	-	0.592	1 - 6	-	-	0.040 - 0.135	-	-	-
(1999)	24		15	1.25	-	1.225	1 - 6	-	-	0.047 - 0.140	-	-	-
			15	1.25	-	2.703	1 - 6	-	-	0.030 - 0.100	-	-	-
			15	1.25	-	7.143	1 - 6	-	-	0.016 - 0.065	-	-	-
Freitas et al.	25	ELALC	60	1.28	-	0.10	3 - 17	12 - 19	-	0.08 - 0.21	-	-	-
(1999)	26	ILALC	60	2.07	-	4.25	3 - 17	30 - 48	-	0.05 - 0.23	-	-	-
Pavko and Charles	27	ELALC	8	1.00	0.3	0.16	0.7 - 6.0	5.5 - 8.5	-	-	-	-	-
(1988)	28	ILALC	8	1.50	-	1.61	0.7 - 6.0	35 - 95	-	-	-	-	-
Merchuk et al.	29	ILALC	30	2	-	1.06	0.2 - 4.5	12 - 42	15 - 35	0.01 - 0.145	-	-	-
(1996)	30		300	4.1	-	1.17	0.2 - 2.5	20 - 25	-	0.01 - 0.095	-	-	-
Wu et al.		ILALC	15	1.13	-	3.000	0.3 - 6.0	-	-	-	-	-	0.0075 - 0.045
(1992)			15	1.13	-	1.642	0.3 - 6.0	-	-	-	-	-	0.0085 - 0.044
	31		15	1.13	-	1.087	0.3 - 6.0	-	-	-	-	-	0.0090 - 0.042
	32		15	1.13	-	0.563	0.3 - 6.0	-	-	-	-	-	0.0095 - 0.037
Hsiun and Wu		ILALC	6.1	0.90	-	3.000	2.5 - 9.0	-	-	-	-	-	0.0222 - 0.070
(1995)			6.1	0.90	-	0.917	2.5 - 9.0	-	-	-	-	-	0.0170 - 0.065
			15.3	1.00	-	3.000	1.3 - 10.0	-	-	-	-	-	0.0095 - 0.070
			15.3	1.00	-	1.086	1.3 - 10.0	-	-	-	-	-	0.0080 - 0.065
			48.2	1.40	-	3.996	1.2 - 10.0	-	-	-	-	-	0.0100 - 0.080
			48.2	1.40	-	1.983	1.2 - 10.0	-	-	-	-	-	0.0100 - 0.080
			125.5	1.60	-	7.410	1.0 - 11.0	-	-	-	-	-	0.0095 - 0.083
			125.5	1.60	-	2.737	1.0 - 11.0	-	-	-	-	-	0.0090 - 0.082
Gavrilescu and	33	ILALC	70	1.68	-	0.9	3 - 10	-	-	-	-	-	0.090 - 0.210
Tudose (1998)			2500	8.26	-	1.0	2.5 - 10	-	-	-	-	-	0.065 - 0.096

Table 2.1 (cont.)

Author (year)	Reference No.	Details	volume (l)	h (m)	L_c (m)	A_d/A_r (-)	u_{sg} (cm/s)	v_{lr} (cm/s)	v_{ld} (cm/s)	riser gas holdup (-)	downcomer gas holdup (-)	overall gas holdup (-)	k_la (1/s)
Gavrilescu and Tudose (1998)	33	ILALC	5200	6.53	-	1.0	2.5 - 10	-	-	-	-	-	0.061 - 0.088
Kockbeck and		ILALC, annulus sparged	56	1.90	-	0.060	1 - 7	2.5 - 6.0	-	-	-	-	-
Hempel (1994)			56	1.90	-	0.057	1 - 7	3.5 - 6.5	-	-	-	-	-
	34		56	1.90	-	0.101	1 - 7	6.0 - 13.5	-	-	-	-	-
			56	1.90	-	0.156	1 - 7	10.0 - 19.5	-	-	-	-	-
Kawalec and	35	ILALC	49	0.52	-	0.72	0.6 - 8	18 - 52	-	0.023 - 0.142	-	-	0.003 - 0.050
Holowacz (1998)			91	0.95	-	0.72	0.6 - 8	-	-	0.008 - 0.120	0.017 -0.110		-
	36		105	1.10	-	0.72	0.6 - 8	30 - 55	-	0.008 - 0.112	-	-	0.003 - 0.040
			172	1.80	-	0.72	0.6 - 8	-	-	0.007 - 0.100	-	-	0.003 - 0.033
Blazej (2004)	37	ILALC	10.5	1.26	-	1.23	0.5 - 3.0	22 - 37	-	0.017 - 0.070	0.005 - 0.038	-	-
	39		32	1.82	-	0.95	0.5 - 3.0	23 - 35	-	0.017 - 0.090	0.009 - 0.055	-	-
			200	2.94	-	1.01	0.5 - 3.0	32 - 46	-	0.018 - 0.097	0.010 - 0.080	-	-

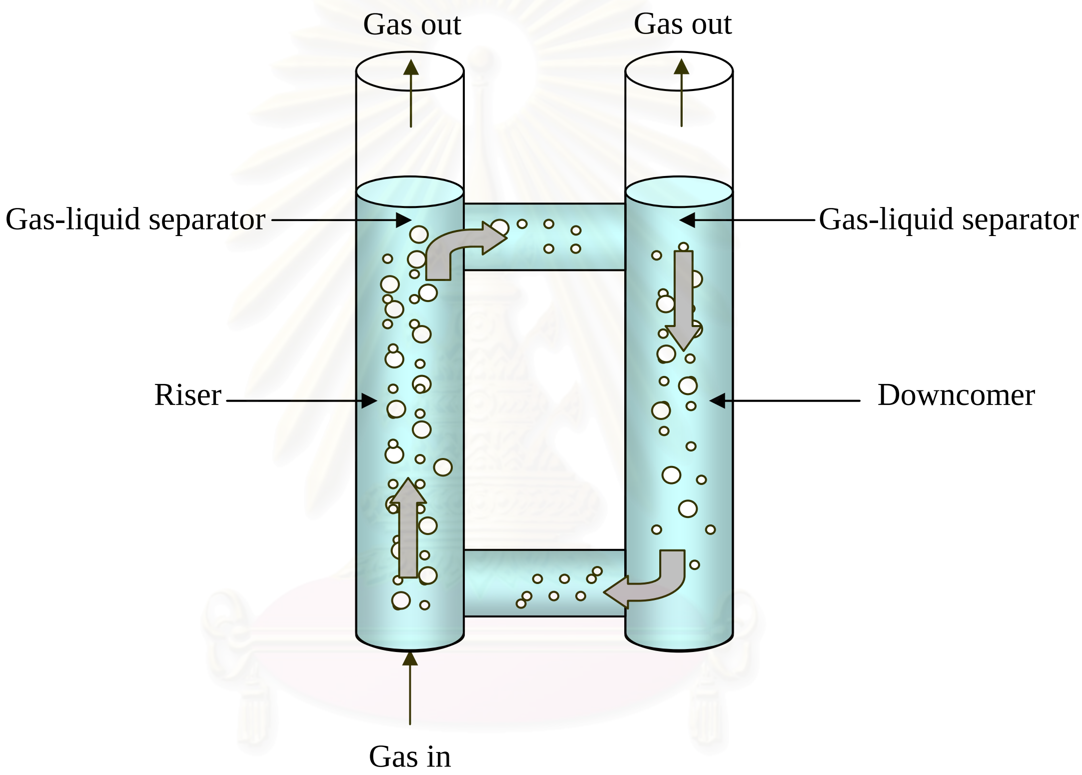

Figure 2.1 Schematic flow directions in airlift system

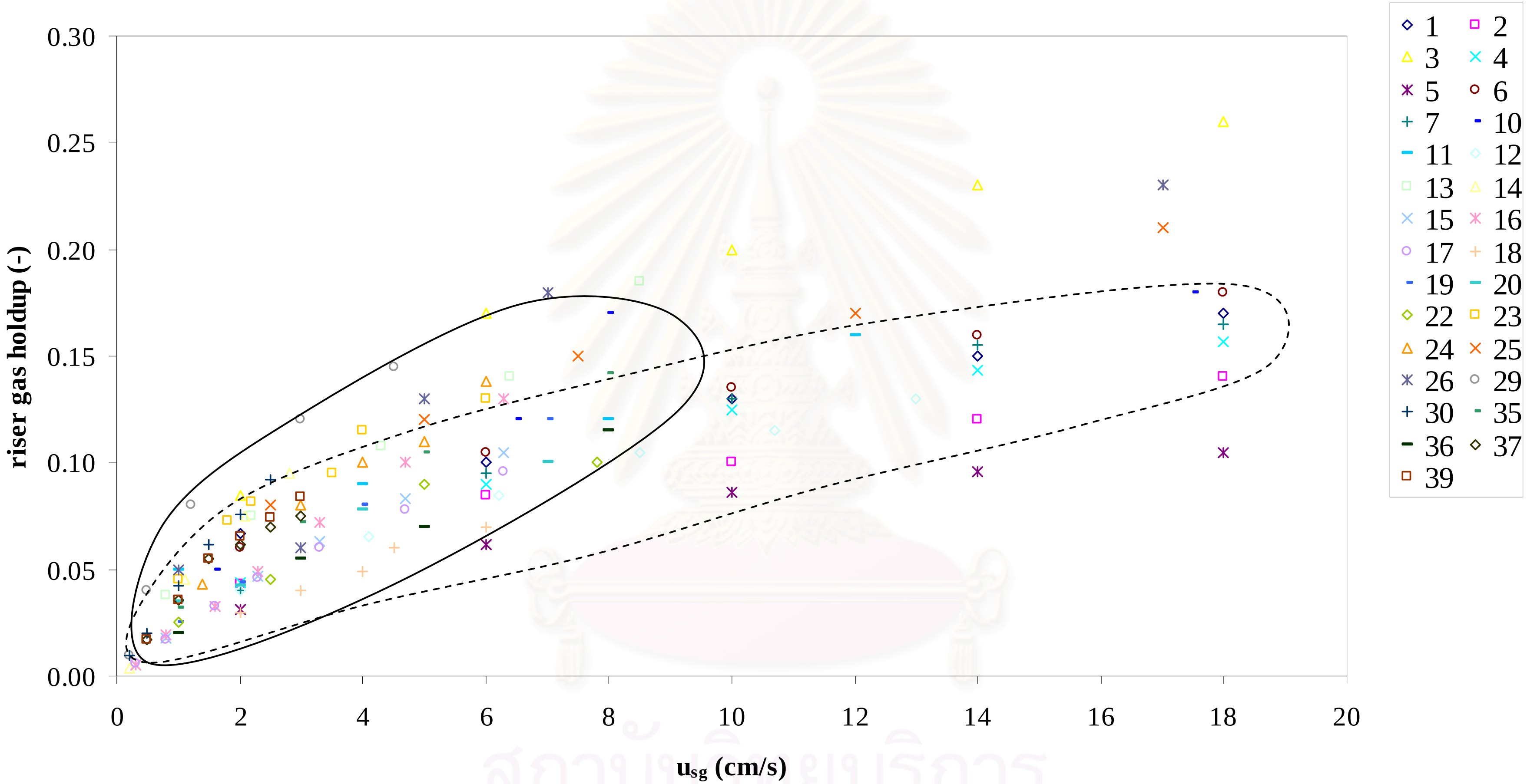

Figure 2.2 Riser gas holdup in airlift contactors (----, external loop; —–, internal loop). (Citation index as indicated in Table 2.1)

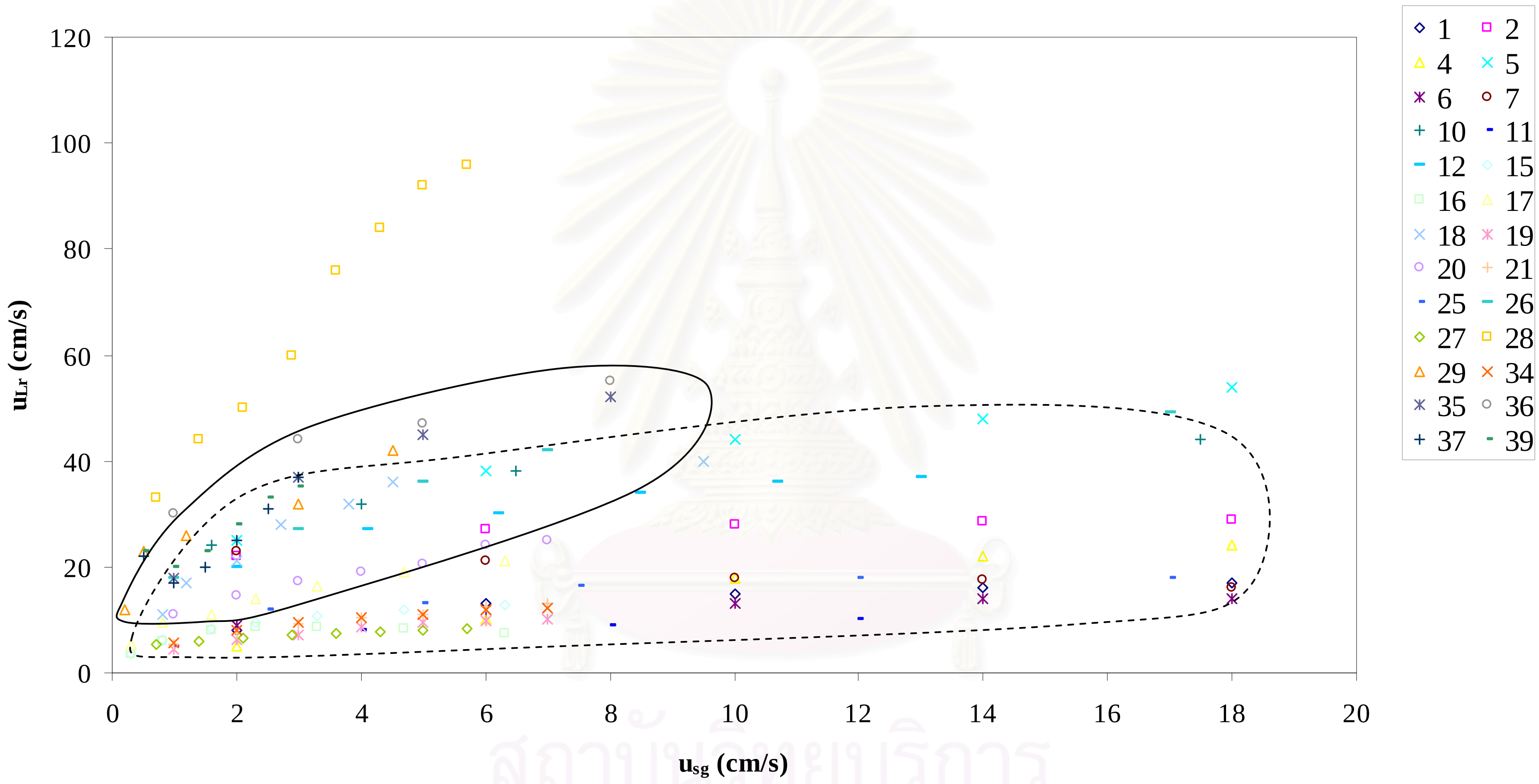

Figure 2.3 Riser liquid velocity in airlift contactors (----, external loop; ——, internal loop). (Citation index as indicated in Table 2.1)

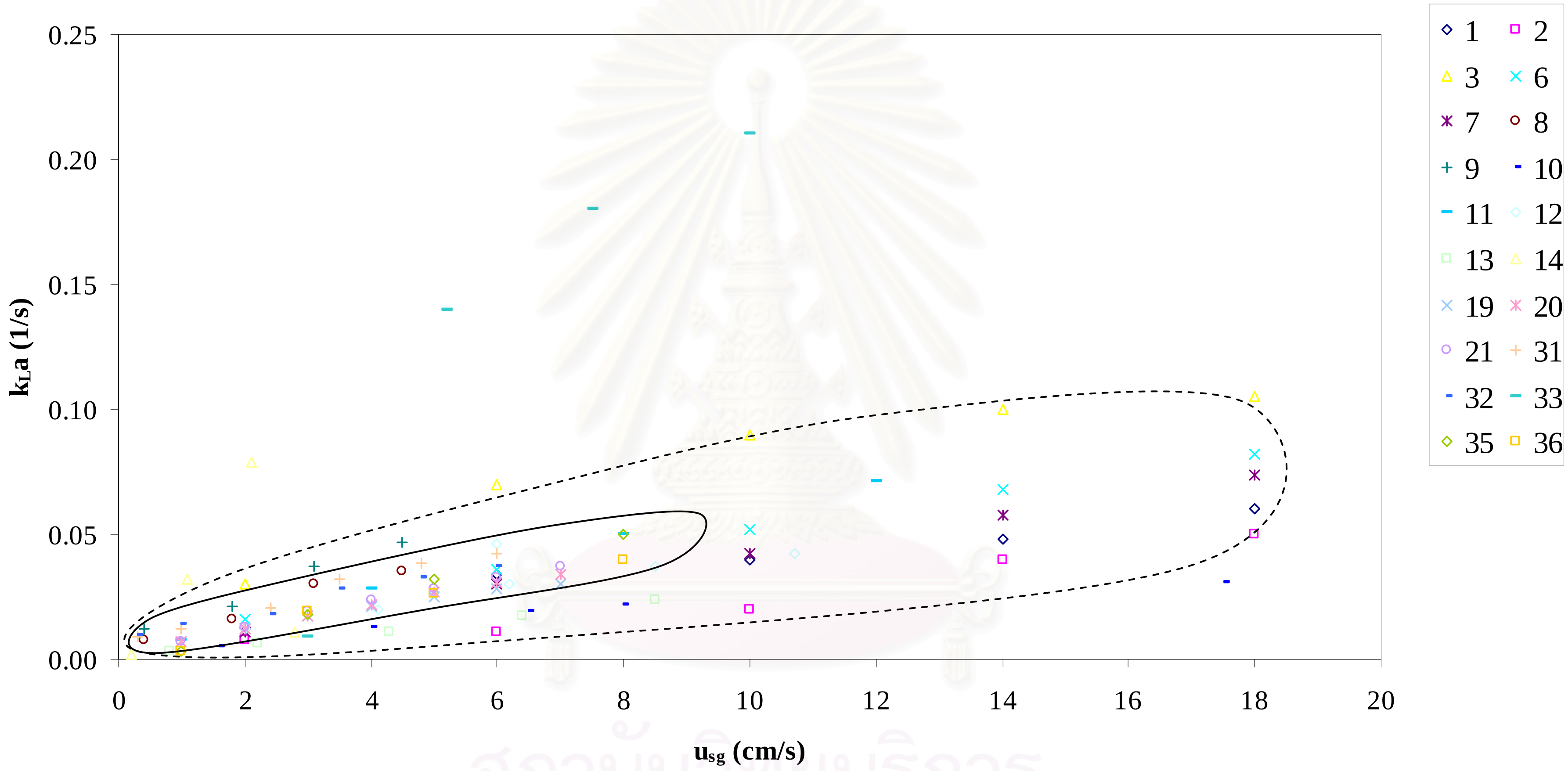

Figure 2.4 Overall volumetric mss transfer coefficient *(k_La)* in airlift contactors (----, external loop; ——, internal loop). (Citation index as indicated in Table 2.1)

CHAPTER III

MATERIALS & METHODS

3.1 Experimental setup

A schematic diagram of experimental system employed in this work is shown in Figure 3.1. The external loop airlift contactor (ELALC) consists of two vertical tubes both with a height (h_1) of 200 cm and inside diameters of riser and downcomer of 10.4 and 5.4 cm, respectively. The riser and the downcomer were connected by connection tubes at the top and bottom sections of column. These connection tubes have an inside diameter of 5.4 cm with a variable length between 10 to 40 cm (L_c). The riser, downcomer and connection tubes have the wall thickness of 3 mm and were designed to be assembled by several pieces to allow an alternation of configuration of external loop airlift contactor as shown in Figure 3.2. All parts of external loop airlift contactor are made from transparent acrylic plastic to allow visual observation and to record the movement of the color tracer for the liquid velocity measurement. The gas is dispersed through the porous sparger installed at the base of the riser where the gas flow rate, as measured in terms of superficial velocity, is regulated in the range from 0 to 10 cm/s by a calibrated rotameter. A dissolved oxygen (DO) meter (METTLER TOLEDO O_2 Transmitter 4500) is used to measure dissolved oxygen in the dispersion for the estimation of mass transfer rate. The experiment is performed as a gas-liquid system with water as liquid phase and air as gas phase. During the experiment, the medium is pumped continuously into the column until the liquid level is 3 cm above the top edge of the top connection tube (h_L) after which air is supplied into the system. Dimension of the various parts of the system is displayed in Table 3.1. The system is then left running for a certain period of time to ensure a steady state operation before taking further measurement.

3.2 Experimental procedures

3.2.1 Gas holdup measurement

The riser gas holdup is determined by the volume expansion method. The downcomer gas holdup is determined by the manometric method. The experimental steps are detailed as follows:

Procedure

1. Fill tap water into the column until the liquid level (h_5) is 3 cm above the top edge of the top connection tube.
2. Open valve to continuously disperse compressed air from an air compressor through the sparger to the column
3. Adjust superficial velocity (u_{sg}) to the desired value by using calibrated rotameter
4. Read the liquid dispersion height (h_D) to evaluate the riser gas holdup in the airlift contactor
5. Measure the pressure difference between the two positions (ΔP) in the downcomer section using the attached water manometer to evaluate the downcomer gas holdup. The calculation is then performed according to Equations 3.3 and 3.11.

3.2.2 Liquid velocity measurement

The liquid velocity is determined by a dye tracer method with detail as follows:

Procedure

1. Fill tap water into the column until the liquid level (h_5) reaches the level 3 cm above the top edge of the top connection tube
2. Open valve to continuously disperse compressed air from an air compressor through the sparger to the column
3. Inject dye tracer directly into the measuring port and meansure the traveling time of the dye between any two vertical positions. This is performed for both riser and downcomer.
4. Calculate riser and downcomer liquid velocities following Equations 3.12 and 3.13
5. Repeat Steps 1 to 4 with other geometric and/or operating parameters

3.2.3 Mass transfer coefficient measurement

The overall volumetric mass transfer coefficient (k_La) is determined by using the dynamic gassing method (Bailey and Ollis, 1986). A dissolved oxygen (DO) meter (METTLER TOLEDO O_2 Transmitter 4500) is used to measure and record the changes in dissolved oxygen concentration in a batch of water. The experimental steps are detailed as follows:

Procedure

1. Fill tap water into the concentric column until the liquid level (h_5) is 3 cm above the top edge of the top connection tube
2. Immerge the dissolved oxygen probe into the water in the column as shown in Figure 3.1 for measuring the dissolved oxygen concentration in the water by dissolved oxygen meter to ensure that all of the oxygen has been removed
3. Disperse nitrogen gas through the base of the contactor to the column to purge out dissolved oxygen from the water
4. Stop the nitrogen gas flow when the dissolved oxygen concentration reaches zero
5. Distribute compressed air from an air compressor continuously, at predetermined flow rate
6. Record the time profile of dissolved oxygen concentration until the water is saturated with oxygen
7. Calculate the overall volumetric mass transfer coefficient using Equation. 3.14
8. Repeat Steps 1 to 6 with other geometric and/or operating parameters

3.3 Calculations

3.3.1 Gas holdup

The riser and downcomer gas holdups are estimated by measuring the pressure difference between two measuring ports of the columns.

Firstly,
$$\Delta P_{\text{column}} = \Delta P_{\text{manometer}} \tag{3.1}$$

$$\rho g \Delta H = \rho_L g \Delta Z \tag{3.2}$$

$$(\rho_L \varepsilon_L + \rho_g \varepsilon_g) g \Delta H = \rho_L g \Delta Z \tag{3.3}$$

Neglecting the wall friction loss and $\rho_L >> \rho_G$, the gas holdup can be calculated from the following equations:

$$(\rho_L \varepsilon_L) g \Delta H = \rho_L g \Delta Z \tag{3.4}$$

$$\varepsilon_L = \frac{\rho_L g \Delta Z}{\rho_L g \Delta H} \tag{3.5}$$

since $$\varepsilon_L = 1 - \varepsilon_G \tag{3.6}$$

so $$1 - \varepsilon_G = \frac{\rho_L g \Delta Z}{\rho_L g \Delta H} \tag{3.7}$$

finally, $$\varepsilon_{G,i} = 1 - \frac{\Delta Z}{\Delta H_i} \tag{3.8}$$

where $i = r$ for riser, $i = d$ for downcomer

ΔP = pressure difference of defined liquid level in the column [g/cm.s^2]

ΔH = height of defined liquid level in the column [cm]

ΔZ = height of liquid level in the manometer [cm]

ρ_G = gas density [g/cm^3]

ρ_L = liquid density [g/cm^3]

g = gravitational acceleration [cm/s^2].

The overall gas holdup is calculated from riser and downcomer gas holdups as follows:

$$\varepsilon_o = \frac{A_r \varepsilon_r + A_d \varepsilon_d}{A_r + A_d} \tag{3.9}$$

3.3.2 Liquid velocity

The liquid velocities both in riser and downcomer are measured by the tracer injection method where the measured times which the color uses in traveling between any two fixed positions is used for the calculation:

$$v_r = \frac{L_r}{t_r} \tag{3.10}$$

$$v_d = \frac{L_d}{t_d} \tag{3.11}$$

where v = liquid velocity [cm/s]

L = distance between any two fixed position [cm]

t = time for any two fixed positions [s].

3.3.3 Overall volumetric mass transfer coefficient

The overall volumetric mass transfer coefficient (k_La) is determined by using the dynamic method. The time profile of dissolved oxygen concentration in the solution is measured and recorded until equilibrium concentration is reached. The k_La can then be calculated from the slope of this equation:

$$\ln\frac{(c^* - c_o)}{(c^* - c_L)} = k_L a t \tag{3.12}$$

where c^* = saturated oxygen concentration [mg/L]

c_o = initial oxygen concentration [mg/L]

c_L = oxygen concentration in liquid phases [mg/L]

k_La = overall volumetric mass transfer coefficient [1/s]

t = time profile [s].

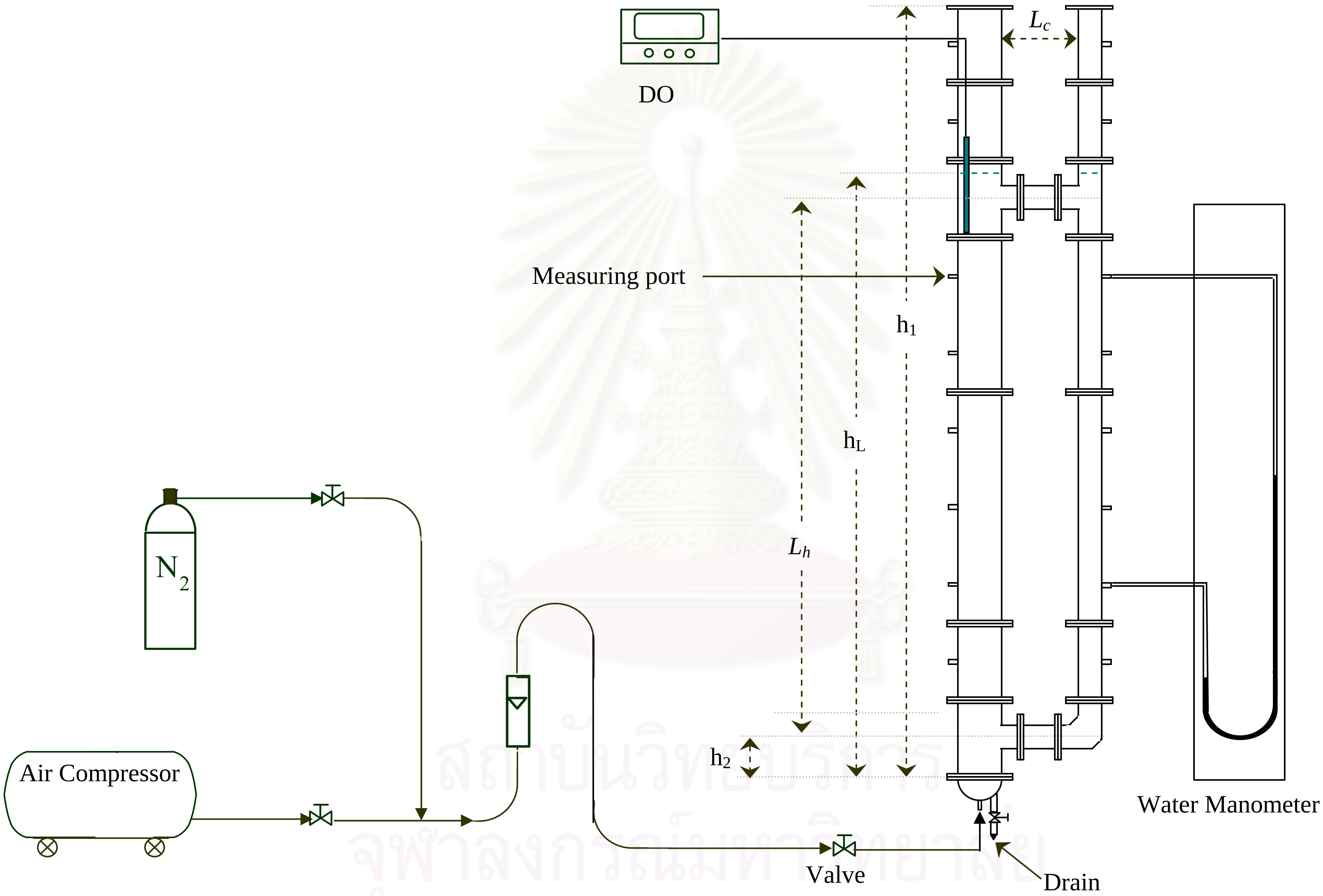

Figure 3.1 Schematic diagram of the external loop airlift contactor employed in this work

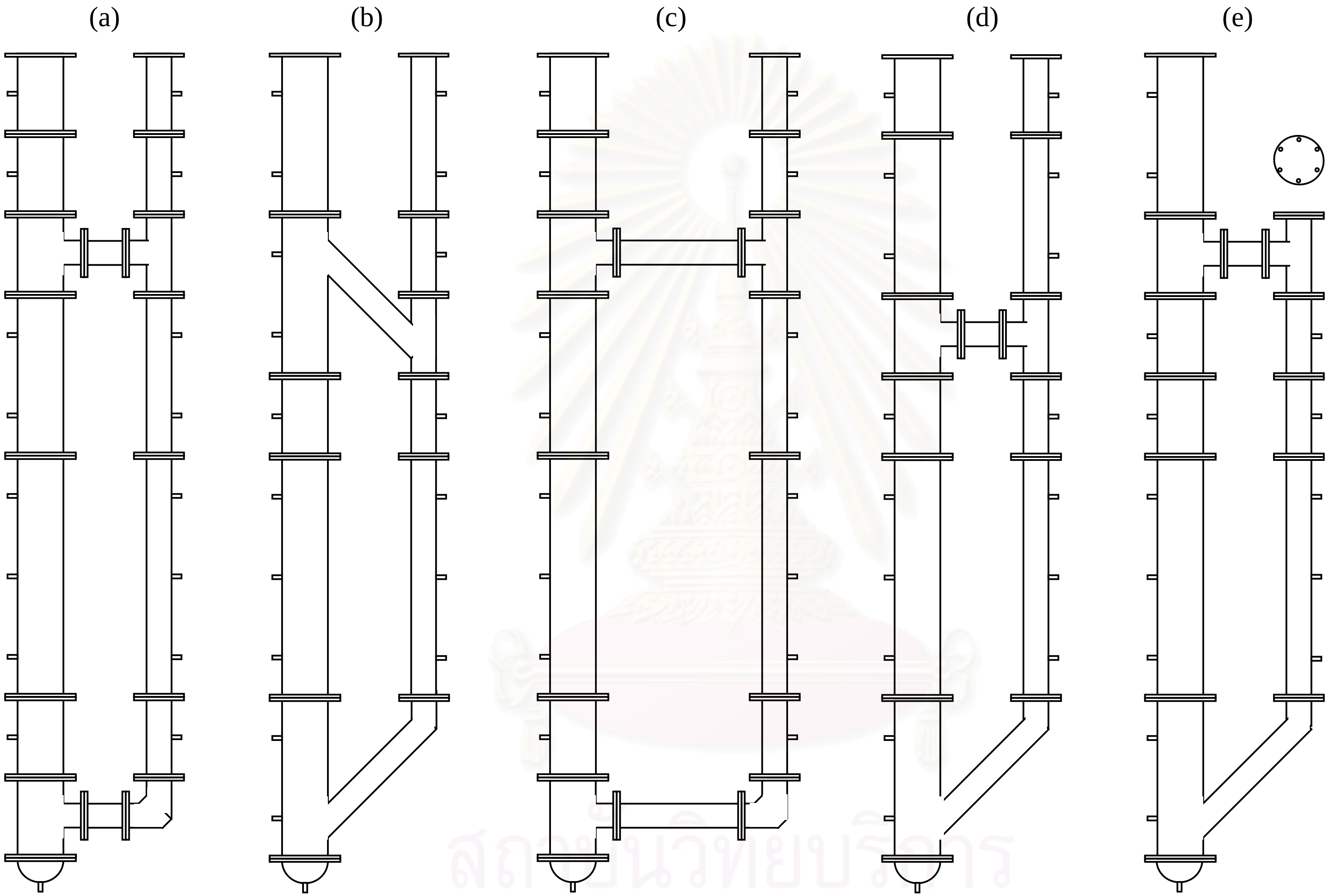

Figure 3.2 Various configurations of external loop airlift contactor (ELALC): (a) ELALC with a connection tubes length of 20 cm (b) ELALC with inclined the connection tubes (c) ELALC with the connection tubes length of 40 cm (d) ELALC with lower top connection tube (e) ELALC with closed downcomer gas liquid separator.

Table 3.1 Dimensions of ELALC (parameters as shown in Fig. 3.1)

Configurations	h_1 [cm]	L_c [cm]	L_h [cm]	h_2 [cm]	h_L [cm]
A	200	20	150	10	156
B	200	20	150	10	156
C	200	40	150	10	156
D	200	20	130	10	156
E	200	20	150	10	156

Remarks

h_1 = total height of ELALC [cm]

L_c = connection tubes length between riser and downcomer [cm]

L_h = height of riser and downcomer [cm]

h_2 = height of bottom connection tube [cm]

h_L = water level [cm]

CHAPTER IV

RESULTS & DISCUSSION

4.1 Gas holdups

4.1.1 Effect of length of connection tubes on gas holdups in ELALC

To investigate the effect of length of connection tubes (L_c), the experiment was operated in a regular ELALC with three different lengths of connection tubes. These were labeled as “1 L20 O” for L_c = 20 cm, “1 L30 O” for L_c = 30 cm and “1 L40 O” for L_c = 40 cm as indicated in Table 4.1. Figures 4.1 shows that overall, riser and downcomer gas holdups increased with u_{sg} for all L_c configurations. However, an increase in L_c seemed to reduce the gas holdups in all sections of ELALC. Figure 4.2 displays that an increase in the length of the horizontal connection tubes (as shown in Figure 4.2) provided more time for gas bubbles to move up the tube, coalesce to large bubbles and finally separate from the liquid phase at the gas-liquid separator, reducing the gas fraction resided in the liquid. Consequently, lesser gas holdup was observed.

4.1.2 Effect of height of riser and downcomer on gas holdups in ELALC

The height of riser and downcomer (L_h) of the ELALC is among one of the most important design parameters. In this experiment, two different configurations of ELALC were designed to investigate the effect of L_h on gas holdups. One configuration is the regular ELALC with L_h = 140 cm (1 L20 O), L_h = 120 cm (2 L20 O), L_h = 100 cm (3 L20 O). The other is ELALC with inclined connection tubes with L_h = 140 cm (1 S O), L_h = 120 cm (2 S O), L_h = 100 cm (3 S O). The results demonstrated that increasing L_h decreased riser gas holdups. In fact, an increase in L_h allowed a longer contact time between the gas bubbles and the liquid, promoting the liquid velocity through energy transfer. As gas bubbles moved at higher relative velocity (bubble velocity plus liquid velocity), they left the system more rapidly and reducing the gas holdup especially in riser. On the other hand, a faster liquid velocity carried enough inertia to bring bubbles of large sizes into downcomer. Therefore a larger downcomer gas holdup was observed with an increase in the column height.

This was true for both regular and inclined connection tube ELALC (see Figures 4.3 and 4.4).

It is interesting to note that, as the overall gas holdup is the sum of riser and downcomer gas holdups. The changes in L_h had an opposite effects on riser and downcomer gas holdups, i.e. an increase in L_h led to a lower riser gas holdup and a higher downcomer gas holdup. Therefore, the effect of L_h on the overall gas holdup was somewhat difficult to predict. Experiments as illustrated in Figures 4.3 and 4.4 suggested that the overall gas holdup remained approximately constant regardless of the liquid height.

A large number of experiments were carried out to statistically examine the relationship between the various system behaviors such as gas holdups and the designed and operating variables where the following empirical equations could be proposed for the estimate of gas holdups.

For the regular ELALC:

$$\varepsilon_r = 0.49 u_{SG}^{0.67} L_c^{-0.15} L_h^{-0.22} \tag{4.1}$$

$$\varepsilon_d = 0.29 u_{SG} \frac{L_h^{0.39}}{L_c^{0.45}} \tag{4.2}$$

For the inclined connection tube ELALC:

$$\varepsilon_r = 0.53 u_{SG}^{0.54} (L_h)^{-0.41} \tag{4.3}$$

$$\varepsilon_d = 0.93 u_{SG}^{0.88} (L_h)^{0.47} \tag{4.4}$$

4.1.3 Effect of designed configurations of ELALC on gas holdups

Comparison of the performance of ELALC with three different configurations was carried out. The three included the regular ELALC (1 L20 O), ELALC with inclined connection tubes (1 S O) and ELALC with close downcomer gas liquid separator (1 L20 C). It was found that ELALC with inclined connection tubes and ELALC with close downcomer gas liquid separator exhibited higher gas holdups than regular ELALC. Visual observation clearly revealed that there existed large chunks of gas bubbles in ELALCs (1 S O) and (1 L20 C). For the inclined connection tube ELALC (1 S O), this coalescence occurred at the entrance of the top connection tube, whereas for the ELALC (1 L20 C), this occurred at the end of top connection tube and this was extended through the gas-liquid separator section as shown in Figure 4.5 (b) and (a). These groups of gas bubbles were large enough to reduce the cross sectional area of fluid flows causing an interruption for the gas and liquid circulation in the airlift. This reduction in liquid velocity allowed more time for the gas bubbles to stay in the system, and hence, higher gas holdups were observed than that in regular ELALC (Figure 4.6).

Figure 4.6 also illustrates that the closed downcomer ELALC had the highest quantity of bubbles in the riser whilst the inclined tube ELALC had its highest in the downcomer. In the next section, it was shown that the liquid velocity in the closed downcomer ELALC would be the lowest. Therefore it allowed gas bubbles in the riser to stay longer in the system (Figure 4.5 (b)). However, as the liquid velocity was low, this system had low inertia for dragging the bubbles to downcomer, and hence, lesser downcomer gas holdup than that of inclined connection tube ELALC was observed in the closed downcomer ELALC.

Similar to the discussion in the previous section, the overall gas holdup depended on the trade-off between riser and downcomer gas holdups. With the systems examined in the work, the overall gas holdups for the inclined connection tubes and the closed downcomer ELALCs took almost the same value, whereas the overall gas holdup of the regular ELALC was much lower (as both riser and downcomer gas holdups in regular ELALC were much lower than the other two systems).

The relationship between gas holdups and superficial velocity in the systems evaluated in this section could be formulated with a proper design of experiments and statistics consideration. This was summarized as follows:

For close downcomer gas liquid separator ELALC:

$$\varepsilon_r = 0.7u_{SG}^{0.66} \qquad (4.5)$$

$$\varepsilon_d = 0.39u_{SG}^{0.53} \qquad (4.6)$$

4.2 Liquid velocity

4.2.1 Effect of length of connection tubes on liquid velocity

Liquid velocity was induced due to two reasons. Firstly, the gas dispersed through airlift system carried with it kinetic energy which was transferred to liquid as the bubbles moved along the length of the riser. The second was the hydrostatic pressure due to the difference in the apparent densities of the fluids in the riser and downcomer.

The measurement of liquid velocity was performed both in riser and downcomer. The results indicated that these two liquid velocities were not corresponded well when they were compared according to the mass balance (Eqs. 4.7 and 4.8). Note that this experiment was conducted with the area of riser larger than that of downcomer (A_d/A_r = 0.262). It could therefore be that there existed internal liquid circulation in riser as suggested by Wongsuchoto et al. (2004). In other words, the liquid in the riser could also move downward by turbulence and the measurement of liquid velocity in the riser was only for the fraction that moved upwards causing the conservation equation to be unbalanced. Therefore in this study, only the downcomer liquid velocity was used to describe the effect of various configuration of ELALC and the average liquid velocity in the riser is estimated from:

$$Q_{Lr} = Q_{Ld} \tag{4.7}$$

$$v_{Lr}(1-\varepsilon_r)A_r = v_{Ld}(1-\varepsilon_d)A_d \tag{4.8}$$

The lengths of connection tubes are important in defining the liquid velocity. Figure 4.7 illustrates the effect of length connection tubes (L_c between 20-40 cm) on downcomer liquid velocities. Large L_c meant that the length of connection tubes was high. In this case, gas and liquid in the top connection tube had more time to separate themselves causing lesser gas holdup in the downcomer. Note that liquid in the downcomer would have to flow through the gas bubbles and losing its kinetic energy through the contact with the moving up bubbles. Therefore lesser gas fraction in the downcomer section could rightly allow liquid velocity to move at higher speed.

In addition, a reducing downcomer gas holdup with an increase in L_c exhibited a larger different hydrostatic density between riser and downcomer. This would, to some extent, cause liquid to move at a higher speed.

It is interesting to examine Figure 4.7 in more detail. For all cases, when increased superficial gas velocity (u_{sg}), the liquid velocity increased but tended to reach a constant value at u_{sg} equal to 5 cm/s, after which the liquid velocity remained constant. This could be due to the balance between the momentum transfer between bubbles and liquid both in riser and downcomer. An increase in u_{sg} added more kinetic energy into the system resulting in an increase in liquid velocity. However, at this condition, more bubbles also were dragged into the downcomer. These bubbles in the downcomer counteracted the movement of the liquid as these bubbles tried to move upwards, lessening the kinetic energy carried along with the liquid. It could be that, a balance between the positive and negative effects acting on liquid occurred at u_{sg} of around 5 cm/s.

4.2.2 Effect of height of riser and downcomer on liquid velocity

To investigate the effect of height of riser and downcomer (L_h) on liquid velocity, this experiment was operated with two types of ELALC: (i) regular ELALC (1 L20 O, 2 L20 O and 3 L20 O) and (ii) inclined connection tube ELALC (1 S O, 2 S O and 3 S O). The airlift was operated with the 20 cm long connection tube (L_c) whereas the height of riser and downcomer were varied from 100-140 cm. The effect of L_h is illustrated in Figure 4.8 for regular ELALC and Figure 4.9 for ELALC with inclined connection tubes. The results showed that, large L_h induced more liquid velocity. This was because a large L_h or longer riser section allow a longer contact time between gas and liquid allowing more energy/momentum transfer, and a higher liquid velocity was yielded. It should be mentioned that, in fact, an increase in L_h, despite inducing higher liquid velocity, also allowed more bubbles to be dragged down into the downcomer. This reduced the hydrostatic pressure difference between the riser and downcomer which then retarded the movement of the liquid. It was obvious from the experiment data that the momentum transfer factor was stronger than the hydrostatic pressure difference in dictating the liquid velocity, in this case.

Similar to the previous case, the liquid velocity, at certain point, did not further increase with an increase in superficial gas velocity (u_{sg}). In regular ELALC, this threshold u_{sg} was found to be 5 cm/s whereas in ELALC with inclined connection tubes, this occurred at u_{sg} greater than 2 cm/s.

The following empirical equations in estimating downcomer liquid velocity were obtained from the experimental data in this section.

For regular ELALC:

$$v_{Ld} = 129 u_{SG}^{0.197} L_c^{0.147} L_h^{0.177} \tag{4.9}$$

For ELALC with inclined connection tubes:

$$v_{Ld} = \frac{312 u_{SG}^{0.05}}{7.6 + u_{SG}^{2.16}} L_h^{0.68} \tag{4.10}$$

4.2.3 Effect of various configurations of ELALC on liquid velocity

The difference in configurations of ELALC had been proven to have marked influence on the behavior on the ELALC. Figure 4.10 demonstrates the effect of configuration on downcomer liquid velocity. In this figure, the notation "1 L20 O" represents the regular ELALC, "1 S O" for the ELALC with inclined connection tubes and "1 L20 C" for the ELALC with closed downcomer gas liquid separator. The regular ELALC gave the highest downcomer liquid velocity as this configuration did not have bubble coalescence which would, otherwise, obstruct the liquid flow. On the other hand, ELALC with closed downcomer gas liquid separator allowed the formation of large coalescence gas at the top connection tube, blocking the liquid flow, and therefore liquid moved significantly more slowly than other configurations. The inclined connection tube ELALC also had bubble coalescence forming at the connection tube, hindering the movement of liquid, but not as much as that for the closed downcomer case.

Empirical equations for the prediction of liquid velocity are given above in Figures 4.10, the following correlation was for the closed downcomer ELALC:

$$v_{Ld} = \frac{263 u_{SG}^{0.045}}{7.1 + u_{SG}^{2.66}} \tag{4.11}$$

4.3 Gas-liquid mass transfer

4.3.1 Effect of length of connection tube on overall volumetric gas-liquid mass transfer

The effect of the length of connection tubes (L_c) on the overall volumetric gas-liquid mass transfer (k_La) is illustrated in Figure 4.11. The result showed that k_La decreased with an increase in L_c. This was because, for the case with long connection tubes (large L_c), gas was better separated from the liquid particularly at the top connection tube, leading to a lower gas fraction in the system. The amount of gas in the system was the major parameter for the determination of specific interfacial area "a". according to:

$$a = \frac{6\varepsilon}{d_b(1-\varepsilon)} \tag{4.13}$$

where d_b is the Sauter mean diameter and ε the gas holdup. The evaluation of "k_L" could then be made from the quantities of a and k_La as follows:

$$k_L = \frac{k_L a}{a} \tag{4.12}$$

As the bubble size (d_b) in such airlift was difficult to measure, the determination of bubble size was indirectly determined from the information on bubble velocity. It was assumed here that the bubbles freely floated in the fluid at their terminal velocities (u_t) where
terminal velocity of u_t could be calculated from:

$$u_t = \sqrt{\frac{4g(\rho_p - \rho)d_b}{3C_D\rho}} \tag{4.14}$$

Eq. 4.14 can be used to determine the bubble size according to:

$$d_b = \frac{3u_t^2 C_D \rho}{4g(\rho_p - \rho)} \tag{4.15}$$

where C_D is the drag coefficient, ρ and ρ_p the density of fluid and density of bubble, respectively, and g the gravitational acceleration.

It was further assumed that the bubble size was homogeneous in riser and in downcomer and each of the terminal velocities must be determined. The bubble terminal velocity in riser (u_{tr}) could be determined from Eq 4.16. And bubble terminal velocity in downcomer (u_{td}) was assumed to equal downcomer liquid velocity (Eq 4.17).

$$u_{tr} = \frac{Q_{g,in}}{A_r \varepsilon_r} - v_{lr,balance} \tag{4.16}$$

$$u_{td} = v_{Ld} \tag{4.17}$$

where $Q_{g,in}$ is the gas flow rate and $v_{lr,balance}$ the actual liquid velocity obtained from Eq 4.8.

The results on the bubble sizes in riser (d_{br}) and downcomer (d_{bd}) are given in Table 4.2. These values were employed to calculate k_L as shown in Figure 4.12. This figure illustrates that k_L increased with increasing u_{sg} because this increase in u_{sg} would enhance the bubble velocity in riser and downcomer. As a consequence, an increase in the relative velocity or slip velocity between bubbles and liquid was resulted. From the two film theory, an increase in slip velocity induced more shear force, which reduced the film thickness at the interface between gas and liquid. Hence, a higher mass transfer coefficient (k_G and k_L) was obtained, and this was experimentally proven as illustrated in Figure 4.13.

4.3.2 Effect of contactor height on overall volumetric gas-liquid mass transfer

To investigate the effect of contactor height (L_h), the experiment was conducted in the two airlift contactors, i.e. regular ELALC and ELALC with inclined connection tubes. Similar results were obtained from both configurations where an increase in L_h decreased k_La as illustrated in Figures 4.14 and 4.15. This was because an increase in L_h caused a decrease in the riser gas holdup but an increase in the downcomer gas holdup. However, in the airlift employed in this work, the volume of riser was much larger than that of downcomer (area of riser was about 4 times larger than that of downcomer). Therefore, the changes in k_La seemed to follow the variation in the riser gas holdup more closely than the downcomer.

To prove the conclusion as stated above, Eqs 4.13 – 4.17 were employed to calculate bubble sizes in riser (d_{br}) and downcomer (d_{bd}). These were then further used to calculate specific interfacial area in riser (a_r) and downcomer (a_d). The results from this calculation are shown in Table 4.3 where a_r was found to be 8 times more than a_d in regular ELALC and about 4 times in inclined connection tube ELALC. This implied that the value of a_r was more dominant for the estimate of a (in k_La) than the a_d. The value of k_L could then be estimated by dividing k_La with a and this demonstrated that an increase in L_h also increased the value of k_L.

From the results as shown in Figures 4.11, 4.14 and 4.15, the following empirical equations could be formulated for the estimation of k_La.

For regular ELALC:

$$k_l a = 0.149 u_{SG}^{0.45} L_c^{-0.17} L_h^{-0.30} \qquad (4.18)$$

For ELALC with inclined connection tubes:

$$k_l a = 0.23 u_{SG}^{0.48} \left(L_h\right)^{-0.51} \qquad (4.19)$$

4.3.3 Effect of various configurations of ELALC on overall volumetric gas-liquid mass transfer

The three configurations of ELALC were investigated for the effect of configurations of ELALC on k_La. Figure 4.16 demonstrates that, ELALC with inclined connection tube (1 S O) and ELALC with closed downcomer gas liquid separator (1 L20 C) exhibited higher k_La than the regular ELALC (1 L20 O). This could be because the two configurations induced higher bubble coalescence gas which obstructed fluid flow and consequently reduced liquid velocities. Gas in the systems therefore stayed longer in the riser and downcomer, providing a larger mass transfer area for the gas bubbles. Consequently, more k_La was observed.

The following empirical equation was proposed for estimating k_La in the ELALC with closed downcomer:

$$k_l a = 0.21 u_{SG}^{0.49} \qquad (4.20)$$

4.4 Accuracy of empirical models for the estimate of various parameters in ELALC

The accuracy of all empirical models proposed in this work was verified with experimental data. This was illustrated in Figures 4.17 to 4.20. It could be seen that the model could predict experimental data within the deviation range of $\pm$10 to $\pm$20%.

Table 4.1 Dimensions of ELALC (parameters as shown in Fig. 3.1)

Configurations	L_c [cm]	L_h [cm]	Connection tubes	Downcomer gas liquid separator
1 L20 O	20	140	horizontal	Open
1 L30 O	30	140	horizontal	Open
1 L40 O	40	140	horizontal	Open
2 L20 O	20	120	horizontal	Open
3 L20 O	20	100	horizontal	Open
1 S O	20	140	inclined	Open
2 S O	20	120	inclined	Open
3 S O	20	100	inclined	Open
1 L20 C	20	140	horizontal	Close

Table 4.2 Estimates of a and k_L in ELALC operating at various superficial gas velocities (u_{sg})

u_{sg} (cm/s)	k_La (1/s)	u_{tr} (cm/s)	u_{td} (cm/s)	d_{br} (cm)	d_{bd} (cm)	a (cm^2/cm^3)	k_L (cm/s)
0.5	0.017	17.7	35.4	0.11	0.42	0.89	0.019
1.0	0.022	29.1	44.4	0.28	0.66	0.43	0.052
2.0	0.032	31.6	51.4	0.34	0.89	0.64	0.051
5.0	0.047	43.4	62.1	0.64	1.30	0.65	0.073
10.0	0.060	70.1	61.0	1.68	1.24	0.42	0.145

Table 4.3 Estimates of a_r and a_d in two ELALC with different configurations, all at u_{sg} = 5 cm/s.

configuration	L_h (cm)	k_La (1/s)	d_{br} (cm)	d_{bd} (cm)	a_r (cm^2/cm^3)	a_d (cm^2/cm^3)	k_L (cm/s)
regular ELALC	140	0.047	0.64	1.30	0.85	0.18	0.073
	120	0.050	0.54	1.16	1.10	0.16	0.060
	100	0.052	0.49	1.16	1.26	0.14	0.053
inclined connection tubes ELALC	140	0.053	0.40	0.59	1.85	0.53	0.044
	120	0.057	0.42	0.54	1.73	0.41	0.043
	100	0.059	0.37	0.42	2.15	0.33	0.039

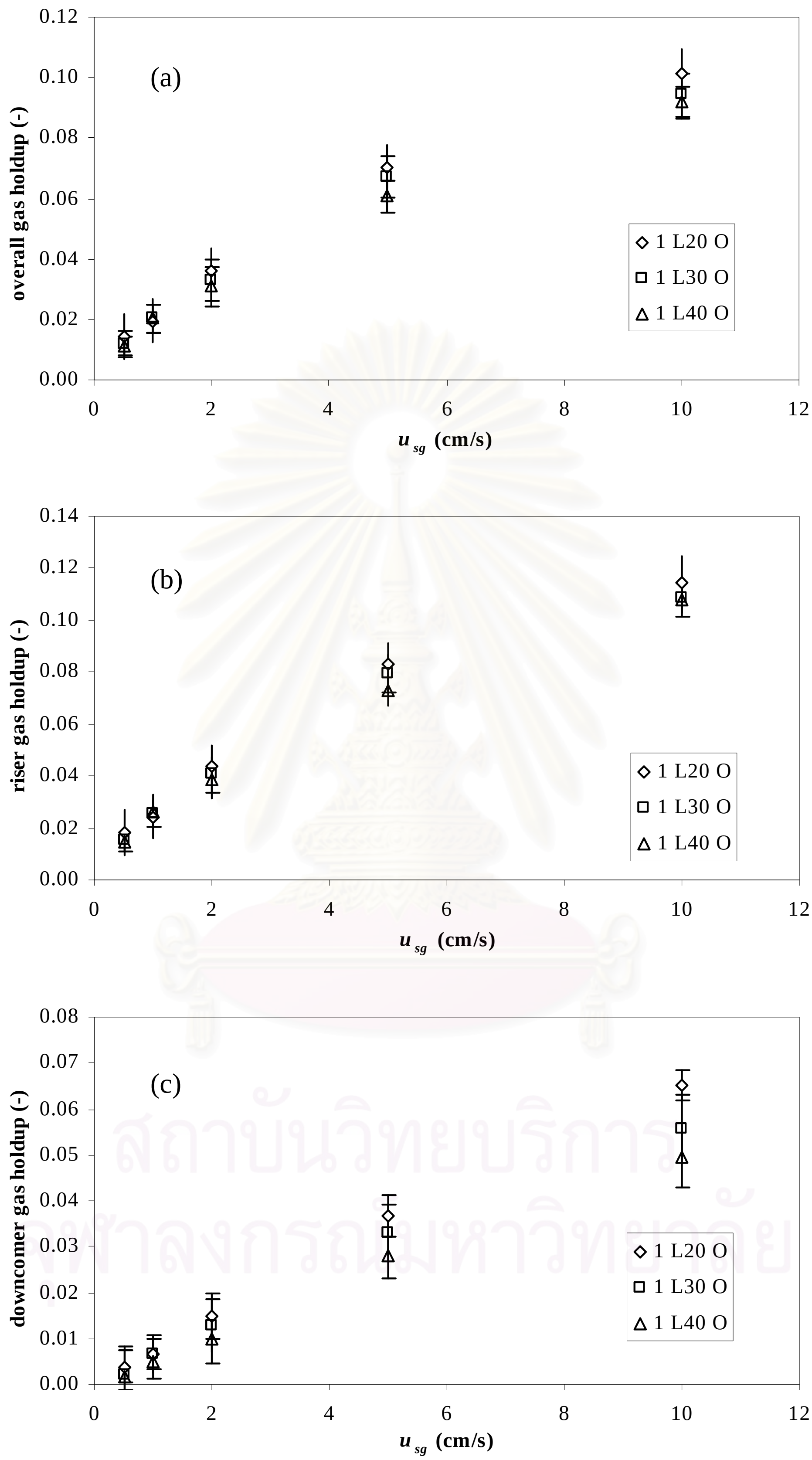

Figure 4.1 Effect of L_c on gas holdups in regular ELALC: (a) overall (b) riser (c) downcomer

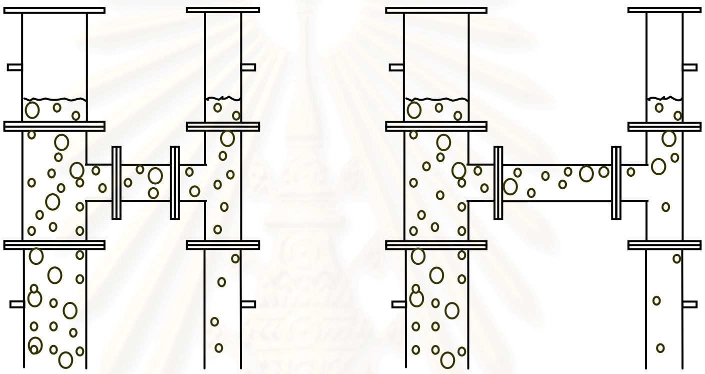

Figure 4.2 Schematic of regular ELALC with different L_c: (a) L_c = 20 cm (b) L_c = 40 cm

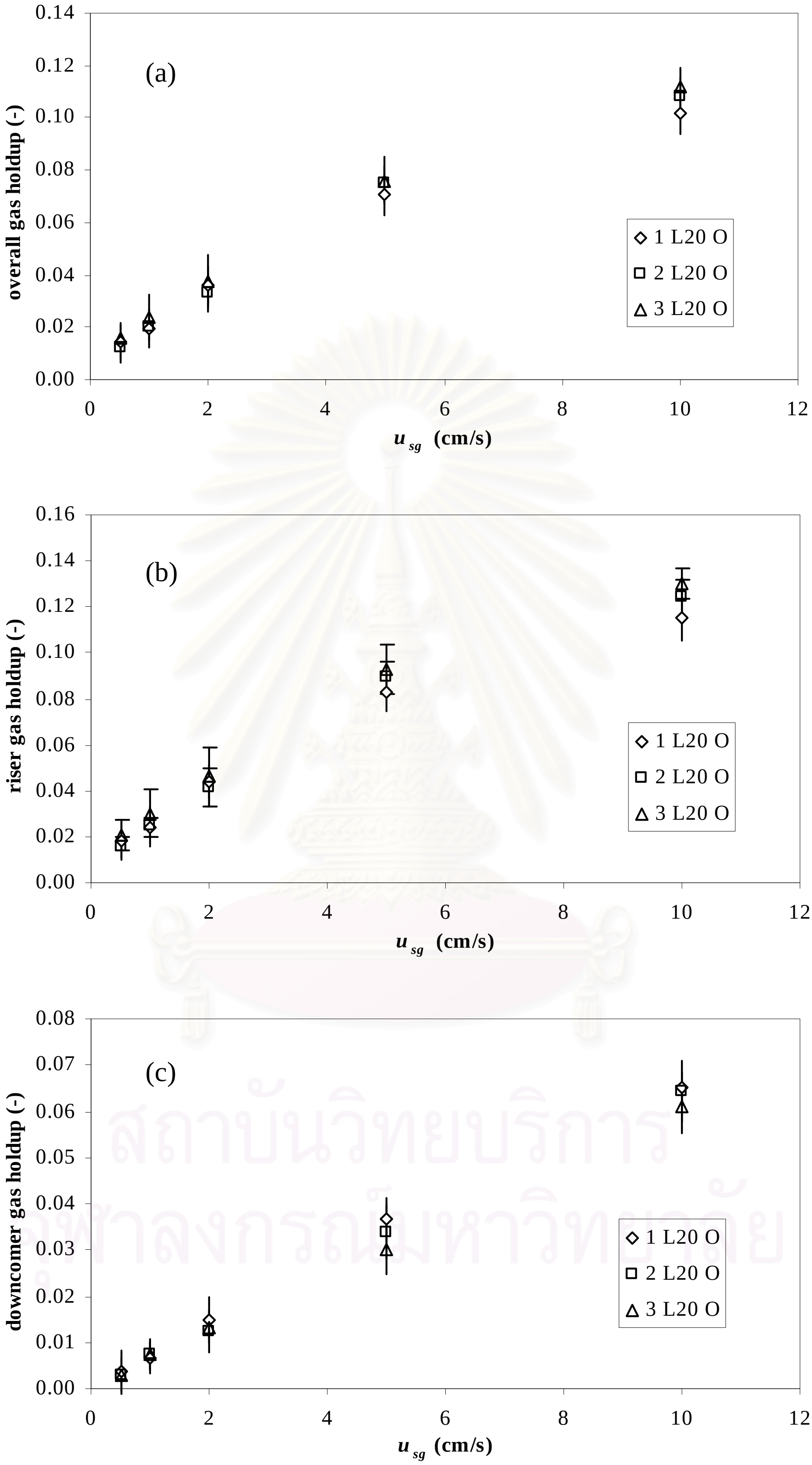

Figure 4.3 Effect of L_h on gas holdups in regular ELALC (a) overall (b) riser (c) downcomer

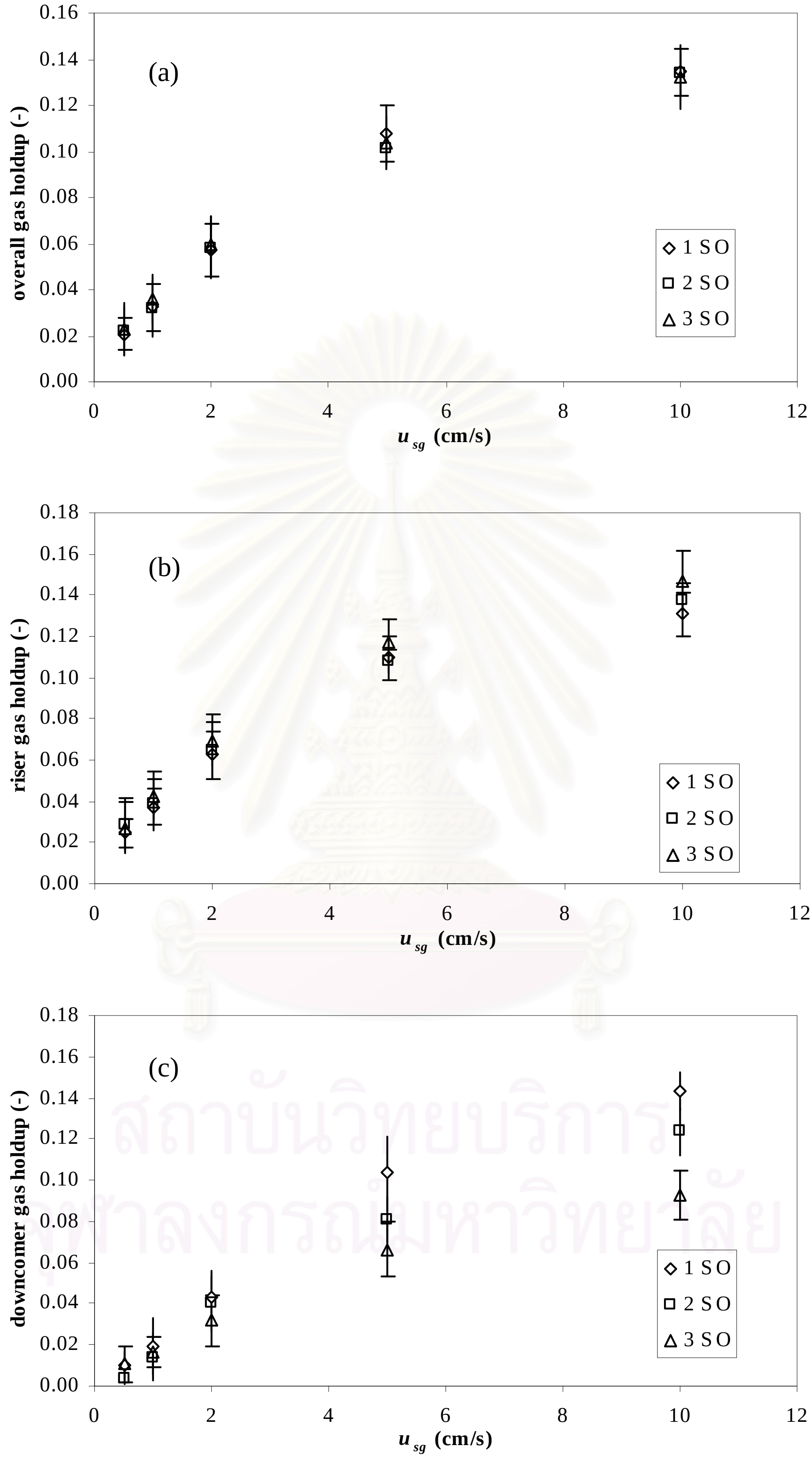

Figure 4.4 Effect of L_h on gas holdups in inclined connection tube ELALCs: (a) overall (b) riser (c) downcomer

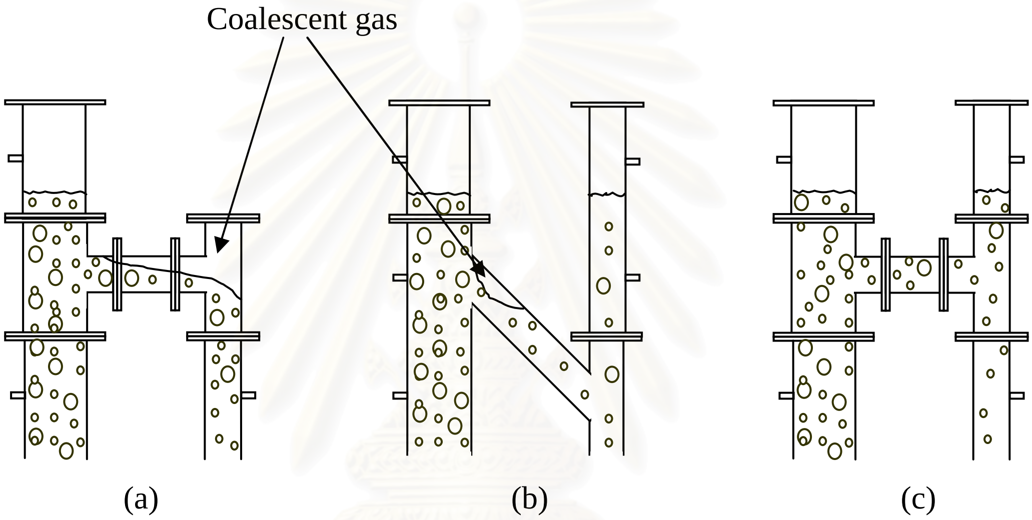

Figure 4.5 Illustration of coalescence of gas bubbles in ELALC with (a) close downcomer gas-liquid separator, (b) inclined connection tubes and (c) regular ELALC with no bubble coalescence

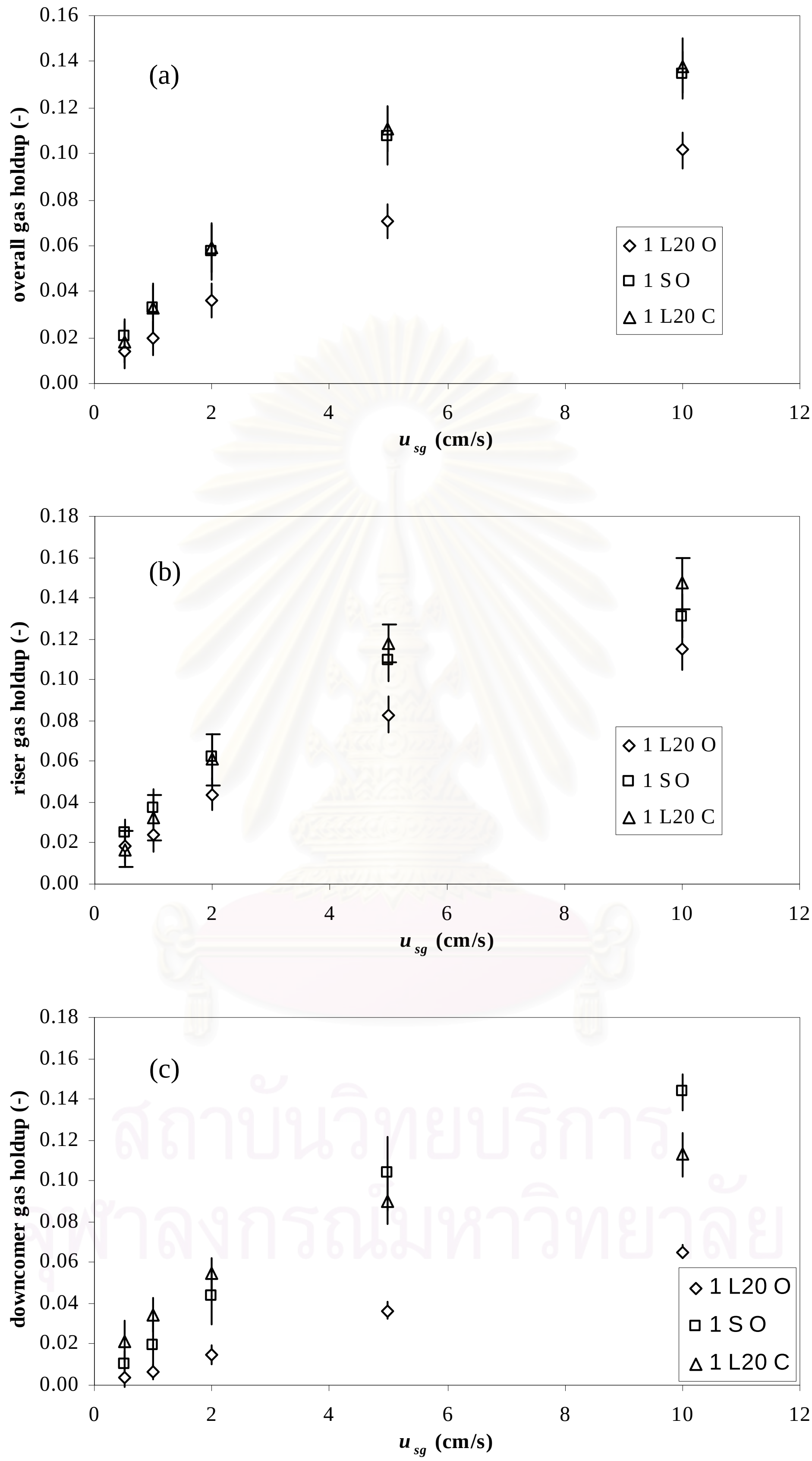

Figure 4.6 Effect of contactor configuration on gas holdups: (a) overall (b) riser (c) downcomer

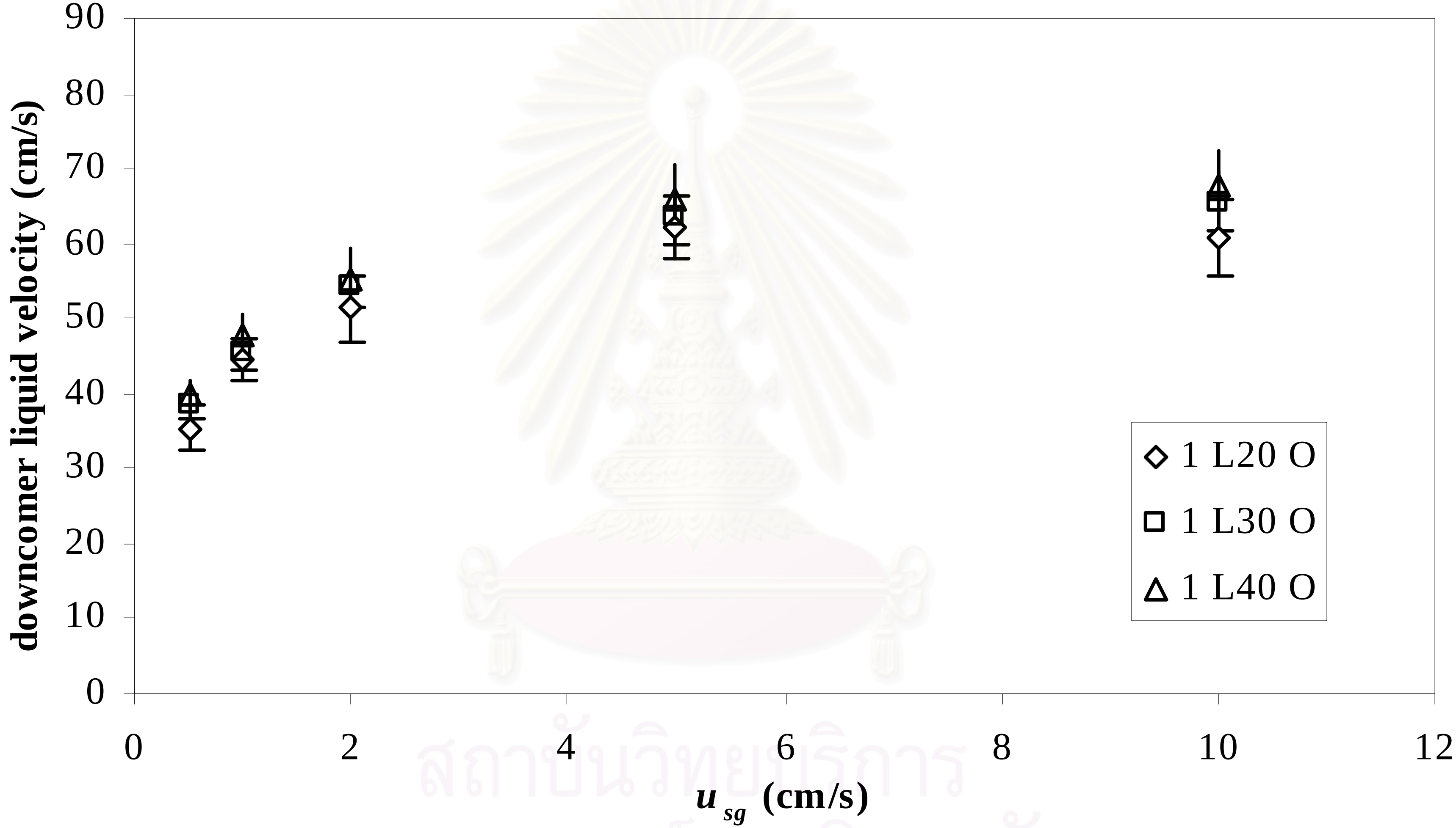

Figure 4.7 Effect of L_c on downcomer liquid velocity

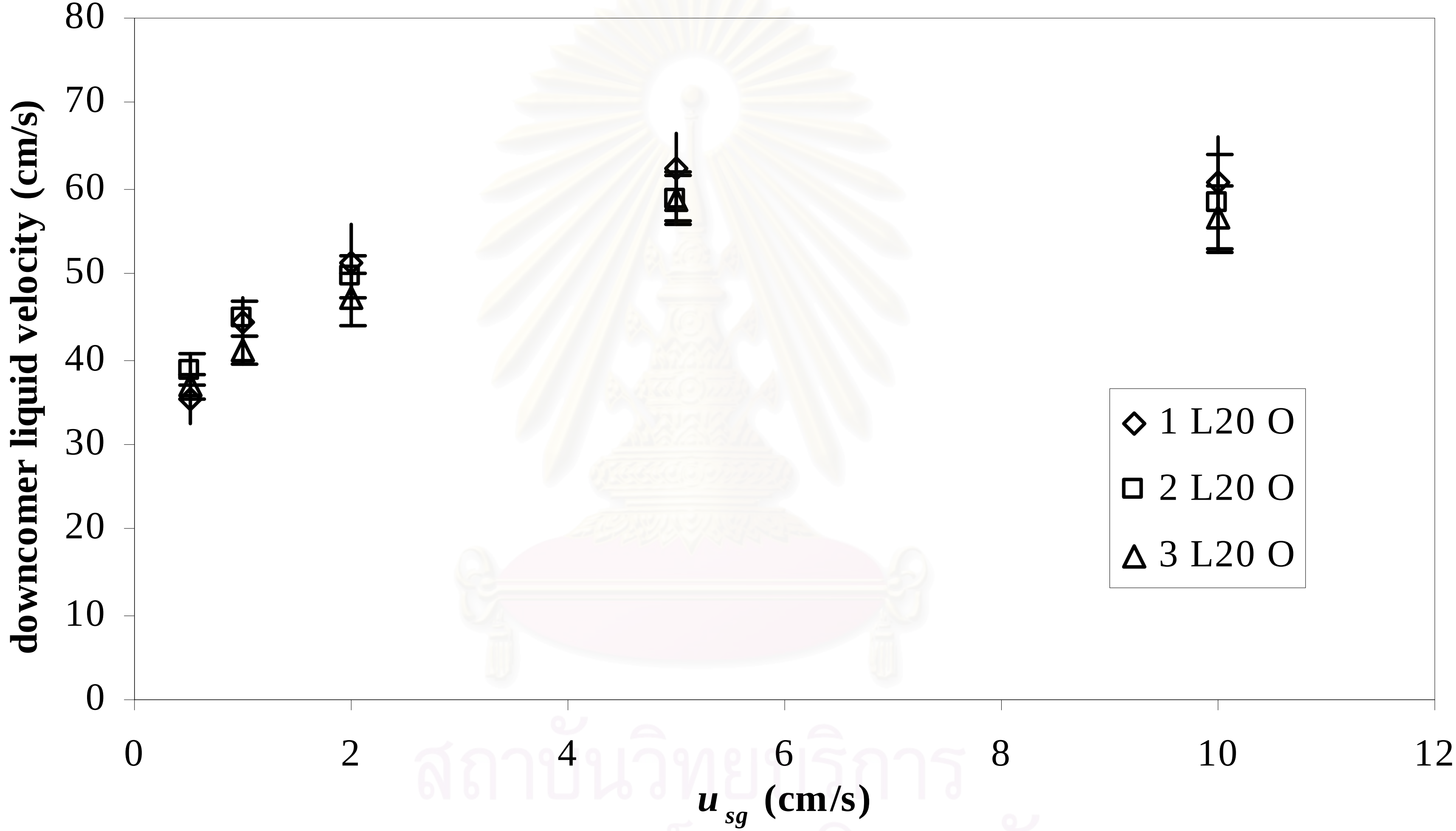

Figure 4.8 Effect of L_h on downcomer liquid velocity in regular ELALC

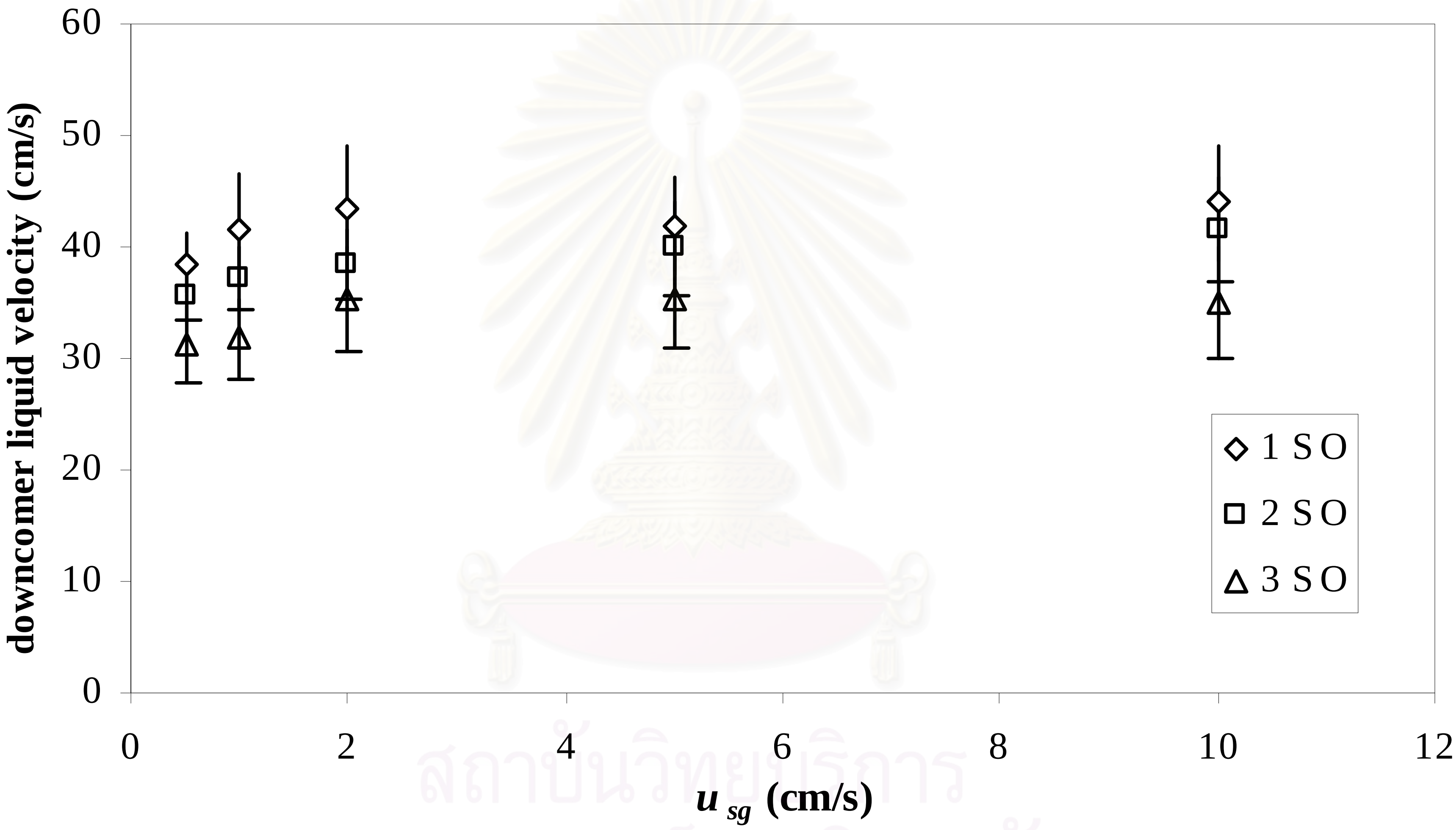

Figure 4.9 Effect of L_h on downcomer liquid velocity in ELALC with inclined connection tubes

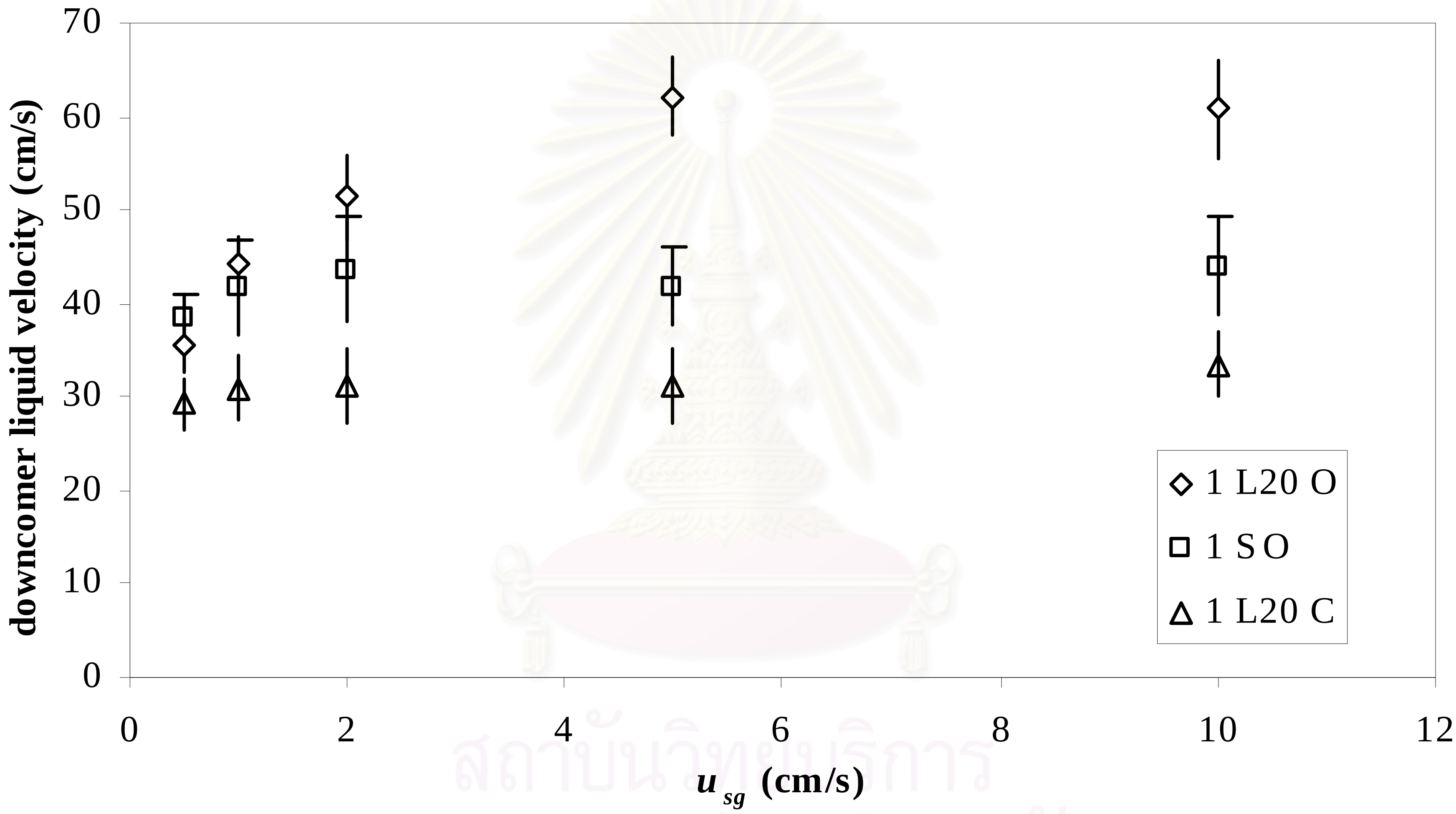

Figure 4.10 Effect of designed configurations on downcomer liquid velocity

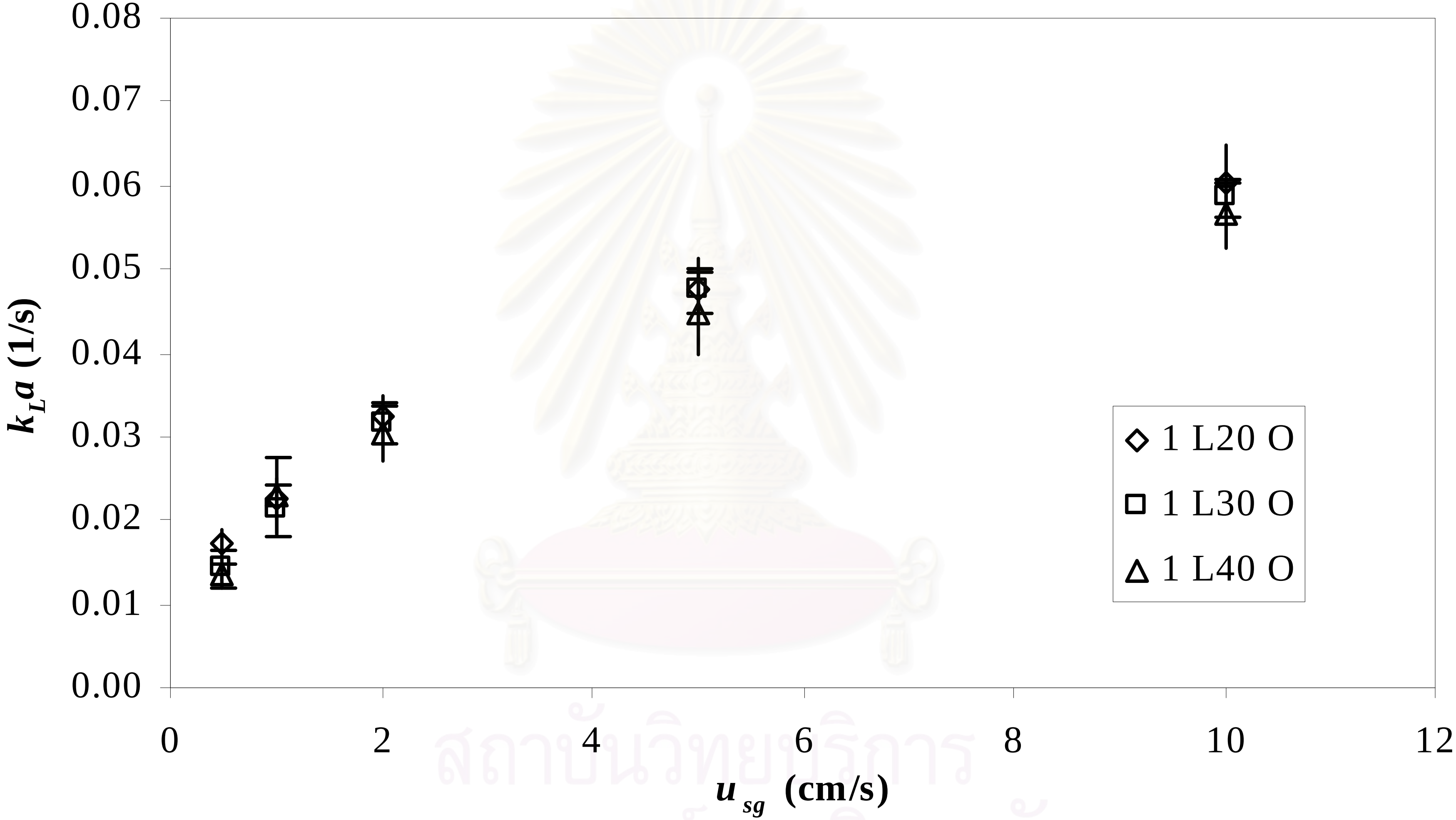

Figure 4.11 Effect of L_c on k_La

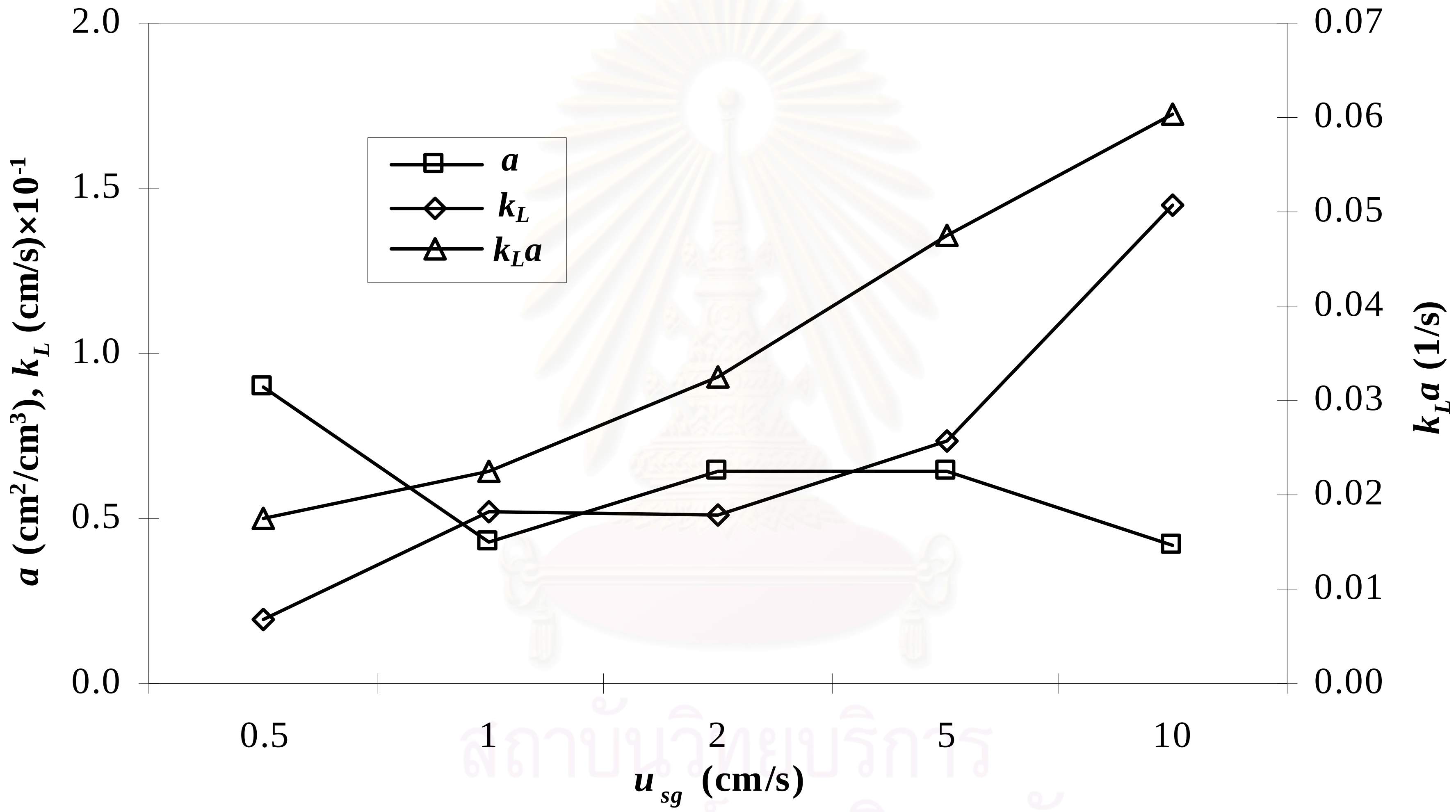

Figure 4.12 Effect of u_{sg} on a, k_L and k_La in ELALC with 1 L20 O configuration

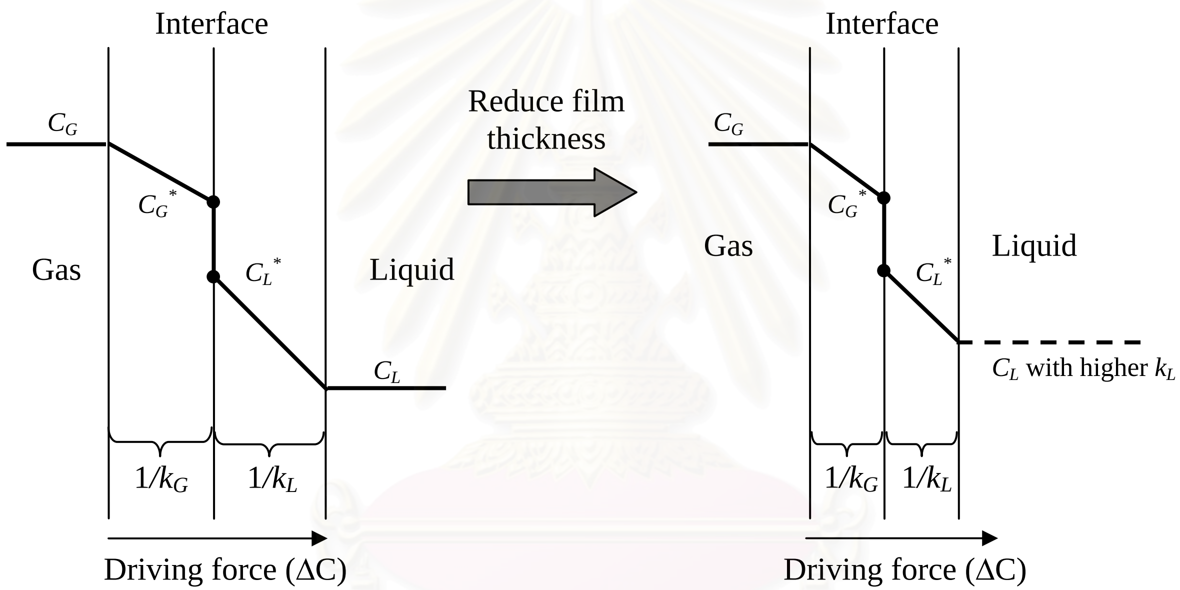

Figure 4.13 The two film theory

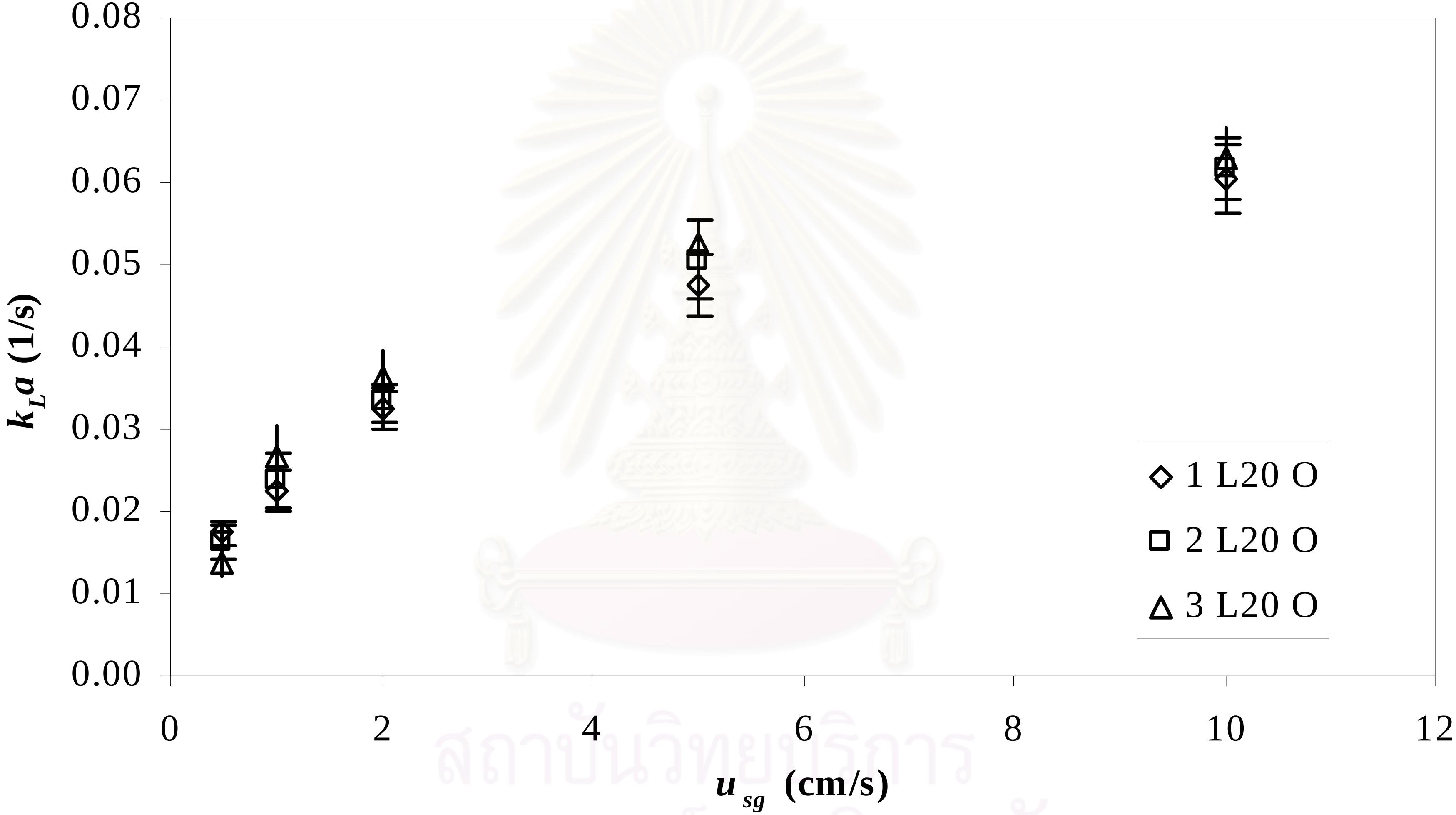

Figure 4.14 Effect of L_h on k_La in regular ELALC

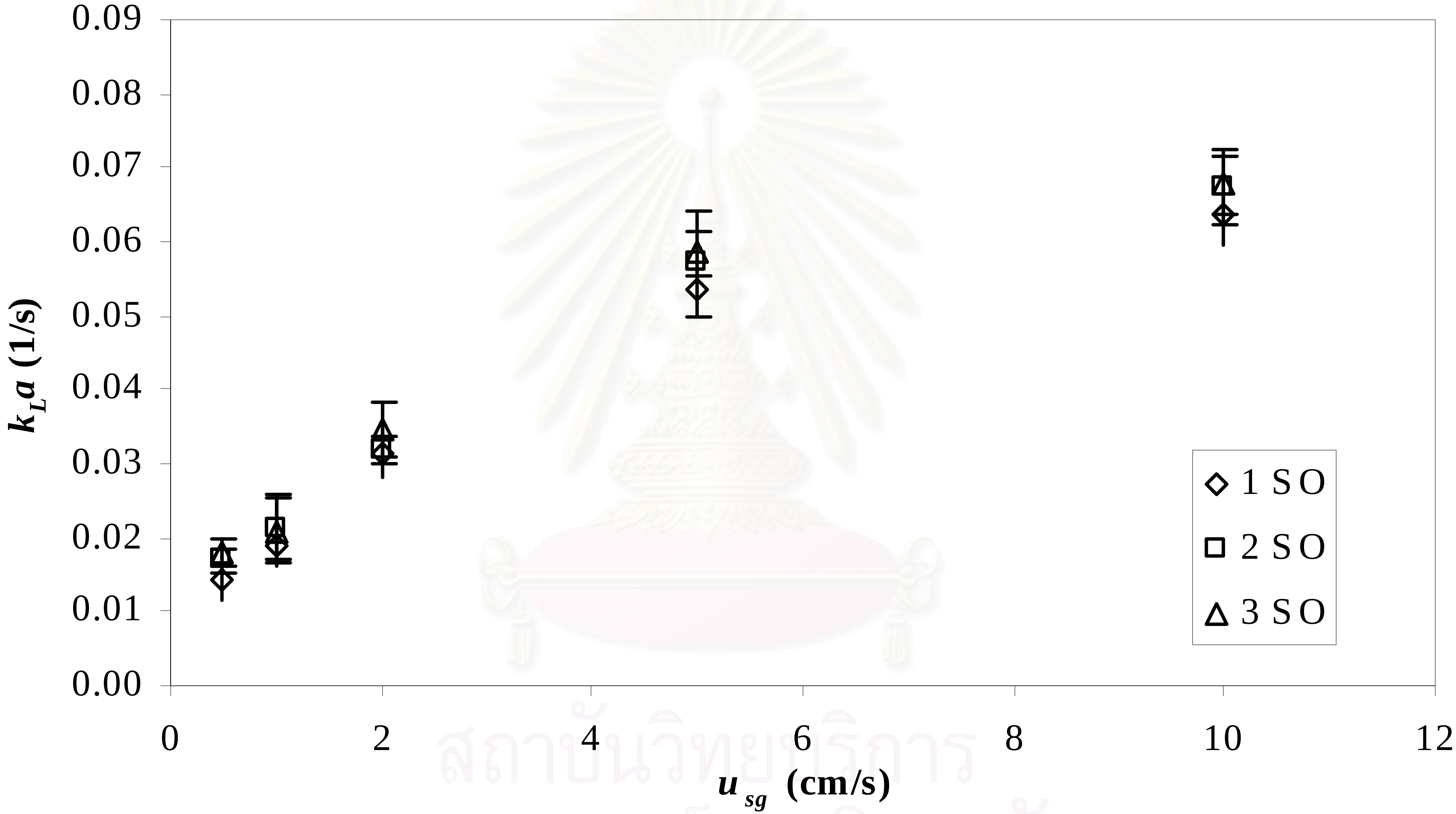

Figure 4.15 Effect of L_h on k_La in ELALC with inclined connection tubes

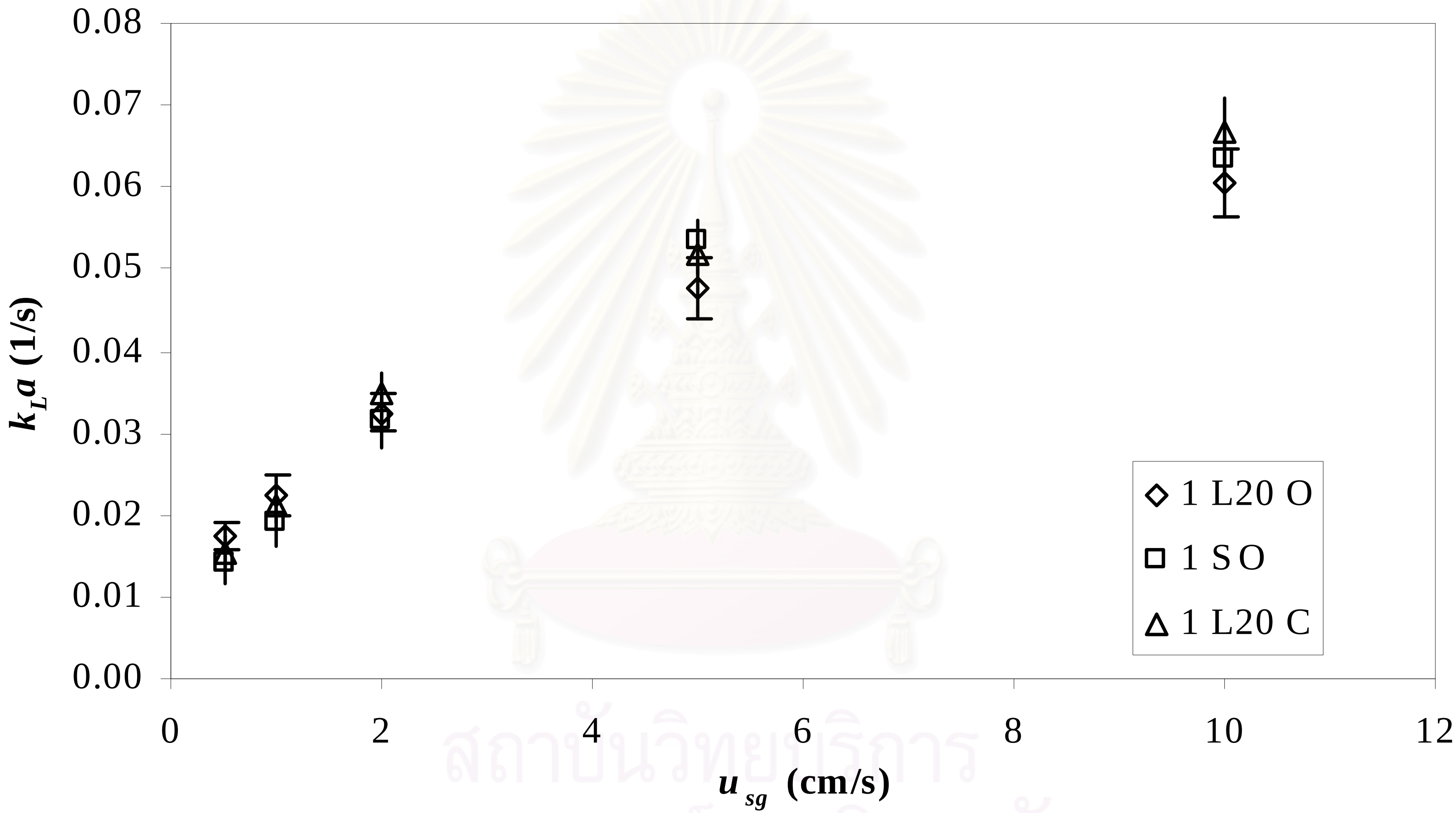

Figure 4.16 Effect of designed configuration on k_La

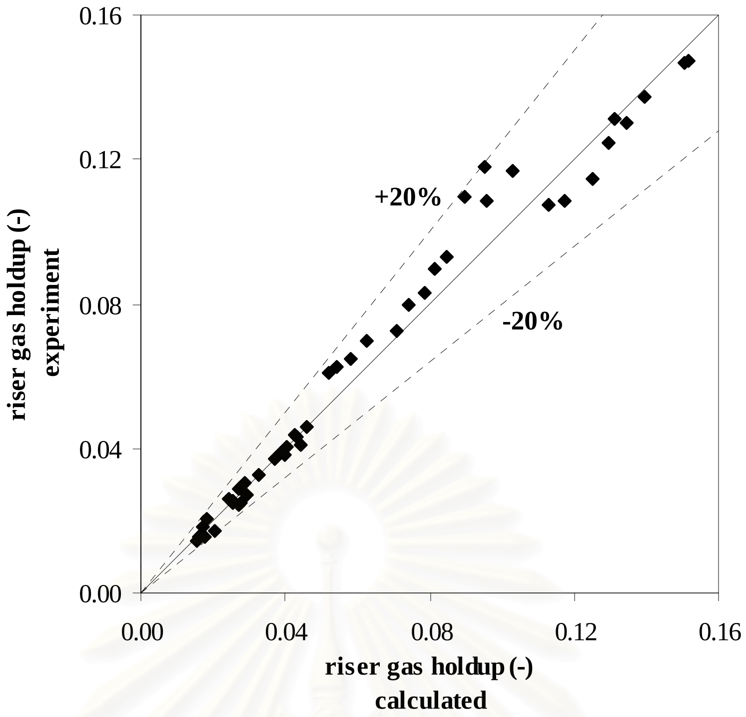

Figure 4.17 Comparison of riser gas holdup data using Eqs. 4.1, 4.3 and 4.5

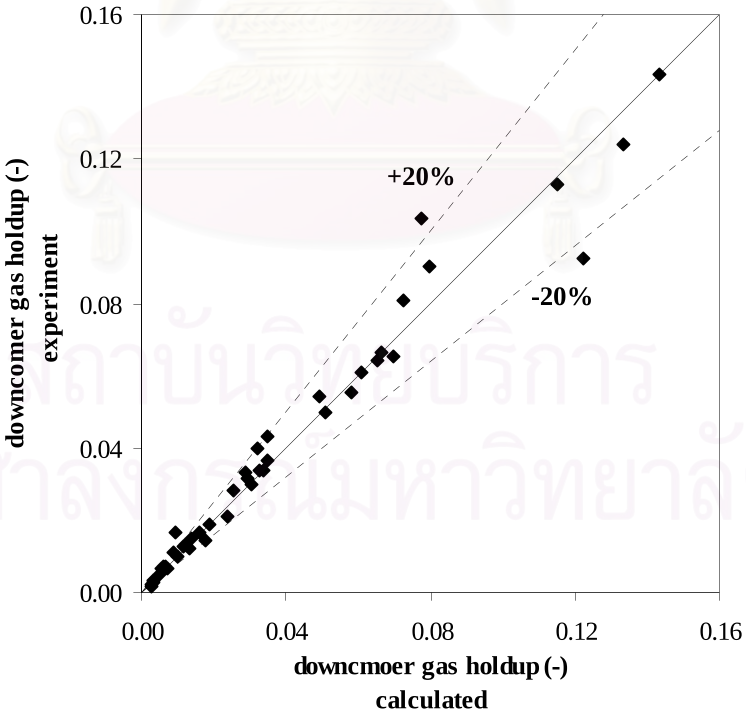

Figure 4.18 Comparison of downcomer gas holdup data using Eqs. 4.2, 4.4 and 4.6

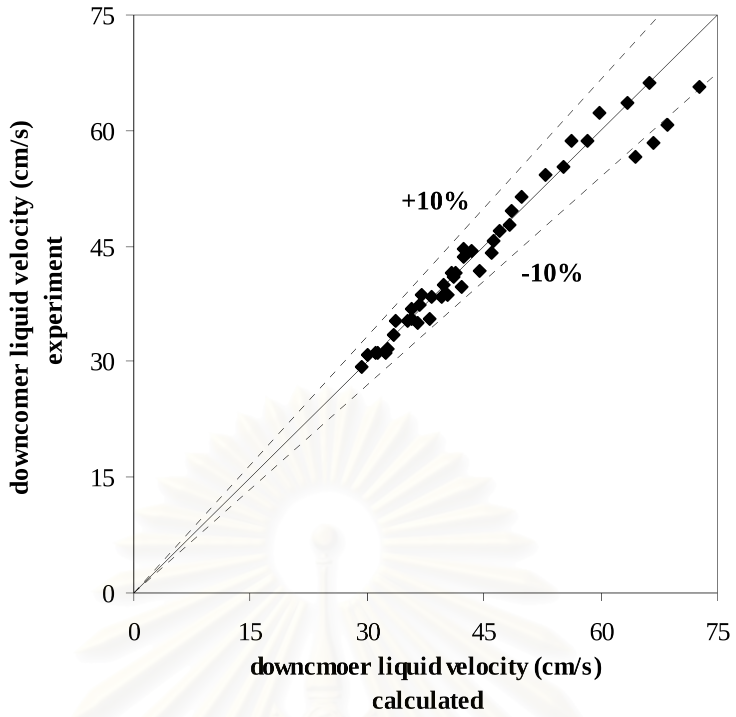

Figure 4.19 Comparison of downcomer liquid velocity data using Eqs. 4.9-4.11

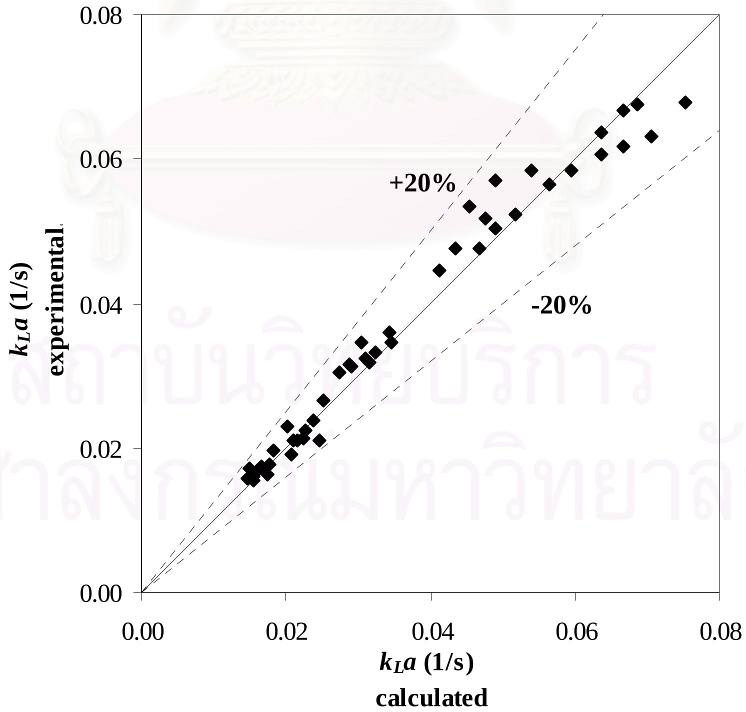

Figure 4.20 Comparison of k_La data using Eqs. 4.18-4.20

CHAPTER V

MATHEMATICAL MODEL FOR THE PREDICTION OF GAS-LIQUID MASS TRANSFER IN ALCs

5.1 Mathematical Model Development

The ALC is assumed to consist of six main sections as shown in a schematic diagram in Figure 5.1. Basically the components in the airlift are: riser – gas separator1 – top connection tube – gas separator2 – downcomer – bottom connection tube. The gas is supplied only at riser section and the gas is disengaged from the system at the two top sections, i.e. gas separators 1 and 2 which are located at the top-left and top-right of the contactor respectively. The fluid content in the bottom section re-enters the riser at the bottom column with the inlet gas.

To construct a mathematical model for this system, each part of the ALC is considered separately as illustrated in the right side of Figure 5.1. The riser, downcomer, top connection and bottom connection tubes are represented by the dispersion model with the exchange of oxygen between gas and liquid phases in each volume element. No liquid is added or removed from the system, whereas gas enters the system only at the bottom section of the riser and leaves the contactor at the gas separators. The behavior of the gas separators is assumed to be well mixed and is represented by the stirred-tank model. Hence, the overall model is represented by a series of various types of reactors, i.e. dispersion-stirred tank-dispersion-stirred tank-dispersion-dispersion.

For simplicity, the model is developed by considering the following additional assumptions:

1. The effect of hydrostatic head on solubility of oxygen is negligible. This is reasonable for small-scale systems.

2. The overall volumetric mass transfer coefficient is uniform for all sections in the contactor.
3. The gas holdup is uniform within each individual compartment.
4. The system is isothermal.
5. There is no radial effect in the ALC.
6. Oxygen is sparingly soluble in water and Henry's law can be applied to explain the solubility of oxygen.
7. The behavior of the gas in the system is ideal.
8. The operating parameters, e.g. gas holdups, liquid circulation flowrate, are not a function of time and space.
9. The gas velocity in downcomer section is assumed to be zero as the bubble velocity is assumed to be equal to local liquid velocity in these sections.

Following the continuity equation principal, the following set of equations is obtained:

For gas phase oxygen concentration in the riser, downcomer top connection tube and bottom connection tube:

At $0 < z_i < L_i$:

$$\frac{\partial O_{Gi}(z_i,t)}{\partial t} = -v_{Gi}\frac{\partial O_{Gi}(z_i,t)}{\partial z_i} + D_{Gi}\frac{\partial^2 O_{Gi}(z_i,t)}{\partial {z_i}^2} - \frac{(1-\varepsilon_{Gi})}{\varepsilon_{Gi}} K_L a\left(O_{Gi}(z_i,t) - O_{Li}(z_i,t)H\right) \tag{5.1}$$

For liquid phase oxygen concentration in the riser, downcomer top connection tube and bottom connection tube:

At $0 < z_i < L_i$:

$$\frac{\partial O_{Li}(z_i,t)}{\partial t} = -v_{Li}\frac{\partial O_{Li}(z_i,t)}{\partial z_i} + D_{Li}\frac{\partial^2 O_{Li}(z_i,t)}{\partial {z_i}^2} + K_L a\left(\frac{O_{Gi}(z_i,t)}{H} - O_{Li}(z_i,t)\right) \tag{5.2}$$

where $i = r$ for riser, $i = d$ for downcomer, $i = ct$ for top connection tube, $i = cb$ for bottom connection tube, H is the Henry's law constant for oxygen in water at STP.

For gas phase oxygen concentration in the gas-liquid separator1:

$$\frac{dO_{Gs1}}{dt}=\frac{\varepsilon_{Gr}A_r v_{Gr}O_{Gr}(z_r=L_r)-\varepsilon_{Gct}A_{ct}v_{Gct}O_{Gct}(z_{ct}=0)-Q_{G,out1}O_{Gs1}}{\varepsilon_{Gs1}V_{s1}}-\left(\frac{1-\varepsilon_{Gs1}}{\varepsilon_{Gs1}}\right)K_L a(O_{Gs1}-O_{Ls1}H) \tag{5.3}$$

For liquid phase oxygen concentration in the gas-liquid separator1:

$$\frac{dO_{Ls1}}{dt}=\frac{(1-\varepsilon_{Gr})A_r v_{Lr}O_{Lr}(z_r=L_r)-(1-\varepsilon_{Gct})A_{ct}v_{Lct}O_{Lct}(z_{ct}=0)}{(1-\varepsilon_{Gs1})V_{s1}}+K_L a\left(\frac{O_{Gs1}}{H}-O_{Ls1}\right) \tag{5.4}$$

For gas phase oxygen concentration in the gas-liquid separator2:

$$\frac{dO_{Gs2}}{dt}=\frac{\varepsilon_{Gct}A_{ct}v_{Gct}O_{Gct}(z_{ct}=L_{ct})-\varepsilon_{Gd}A_d v_{Gd}O_{Gd}(z_d=0)-Q_{G,out2}O_{Gs2}}{\varepsilon_{Gs2}V_{s2}}-\left(\frac{1-\varepsilon_{Gs2}}{\varepsilon_{Gs2}}\right)K_L a(O_{Gs2}-O_{Ls2}H) \tag{5.5}$$

For liquid phase oxygen concentration in the gas-liquid separator2:

$$\frac{dO_{Ls2}}{dt}=\frac{(1-\varepsilon_{Gct})A_{ct}v_{Lct}O_{Lct}(z_{ct}=L_{ct})-(1-\varepsilon_{Gd})A_d v_{Ld}O_{Ld}(z_d=0)}{(1-\varepsilon_{Gs2})V_{s2}}+K_L a\left(\frac{O_{Gs2}}{H}-O_{Ls2}\right) \tag{5.6}$$

Boundary and initial conditions for this set of equations in term of dimensional are given in Table 5.1, Eqs. (5.7) - (5.28). This set of equations is made dimensionless by introducing the following dimensionless variables:

$$\left.\begin{aligned}\bar{t}&=\frac{V_r}{v_{ld}A_d}\\ \tau&=\frac{t}{\bar{t}}\\ Z&=\frac{z}{L}\\ \overline{O}_L&=\frac{O_L}{O_L^*}\\ \overline{O}_G&=\frac{O_G}{O_{G,in}}\end{aligned}\right\} \tag{5.29}$$

Substitute Eq. (5.29) into Eqs (5.1) – (5.6) yields the set of equations of dimensionless variables in Eqs. (5.31) – (5.36).

For gas phase oxygen concentration in the riser, downcomer, top connection tube and bottom connection tube:

$$\frac{d\overline{O}_{Gi}}{d\tau_i} = -\frac{v_{Gi}\bar{t}}{L_i}\frac{\partial \overline{O}_{Gi}}{\partial z_i} + \frac{D_{Gi}\bar{t}}{L_i^2}\frac{\partial^2 \overline{O}_{Gi}}{\partial z_i^2} - \frac{(1-\varepsilon_{Gi})K_L a\bar{t}}{\varepsilon_{Gi} H}(\overline{O}_{Gi}(Z_i) - \overline{O}_{Li}(Z_i)) \tag{5.31}$$

For liquid phase oxygen concentration in the riser, downcomer, top connection tube and bottom connection tube:

$$\frac{d\overline{O}_{Li}}{d\tau_i} = -\frac{v_{Li}\bar{t}}{L_i}\frac{\partial \overline{O}_{Li}}{\partial z_i} + \frac{D_{Li}\bar{t}}{L_i^2}\frac{\partial^2 \overline{O}_{Li}}{\partial z_i^2} - K_L a\bar{t}(\overline{O}_{Gi}(Z_i) - \overline{O}_{Li}(Z_i)) \tag{5.32}$$

where i = r for riser, i = d for downcomer, i = ct for top connection tube, i = cb for bottom connection tube.

For gas phase oxygen concentration in the gas-liquid separator1:

$$\frac{d\overline{O}_{Gs1}}{d\tau_{s1}} = \frac{\bar{t}Q_{Gr}}{V_{s1}\varepsilon_{Gs1}}\overline{O}_{Gr}(Z_r = L_r) - \frac{\bar{t}Q_{G,out1}}{V_{s1}\varepsilon_{Gs1}}\overline{O}_{Gs1} - \frac{\bar{t}Q_{Gct}}{V_{s1}\varepsilon_{Gs1}}\overline{O}_{Gs1} - \frac{K_L a(1-\varepsilon_{Gs1})\bar{t}}{\varepsilon_{Gs1}H}(\overline{O}_{Gs1} - \overline{O}_{Ls1}) \tag{5.33}$$

For liquid phase oxygen concentration in the gas-liquid separator1:

$$\frac{d\overline{O}_{Ls1}}{d\tau_{s1}} = \frac{\bar{t}Q_{Lr}}{V_{s1}(1-\varepsilon_{Gs1})}\overline{O}_{Lr}(Z_r = L_r) - \frac{\bar{t}Q_{Lct}}{V_{s1}(1-\varepsilon_{Gs1})}\overline{O}_{Ls1} + K_L a\bar{t}(\overline{O}_{Gs1} - \overline{O}_{Ls1}) \tag{5.34}$$

For gas phase oxygen concentration in the gas-liquid separator2:

$$\frac{d\overline{O}_{Gs2}}{d\tau_{s2}} = \frac{\bar{t}Q_{Gct}}{V_{s2}\varepsilon_{Gs2}}\overline{O}_{Gct}(Z_{ct} = L_{ct}) - \frac{\bar{t}Q_{G,out2}}{V_{s2}\varepsilon_{Gs2}}\overline{O}_{Gs2} - \frac{\bar{t}Q_{Gd}}{V_{s2}\varepsilon_{Gs2}}\overline{O}_{Gs2} - \frac{K_L a(1-\varepsilon_{Gs2})\bar{t}}{\varepsilon_{Gs2}H}(\overline{O}_{Gs2} - \overline{O}_{Ls2}) \tag{5.35}$$

For liquid phase oxygen concentration in the gas-liquid separator2:

$$\frac{d\overline{O}_{Ls2}}{d\tau_{s2}} = \frac{\bar{t}Q_{Lct}}{V_{s2}(1-\varepsilon_{Gs2})}\overline{O}_{Lct}(Z_{ct} = L_{ct}) - \frac{\bar{t}Q_{Ld}}{V_{s2}(1-\varepsilon_{Gs2})}\overline{O}_{Ls2} + K_L a\bar{t}(\overline{O}_{Gs2} - \overline{O}_{Ls2}) \tag{5.36}$$

Similarly, Eq. (5.29) is also substituted into initial and boundary conditions, Eqs. (5.7) – (5.28), to yield dimensionless initial and boundary conditions. The dimensional and dimensionless forms of this set of equations are given in Table 5.1.

The resulting dimensionless equations, Eqs. (5.31) to (5.36), are partial differential equations and therefore are discretized using the finite difference method. This leads to a set of ordinary difference equations that could be integrated via the 4th order Runge-Kutta integration method (Chapra and Canale, 1998).

5.2 Parameter Estimations

To predict oxygen concentration in the gas and liquid phases according to the proposed model, hydrodynamic and mass transfer parameters including gas holdups (ε_G), liquid velocities (v_L) and overall volumetric gas-liquid mass transfer coefficient (k_La) have to be known in a priori. Other parameters could subsequently be calculated using the following equations.

Firstly, liquid velocity of riser, top and bottom connection tubes are calculated using the continuity equation:

$$v_{Ld} A_d (1-\varepsilon_{Gd}) = v_{Li} A_i (1-\varepsilon_{Gi}) \tag{5.59}$$

where $i = r$ for riser, $i = ct$ for top connection tube, $i = cb$ for bottom connection tube. Riser gas velocity (v_{Gr}) is calculated from gas flow rate inlet ($Q_{G,in}$), ε_r and A_r as follows:

$$v_{Gr} = \frac{Q_{G,in}}{\varepsilon_r A_r} \tag{5.60}$$

According to Assumption # 9, the downcomer gas velocity (v_{Gd}) is assumed to be zero. This virtually means that the volume of bubbles resided within the downcomer section remained constant. For this to be true, the amounts of bubbles in top and bottom connection tubes also need to be constant, i.e. the net movement of gas bubbles in and out of this section is zero.

Generally, gas input to ALC is equal to gas leaving gas-liquid separator 1 ($Q_{G,out1}$) plus that from gas-liquid separator 2 ($Q_{G,out2}$), as described in Eq. 5.61.

$$Q_{G,in} = Q_{G,out1} + Q_{G,out2} \tag{5.61}$$

The flow rates of gas leaving the system gas-liquid separators 1 and 2 must be estimated, and in this work, the ratio between these two quantities are assumed to be proportional to the area occupied by gas phase in the gas-liquid separators 1 and 2, respectively. Therefore, gas leaving each sections is equal to the flow rate of gas input multiplied by the area in each section divided by the total area of gas phase in gas-liquid separators.

$$Q_{G,out1} = Q_{G,in} \frac{A_{s1}\varepsilon_{s1}}{(A_{s1}\varepsilon_{s1} + A_{s2}\varepsilon_{s2})} \tag{5.62}$$

$$Q_{G,out2} = Q_{G,in} \frac{A_{s2}\varepsilon_{s2}}{(A_{s1}\varepsilon_{s1} + A_{s2}\varepsilon_{s2})} \tag{5.63}$$

The gas velocities in the top connection tube (v_{Gct}) are calculated using the continuity equation, where:

$$v_{Gct} = \frac{v_{Gr}A_r\varepsilon_r - Q_{G,out1}}{A_{ct}\varepsilon_{Gct}} \tag{5.64}$$

At this point, axial dispersion coefficients in gas and liquid phases in all sections of ALC (D_{Gr}, D_{Gd}, D_{Lr}, D_{Ld}, D_{Gct}, D_{Gcb}, D_{Lct}, D_{Lcb}) remain still unknown. Liquid phase dispersion coefficient (D_L) was reported by Verlaan (1989) to be in a range from 0.01 to 0.12 m^2/s for ELALC with u_{sg} between 1-14 cm/s. Gas phase coefficient (D_G) was reported to be 2-5 m^2/s for ILALC with u_{sg} between 1-10 cm/s (Ruffer et al., 1994). Wongsuchoto and Pavasant, (2004) developed the mathematical model for the internal loop ALC and employed D_L and D_G within these reported range to successfully describe the oxygen gas-liquid mass transfer in the airlift.

It can be seen that the values of dispersion coefficients from literature were quite variable. To select suitable dispersion coefficients for simulation, a simple sensitivity test was performed with the range of the report indicated above. The simulation demonstrated that, within the range investigated, the results were not significantly different from each other. In other words, the range of dispersion coefficients as obtained from previous reports provided similar responding time for the system to reach gas-liquid equilibrium. Therefore the value of D_L and D_G in all sections of ELALC were arbitrarily selected from this range of magnitude at 0.06 and 3 m^2/s, respectively.

5.3 Model Verification

To verify the model, the simulation results were compared with experimental data. Detail of the design parameters for the ELALC employed in this experiment is shown in Table 4.1. For regular ELALCs, Eqs. (4.1, 4.2, 4.9 and 4.12) obtained from experiments in Chapter 4, were used to estimate the hydrodynamics and mass transfer parameters in the model. Figure 5.2 illustrates the comparisons between the simulation results and experimental data on liquid phase oxygen concentration in the riser (O_{Lr}) in the system at three different superficial gas velocities (u_{sg} = 0.5, 2.0 and 10.0 cm/s). The simulation results and experimental data demonstrated that the model produced results with reasonable accuracy when compared with experimental data for the range of u_{sg} examined here (0.5-10.0 cm/s). Overall, the oxygen concentration profile reached equilibrium concentration more quickly at high u_{sg}.

Prediction of O_{Lr} in regular ELALC at various L_c and L_h are illustrated in Figures 5.3 and 5.4, respectively. These results agreed well with experiment data, and it was found that the model was able to accurately predict the behavior of liquid phase oxygen concentration. The effects of various configurations on O_{Lr} were also compared. Eqs. (4.3, 4.4, 4.10 and 4.13) were used in estimate the hydrodynamics and mass transfer parameters in ELALC with inclined connection tubes and Eqs. (4.5, 4.6, 4.11 and 4.14) in ELALC with closed downcomer gas-liquid separator. The simulation results and experiment data of different configurations on O_{Lr} were compared in Figure 5.5 where good agreements between simulation and experiments could be obtained.

5.4 Concluding remarks

The developed model for the prediction of dissolve oxygen concentration profiles in the external loop ALC with various conditions was found to be reasonably accurate by a series of mathematical equations where the riser, downcomer, top and bottom connection tubes were represented by the dispersion model and the two gas separators represented by the stirred tank model. This model can then be used with reasonable accuracy for the estimate of transport phenomena which could occur in the airlift reactor or bioreactor where gas-liquid mass transfer is important.

Table 5.1 Initial and boundary conditions in each section of ELALC

			Dimensional form	Dimensionless form
Riser	I.C.	Gas	$O_{Gr}\left(0 \le z_r \le L_r, t=0\right)=0.21atm_{O_2}$ (5.7)	$\overline{O}_{Gr}\left(0 \le Z_r \le L_r, \tau=0\right)=1$ (5.37)
	B.C.1		$O_{Gr}\left(z_r=0, t>0\right) = \left(\dfrac{\varepsilon_{cb} v_{Gcb} A_{cb} O_{Gcb}\left(z_{cb}=L_{cb}, t>0\right)+Q_{G,in} O_{G,in}}{\varepsilon_r v_{Gr} A_r}\right)$ (5.8) $Q_{G,in}$ = inlet gas flowrate (m^{3}s^{-1})	$\overline{O}_{Gr}\left(Z_r=0, \tau>0\right) = \left(\dfrac{\varepsilon_{cb} v_{Gcb} A_{cb} \overline{O}_{Gcb}\left(Z_{cb}=L_{cb}, \tau>0\right)+Q_{G,in} \overline{O}_{G,in}}{\varepsilon_r v_{Gr} A_r}\right)$ (5.38) $Q_{G,in}$ = inlet gas flowrate (m^{3}s^{-1})
	B.C.2		$O_{Gr}\left(z_r=L_r, t>0\right)=O_{Gs1}\left(t>0\right)$ (5.9)	$\overline{O}_{Gr}\left(Z_r=L_r, \tau>0\right)=\overline{O}_{Gs1}\left(\tau>0\right)$ (5.39)
	I.C.	Liquid	$O_{Lr}\left(0 \le z_r \le L_r, t=0\right)=0$ (5.10)	$\overline{O}_{Lr}\left(0 \le Z_r \le 1, \tau=0\right)=0$ (5.40)
	B.C.1		$O_{Lr}\left(z_r=0_r, t>0\right)=O_{Lcb}\left(z_{cb}=L_{cb}, t>0\right)$ (5.11)	$\overline{O}_{Lr}\left(Z_r=0_r, \tau>0\right)=\overline{O}_{Lcb}\left(Z_{cb}=L_{cb}, \tau>0\right)$ (5.41)
	B.C.2		$O_{Lr}\left(z_r=L_r, t>0\right)=O_{Ls1}\left(t>0\right)$ (5.12)	$\overline{O}_{Lr}\left(Z_r=L_r, \tau>0\right)=\overline{O}_{Ls1}\left(\tau>0\right)$ (5.42)
Downcomer	I.C.	Gas	$O_{Gd}\left(0 \le z_d \le L_d, t=0\right)=0.21atm_{O_2}$ (5.13)	$\overline{O}_{Gd}\left(0 \le Z_d \le L_d, \tau=0\right)=1$ (5.43)
	B.C.		$O_{Gd}\left(z_d=0, t>0\right)=O_{Gs2}\left(t>0\right)$ (5.14)	$\overline{O}_{Gd}\left(Z_d=0, \tau>0\right)=\overline{O}_{Gs2}\left(\tau>0\right)$ (5.44)
	I.C.	Liquid	$O_{Ld}\left(0 \le z_d \le L_d, t=0\right)=0$ (5.15)	$\overline{O}_{Ld}\left(0 \le Z_d \le 1, \tau=0\right)=0$ (5.45)
	B.C.		$O_{Ld}\left(z_d=0, t>0\right)=O_{Ls2}\left(t>0\right)$ (5.16)	$\overline{O}_{Ld}\left(Z_d=0, \tau>0\right)=\overline{O}_{Ls2}\left(\tau>0\right)$ (5.46)
Gas-liquid separator1	I.C.	Gas	$O_{Gs1}\left(t=0\right)=0.21atm_{O_2}$ (5.17)	$\overline{O}_{Gs1}\left(\tau=0\right)=1$ (5.47)
		Liquid	$O_{Ls1}\left(t=0\right)=0$ (5.18)	$\overline{O}_{Ls1}\left(\tau=0\right)=0$ (5.48)

*I.C.—Initial Condition, B.C.—Boundary Condition

Table 5.1 (cont.)

			Dimensional form	Dimensionless form
Gas-liquid separator2	I.C.	Gas	$O_{Gs2}(t=0)=0.21atm_{O_2}$ (5.19)	$\overline{O}_{Gs2}(\tau=0)=1$ (5.49)
		Liquid	$O_{Ls2}(t=0)=0$ (5.20)	$\overline{O}_{Ls2}(\tau=0)=0$ (5.50)
Top connection tube	I.C.	Gas	$O_{Gct}(0\le z_{ct}\le L_{ct},t=0)=0.21atm_{O_2}$ (5.21)	$\overline{O}_{Gct}(0\le Z_{ct}\le L_{ct},\tau=0)=1$ (5.51)
	B.C.		$O_{Gct}(z_{ct}=0,t>0)=O_{Gs1}(t>0)$ (5.22)	$\overline{O}_{Gct}(Z_{ct}=0,\tau>0)=\overline{O}_{Gs1}(\tau>0)$ (5.52)
	I.C.	Liquid	$O_{Lct}(0\le z_{ct}\le L_{ct},t=0)=0$ (5.23)	$\overline{O}_{Lct}(0\le Z_{ct}\le L_{ct},\tau=0)=0$ (5.53)
	B.C.		$O_{Lct}(z_{ct}=0,t>0)=O_{Ls1}(t>0)$ (5.24)	$\overline{O}_{Lct}(Z_{ct}=0,\tau>0)=\overline{O}_{Ls1}(\tau>0)$ (5.54)
Bottom connection tube	I.C.	Gas	$O_{Gcb}(0\le z_{cb}\le L_{cb},t=0)=0.21atm_{O_2}$ (5.25)	$\overline{O}_{Gcb}(0\le Z_{cb}\le L_{cb},\tau=0)=1$ (5.55)
	B.C.		$O_{Gcb}(z_{cb}=0,t>0)=O_{Gd}(z_d=L_d,t>0)$ (5.26)	$\overline{O}_{Gcb}(Z_{cb}=0,\tau>0)=\overline{O}_{Gd}(Z_d=L_d,\tau>0)$ (5.56)
	I.C.	Liquid	$O_{Lcb}(0\le z_{cb}\le L_{cb},t=0)=0$ (5.27)	$\overline{O}_{Lcb}(0\le Z_{cb}\le L_{cb},\tau=0)=0$ (5.57)
	B.C.		$O_{Lcb}(z_{cb}=0,t>0)=O_{Ld}(z_d=L_d,t>0)$ (5.28)	$\overline{O}_{Lcb}(Z_{cb}=0,\tau>0)=\overline{O}_{Lb}(Z_d=L_d,\tau>0)$ (5.58)

*I.C.—Initial Condition, B.C.—Boundary Condition

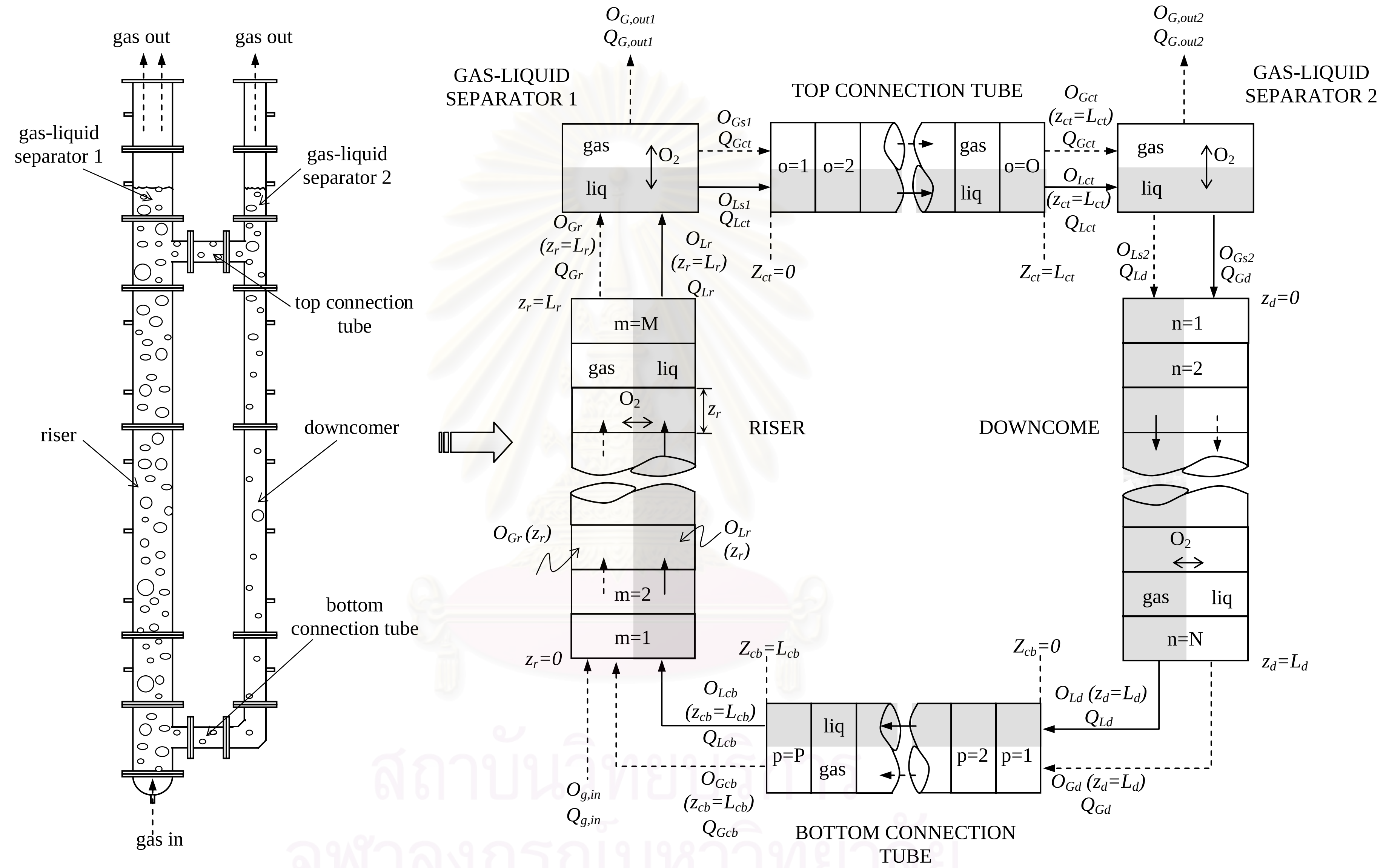

Figure 5.1 Block flow representation of the external loop ALC

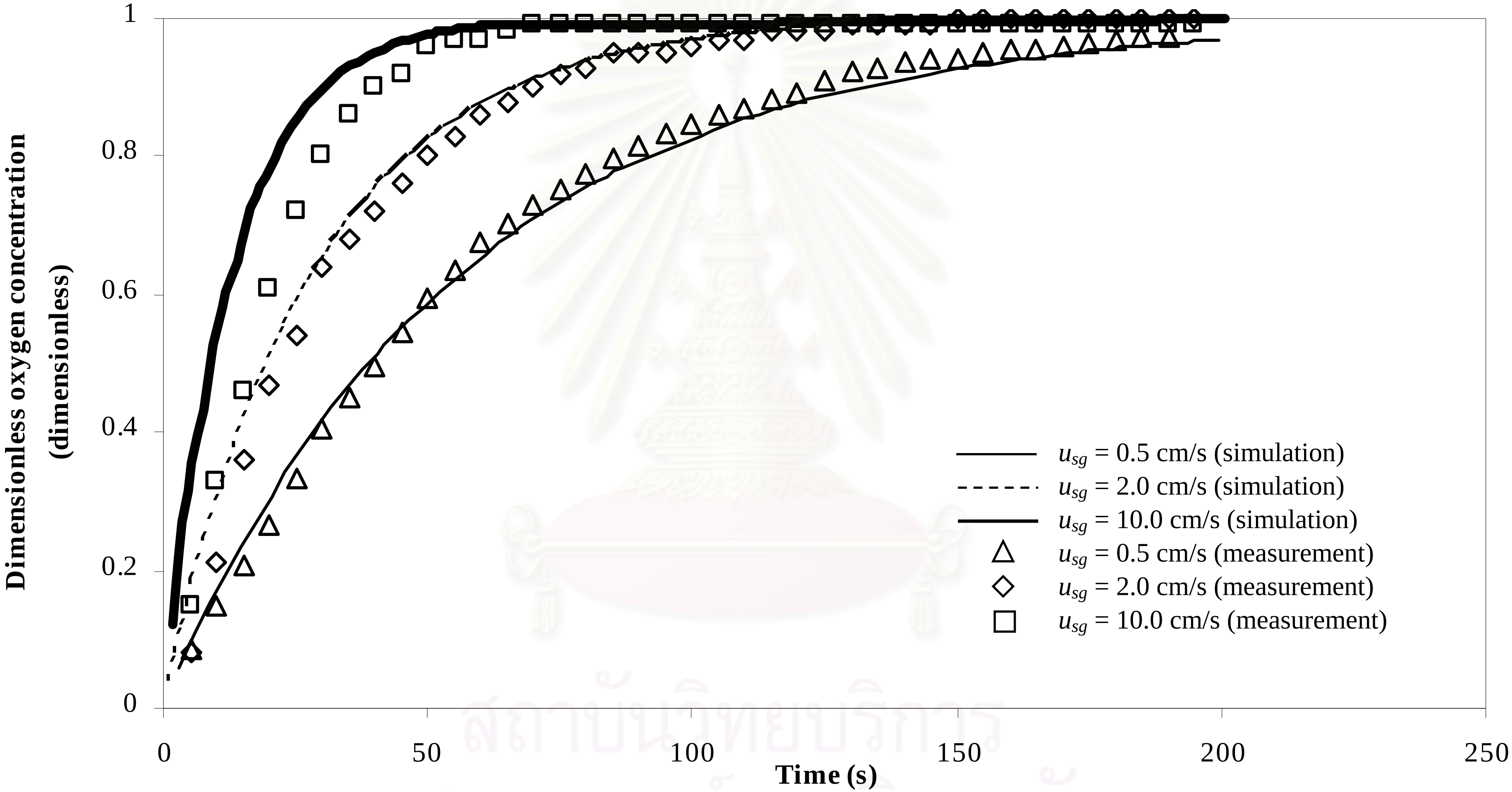

Figure 5.2 Comparison between simulation results and measurement of O_{lr} time profile in ELALC: Effect of u_{sg}

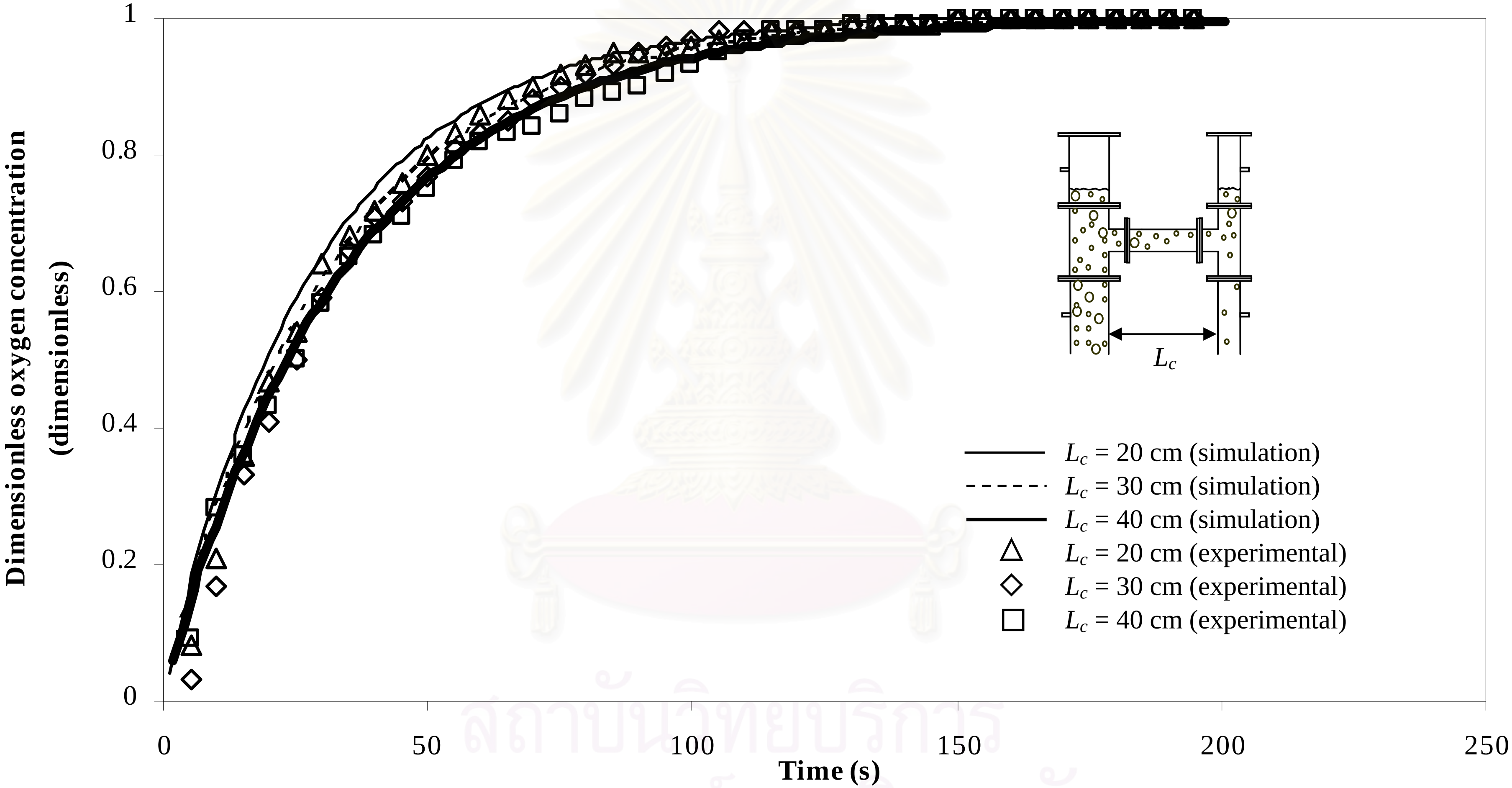

Figure 5.3 Comparison between simulation results and measurement of O_{lr} time profile in ELALC: Effect of L_c

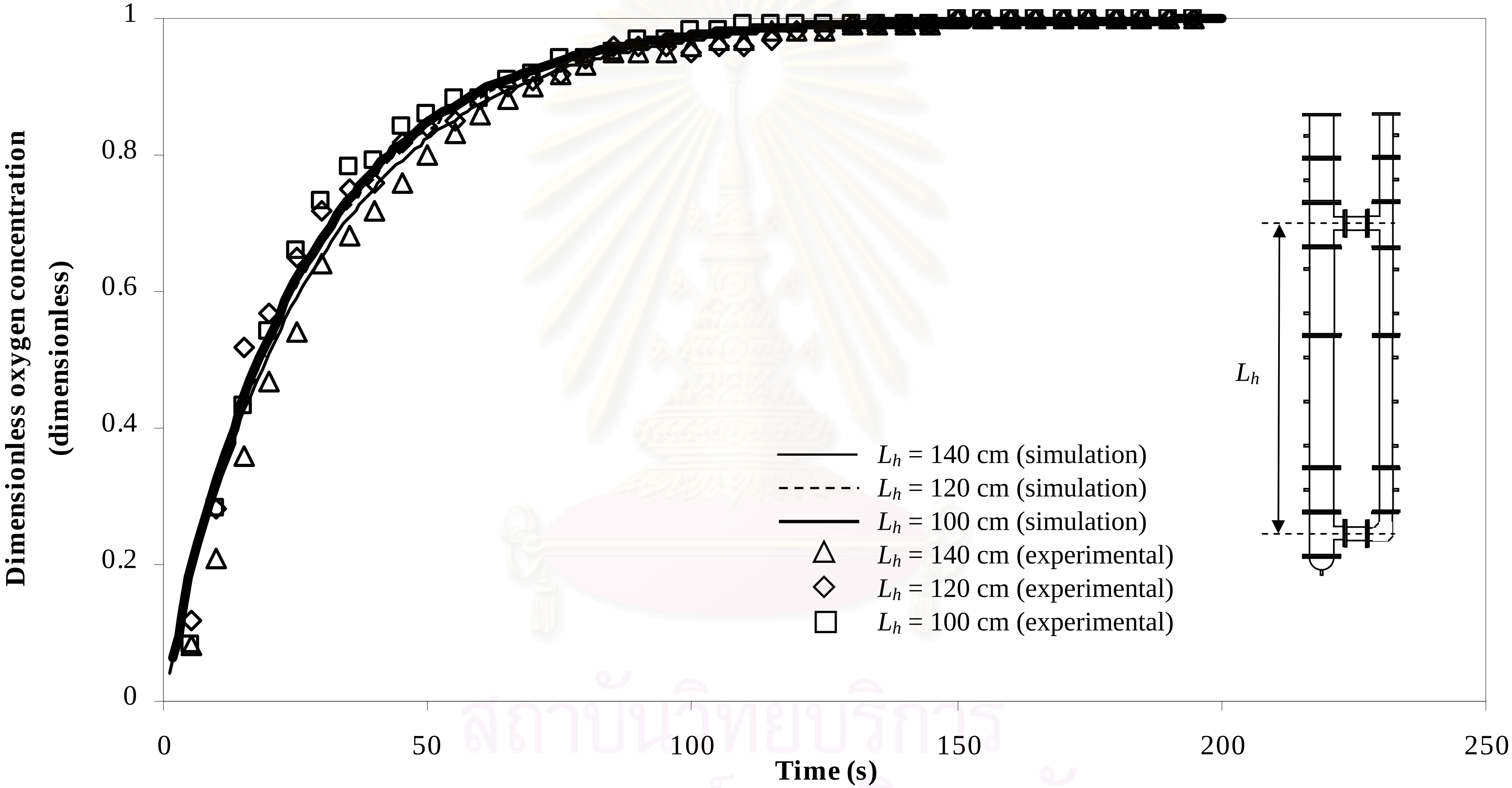

Figure 5.4 Comparison between simulation results and measurement of O_{lr} time profile in regular ELALC: Effect of L_h

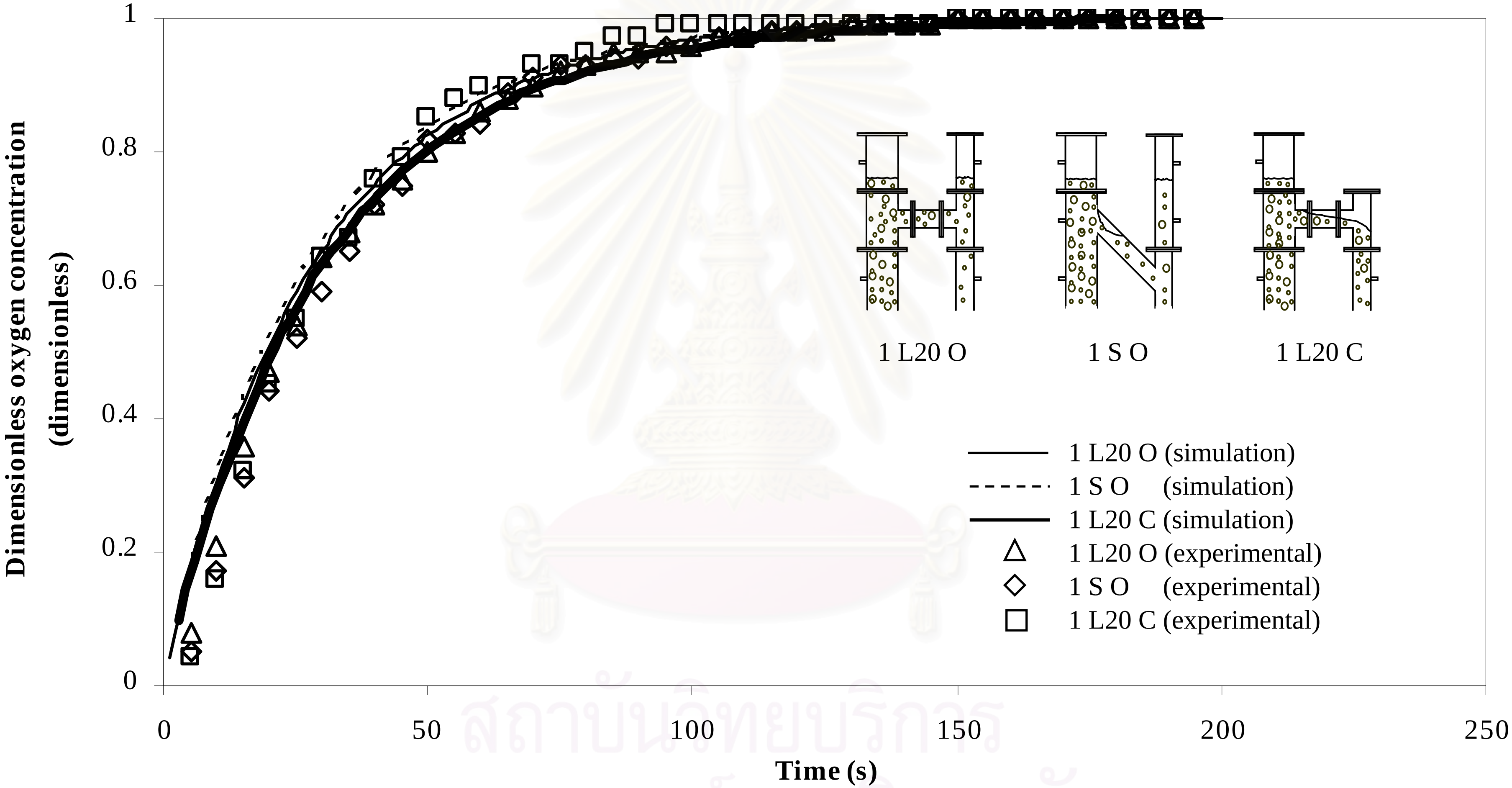

Figure 5.5 Comparison between simulation result and measurement of O_{lr} time profile in ELALC : Effect of configurations of ELALC; 1 L20 O for regular ELALC, 1 S O for ELALC with inclined connection tubes, 1 L20 C for ELALC with closed downcomer gas-liquid separator

CHAPTER VI

CONCLUSIONS & RECOMMENDATIONS

6.1 Achievements

External loop airlift contactors (ELALCs) has separate riser and downcomer compartments which allows easier design and maintenance than the internal loop systems. This thesis aimed to provide the performance of the different configurations of ELALC regarding hydrodynamics and gas-liquid mass transfer behaviors. The summary of the findings from this work is given in Table 6.1. For regular ELALC, an increase in the length of connection tubes (L_c) or increased height of riser and dowcomer (L_h) increased liquid velocity but reduced riser gas holdup and gas-liquid mass transfer coefficient. However, increasing L_c resulted in a decrease in downcomer gas holdup whereas an increase in L_h increased the downcomer gas holdup.

The effect of configurations on ELALC performance was also investigated. The ELALC with inclined connection tubes and closed downcomer gas liquid separator were compared with regular ELALC where the first two allowed the formation of large chunks of gas bubbles within the top connection tubes. This resulted in a larger gas holdup and slower downcomer liquid velocity when compared with regular configuration of ELALC.

The transfer of oxygen between gas and liquid phases was well predicted by a mathematical model based on principals of material conservation. It was shown that ELALC with various configurations could be well described by a simple continuity equation where each part of the ELALC could be represented by different types of fundamental reactors, i.e riser, downcomer, top and bottom connection tubes were described by plug flow reactor model, whilst gas-liquid separators by continuous stirred tank reactor model. The simulation results agreed well with experimental data.

Table 6.1 Summation of characteristics in airlift contactor with different configurations.

Parameter	L_c [cm]	L_h [cm]	L_h [cm] on S configuration	configurations (u_{sg} 0.5-2.0 cm/s)	configurations (u_{sg} 5.0-10.0 cm/s)
ε_r	20>30>40	100>120>140	100>120>140	S > C > R	C > S > R
ε_d	20>30>40	140>120>100	140>120>100	C > S > R	S > C > R
ε_o	20>30>40	100>120>140	no trend	C ≈ S > R	C ≈ S > R
v_{ld}	40>30>20	140>120>100	140>120>100	R > S > C	R > S > C
k_La	20>30>40	100>120>140	100>120>140	C ≈ S > R	C ≈ S > R

Remarks

R = regular ELALC

S = inclined connection tube ELALC

C = closed downcomer gas liquid separator ELALC

6.2 Contributions

As mentioned earlier, ELALCs have the structure which is easy to design and maintenance. It also allows simple modification of the reactor to suit each application. It is surprising to learn that, despite all the advantages, this type of airlift was not as much investigated as the internal loop airlift. This thesis was among the few attempts to describe the performance of ELALCs regarding their hydrodynamic and gas-liquid mass transfer behaviors. With the information as presented in this work, ones could design simple ELALC to suit their own needs. For instance, if this is to use in biotechnological applications where cells are to be cultivated in the airlift. If oxygen mass transfer is important, one might choose conditions which provides high gas-liquid mass transfer, or if the application requires that liquid moves at high speed, one could also design the system to best answer this demand, e.g. by the adjustment of the length of connection tubes or contactor height.

This thesis also proved that the ELALC could be described relatively accurately with a simple mass-balance mathematical model. This is a good starting point to further design or predicts the performance of the system without having tedious work in arranging experiments which could be time and resources consuming.

6.3 Recommendations

- On the inclined connection tubes ELALC configuration, the design of top connection tube was inclined down to downcomer. Experiment demonstrated that the gas bubbles coalesced at the entrance of the top connection tube. To remove this formation of bubbles, it might be that this slope inclined up the downcomer, and this has to be investigated.

- The value of A_d/A_r in this thesis was constant at 0.269. This has to be mentioned if the proposed empirical models are to be used for future reference.

- The height of water in this ELALC remained constant, even in experiments with changes in the height of riser and downcomer (L_h). It would be interesting to examine the performance of the airlift reactor with different liquid height to complete the scale up work.

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APPENDIX

Program source codes

This appendix presents all of the main programs used in this work. The program was written in MATLAB (VERSION 7.4)

```
%File Name:DISPERSION MODEL
%Programe PFR with or without Dispersion term in RISER and DOWNCOMER

clear

%Design parameter
D1=0.104;
D2=0.054;
D3=0.104;
D4=0.054;
D5=0.054;
D6=0.054;

%Cross-sectional area of riser (Ar)
A1=(pi/4)*(D1^2);
%Cross-sectional area of downcomer (A2)
A2=(pi/4)*(D2^2);
%Cross-sectional area of separator1 (A3)
A3=(pi/4)*(D3^2);
%Cross-sectional area of separator2 (A4)
A4=(pi/4)*(D4^2);
%Cross-sectional area of top connection tube (A5)
A5=(pi/4)*(D5^2);
%Cross-sectional area of bottom connection tube (A6)
A6=(pi/4)*(D6^2);


%Ad/Ar
ratio=A2/A1;

%Draft tube height (H1)
H1=input('enter height of riser (m) = ');
H2=input('enter height of downcomer (m) = ');
H5=input('enter length of top connection (m) = ');
H6=input('enter length of bottom connection (m) = ');

%Superficial gas velocity (m/s)
usg=input('Enter inlet superficial gas velocity (m/s)=');
Qg_in=usg*A1;

%H7=input('Enter unaerated liquid height (m) = ');
H7=1.5;

%e1=input('enter riser gas holdup cal (-) = ');
%e5=input('enter downcomer gas holdup cal (-) = ');
%e1=0.47*usg^0.6*(H1*H5)^-0.23*1.00;
e1=0.49*usg^0.67*H1^-0.15*H2^-0.22;
e2=0.29*usg*(H5^0.39/H1^0.49);
%e1=1.306*ratio^-0.21*usg^0.92;
%e2=0.865*e1-0.0038;
%incliend
%e1=0.53*usg^0.54*H5^-0.41;
%e2=0.93*usg^0.88*H5^-0.47;
%close
%e1=0.7*usg^0.66;
%e2=0.39*usg^0.53;

e3=e1;
e4=e3;
Hd=H7/(1-e3);
H3=Hd-H1;
H4=H3;
%H4=H3+0.2;
%H4=0.13;
e5=(e1+e2)/2;
e6=e2;

%Volume of riser (V1), gas-liquid separator1 (V3) and downcomer (V2)
V1=A1*H1;
V2=A2*H2;
V3=A3*H3;
V4=A4*H4;
V5=A5*H5;
V6=A6*H6;

kla1=0.149*usg^0.45*(H1^-0.17*H5^-0.30);
%kla1=0.288*ratio^-0.17*usg^0.74;poo
%kla1=0.23*usg^0.48*(H1)^-0.51;
%kla1=0.21*usg^0.49;
kla2=kla1;
kla3=kla1;
kla4=kla1;
kla5=kla1;
kla6=kla1;

%v_slip=0.25;

%vl_1=0.48*usg^0.104*(H1*H5)^0.134*1;
%vl_1=0.241+0.604*ratio^1.142*usg^0.324;
%vl_2=vl_1*A1*(1-e1)/A2/(1-e2)*1;
vl_2=1.29*usg^0.197*(H1^0.147*H5^0.177);
%vl_2=3.12*usg^0.05*H5^0.68/(7.6+usg^2.16);
%vl_2=2.63*usg^0.045/(7.1+usg^2.66);
vl_1=vl_2*A2*(1-e2)/A1/(1-e1);
vl_5=vl_2*A2*(1-e2)/A5/(1-e5);
vl_6=vl_2*A2*(1-e2)/A6/(1-e6);

vg_1=Qg_in/e1/A1;
%vg_1=vl_1+v_slip;
%vg_2=vl_2+v_slip;
vg_2=(vg_1*A1*e1-Qg_in)/(A2*e2);
%vg_2=vl_2*2/5;*A3/(A3+A4)
vg_5=(vg_1*A1*e1-Qg_in*A3/(A3+A4))/(A5*e5);
%vg_5=(vg_1+vg_2)/2;
vg_6=vg_2;

interval=20;

m=interval;
n=interval;
o=interval;
p=interval;

t_factor=V1/(A2*vl_2);
z_factor=H1;

del_z1=1/m;
del_z2=1/n;
del_z5=1/o;
del_z6=1/p;

Henry=23.6;

disp_g=3;
disp_l=0.06;
```

```
%COEFICIENT OF RISER(Dispersion)
    %GAS PHASE COEFICIENT
eq1g_1=-vg_1*t_factor/z_factor;
eq1g_2=eq1g_1/del_z1;
eq1g_3=eq1g_1/2/del_z1;
eq1g_4=disp_g*t_factor/z_factor^2;
eq1g_5=eq1g_4/del_z1^2;
eq1g_6=-(1-e1)*kla1*t_factor/e1/Henry;

    %LIQUID PHASE COEFICIENT
eq1l_1=-vl_1*t_factor/z_factor;
eq1l_2=eq1l_1/del_z1;
eq1l_3=eq1l_1/2/del_z1;
eq1l_4=disp_l*t_factor/z_factor^2;
eq1l_5=eq1l_4/del_z1^2;
eq1l_6=kla1*t_factor;

%COEFICIENT OF DOWNCOMER(Dispersion)
    %GAS PHASE COEFICIENT
eq2g_1=-vg_2*t_factor/z_factor;
eq2g_2=eq2g_1/del_z1;
eq2g_3=eq2g_1/2/del_z1;
eq2g_4=disp_g*t_factor/z_factor^2;
eq2g_5=eq2g_4/del_z1^2;
eq2g_6=-(1-e2)*kla2*t_factor/e2/Henry;

    %LIQUID PHASE COEFICIENT
eq2l_1=-vl_2*t_factor/z_factor;
eq2l_2=eq2l_1/del_z1;
eq2l_3=eq2l_1/2/del_z1;
eq2l_4=disp_l*t_factor/z_factor^2;
eq2l_5=eq2l_4/del_z1^2;
eq2l_6=kla2*t_factor;

%COEFICIENT OF GAS SEPARATOR 1(CSTR)
    %GAS PHASE COEFICIENT
%Input from riser
eq3g_1=e1*A1*vg_1*t_factor/(e3*V3);
%Output to downcomer
eq3g_2=-vg_5*A5*e5*t_factor/(e3*V3);
%Output by mass transfer
eq3g_3=-(1-e3)*kla3*t_factor/e3/Henry;
%Output to atm
eq3g_4=-Qg_in*A3/(A3+A4)*t_factor/(e3*V3);

    %LIQUID PHASE COEFICIENT
eq3l_1=(1-e1)*A1*vl_1*t_factor/(1-e3)/V3;
eq3l_2=-vl_5*A5*(1-e5)*t_factor/(1-e3)/V3;
eq3l_3=kla3*t_factor;

%COEFICIENT OF GAS SEPARATOR 2(CSTR)
    %GAS PHASE COEFICIENT
%Input from top connection
eq4g_1=e5*A5*vg_5*t_factor/(e4*V4);
%Output to downcomer
eq4g_2=-vg_2*A2*e2*t_factor/(e4*V4);
%Output by mass transfer
eq4g_3=-(1-e4)*kla4*t_factor/e4/Henry;
%Output to atm
eq4g_4=-Qg_in*A4/(A3+A4)*t_factor/(e4*V4);

    %LIQUID PHASE COEFICIENT
eq4l_1=(1-e5)*A5*vl_5*t_factor/(1-e4)/V4;
eq4l_2=-vl_2*A2*(1-e2)*t_factor/(1-e4)/V4;
eq4l_3=kla4*t_factor;

%COEFICIENT OF TOP CONNECTION TUBE(Dispersion)
    %GAS PHASE COEFICIENT
eq5g_1=-vg_5*t_factor/z_factor;
eq5g_2=eq5g_1/del_z2;
eq5g_3=eq5g_1/2/del_z2;
eq5g_4=disp_g*t_factor/z_factor^2;
eq5g_5=eq5g_4/del_z1^2;
eq5g_6=-(1-e5)*kla5*t_factor/e5/Henry;

    %LIQUID PHASE COEFICIENT
eq5l_1=-vl_5*t_factor/z_factor;
eq5l_2=eq5l_1/del_z2;
eq5l_3=eq5l_1/2/del_z2;
eq5l_4=disp_l*t_factor/z_factor^2;
eq5l_5=eq5l_4/del_z2^2;
eq5l_6=kla5*t_factor;

%COEFICIENT OF BOTTOM CONNECTION TUBE(Dispersion)
    %GAS PHASE COEFICIENT
eq6g_1=-vg_6*t_factor/z_factor;
eq6g_2=eq6g_1/del_z2;
eq6g_3=eq6g_1/2/del_z2;
eq6g_4=disp_g*t_factor/z_factor^2;
eq6g_5=eq6g_4/del_z2^2;
eq6g_6=-(1-e5)*kla6*t_factor/e6/Henry;

    %LIQUID PHASE COEFICIENT
eq6l_1=-vl_6*t_factor/z_factor;
eq6l_2=eq6l_1/del_z2;
eq6l_3=eq6l_1/2/del_z2;
eq6l_4=disp_l*t_factor/z_factor^2;
eq6l_5=eq6l_4/del_z2^2;
eq6l_6=kla6*t_factor;

%SET UP INITIAL AND BOUNDARY CONDITIONS
for j=1:1:m
Og_riser(1,j)=1;
Ol_riser(1,j)=0;
end

for j=1:1:n
Og_downcomer(1,j)=1;
Ol_downcomer(1,j)=0;
end

Og_sep1(1)=1;
Ol_sep1(1)=0;
Og_sep2(1)=1;
Ol_sep2(1)=0;

for j=1:1:o
Og_con_t(1,j)=1;
Ol_con_t(1,j)=0;
end

for j=1:1:p
Og_con_b(1,j)=1;
Ol_con_b(1,j)=0;
end
```

```
%SOLVE FOR OXYGEN RESPONSE
%Programe Beginning

%FIND OXYGEN CONCENTRATION WITH TIME

del_t1=0.00001;
del_t2=0.00001;

%del_t1=input('enter step size at the 1st period of time = ');
%del_t2=input('enter step size at the 2nd period of time = ');

z=1;
k=1;
c1=1;
c2=1;
round=2000000;

while ((Ol_riser(1,m)<0.99) && (z<round));
%Ol_riser(1,m)<1.10
%z<50000
   if Ol_riser(1,m)<0.8;
      h=del_t1;
      c1=z;
   else
      h=del_t2;
      c2=z;
    end

%Runge Kutta
%SET UP k l m n o p
%k1*************************************************************************************
******************************
%riser gas
      k11(1)=eq1g_2*(Og_riser(1,2)-Og_riser(1,1));
      k12(1)=eq1g_5*(Og_riser(1,3)-2*Og_riser(1,2)+Og_riser(1,1));
      k13(1)=eq1g_6*(Og_riser(1,1)-Ol_riser(1,1));
      k1(1)=h*(k11(1)+k12(1)+k13(1));

      k11(2)=eq1g_3*(Og_riser(1,3)-Og_riser(1,1));
      k12(2)=eq1g_5*(Og_riser(1,3)-2*Og_riser(1,2)+Og_riser(1,1));
      k13(2)=eq1g_6*(Og_riser(1,2)-Ol_riser(1,2));
      k1(2)=h*(k11(2)+k12(2)+k13(2));

    for j=3:1:m-2
      k11(j)=eq1g_3*(Og_riser(1,j+1)-Og_riser(1,j-1));
      k12(j)=eq1g_5*(Og_riser(1,j+1)-2*Og_riser(1,j)+Og_riser(1,j-1));
      k13(j)=eq1g_6*(Og_riser(1,j)-Ol_riser(1,j));
      k1(j)=h*(k11(j)+k12(j)+k13(j));
    end

      k11(m-1)=eq1g_3*(Og_riser(1,m)-Og_riser(1,m-2));
      k12(m-1)=eq1g_5*(Og_riser(1,m)-2*Og_riser(1,m-1)+Og_riser(1,m-2));
      k13(m-1)=eq1g_6*(Og_riser(1,m-1)-Ol_riser(1,m-1));
      k1(m-1)=h*(k11(m-1)+k12(m-1)+k13(m-1));

      k11(m)=eq1g_2*(Og_riser(1,m)-Og_riser(1,m-1));
      k12(m)=eq1g_5*(Og_riser(1,m)-2*Og_riser(1,m-1)+Og_riser(1,m-2));
      k13(m)=eq1g_6*(Og_riser(1,m)-Ol_riser(1,m));
      k1(m)=h*(k11(m)+k12(m)+k13(m));
%riser liquid
      l11(1)=eq1l_2*(Ol_riser(1,2)-Ol_riser(1,1));
      l12(1)=eq1l_5*(Ol_riser(1,3)-2*Ol_riser(1,2)+Ol_riser(1,1));
      l13(1)=eq1l_6*(Og_riser(1,1)-Ol_riser(1,1));
      l1(1)=h*(l11(1)+l12(1)+l13(1));

      l11(2)=eq1l_3*(Ol_riser(1,3)-Ol_riser(1,1));
      l12(2)=eq1l_5*(Ol_riser(1,3)-2*Ol_riser(1,2)+Ol_riser(1,1));
      l13(2)=eq1l_6*(Og_riser(1,2)-Ol_riser(1,2));
      l1(2)=h*(l11(2)+l12(2)+l13(2));


  for j=3:1:m-2
      l11(j)=eq1l_3*(Ol_riser(1,j+1)-Ol_riser(1,j-1));
      l12(j)=eq1l_5*(Ol_riser(1,j+1)-2*Ol_riser(1,j)+Ol_riser(1,j-1));
      l13(j)=eq1l_6*(Og_riser(1,j)-Ol_riser(1,j));
      l1(j)=h*(l11(j)+l12(j)+l13(j));
  end

      l11(m-1)=eq1l_3*(Ol_riser(1,m)-Ol_riser(1,m-2));
      l12(m-1)=eq1l_5*(Ol_riser(1,m)-2*Ol_riser(1,m-1)+Ol_riser(1,m-2));
      l13(m-1)=eq1l_6*(Og_riser(1,m-1)-Ol_riser(1,m-1));
      l1(m-1)=h*(l11(m-1)+l12(m-1)+l13(m-1));

      l11(m)=eq1l_2*(Ol_riser(1,m)-Ol_riser(1,m-1));
      l12(m)=eq1l_5*(Ol_riser(1,m)-2*Ol_riser(1,m-1)+Ol_riser(1,m-2));
      l13(m)=eq1l_6*(Og_riser(1,m)-Ol_riser(1,m));
      l1(m)=h*(l11(m)+l12(m)+l13(m));
%separator1
      m1=h*(eq3g_1*Og_riser(1,m)+eq3g_2*Og_sep1(1)+eq3g_3*(Og_sep1(1)-
Ol_sep1(1))+eq3g_4*Og_sep1(1));
      n1=h*(eq3l_1*Ol_riser(1,m)+eq3l_2*Ol_sep1(1)+eq3l_3*(Og_sep1(1)-Ol_sep1(1)));

%connection t gas
      o11(1)=eq5g_2*(Og_con_t(1,2)-Og_con_t(1,1));
      o12(1)=eq5g_5*(Og_con_t(1,3)-2*Og_con_t(1,2)+Og_con_t(1,1));
      o13(1)=eq5g_6*(Og_con_t(1,1)-Ol_con_t(1,1));
      o1(1)=h*(o11(1)+o12(1)+o13(1));

      o11(2)=eq5g_3*(Og_con_t(1,3)-Og_con_t(1,1));
      o12(2)=eq5g_5*(Og_con_t(1,3)-2*Og_con_t(1,2)+Og_con_t(1,1));
      o13(2)=eq5g_6*(Og_con_t(1,2)-Ol_con_t(1,2));
      o1(2)=h*(o11(2)+o12(2)+o13(2));

      for j=3:1:o-2
      o11(j)=eq5g_3*(Og_con_t(1,j+1)-Og_con_t(1,j-1));
      o12(j)=eq5g_5*(Og_con_t(1,j+1)-2*Og_con_t(1,j)+Og_con_t(1,j-1));
      o13(j)=eq5g_6*(Og_con_t(1,j)-Ol_con_t(1,j));
      o1(j)=h*(o11(j)+o12(j)+o13(j));
      end

      o11(o-1)=eq5g_3*(Og_con_t(1,o)-Og_con_t(1,o-2));
      o12(o-1)=eq5g_5*(Og_con_t(1,o)-2*Og_con_t(1,o-1)+Og_con_t(1,o-2));
      o13(o-1)=eq5g_6*(Og_con_t(1,o-1)-Ol_con_t(1,o-1));
      o1(o-1)=h*(o11(o-1)+o12(o-1)+o13(o-1));

      o11(o)=eq5g_2*(Og_con_t(1,o)-Og_con_t(1,o-1));
      o12(o)=eq5g_5*(Og_con_t(1,o)-2*Og_con_t(1,o-1)+Og_con_t(1,o-2));
      o13(o)=eq5g_6*(Og_con_t(1,o)-Ol_con_t(1,o));
      o1(o)=h*(o11(o)+o12(o)+o13(o));
%connection t liquid
      p11(1)=eq5l_2*(Ol_con_t(1,2)-Ol_con_t(1,1));
      p12(1)=eq5l_5*(Ol_con_t(1,3)-2*Ol_con_t(1,2)+Ol_con_t(1,1));
      p13(1)=eq5l_6*(Og_con_t(1,1)-Ol_con_t(1,1));
      p1(1)=h*(p11(1)+p12(1)+p13(1));

      p11(2)=eq5l_3*(Ol_con_t(1,3)-Ol_con_t(1,1));
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      p12(2)=eq5l_5*(Ol_con_t(1,3)-2*Ol_con_t(1,2)+Ol_con_t(1,1));
      p13(2)=eq5l_6*(Og_con_t(1,2)-Ol_con_t(1,2));
      p1(2)=h*(p11(2)+p12(2)+p13(2));

     for j=3:1:o-2
      p11(j)=eq5l_3*(Ol_con_t(1,j+1)-Ol_con_t(1,j-1));
      p12(j)=eq5l_5*(Ol_con_t(1,j+1)-2*Ol_con_t(1,j)+Ol_con_t(1,j-1));
      p13(j)=eq5l_6*(Og_con_t(1,j)-Ol_con_t(1,j));
      p1(j)=h*(p11(j)+p12(j)+p13(j));
     end

      p11(o-1)=eq5l_3*(Ol_con_t(1,o)-Ol_con_t(1,o-2));
      p12(o-1)=eq5l_5*(Ol_con_t(1,o)-2*Ol_con_t(1,o-1)+Ol_con_t(1,o-2));
      p13(o-1)=eq5l_6*(Og_con_t(1,o-1)-Ol_con_t(1,o-1));
      p1(o-1)=h*(p11(o-1)+p12(o-1)+p13(o-1));

      p11(o)=eq5l_2*(Ol_con_t(1,o)-Ol_con_t(1,o-1));
      p12(o)=eq5l_5*(Ol_con_t(1,o)-2*Ol_con_t(1,o-1)+Ol_con_t(1,o-2));
      p13(o)=eq5l_6*(Og_con_t(1,o)-Ol_con_t(1,o));
      p1(o)=h*(p11(o)+p12(o)+p13(o));
%separator2
      q1=h*(eq4g_1*Og_con_t(1,o)+eq4g_2*Og_sep2(1)+eq4g_3*(Og_sep2(1)-
Ol_sep2(1))+eq4g_4*Og_sep2(1));
      r1=h*(eq4l_1*Ol_con_t(1,o)+eq4l_2*Ol_sep2(1)+eq4l_3*(Og_sep2(1)-Ol_sep2(1)));

%downcomer gas
      s11(1)=eq2g_2*(Og_downcomer(1,2)-Og_downcomer(1,1));
      s12(1)=eq2g_5*(Og_downcomer(1,3)-2*Og_downcomer(1,2)+Og_downcomer(1,1));
      s13(1)=eq2g_6*(Og_downcomer(1,1)-Ol_downcomer(1,1));
      s1(1)=h*(s11(1)+s12(1)+s13(1));

      s11(2)=eq2g_3*(Og_downcomer(1,3)-Og_downcomer(1,1));
      s12(2)=eq2g_5*(Og_downcomer(1,3)-2*Og_downcomer(1,2)+Og_downcomer(1,1));
      s13(2)=eq2g_6*(Og_downcomer(1,2)-Ol_downcomer(1,2));
      s1(2)=h*(s11(2)+s12(2)+s13(2));

     for j=3:1:n-2
      s11(j)=eq2g_3*(Og_downcomer(1,j+1)-Og_downcomer(1,j-1));
      s12(j)=eq2g_5*(Og_downcomer(1,j+1)-2*Og_downcomer(1,j)+Og_downcomer(1,j-1));
      s13(j)=eq2g_6*(Og_downcomer(1,j)-Ol_downcomer(1,j));
      s1(j)=h*(s11(j)+s12(j)+s13(j));
     end

      s11(n-1)=eq2g_3*(Og_downcomer(1,n)-Og_downcomer(1,n-2));
      s12(n-1)=eq2g_5*(Og_downcomer(1,n)-2*Og_downcomer(1,n-1)+Og_downcomer(1,n-2));
      s13(n-1)=eq2g_6*(Og_downcomer(1,n-1)-Ol_downcomer(1,n-1));
      s1(n-1)=h*(s11(n-1)+s12(n-1)+s13(n-1));

      s11(n)=eq2g_2*(Og_downcomer(1,n)-Og_downcomer(1,n-1));
      s12(n)=eq2g_5*(Og_downcomer(1,n)-2*Og_downcomer(1,n-1)+Og_downcomer(1,n-2));
      s13(n)=eq2g_6*(Og_downcomer(1,n)-Ol_downcomer(1,n));
      s1(n)=h*(s11(n)+s12(n)+s13(n));
%downcomer liquid
      t11(1)=eq2l_2*(Ol_downcomer(1,2)-Ol_downcomer(1,1));
      t12(1)=eq2l_5*(Ol_downcomer(1,3)-2*Ol_downcomer(1,2)+Ol_downcomer(1,1));
      t13(1)=eq2l_6*(Og_downcomer(1,1)-Ol_downcomer(1,1));
      t1(1)=h*(t11(1)+t12(1)+t13(1));

      t11(2)=eq2l_3*(Ol_downcomer(1,3)-Ol_downcomer(1,1));
      t12(2)=eq2l_5*(Ol_downcomer(1,3)-2*Ol_downcomer(1,2)+Ol_downcomer(1,1));
      t13(2)=eq2l_6*(Og_downcomer(1,2)-Ol_downcomer(1,2));
      t1(2)=h*(t11(2)+t12(2)+t13(2));

      for j=3:1:n-2
      t11(j)=eq2l_3*(Ol_downcomer(1,j+1)-Ol_downcomer(1,j-1));
      t12(j)=eq2l_5*(Ol_downcomer(1,j+1)-2*Ol_downcomer(1,j)+Ol_downcomer(1,j-1));
      t13(j)=eq2l_6*(Og_downcomer(1,j)-Ol_downcomer(1,j));
      t1(j)=h*(t11(j)+t12(j)+t13(j));
      end

      t11(n-1)=eq2l_3*(Ol_downcomer(1,n)-Ol_downcomer(1,n-2));
      t12(n-1)=eq2l_5*(Ol_downcomer(1,n)-2*Ol_downcomer(1,n-1)+Ol_downcomer(1,n-2));
      t13(n-1)=eq2l_6*(Og_downcomer(1,n-1)-Ol_downcomer(1,n-1));
      t1(n-1)=h*(t11(n-1)+t12(n-1)+t13(n-1));

      t11(n)=eq2l_2*(Ol_downcomer(1,n)-Ol_downcomer(1,n-1));
      t12(n)=eq2l_5*(Ol_downcomer(1,n)-2*Ol_downcomer(1,n-1)+Ol_downcomer(1,n-2));
      t13(n)=eq2l_6*(Og_downcomer(1,n)-Ol_downcomer(1,n));
      t1(n)=h*(t11(n)+t12(n)+t13(n));

%connection b gas
      u11(1)=eq6g_2*(Og_con_b(1,2)-Og_con_b(1,1));
      u12(1)=eq6g_5*(Og_con_b(1,3)-2*Og_con_b(1,2)+Og_con_b(1,1));
      u13(1)=eq6g_6*(Og_con_b(1,1)-Ol_con_b(1,1));
      u1(1)=h*(u11(1)+u12(1)+u13(1));

      u11(2)=eq6g_3*(Og_con_b(1,3)-Og_con_b(1,1));
      u12(2)=eq6g_5*(Og_con_b(1,3)-2*Og_con_b(1,2)+Og_con_b(1,1));
      u13(2)=eq6g_6*(Og_con_b(1,2)-Ol_con_b(1,2));
      u1(2)=h*(u11(2)+u12(2)+u13(2));

    for j=3:1:p-2
      u11(j)=eq6g_3*(Og_con_b(1,j+1)-Og_con_b(1,j-1));
      u12(j)=eq6g_5*(Og_con_b(1,j+1)-2*Og_con_b(1,j)+Og_con_b(1,j-1));
      u13(j)=eq6g_6*(Og_con_b(1,j)-Ol_con_b(1,j));
      u1(j)=h*(u11(j)+u12(j)+u13(j));
    end

      u11(p-1)=eq6g_3*(Og_con_b(1,p)-Og_con_b(1,p-2));
      u12(p-1)=eq6g_5*(Og_con_b(1,p)-2*Og_con_b(1,p-1)+Og_con_b(1,p-2));
      u13(p-1)=eq6g_6*(Og_con_b(1,p-1)-Ol_con_b(1,p-1));
      u1(p-1)=h*(u11(p-1)+u12(p-1)+u13(p-1));

      u11(p)=eq6g_2*(Og_con_b(1,p)-Og_con_b(1,p-1));
      u12(p)=eq6g_5*(Og_con_b(1,p)-2*Og_con_b(1,p-1)+Og_con_b(1,p-2));
      u13(p)=eq6g_6*(Og_con_b(1,p)-Ol_con_b(1,p));
      u1(p)=h*(u11(p)+u12(p)+u13(p));
%connection b liquid
      v11(1)=eq6l_2*(Ol_con_b(1,2)-Ol_con_b(1,1));
      v12(1)=eq6l_5*(Ol_con_b(1,3)-2*Ol_con_b(1,2)+Ol_con_b(1,1));
      v13(1)=eq6l_6*(Og_con_b(1,1)-Ol_con_b(1,1));
      v1(1)=h*(v11(1)+v12(1)+v13(1));

      v11(2)=eq6l_3*(Ol_con_b(1,3)-Ol_con_b(1,1));
      v12(2)=eq6l_5*(Ol_con_b(1,3)-2*Ol_con_b(1,2)+Ol_con_b(1,1));
      v13(2)=eq6l_6*(Og_con_b(1,2)-Ol_con_b(1,2));
      v1(2)=h*(v11(2)+v12(2)+v13(2));

   for j=3:1:p-2
      v11(j)=eq6l_3*(Ol_con_b(1,j+1)-Ol_con_b(1,j-1));
      v12(j)=eq6l_5*(Ol_con_b(1,j+1)-2*Ol_con_b(1,j)+Ol_con_b(1,j-1));
      v13(j)=eq6l_6*(Og_con_b(1,j)-Ol_con_b(1,j));
      v1(j)=h*(v11(j)+v12(j)+v13(j));
   end

      v11(p-1)=eq6l_3*(Ol_con_b(1,p)-Ol_con_b(1,p-2));
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      v12(p-1)=eq6l_5*(Ol_con_b(1,p)-2*Ol_con_b(1,p-1)+Ol_con_b(1,p-2));
      v13(p-1)=eq6l_6*(Og_con_b(1,p-1)-Ol_con_b(1,p-1));
      v1(p-1)=h*(v11(p-1)+v12(p-1)+v13(p-1));

      v11(p)=eq6l_2*(Ol_con_b(1,p)-Ol_con_b(1,p-1));
      v12(p)=eq6l_5*(Ol_con_b(1,p)-2*Ol_con_b(1,p-1)+Ol_con_b(1,p-2));
      v13(p)=eq6l_6*(Og_con_b(1,p)-Ol_con_b(1,p));
      v1(p)=h*(v11(p)+v12(p)+v13(p));

%k2*********************************************************************************
********************************
%riser gas
      k21(1)=eq1g_2*(Og_riser(1,2)+k1(2)/2-Og_riser(1,1)-k1(1)/2);
      k22(1)=eq1g_5*(Og_riser(1,3)+k1(3)/2-
2*(Og_riser(1,2)+k1(2)/2)+Og_riser(1,1)+k1(1)/2);
      k23(1)=eq1g_6*(Og_riser(1,1)+k1(1)/2-Ol_riser(1,1)-l1(1)/2);
      k2(1)=h*(k21(1)+k22(1)+k23(1));

      k21(2)=eq1g_3*(Og_riser(1,3)+k1(3)/2-Og_riser(1,1)-k1(1)/2);
      k22(2)=eq1g_5*(Og_riser(1,3)+k1(3)/2-
2*(Og_riser(1,2)+k1(2)/2)+Og_riser(1,1)+k1(1)/2);
      k23(2)=eq1g_6*(Og_riser(1,2)+k1(2)/2-Ol_riser(1,2)-l1(2)/2);
      k2(2)=h*(k21(2)+k22(2)+k23(2));

    for j=3:1:m-2
      k21(j)=eq1g_3*(Og_riser(1,j+1)+k1(j+1)/2-Og_riser(1,j-1)-k1(j-1)/2);
      k22(j)=eq1g_5*(Og_riser(1,j+1)+k1(j+1)/2-
2*(Og_riser(1,j)+k1(j)/2)+Og_riser(1,j-1)+k1(j-1)/2);
      k23(j)=eq1g_6*(Og_riser(1,j)+k1(j)/2-Ol_riser(1,j)-l1(j)/2);
      k2(j)=h*(k21(j)+k22(j)+k23(j));
    end

      k21(m-1)=eq1g_3*(Og_riser(1,m)+k1(m)/2-Og_riser(1,m-2)-k1(m-2)/2);
      k22(m-1)=eq1g_5*(Og_riser(1,m)+k1(m)/2-2*(Og_riser(1,m-1)+k1(m-
1)/2)+Og_riser(1,m-2)+k1(m-2)/2);
      k23(m-1)=eq1g_6*(Og_riser(1,m-1)+k1(m-1)/2-Ol_riser(1,m-1)-l1(m-1)/2);
      k2(m-1)=h*(k21(m-1)+k22(m-1)+k23(m-1));

      k21(m)=eq1g_2*(Og_riser(1,m)+k1(m)/2-Og_riser(1,m-1)-k1(m-1)/2);
      k22(m)=eq1g_5*(Og_riser(1,m)+k1(m)/2-2*(Og_riser(1,m-1)+k1(m-
1)/2)+Og_riser(1,m-2)+k1(m-2)/2);
      k23(m)=eq1g_6*(Og_riser(1,m)+k1(m)/2-Ol_riser(1,m)-l1(m)/2);
      k2(m)=h*(k21(m)+k22(m)+k23(m));
%riser liquid
      l21(1)=eq1l_2*(Ol_riser(1,2)+l1(2)/2-Ol_riser(1,1)-l1(1)/2);
      l22(1)=eq1l_5*(Ol_riser(1,3)+l1(3)/2-
2*(Ol_riser(1,2)+l1(2)/2)+Ol_riser(1,1)+l1(1)/2);
      l23(1)=eq1l_6*(Og_riser(1,1)+k1(1)/2-Ol_riser(1,1)-l1(1)/2);
      l2(1)=h*(l21(1)+l22(1)+l23(1));

      l21(2)=eq1l_3*(Ol_riser(1,3)+l1(3)/2-Ol_riser(1,1)-l1(1)/2);
      l22(2)=eq1l_5*(Ol_riser(1,3)+l1(3)/2-
2*(Ol_riser(1,2)+l1(2)/2)+Ol_riser(1,1)+l1(1)/2);
      l23(2)=eq1l_6*(Og_riser(1,2)+k1(2)/2-Ol_riser(1,2)-l1(2)/2);
      l2(2)=h*(l21(2)+l22(2)+l23(2));

      for j=3:1:m-2
      l21(j)=eq1l_3*(Ol_riser(1,j+1)+l1(j+1)/2-Ol_riser(1,j-1)-l1(j-1)/2);
      l22(j)=eq1l_5*(Ol_riser(1,j+1)+l1(j+1)/2-
2*(Ol_riser(1,j)+l1(j)/2)+Ol_riser(1,j-1)+l1(j-1)/2);
      l23(j)=eq1l_6*(Og_riser(1,j)+k1(j)/2-Ol_riser(1,j)-l1(j)/2);
      l2(j)=h*(l21(j)+l22(j)+l23(j));
      end

      l21(m-1)=eq1l_3*(Ol_riser(1,m)+l1(m)/2-Ol_riser(1,m-2)-l1(m-2)/2);
      l22(m-1)=eq1l_5*(Ol_riser(1,m)+l1(m)/2-2*(Ol_riser(1,m-1)+l1(m-
1)/2)+Ol_riser(1,m-2)+l1(m-2)/2);
      l23(m-1)=eq1l_6*(Og_riser(1,m-1)+k1(m-1)/2-Ol_riser(1,m-1)-l1(m-1)/2);
      l2(m-1)=h*(l21(m-1)+l22(m-1)+l23(m-1));

      l21(m)=eq1l_2*(Ol_riser(1,m)+l1(m)/2-Ol_riser(1,m-1)-l1(m-1)/2);
      l22(m)=eq1l_5*(Ol_riser(1,m)+l1(m)/2-2*(Ol_riser(1,m-1)+l1(m-
1)/2)+Ol_riser(1,m-2)+l1(m-2)/2);
      l23(m)=eq1l_6*(Og_riser(1,m)+k1(m)/2-Ol_riser(1,m)-l1(m)/2);
      l2(m)=h*(l21(m)+l22(m)+l23(m));
%separator1

m2=h*(eq3g_1*(Og_riser(1,m)+k1(m)/2)+eq3g_2*(Og_sep1(1)+m1/2)+eq3g_3*((Og_sep1(1)+m1/
2)-(Ol_sep1(1)+n1/2))+eq3g_4*(Og_sep1(1)+m1/2));

n2=h*(eq3l_1*(Ol_riser(1,m)+l1(m)/2)+eq3l_2*(Ol_sep1(1)+n1/2)+eq3l_3*((Og_sep1(1)+m1/
2)-(Ol_sep1(1)+n1/2)));

%connection t gas
      o21(1)=eq5g_2*(Og_con_t(1,2)+o1(2)/2-Og_con_t(1,1)-o1(1)/2);
      o22(1)=eq5g_5*(Og_con_t(1,3)+o1(3)/2-
2*(Og_con_t(1,2)+o1(2)/2)+Og_con_t(1,1)+o1(1)/2);
      o23(1)=eq5g_6*(Og_con_t(1,1)+o1(1)/2-Ol_con_t(1,1)-p1(1)/2);
      o2(1)=h*(o21(1)+o22(1)+o23(1));

      o21(2)=eq5g_3*(Og_con_t(1,3)+o1(3)/2-Og_con_t(1,1)-o1(1)/2);
      o22(2)=eq5g_5*(Og_con_t(1,3)+o1(3)/2-
2*(Og_con_t(1,2)+o1(2)/2)+Og_con_t(1,1)+o1(1)/2);
      o23(2)=eq5g_6*(Og_con_t(1,2)+o1(2)/2-Ol_con_t(1,2)-p1(2)/2);
      o2(2)=h*(o21(2)+o22(2)+o23(2));

      for j=3:1:o-2
      o21(j)=eq5g_3*(Og_con_t(1,j+1)+o1(j+1)/2-Og_con_t(1,j-1)-o1(j-1)/2);
      o22(j)=eq5g_5*(Og_con_t(1,j+1)+o1(j+1)/2-
2*(Og_con_t(1,j)+o1(j)/2)+Og_con_t(1,j-1)+o1(j-1)/2);
      o23(j)=eq5g_6*(Og_con_t(1,j)+o1(j)/2-Ol_con_t(1,j)-p1(j)/2);
      o2(j)=h*(o21(j)+o22(j)+o23(j));
      end

      o21(o-1)=eq5g_3*(Og_con_t(1,o)+o1(o)/2-Og_con_t(1,o-2)-o1(o-2)/2);
      o22(o-1)=eq5g_5*(Og_con_t(1,o)+o1(o)/2-2*(Og_con_t(1,o-1)+o1(o-
1)/2)+Og_con_t(1,o-2)+o1(o-2)/2);
      o23(o-1)=eq5g_6*(Og_con_t(1,o-1)+o1(o-1)/2-Ol_con_t(1,o-1)-p1(o-1)/2);
      o2(o-1)=h*(o21(o-1)+o22(o-1)+o23(o-1));

      o21(o)=eq5g_2*(Og_con_t(1,o)+o1(o)/2-Og_con_t(1,o-1)-o1(o-1)/2);
      o22(o)=eq5g_5*(Og_con_t(1,o)+o1(o)/2-2*(Og_con_t(1,o-1)+o1(o-
1)/2)+Og_con_t(1,o-2)+o1(o-2)/2);
      o23(o)=eq5g_6*(Og_con_t(1,o)+o1(o)/2-Ol_con_t(1,o)-p1(o)/2);
      o2(o)=h*(o21(o)+o22(o)+o23(o));
%connection t liquid
      p21(1)=eq5l_2*(Ol_con_t(1,2)+p1(2)/2-Ol_con_t(1,1)-p1(1)/2);
      p22(1)=eq5l_5*(Ol_con_t(1,3)+p1(3)/2-
2*(Ol_con_t(1,2)+p1(2)/2)+Ol_con_t(1,1)+p1(1)/2);
      p23(1)=eq5l_6*(Og_con_t(1,1)+o1(1)/2-Ol_con_t(1,1)-p1(1)/2);
      p2(1)=h*(p21(1)+p22(1)+p23(1));

      p21(2)=eq5l_3*(Ol_con_t(1,3)+p1(3)/2-Ol_con_t(1,1)-p1(1)/2);
      p22(2)=eq5l_5*(Ol_con_t(1,3)+p1(3)/2-
2*(Ol_con_t(1,2)+p1(2)/2)+Ol_con_t(1,1)+p1(1)/2);
      p23(2)=eq5l_6*(Og_con_t(1,2)+o1(2)/2-Ol_con_t(1,2)-p1(2)/2);
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      p2(2)=h*(p21(2)+p22(2)+p23(2));

   for j=3:1:o-2
      p21(j)=eq5l_3*(Ol_con_t(1,j+1)+p1(j+1)/2-Ol_con_t(1,j-1)-p1(j-1)/2);
      p22(j)=eq5l_5*(Ol_con_t(1,j+1)+p1(j+1)/2-
2*(Ol_con_t(1,j)+p1(j)/2)+Ol_con_t(1,j-1)+p1(j-1)/2);
      p23(j)=eq5l_6*(Og_con_t(1,j)+o1(j)/2-Ol_con_t(1,j)-p1(j)/2);
      p2(j)=h*(p21(j)+p22(j)+p23(j));
   end

      p21(o-1)=eq5l_3*(Ol_con_t(1,o)+p1(o)/2-Ol_con_t(1,o-2)-p1(o-2)/2);
      p22(o-1)=eq5l_5*(Ol_con_t(1,o)+p1(o)/2-2*(Ol_con_t(1,o-1)+p1(o-
1)/2)+Ol_con_t(1,o-2)+p1(o-2)/2);
      p23(o-1)=eq5l_6*(Og_con_t(1,o-1)+o1(o-1)/2-Ol_con_t(1,o-1)-p1(o-1)/2);
      p2(o-1)=h*(p21(o-1)+p22(o-1)+p23(o-1));

      p21(o)=eq5l_2*(Ol_con_t(1,o)+p1(o)/2-Ol_con_t(1,o-1)-p1(o-1)/2);
      p22(o)=eq5l_5*(Ol_con_t(1,o)+p1(o)/2-2*(Ol_con_t(1,o-1)+p1(o-
1)/2)+Ol_con_t(1,o-2)+p1(o-2)/2);
      p23(o)=eq5l_6*(Og_con_t(1,o)+o1(o)/2-Ol_con_t(1,o)-p1(o)/2);
      p2(o)=h*(p21(o)+p22(o)+p23(o));
%separator2

q2=h*(eq4g_1*(Og_con_t(1,o)+o1(o)/2)+eq4g_2*(Og_sep2(1)+q1/2)+eq4g_3*((Og_sep2(1)+q1/
2)-(Ol_sep2(1)+r1/2))+eq4g_4*(Og_sep2(1)+q1/2));

r2=h*(eq4l_1*(Ol_con_t(1,o)+p1(o)/2)+eq4l_2*(Ol_sep2(1)+r1/2)+eq4l_3*((Og_sep2(1)+q1/
2)-(Ol_sep2(1)+r1/2)));

%downcomer gas
      s21(1)=eq2g_2*(Og_downcomer(1,2)+s1(2)/2-Og_downcomer(1,1)-s1(1)/2);
      s22(1)=eq2g_5*(Og_downcomer(1,3)+s1(3)/2-
2*(Og_downcomer(1,2)+s1(2)/2)+Og_downcomer(1,1)+s1(1)/2);
      s23(1)=eq2g_6*(Og_downcomer(1,1)+s1(1)/2-Ol_downcomer(1,1)-t1(1)/2);
      s2(1)=h*(s21(1)+s22(1)+s23(1));

      s21(2)=eq2g_3*(Og_downcomer(1,3)+s1(3)/2-Og_downcomer(1,1)-s1(1)/2);
      s22(2)=eq2g_5*(Og_downcomer(1,3)+s1(3)/2-
2*(Og_downcomer(1,2)+s1(2)/2)+Og_downcomer(1,1)+s1(1)/2);
      s23(2)=eq2g_6*(Og_downcomer(1,2)+s1(2)/2-Ol_downcomer(1,2)-t1(2)/2);
      s2(2)=h*(s21(2)+s22(2)+s23(2));

  for j=3:1:n-2
      s21(j)=eq2g_3*(Og_downcomer(1,j+1)+s1(j+1)/2-Og_downcomer(1,j-1)-s1(j-1)/2);
      s22(j)=eq2g_5*(Og_downcomer(1,j+1)+s1(j+1)/2-
2*(Og_downcomer(1,j)+s1(j)/2)+Og_downcomer(1,j-1)+s1(j-1)/2);
      s23(j)=eq2g_6*(Og_downcomer(1,j)+s1(j)/2-Ol_downcomer(1,j)-t1(j)/2);
      s2(j)=h*(s21(j)+s22(j)+s23(j));
  end

      s21(n-1)=eq2g_3*(Og_downcomer(1,n)+s1(n)/2-Og_downcomer(1,n-2)-s1(n-2)/2);
      s22(n-1)=eq2g_5*(Og_downcomer(1,n)+s1(n)/2-2*(Og_downcomer(1,n-1)+s1(n-
1)/2)+Og_downcomer(1,n-2)+s1(n-2)/2);
      s23(n-1)=eq2g_6*(Og_downcomer(1,n-1)+s1(n-1)/2-Ol_downcomer(1,n-1)-t1(n-1)/2);
      s2(n-1)=h*(s21(n-1)+s22(n-1)+s23(n-1));

      s21(n)=eq2g_2*(Og_downcomer(1,n)+s1(n)/2-Og_downcomer(1,n-1)-s1(n-1)/2);
      s22(n)=eq2g_5*(Og_downcomer(1,n)+s1(n)/2-2*(Og_downcomer(1,n-1)+s1(n-
1)/2)+Og_downcomer(1,n-2)+s1(n-2)/2);
      s23(n)=eq2g_6*(Og_downcomer(1,n)+s1(n)/2-Ol_downcomer(1,n)-t1(n)/2);
      s2(n)=h*(s21(n)+s22(n)+s23(n));
%downcomer liquid
      t21(1)=eq2l_2*(Ol_downcomer(1,2)+t1(2)/2-Ol_downcomer(1,1)-t1(1)/2);
      t22(1)=eq2l_5*(Ol_downcomer(1,3)+t1(3)/2-
2*(Ol_downcomer(1,2)+t1(2)/2)+Ol_downcomer(1,1)+t1(1)/2);
      t23(1)=eq2l_6*(Og_downcomer(1,1)+s1(1)/2-Ol_downcomer(1,1)-t1(1)/2);
      t2(1)=h*(t21(1)+t22(1)+t23(1));

      t21(2)=eq2l_3*(Ol_downcomer(1,3)+t1(3)/2-Ol_downcomer(1,1)-t1(1)/2);
      t22(2)=eq2l_5*(Ol_downcomer(1,3)+t1(3)/2-
2*(Ol_downcomer(1,2)+t1(2)/2)+Ol_downcomer(1,1)+t1(1)/2);
      t23(2)=eq2l_6*(Og_downcomer(1,2)+s1(2)/2-Ol_downcomer(1,2)-t1(2)/2);
      t2(2)=h*(t21(2)+t22(2)+t23(2));

      for j=3:1:n-2
      t21(j)=eq2l_3*(Ol_downcomer(1,j+1)+t1(j+1)/2-Ol_downcomer(1,j-1)-t1(j-1)/2);
      t22(j)=eq2l_5*(Ol_downcomer(1,j+1)+t1(j+1)/2-
2*(Ol_downcomer(1,j)+t1(j)/2)+Ol_downcomer(1,j-1)+t1(j-1)/2);
      t23(j)=eq2l_6*(Og_downcomer(1,j)+s1(j)/2-Ol_downcomer(1,j)-t1(j)/2);
      t2(j)=h*(t21(j)+t22(j)+t23(j));
      end

      t21(n-1)=eq2l_3*(Ol_downcomer(1,n)+t1(n)/2-Ol_downcomer(1,n-2)-t1(n-2)/2);
      t22(n-1)=eq2l_5*(Ol_downcomer(1,n)+t1(n)/2-2*(Ol_downcomer(1,n-1)+t1(n-
1)/2)+Ol_downcomer(1,n-2)+t1(n-2)/2);
      t23(n-1)=eq2l_6*(Og_downcomer(1,n-1)+s1(n-1)/2-Ol_downcomer(1,n-1)-t1(n-1)/2);
      t2(n-1)=h*(t21(n-1)+t22(n-1)+t23(n-1));

      t21(n)=eq2l_2*(Ol_downcomer(1,n)+t1(n)/2-Ol_downcomer(1,n-1)-t1(n-1)/2);
      t22(n)=eq2l_5*(Ol_downcomer(1,n)+t1(n)/2-2*(Ol_downcomer(1,n-1)+t1(n-
1)/2)+Ol_downcomer(1,n-2)+t1(n-2)/2);
      t23(n)=eq2l_6*(Og_downcomer(1,n)+s1(n)/2-Ol_downcomer(1,n)-t1(n)/2);
      t2(n)=h*(t21(n)+t22(n)+t23(n));

%connection b gas
      u21(1)=eq6g_2*(Og_con_b(1,2)+u1(2)/2-Og_con_b(1,1)-u1(1)/2);
      u22(1)=eq6g_5*(Og_con_b(1,3)+u1(3)/2-
2*(Og_con_b(1,2)+u1(2)/2)+Og_con_b(1,1)+u1(1)/2);
      u23(1)=eq6g_6*(Og_con_b(1,1)+u1(1)/2-Ol_con_b(1,1)-v1(1)/2);
      u2(1)=h*(u21(1)+u22(1)+u23(1));

      u21(2)=eq6g_3*(Og_con_b(1,3)+u1(3)/2-Og_con_b(1,1)-u1(1)/2);
      u22(2)=eq6g_5*(Og_con_b(1,3)+u1(3)/2-
2*(Og_con_b(1,2)+u1(2)/2)+Og_con_b(1,1)+u1(1)/2);
      u23(2)=eq6g_6*(Og_con_b(1,2)+u1(2)/2-Ol_con_b(1,2)-v1(2)/2);
      u2(2)=h*(u21(2)+u22(2)+u23(2));

       for j=3:1:p-2
      u21(j)=eq6g_3*(Og_con_b(1,j+1)+u1(j+1)/2-Og_con_b(1,j-1)-u1(j-1)/2);
      u22(j)=eq6g_5*(Og_con_b(1,j+1)+u1(j+1)/2-
2*(Og_con_b(1,j)+u1(j)/2)+Og_con_b(1,j-1)+u1(j-1)/2);
      u23(j)=eq6g_6*(Og_con_b(1,j)+u1(j)/2-Ol_con_b(1,j)-v1(j)/2);
      u2(j)=h*(u21(j)+u22(j)+u23(j));
       end

      u21(p-1)=eq6g_3*(Og_con_b(1,p)+u1(p)/2-Og_con_b(1,p-2)-u1(p-2)/2);
      u22(p-1)=eq6g_5*(Og_con_b(1,p)+u1(p)/2-2*(Og_con_b(1,p-1)+u1(p-
1)/2)+Og_con_b(1,p-2)+u1(p-2)/2);
      u23(p-1)=eq6g_6*(Og_con_b(1,p-1)+u1(p-1)/2-Ol_con_b(1,p-1)-v1(p-1)/2);
      u2(p-1)=h*(u21(p-1)+u22(p-1)+u23(p-1));

      u21(p)=eq6g_2*(Og_con_b(1,p)+u1(p)/2-Og_con_b(1,p-1)-u1(p-1)/2);
      u22(p)=eq6g_5*(Og_con_b(1,p)+u1(p)/2-2*(Og_con_b(1,p-1)+u1(p-
1)/2)+Og_con_b(1,p-2)+u1(p-2)/2);
      u23(p)=eq6g_6*(Og_con_b(1,p)+u1(p)/2-Ol_con_b(1,p)-v1(p)/2);
      u2(p)=h*(u21(p)+u22(p)+u23(p));
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```
%connection b liquid
      v21(1)=eq6l_2*(Ol_con_b(1,2)+v1(2)/2-Ol_con_b(1,1)-v1(1)/2);
      v22(1)=eq6l_5*(Ol_con_b(1,3)+v1(3)/2-
2*(Ol_con_b(1,2)+v1(2)/2)+Ol_con_b(1,1)+v1(1)/2);
      v23(1)=eq6l_6*(Og_con_b(1,1)+u1(1)/2-Ol_con_b(1,1)-v1(1)/2);
      v2(1)=h*(v21(1)+v22(1)+v23(1));

      v21(2)=eq6l_3*(Ol_con_b(1,3)+v1(3)/2-Ol_con_b(1,1)-v1(1)/2);
      v22(2)=eq6l_5*(Ol_con_b(1,3)+v1(3)/2-
2*(Ol_con_b(1,2)+v1(2)/2)+Ol_con_b(1,1)+v1(1)/2);
      v23(2)=eq6l_6*(Og_con_b(1,2)+u1(2)/2-Ol_con_b(1,2)-v1(2)/2);
      v2(2)=h*(v21(2)+v22(2)+v23(2));

      for j=3:1:p-2
      v21(j)=eq6l_3*(Ol_con_b(1,j+1)+v1(j+1)/2-Ol_con_b(1,j-1)-v1(j-1)/2);
      v22(j)=eq6l_5*(Ol_con_b(1,j+1)+v1(j+1)/2-
2*(Ol_con_b(1,j)+v1(j)/2)+Ol_con_b(1,j-1)+v1(j-1)/2);
      v23(j)=eq6l_6*(Og_con_b(1,j)+u1(j)/2-Ol_con_b(1,j)-v1(j)/2);
      v2(j)=h*(v21(j)+v22(j)+v23(j));
      end

      v21(p-1)=eq6l_3*(Ol_con_b(1,p)+v1(p)/2-Ol_con_b(1,p-2)-v1(p-2)/2);
      v22(p-1)=eq6l_5*(Ol_con_b(1,p)+v1(p)/2-2*(Ol_con_b(1,p-1)+v1(p-
1)/2)+Ol_con_b(1,p-2)+v1(p-2)/2);
      v23(p-1)=eq6l_6*(Og_con_b(1,p-1)+u1(p-1)/2-Ol_con_b(1,p-1)-v1(p-1)/2);
      v2(p-1)=h*(v21(p-1)+v22(p-1)+v23(p-1));

      v21(p)=eq6l_2*(Ol_con_b(1,p)+v1(p)/2-Ol_con_b(1,p-1)-v1(p-1)/2);
      v22(p)=eq6l_5*(Ol_con_b(1,p)+v1(p)/2-2*(Ol_con_b(1,p-1)+v1(p-
1)/2)+Ol_con_b(1,p-2)+v1(p-2)/2);
      v23(p)=eq6l_6*(Og_con_b(1,p)+u1(p)/2-Ol_con_b(1,p)-v1(p)/2);
      v2(p)=h*(v21(p)+v22(p)+v23(p));

%k3*********************************************************************************
********************************
%riser gas
      k31(1)=eq1g_2*(Og_riser(1,2)+k2(2)/2-Og_riser(1,1)-k2(1)/2);
      k32(1)=eq1g_5*(Og_riser(1,3)+k2(3)/2-
2*(Og_riser(1,2)+k2(2)/2)+Og_riser(1,1)+k2(1)/2);
      k33(1)=eq1g_6*(Og_riser(1,1)+k2(1)/2-Ol_riser(1,1)-l2(1)/2);
      k3(1)=h*(k31(1)+k32(1)+k33(1));

      k31(2)=eq1g_3*(Og_riser(1,3)+k2(3)/2-Og_riser(1,1)-k2(1)/2);
      k32(2)=eq1g_5*(Og_riser(1,3)+k2(3)/2-
2*(Og_riser(1,2)+k2(2)/2)+Og_riser(1,1)+k2(1)/2);
      k33(2)=eq1g_6*(Og_riser(1,2)+k2(2)/2-Ol_riser(1,2)-l2(2)/2);
      k3(2)=h*(k31(2)+k32(2)+k33(2));

      for j=3:1:m-2
      k31(j)=eq1g_3*(Og_riser(1,j+1)+k2(j+1)/2-Og_riser(1,j-1)-k2(j-1)/2);
      k32(j)=eq1g_5*(Og_riser(1,j+1)+k2(j+1)/2-
2*(Og_riser(1,j)+k2(j)/2)+Og_riser(1,j-1)+k2(j-1)/2);
      k33(j)=eq1g_6*(Og_riser(1,j)+k2(j)/2-Ol_riser(1,j)-l2(j)/2);
      k3(j)=h*(k31(j)+k32(j)+k33(j));
      end

      k31(m-1)=eq1g_3*(Og_riser(1,m)+k2(m)/2-Og_riser(1,m-2)-k2(m-2)/2);
      k32(m-1)=eq1g_5*(Og_riser(1,m)+k2(m)/2-2*(Og_riser(1,m-1)+k2(m-
1)/2)+Og_riser(1,m-2)+k2(m-2)/2);
      k33(m-1)=eq1g_6*(Og_riser(1,m-1)+k2(m-1)/2-Ol_riser(1,m-1)-l2(m-1)/2);
      k3(m-1)=h*(k31(m-1)+k32(m-1)+k33(m-1));

      k31(m)=eq1g_2*(Og_riser(1,m)+k2(m)/2-Og_riser(1,m-1)-k2(m-1)/2);
      k32(m)=eq1g_5*(Og_riser(1,m)+k2(m)/2-2*(Og_riser(1,m-1)+k2(m-
1)/2)+Og_riser(1,m-2)+k2(m-2)/2);
      k33(m)=eq1g_6*(Og_riser(1,m)+k2(m)/2-Ol_riser(1,m)-l2(m)/2);
      k3(m)=h*(k31(m)+k32(m)+k33(m));
%riser liquid
      l31(1)=eq1l_2*(Ol_riser(1,2)+l2(2)/2-Ol_riser(1,1)-l2(1)/2);
      l32(1)=eq1l_5*(Ol_riser(1,3)+l2(3)/2-
2*(Ol_riser(1,2)+l2(2)/2)+Ol_riser(1,1)+l2(1)/2);
      l33(1)=eq1l_6*(Og_riser(1,1)+k2(1)/2-Ol_riser(1,1)-l2(1)/2);
      l3(1)=h*(l31(1)+l32(1)+l33(1));

      l31(2)=eq1l_3*(Ol_riser(1,3)+l2(3)/2-Ol_riser(1,1)-l2(1)/2);
      l32(2)=eq1l_5*(Ol_riser(1,3)+l2(3)/2-
2*(Ol_riser(1,2)+l2(2)/2)+Ol_riser(1,1)+l2(1)/2);
      l33(2)=eq1l_6*(Og_riser(1,2)+k2(2)/2-Ol_riser(1,2)-l2(2)/2);
      l3(2)=h*(l31(2)+l32(2)+l33(2));

      for j=3:1:m-2
      l31(j)=eq1l_3*(Ol_riser(1,j+1)+l2(j+1)/2-Ol_riser(1,j-1)-l2(j-1)/2);
      l32(j)=eq1l_5*(Ol_riser(1,j+1)+l2(j+1)/2-
2*(Ol_riser(1,j)+l2(j)/2)+Ol_riser(1,j-1)+l2(j-1)/2);
      l33(j)=eq1l_6*(Og_riser(1,j)+k2(j)/2-Ol_riser(1,j)-l2(j)/2);
      l3(j)=h*(l31(j)+l32(j)+l33(j));
      end

      l31(m-1)=eq1l_3*(Ol_riser(1,m)+l2(m)/2-Ol_riser(1,m-2)-l2(m-2)/2);
      l32(m-1)=eq1l_5*(Ol_riser(1,m)+l2(m)/2-2*(Ol_riser(1,m-1)+l2(m-
1)/2)+Ol_riser(1,m-2)+l2(m-2)/2);
      l33(m-1)=eq1l_6*(Og_riser(1,m-1)+k2(m-1)/2-Ol_riser(1,m-1)-l2(m-1)/2);
      l3(m-1)=h*(l31(m-1)+l32(m-1)+l33(m-1));

      l31(m)=eq1l_2*(Ol_riser(1,m)+l2(m)/2-Ol_riser(1,m-1)-l2(m-1)/2);
      l32(m)=eq1l_5*(Ol_riser(1,m)+l2(m)/2-2*(Ol_riser(1,m-1)+l2(m-
1)/2)+Ol_riser(1,m-2)+l2(m-2)/2);
      l33(m)=eq1l_6*(Og_riser(1,m)+k2(m)/2-Ol_riser(1,m)-l2(m)/2);
      l3(m)=h*(l31(m)+l32(m)+l33(m));
%separator1

m3=h*(eq3g_1*(Og_riser(1,m)+k2(m)/2)+eq3g_2*(Og_sep1(1)+m2/2)+eq3g_3*((Og_sep1(1)+m2/
2)-(Ol_sep1(1)+n2/2))+eq3g_4*(Og_sep1(1)+m2/2));

n3=h*(eq3l_1*(Ol_riser(1,m)+l2(m)/2)+eq3l_2*(Ol_sep1(1)+n2/2)+eq3l_3*((Og_sep1(1)+m2/
2)-(Ol_sep1(1)+n2/2)));

%connection t gas
      o31(1)=eq5g_2*(Og_con_t(1,2)+o2(2)/2-Og_con_t(1,1)-o2(1)/2);
      o32(1)=eq5g_5*(Og_con_t(1,3)+o2(3)/2-
2*(Og_con_t(1,2)+o2(2)/2)+Og_con_t(1,1)+o2(1)/2);
      o33(1)=eq5g_6*(Og_con_t(1,1)+o2(1)/2-Ol_con_t(1,1)-p2(1)/2);
      o3(1)=h*(o31(1)+o32(1)+o33(1));

      o31(2)=eq5g_3*(Og_con_t(1,3)+o2(3)/2-Og_con_t(1,1)-o2(1)/2);
      o32(2)=eq5g_5*(Og_con_t(1,3)+o2(3)/2-
2*(Og_con_t(1,2)+o2(2)/2)+Og_con_t(1,1)+o2(1)/2);
      o33(2)=eq5g_6*(Og_con_t(1,2)+o2(2)/2-Ol_con_t(1,2)-p2(2)/2);
      o3(2)=h*(o31(2)+o32(2)+o33(2));

      for j=3:1:o-2
      o31(j)=eq5g_3*(Og_con_t(1,j+1)+o2(j+1)/2-Og_con_t(1,j-1)-o2(j-1)/2);
      o32(j)=eq5g_5*(Og_con_t(1,j+1)+o2(j+1)/2-
2*(Og_con_t(1,j)+o2(j)/2)+Og_con_t(1,j-1)+o2(j-1)/2);
      o33(j)=eq5g_6*(Og_con_t(1,j)+o2(j)/2-Ol_con_t(1,j)-p2(j)/2);
      o3(j)=h*(o31(j)+o32(j)+o33(j));
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```
      end

      o31(o-1)=eq5g_3*(Og_con_t(1,o)+o2(o)/2-Og_con_t(1,o-2)-o2(o-2)/2);
      o32(o-1)=eq5g_5*(Og_con_t(1,o)+o2(o)/2-2*(Og_con_t(1,o-1)+o2(o-
1)/2)+Og_con_t(1,o-2)+o2(o-2)/2);
      o33(o-1)=eq5g_6*(Og_con_t(1,o-1)+o2(o-1)/2-Ol_con_t(1,o-1)-p2(o-1)/2);
      o3(o-1)=h*(o31(o-1)+o32(o-1)+o33(o-1));

      o31(o)=eq5g_2*(Og_con_t(1,o)+o2(o)/2-Og_con_t(1,o-1)-o2(o-1)/2);
      o32(o)=eq5g_5*(Og_con_t(1,o)+o2(o)/2-2*(Og_con_t(1,o-1)+o2(o-
1)/2)+Og_con_t(1,o-2)+o2(o-2)/2);
      o33(o)=eq5g_6*(Og_con_t(1,o)+o2(o)/2-Ol_con_t(1,o)-p2(o)/2);
      o3(o)=h*(o31(o)+o32(o)+o33(o));
%connection t liquid
      p31(1)=eq5l_2*(Ol_con_t(1,2)+p2(2)/2-Ol_con_t(1,1)-p2(1)/2);
      p32(1)=eq5l_5*(Ol_con_t(1,3)+p2(3)/2-
2*(Ol_con_t(1,2)+p2(2)/2)+Ol_con_t(1,1)+p2(1)/2);
      p33(1)=eq5l_6*(Og_con_t(1,1)+o2(1)/2-Ol_con_t(1,1)-p2(1)/2);
      p3(1)=h*(p31(1)+p32(1)+p33(1));

      p31(2)=eq5l_3*(Ol_con_t(1,3)+p2(3)/2-Ol_con_t(1,1)-p2(1)/2);
      p32(2)=eq5l_5*(Ol_con_t(1,3)+p2(3)/2-
2*(Ol_con_t(1,2)+p2(2)/2)+Ol_con_t(1,1)+p2(1)/2);
      p33(2)=eq5l_6*(Og_con_t(1,2)+o2(2)/2-Ol_con_t(1,2)-p2(2)/2);
      p3(2)=h*(p31(2)+p32(2)+p33(2));

      for  j=3:1:o-2
      p31(j)=eq5l_3*(Ol_con_t(1,j+1)+p2(j+1)/2-Ol_con_t(1,j-1)-p2(j-1)/2);
      p32(j)=eq5l_5*(Ol_con_t(1,j+1)+p2(j+1)/2-
2*(Ol_con_t(1,j)+p2(j)/2)+Ol_con_t(1,j-1)+p2(j-1)/2);
      p33(j)=eq5l_6*(Og_con_t(1,j)+o2(j)/2-Ol_con_t(1,j)-p2(j)/2);
      p3(j)=h*(p31(j)+p32(j)+p33(j));
      end

      p31(o-1)=eq5l_3*(Ol_con_t(1,o)+p2(o)/2-Ol_con_t(1,o-2)-p2(o-2)/2);
      p32(o-1)=eq5l_5*(Ol_con_t(1,o)+p2(o)/2-2*(Ol_con_t(1,o-1)+p2(o-
1)/2)+Ol_con_t(1,o-2)+p2(o-2)/2);
      p33(o-1)=eq5l_6*(Og_con_t(1,o-1)+o2(o-1)/2-Ol_con_t(1,o-1)-p2(o-1)/2);
      p3(o-1)=h*(p31(o-1)+p32(o-1)+p33(o-1));

      p31(o)=eq5l_2*(Ol_con_t(1,o)+p2(o)/2-Ol_con_t(1,o-1)-p2(o-1)/2);
      p32(o)=eq5l_5*(Ol_con_t(1,o)+p2(o)/2-2*(Ol_con_t(1,o-1)+p2(o-
1)/2)+Ol_con_t(1,o-2)+p2(o-2)/2);
      p33(o)=eq5l_6*(Og_con_t(1,o)+o2(o)/2-Ol_con_t(1,o)-p2(o)/2);
      p3(o)=h*(p31(o)+p32(o)+p33(o));
%separator2

q3=h*(eq4g_1*(Og_con_t(1,o)+o2(o)/2)+eq4g_2*(Og_sep2(1)+q2/2)+eq4g_3*((Og_sep2(1)+q2/
2)-(Ol_sep2(1)+r2/2))+eq4g_4*(Og_sep2(1)+q2/2));

r3=h*(eq4l_1*(Ol_con_t(1,o)+p2(o)/2)+eq4l_2*(Ol_sep2(1)+r2/2)+eq4l_3*((Og_sep2(1)+q2/
2)-(Ol_sep2(1)+r2/2)));

%downcomer gas
      s31(1)=eq2g_2*(Og_downcomer(1,2)+s2(2)/2-Og_downcomer(1,1)-s2(1)/2);
      s32(1)=eq2g_5*(Og_downcomer(1,3)+s2(3)/2-
2*(Og_downcomer(1,2)+s2(2)/2)+Og_downcomer(1,1)+s2(1)/2);
      s33(1)=eq2g_6*(Og_downcomer(1,1)+s2(1)/2-Ol_downcomer(1,1)-t2(1)/2);
      s3(1)=h*(s31(1)+s32(1)+s33(1));

      s31(2)=eq2g_3*(Og_downcomer(1,3)+s2(3)/2-Og_downcomer(1,1)-s2(1)/2);
      s32(2)=eq2g_5*(Og_downcomer(1,3)+s2(3)/2-
2*(Og_downcomer(1,2)+s2(2)/2)+Og_downcomer(1,1)+s2(1)/2);
      s33(2)=eq2g_6*(Og_downcomer(1,2)+s2(2)/2-Ol_downcomer(1,2)-t2(2)/2);
      s3(2)=h*(s31(2)+s32(2)+s33(2));

      for j=3:1:n-2
      s31(j)=eq2g_3*(Og_downcomer(1,j+1)+s2(j+1)/2-Og_downcomer(1,j-1)-s2(j-1)/2);
      s32(j)=eq2g_5*(Og_downcomer(1,j+1)+s2(j+1)/2-
2*(Og_downcomer(1,j)+s2(j)/2)+Og_downcomer(1,j-1)+s2(j-1)/2);
      s33(j)=eq2g_6*(Og_downcomer(1,j)+s2(j)/2-Ol_downcomer(1,j)-t2(j)/2);
      s3(j)=h*(s31(j)+s32(j)+s33(j));
      end

      s31(n-1)=eq2g_3*(Og_downcomer(1,n)+s2(n)/2-Og_downcomer(1,n-2)-s2(n-2)/2);
      s32(n-1)=eq2g_5*(Og_downcomer(1,n)+s2(n)/2-2*(Og_downcomer(1,n-1)+s2(n-
1)/2)+Og_downcomer(1,n-2)+s2(n-2)/2);
      s33(n-1)=eq2g_6*(Og_downcomer(1,n-1)+s2(n-1)/2-Ol_downcomer(1,n-1)-t2(n-1)/2);
      s3(n-1)=h*(s31(n-1)+s32(n-1)+s33(n-1));

      s31(n)=eq2g_2*(Og_downcomer(1,n)+s2(n)/2-Og_downcomer(1,n-1)-s2(n-1)/2);
      s32(n)=eq2g_5*(Og_downcomer(1,n)+s2(n)/2-2*(Og_downcomer(1,n-1)+s2(n-
1)/2)+Og_downcomer(1,n-2)+s2(n-2)/2);
      s33(n)=eq2g_6*(Og_downcomer(1,n)+s2(n)/2-Ol_downcomer(1,n)-t2(n)/2);
      s3(n)=h*(s31(n)+s32(n)+s33(n));
%downcomer liquid
      t31(1)=eq2l_2*(Ol_downcomer(1,2)+t2(2)/2-Ol_downcomer(1,1)-t2(1)/2);
      t32(1)=eq2l_5*(Ol_downcomer(1,3)+t2(3)/2-
2*(Ol_downcomer(1,2)+t2(2)/2)+Ol_downcomer(1,1)+t2(1)/2);
      t33(1)=eq2l_6*(Og_downcomer(1,1)+s2(1)/2-Ol_downcomer(1,1)-t2(1)/2);
      t3(1)=h*(t31(1)+t32(1)+t33(1));

      t31(2)=eq2l_3*(Ol_downcomer(1,3)+t2(3)/2-Ol_downcomer(1,1)-t2(1)/2);
      t32(2)=eq2l_5*(Ol_downcomer(1,3)+t2(3)/2-
2*(Ol_downcomer(1,2)+t2(2)/2)+Ol_downcomer(1,1)+t2(1)/2);
      t33(2)=eq2l_6*(Og_downcomer(1,2)+s2(2)/2-Ol_downcomer(1,2)-t2(2)/2);
      t3(2)=h*(t31(2)+t32(2)+t33(2));

      for  j=3:1:n-2
      t31(j)=eq2l_3*(Ol_downcomer(1,j+1)+t2(j+1)/2-Ol_downcomer(1,j-1)-t2(j-1)/2);
      t32(j)=eq2l_5*(Ol_downcomer(1,j+1)+t2(j+1)/2-
2*(Ol_downcomer(1,j)+t2(j)/2)+Ol_downcomer(1,j-1)+t2(j-1)/2);
      t33(j)=eq2l_6*(Og_downcomer(1,j)+s2(j)/2-Ol_downcomer(1,j)-t2(j)/2);
      t3(j)=h*(t31(j)+t32(j)+t33(j));
      end

      t31(n-1)=eq2l_3*(Ol_downcomer(1,n)+t2(n)/2-Ol_downcomer(1,n-2)-t2(n-2)/2);
      t32(n-1)=eq2l_5*(Ol_downcomer(1,n)+t2(n)/2-2*(Ol_downcomer(1,n-1)+t2(n-
1)/2)+Ol_downcomer(1,n-2)+t2(n-2)/2);
      t33(n-1)=eq2l_6*(Og_downcomer(1,n-1)+s2(n-1)/2-Ol_downcomer(1,n-1)-t2(n-1)/2);
      t3(n-1)=h*(t31(n-1)+t32(n-1)+t33(n-1));

      t31(n)=eq2l_2*(Ol_downcomer(1,n)+t2(n)/2-Ol_downcomer(1,n-1)-t2(n-1)/2);
      t32(n)=eq2l_5*(Ol_downcomer(1,n)+t2(n)/2-2*(Ol_downcomer(1,n-1)+t2(n-
1)/2)+Ol_downcomer(1,n-2)+t2(n-2)/2);
      t33(n)=eq2l_6*(Og_downcomer(1,n)+s2(n)/2-Ol_downcomer(1,n)-t2(n)/2);
      t3(n)=h*(t31(n)+t32(n)+t33(n));

%connection b gas
      u31(1)=eq6g_2*(Og_con_b(1,2)+u2(2)/2-Og_con_b(1,1)-u2(1)/2);
      u32(1)=eq6g_5*(Og_con_b(1,3)+u2(3)/2-
2*(Og_con_b(1,2)+u2(2)/2)+Og_con_b(1,1)+u2(1)/2);
      u33(1)=eq6g_6*(Og_con_b(1,1)+u2(1)/2-Ol_con_b(1,1)-v2(1)/2);
      u3(1)=h*(u31(1)+u32(1)+u33(1));

      u31(2)=eq6g_3*(Og_con_b(1,3)+u2(3)/2-Og_con_b(1,1)-u2(1)/2);
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      u32(2)=eq6g_5*(Og_con_b(1,3)+u2(3)/2-
2*(Og_con_b(1,2)+u2(2)/2)+Og_con_b(1,1)+u2(1)/2);
      u33(2)=eq6g_6*(Og_con_b(1,2)+u2(2)/2-Ol_con_b(1,2)-v2(2)/2);
      u3(2)=h*(u31(2)+u32(2)+u33(2));

      for j=3:1:p-2
      u31(j)=eq6g_3*(Og_con_b(1,j+1)+u2(j+1)/2-Og_con_b(1,j-1)-u2(j-1)/2);
      u32(j)=eq6g_5*(Og_con_b(1,j+1)+u2(j+1)/2-
2*(Og_con_b(1,j)+u2(j)/2)+Og_con_b(1,j-1)+u2(j-1)/2);
      u33(j)=eq6g_6*(Og_con_b(1,j)+u2(j)/2-Ol_con_b(1,j)-v2(j)/2);
      u3(j)=h*(u31(j)+u32(j)+u33(j));
      end

      u31(p-1)=eq6g_3*(Og_con_b(1,p)+u2(p)/2-Og_con_b(1,p-2)-u2(p-2)/2);
      u32(p-1)=eq6g_5*(Og_con_b(1,p)+u2(p)/2-2*(Og_con_b(1,p-1)+u2(p-
1)/2)+Og_con_b(1,p-2)+u2(p-2)/2);
      u33(p-1)=eq6g_6*(Og_con_b(1,p-1)+u2(p-1)/2-Ol_con_b(1,p-1)-v2(p-1)/2);
      u3(p-1)=h*(u31(p-1)+u32(p-1)+u33(p-1));

      u31(p)=eq6g_2*(Og_con_b(1,p)+u2(p)/2-Og_con_b(1,p-1)-u2(p-1)/2);
      u32(p)=eq6g_5*(Og_con_b(1,p)+u2(p)/2-2*(Og_con_b(1,p-1)+u2(p-
1)/2)+Og_con_b(1,p-2)+u2(p-2)/2);
      u33(p)=eq6g_6*(Og_con_b(1,p)+u2(p)/2-Ol_con_b(1,p)-v2(p)/2);
      u3(p)=h*(u31(p)+u32(p)+u33(p));
%connection b liquid
      v31(1)=eq6l_2*(Ol_con_b(1,2)+v2(2)/2-Ol_con_b(1,1)-v2(1)/2);
      v32(1)=eq6l_5*(Ol_con_b(1,3)+v2(3)/2-
2*(Ol_con_b(1,2)+v2(2)/2)+Ol_con_b(1,1)+v2(1)/2);
      v33(1)=eq6l_6*(Og_con_b(1,1)+u2(1)/2-Ol_con_b(1,1)-v2(1)/2);
      v3(1)=h*(v31(1)+v32(1)+v33(1));

      v31(2)=eq6l_3*(Ol_con_b(1,3)+v2(3)/2-Ol_con_b(1,1)-v2(1)/2);
      v32(2)=eq6l_5*(Ol_con_b(1,3)+v2(3)/2-
2*(Ol_con_b(1,2)+v2(2)/2)+Ol_con_b(1,1)+v2(1)/2);
      v33(2)=eq6l_6*(Og_con_b(1,2)+u2(2)/2-Ol_con_b(1,2)-v2(2)/2);
      v3(2)=h*(v31(2)+v32(2)+v33(2));

    for j=3:1:p-2
      v31(j)=eq6l_3*(Ol_con_b(1,j+1)+v2(j+1)/2-Ol_con_b(1,j-1)-v2(j-1)/2);
      v32(j)=eq6l_5*(Ol_con_b(1,j+1)+v2(j+1)/2-
2*(Ol_con_b(1,j)+v2(j)/2)+Ol_con_b(1,j-1)+v2(j-1)/2);
      v33(j)=eq6l_6*(Og_con_b(1,j)+u2(j)/2-Ol_con_b(1,j)-v2(j)/2);
      v3(j)=h*(v31(j)+v32(j)+v33(j));
    end

      v31(p-1)=eq6l_3*(Ol_con_b(1,p)+v2(p)/2-Ol_con_b(1,p-2)-v2(p-2)/2);
      v32(p-1)=eq6l_5*(Ol_con_b(1,p)+v2(p)/2-2*(Ol_con_b(1,p-1)+v2(p-
1)/2)+Ol_con_b(1,p-2)+v2(p-2)/2);
      v33(p-1)=eq6l_6*(Og_con_b(1,p-1)+u2(p-1)/2-Ol_con_b(1,p-1)-v2(p-1)/2);
      v3(p-1)=h*(v31(p-1)+v32(p-1)+v33(p-1));

      v31(p)=eq6l_2*(Ol_con_b(1,p)+v2(p)/2-Ol_con_b(1,p-1)-v2(p-1)/2);
      v32(p)=eq6l_5*(Ol_con_b(1,p)+v2(p)/2-2*(Ol_con_b(1,p-1)+v2(p-
1)/2)+Ol_con_b(1,p-2)+v2(p-2)/2);
      v33(p)=eq6l_6*(Og_con_b(1,p)+u2(p)/2-Ol_con_b(1,p)-v2(p)/2);
      v3(p)=h*(v31(p)+v32(p)+v33(p));

%k4*********************************************************************************
**********************************
%riser gas
      k41(1)=eq1g_2*(Og_riser(1,2)+k3(2)-Og_riser(1,1)-k3(1));
      k42(1)=eq1g_5*(Og_riser(1,3)+k3(3)-
2*(Og_riser(1,2)+k3(2))+Og_riser(1,1)+k3(1));
      k43(1)=eq1g_6*(Og_riser(1,1)+k3(1)-Ol_riser(1,1)-l3(1));
      k4(1)=h*(k41(1)+k42(1)+k43(1));

      k41(2)=eq1g_3*(Og_riser(1,3)+k3(3)-Og_riser(1,1)-k3(1));
      k42(2)=eq1g_5*(Og_riser(1,3)+k3(3)-
2*(Og_riser(1,2)+k3(2))+Og_riser(1,1)+k3(1));
      k43(2)=eq1g_6*(Og_riser(1,2)+k3(2)-Ol_riser(1,2)-l3(2));
      k4(2)=h*(k41(2)+k42(2)+k43(2));

      for j=3:1:m-2
      k41(j)=eq1g_3*(Og_riser(1,j+1)+k3(j+1)-Og_riser(1,j-1)-k3(j-1));
      k42(j)=eq1g_5*(Og_riser(1,j+1)+k3(j+1)-2*(Og_riser(1,j)+k3(j))+Og_riser(1,j-
1)+k3(j-1));
      k43(j)=eq1g_6*(Og_riser(1,j)+k3(j)-Ol_riser(1,j)-l3(j));
      k4(j)=h*(k41(j)+k42(j)+k43(j));
      end

      k41(m-1)=eq1g_3*(Og_riser(1,m)+k3(m)-Og_riser(1,m-2)-k3(m-2));
      k42(m-1)=eq1g_5*(Og_riser(1,m)+k3(m)-2*(Og_riser(1,m-1)+k3(m-1))+Og_riser(1,m-
2)+k3(m-2));
      k43(m-1)=eq1g_6*(Og_riser(1,m-1)+k3(m-1)-Ol_riser(1,m-1)-l3(m-1));
      k4(m-1)=h*(k41(m-1)+k42(m-1)+k43(m-1));

      k41(m)=eq1g_2*(Og_riser(1,m)+k3(m)-Og_riser(1,m-1)-k3(m-1));
      k42(m)=eq1g_5*(Og_riser(1,m)+k3(m)-2*(Og_riser(1,m-1)+k3(m-1))+Og_riser(1,m-
2)+k3(m-2));
      k43(m)=eq1g_6*(Og_riser(1,m)+k3(m)-Ol_riser(1,m)-l3(m));
      k4(m)=h*(k41(m)+k42(m)+k43(m));
%riser liquid
      l41(1)=eq1l_2*(Ol_riser(1,2)+l3(2)-Ol_riser(1,1)-l3(1));
      l42(1)=eq1l_5*(Ol_riser(1,3)+l3(3)-
2*(Ol_riser(1,2)+l3(2))+Ol_riser(1,1)+l3(1));
      l43(1)=eq1l_6*(Og_riser(1,1)+k3(1)-Ol_riser(1,1)-l3(1));
      l4(1)=h*(l41(1)+l42(1)+l43(1));

      l41(2)=eq1l_3*(Ol_riser(1,3)+l3(3)-Ol_riser(1,1)-l3(1));
      l42(2)=eq1l_5*(Ol_riser(1,3)+l3(3)-
2*(Ol_riser(1,2)+l3(2))+Ol_riser(1,1)+l3(1));
      l43(2)=eq1l_6*(Og_riser(1,2)+k3(2)-Ol_riser(1,2)-l3(2));
      l4(2)=h*(l41(2)+l42(2)+l43(2));

      for j=3:1:m-2
      l41(j)=eq1l_3*(Ol_riser(1,j+1)+l3(j+1)-Ol_riser(1,j-1)-l3(j-1));
      l42(j)=eq1l_5*(Ol_riser(1,j+1)+l3(j+1)-2*(Ol_riser(1,j)+l3(j))+Ol_riser(1,j-
1)+l3(j-1));
      l43(j)=eq1l_6*(Og_riser(1,j)+k3(j)-Ol_riser(1,j)-l3(j));
      l4(j)=h*(l41(j)+l42(j)+l43(j));
      end

      l41(m-1)=eq1l_3*(Ol_riser(1,m)+l3(m)-Ol_riser(1,m-2)-l3(m-2));
      l42(m-1)=eq1l_5*(Ol_riser(1,m)+l3(m)-2*(Ol_riser(1,m-1)+l3(m-1))+Ol_riser(1,m-
2)+l3(m-2));
      l43(m-1)=eq1l_6*(Og_riser(1,m-1)+k3(m-1)-Ol_riser(1,m-1)-l3(m-1));
      l4(m-1)=h*(l41(m-1)+l42(m-1)+l43(m-1));

      l41(m)=eq1l_2*(Ol_riser(1,m)+l3(m)-Ol_riser(1,m-1)-l3(m-1));
      l42(m)=eq1l_5*(Ol_riser(1,m)+l3(m)-2*(Ol_riser(1,m-1)+l3(m-1))+Ol_riser(1,m-
2)+l3(m-2));
      l43(m)=eq1l_6*(Og_riser(1,m)+k3(m)-Ol_riser(1,m)-l3(m));
      l4(m)=h*(l41(m)+l42(m)+l43(m));
%separator1
```

```
m4=h*(eq3g_1*(Og_riser(1,m)+k3(m))+eq3g_2*(Og_sep1(1)+m3)+eq3g_3*((Og_sep1(1)+m3)-
(Ol_sep1(1)+n3))+eq3g_4*(Og_sep1(1)+m3));

n4=h*(eq3l_1*(Ol_riser(1,m)+l3(m))+eq3l_2*(Ol_sep1(1)+n3)+eq3l_3*((Og_sep1(1)+m3)-
(Ol_sep1(1)+n3)));

%connection t gas
      o41(1)=eq5g_2*(Og_con_t(1,2)+o3(2)-Og_con_t(1,1)-o3(1));
      o42(1)=eq5g_5*(Og_con_t(1,3)+o3(3)-
2*(Og_con_t(1,2)+o3(2))+Og_con_t(1,1)+o3(1));
      o43(1)=eq5g_6*(Og_con_t(1,1)+o3(1)-Ol_con_t(1,1)-p3(1));
      o4(1)=h*(o41(1)+o42(1)+o43(1));

      o41(2)=eq5g_3*(Og_con_t(1,3)+o3(3)-Og_con_t(1,1)-o3(1));
      o42(2)=eq5g_5*(Og_con_t(1,3)+o3(3)-
2*(Og_con_t(1,2)+o3(2))+Og_con_t(1,1)+o3(1));
      o43(2)=eq5g_6*(Og_con_t(1,2)+o3(2)-Ol_con_t(1,2)-p3(2));
      o4(2)=h*(o41(2)+o42(2)+o43(2));

    for j=3:1:o-2
      o41(j)=eq5g_3*(Og_con_t(1,j+1)+o3(j+1)-Og_con_t(1,j-1)-o3(j-1));
      o42(j)=eq5g_5*(Og_con_t(1,j+1)+o3(j+1)-2*(Og_con_t(1,j)+o3(j))+Og_con_t(1,j-
1)+o3(j-1));
      o43(j)=eq5g_6*(Og_con_t(1,j)+o3(j)-Ol_con_t(1,j)-p3(j));
      o4(j)=h*(o41(j)+o42(j)+o43(j));
    end

      o41(o-1)=eq5g_3*(Og_con_t(1,o)+o3(o)-Og_con_t(1,o-2)-o3(o-2));
      o42(o-1)=eq5g_5*(Og_con_t(1,o)+o3(o)-2*(Og_con_t(1,o-1)+o3(o-1))+Og_con_t(1,o-
2)+o3(o-2));
      o43(o-1)=eq5g_6*(Og_con_t(1,o-1)+o3(o-1)-Ol_con_t(1,o-1)-p3(o-1));
      o4(o-1)=h*(o41(o-1)+o42(o-1)+o43(o-1));

      o41(o)=eq5g_2*(Og_con_t(1,o)+o3(o)-Og_con_t(1,o-1)-o3(o-1));
      o42(o)=eq5g_5*(Og_con_t(1,o)+o3(o)-2*(Og_con_t(1,o-1)+o3(o-1))+Og_con_t(1,o-
2)+o3(o-2));
      o43(o)=eq5g_6*(Og_con_t(1,o)+o3(o)-Ol_con_t(1,o)-p3(o));
      o4(o)=h*(o41(o)+o42(o)+o43(o));
%connection t liquid
      p41(1)=eq5l_2*(Ol_con_t(1,2)+p3(2)-Ol_con_t(1,1)-p3(1));
      p42(1)=eq5l_5*(Ol_con_t(1,3)+p3(3)-
2*(Ol_con_t(1,2)+p3(2))+Ol_con_t(1,1)+p3(1));
      p43(1)=eq5l_6*(Og_con_t(1,1)+o3(1)-Ol_con_t(1,1)-p3(1));
      p4(1)=h*(p41(1)+p42(1)+p43(1));

      p41(2)=eq5l_3*(Ol_con_t(1,3)+p3(3)-Ol_con_t(1,1)-p3(1));
      p42(2)=eq5l_5*(Ol_con_t(1,3)+p3(3)-
2*(Ol_con_t(1,2)+p3(2))+Ol_con_t(1,1)+p3(1));
      p43(2)=eq5l_6*(Og_con_t(1,2)+o3(2)-Ol_con_t(1,2)-p3(2));
      p4(2)=h*(p41(2)+p42(2)+p43(2));

    for j=3:1:o-2
      p41(j)=eq5l_3*(Ol_con_t(1,j+1)+p3(j+1)-Ol_con_t(1,j-1)-p3(j-1));
      p42(j)=eq5l_5*(Ol_con_t(1,j+1)+p3(j+1)-2*(Ol_con_t(1,j)+p3(j))+Ol_con_t(1,j-
1)+p3(j-1));
      p43(j)=eq5l_6*(Og_con_t(1,j)+o3(j)-Ol_con_t(1,j)-p3(j));
      p4(j)=h*(p41(j)+p42(j)+p43(j));
    end

      p41(o-1)=eq5l_3*(Ol_con_t(1,o)+p3(o)-Ol_con_t(1,o-2)-p3(o-2));
      p42(o-1)=eq5l_5*(Ol_con_t(1,o)+p3(o)-2*(Ol_con_t(1,o-1)+p3(o-1))+Ol_con_t(1,o-
2)+p3(o-2));
      p43(o-1)=eq5l_6*(Og_con_t(1,o-1)+o3(o-1)-Ol_con_t(1,o-1)-p3(o-1));
      p4(o-1)=h*(p41(o-1)+p42(o-1)+p43(o-1));

      p41(o)=eq5l_2*(Ol_con_t(1,o)+p3(o)-Ol_con_t(1,o-1)-p3(o-1));
      p42(o)=eq5l_5*(Ol_con_t(1,o)+p3(o)-2*(Ol_con_t(1,o-1)+p3(o-1))+Ol_con_t(1,o-
2)+p3(o-2));
      p43(o)=eq5l_6*(Og_con_t(1,o)+o3(o)-Ol_con_t(1,o)-p3(o));
      p4(o)=h*(p41(o)+p42(o)+p43(o));
%separator2

q4=h*(eq4g_1*(Og_con_t(1,o)+o3(o))+eq4g_2*(Og_sep2(1)+q3)+eq4g_3*((Og_sep2(1)+q3)-
(Ol_sep2(1)+r3))+eq4g_4*(Og_sep2(1)+q3));

r4=h*(eq4l_1*(Ol_con_t(1,o)+p3(o))+eq4l_2*(Ol_sep2(1)+r3)+eq4l_3*((Og_sep2(1)+q3)-
(Ol_sep2(1)+r3)));

%downcomer gas
      s41(1)=eq2g_2*(Og_downcomer(1,2)+s3(2)-Og_downcomer(1,1)-s3(1));
      s42(1)=eq2g_5*(Og_downcomer(1,3)+s3(3)-
2*(Og_downcomer(1,2)+s3(2))+Og_downcomer(1,1)+s3(1));
      s43(1)=eq2g_6*(Og_downcomer(1,1)+s3(1)-Ol_downcomer(1,1)-t3(1));
      s4(1)=h*(s41(1)+s42(1)+s43(1));

      s41(2)=eq2g_3*(Og_downcomer(1,3)+s3(3)-Og_downcomer(1,1)-s3(1));
      s42(2)=eq2g_5*(Og_downcomer(1,3)+s3(3)-
2*(Og_downcomer(1,2)+s3(2))+Og_downcomer(1,1)+s3(1));
      s43(2)=eq2g_6*(Og_downcomer(1,2)+s3(2)-Ol_downcomer(1,2)-t3(2));
      s4(2)=h*(s41(2)+s42(2)+s43(2));

    for j=3:1:n-2
      s41(j)=eq2g_3*(Og_downcomer(1,j+1)+s3(j+1)-Og_downcomer(1,j-1)-s3(j-1));
      s42(j)=eq2g_5*(Og_downcomer(1,j+1)+s3(j+1)-
2*(Og_downcomer(1,j)+s3(j))+Og_downcomer(1,j-1)+s3(j-1));
      s43(j)=eq2g_6*(Og_downcomer(1,j)+s3(j)-Ol_downcomer(1,j)-t3(j));
      s4(j)=h*(s41(j)+s42(j)+s43(j));
    end

      s41(n-1)=eq2g_3*(Og_downcomer(1,n)+s3(n)-Og_downcomer(1,n-2)-s3(n-2));
      s42(n-1)=eq2g_5*(Og_downcomer(1,n)+s3(n)-2*(Og_downcomer(1,n-1)+s3(n-
1))+Og_downcomer(1,n-2)+s3(n-2));
      s43(n-1)=eq2g_6*(Og_downcomer(1,n-1)+s3(n-1)-Ol_downcomer(1,n-1)-t3(n-1));
      s4(n-1)=h*(s41(n-1)+s42(n-1)+s43(n-1));

      s41(n)=eq2g_2*(Og_downcomer(1,n)+s3(n)-Og_downcomer(1,n-1)-s3(n-1));
      s42(n)=eq2g_5*(Og_downcomer(1,n)+s3(n)-2*(Og_downcomer(1,n-1)+s3(n-
1))+Og_downcomer(1,n-2)+s3(n-2));
      s43(n)=eq2g_6*(Og_downcomer(1,n)+s3(n)-Ol_downcomer(1,n)-t3(n));
      s4(n)=h*(s41(n)+s42(n)+s43(n));
%downcomer liquid
      t41(1)=eq2l_2*(Ol_downcomer(1,2)+t3(2)-Ol_downcomer(1,1)-t3(1));
      t42(1)=eq2l_5*(Ol_downcomer(1,3)+t3(3)-
2*(Ol_downcomer(1,2)+t3(2))+Ol_downcomer(1,1)+t3(1));
      t43(1)=eq2l_6*(Og_downcomer(1,1)+s3(1)-Ol_downcomer(1,1)-t3(1));
      t4(1)=h*(t41(1)+t42(1)+t43(1));

      t41(2)=eq2l_3*(Ol_downcomer(1,3)+t3(3)-Ol_downcomer(1,1)-t3(1));
      t42(2)=eq2l_5*(Ol_downcomer(1,3)+t3(3)-
2*(Ol_downcomer(1,2)+t3(2))+Ol_downcomer(1,1)+t3(1));
      t43(2)=eq2l_6*(Og_downcomer(1,2)+s3(2)-Ol_downcomer(1,2)-t3(2));
      t4(2)=h*(t41(2)+t42(2)+t43(2));

    for j=3:1:n-2
      t41(j)=eq2l_3*(Ol_downcomer(1,j+1)+t3(j+1)-Ol_downcomer(1,j-1)-t3(j-1));
```

```
      t42(j)=eq2l_5*(Ol_downcomer(1,j+1)+t3(j+1)-
2*(Ol_downcomer(1,j)+t3(j))+Ol_downcomer(1,j-1)+t3(j-1));
      t43(j)=eq2l_6*(Og_downcomer(1,j)+s3(j)-Ol_downcomer(1,j)-t3(j));
      t4(j)=h*(t41(j)+t42(j)+t43(j));
    end

      t41(n-1)=eq2l_3*(Ol_downcomer(1,n)+t3(n)-Ol_downcomer(1,n-2)-t3(n-2));
      t42(n-1)=eq2l_5*(Ol_downcomer(1,n)+t3(n)-2*(Ol_downcomer(1,n-1)+t3(n-
1))+Ol_downcomer(1,n-2)+t3(n-2));
      t43(n-1)=eq2l_6*(Og_downcomer(1,n-1)+s3(n-1)-Ol_downcomer(1,n-1)-t3(n-1));
      t4(n-1)=h*(t41(n-1)+t42(n-1)+t43(n-1));

      t41(n)=eq2l_2*(Ol_downcomer(1,n)+t3(n)-Ol_downcomer(1,n-1)-t3(n-1));
      t42(n)=eq2l_5*(Ol_downcomer(1,n)+t3(n)-2*(Ol_downcomer(1,n-1)+t3(n-
1))+Ol_downcomer(1,n-2)+t3(n-2));
      t43(n)=eq2l_6*(Og_downcomer(1,n)+s3(n)-Ol_downcomer(1,n)-t3(n));
      t4(n)=h*(t41(n)+t42(n)+t43(n));

%connection b gas
      u41(1)=eq6g_2*(Og_con_b(1,2)+u3(2)-Og_con_b(1,1)-u3(1));
      u42(1)=eq6g_5*(Og_con_b(1,3)+u3(3)-
2*(Og_con_b(1,2)+u3(2))+Og_con_b(1,1)+u3(1));
      u43(1)=eq6g_6*(Og_con_b(1,1)+u3(1)-Ol_con_b(1,1)-v3(1));
      u4(1)=h*(u41(1)+u42(1)+u43(1));

      u41(2)=eq6g_3*(Og_con_b(1,3)+u3(3)-Og_con_b(1,1)-u3(1));
      u42(2)=eq6g_5*(Og_con_b(1,3)+u3(3)-
2*(Og_con_b(1,2)+u3(2))+Og_con_b(1,1)+u3(1));
      u43(2)=eq6g_6*(Og_con_b(1,2)+u3(2)-Ol_con_b(1,2)-v3(2));
      u4(2)=h*(u41(2)+u42(2)+u43(2));

   for j=3:1:p-2
      u41(j)=eq6g_3*(Og_con_b(1,j+1)+u3(j+1)-Og_con_b(1,j-1)-u3(j-1));
      u42(j)=eq6g_5*(Og_con_b(1,j+1)+u3(j+1)-2*(Og_con_b(1,j)+u3(j))+Og_con_b(1,j-
1)+u3(j-1));
      u43(j)=eq6g_6*(Og_con_b(1,j)+u3(j)-Ol_con_b(1,j)-v3(j));
      u4(j)=h*(u41(j)+u42(j)+u43(j));
   end

      u41(p-1)=eq6g_3*(Og_con_b(1,p)+u3(p)-Og_con_b(1,p-2)-u3(p-2));
      u42(p-1)=eq6g_5*(Og_con_b(1,p)+u3(p)-2*(Og_con_b(1,p-1)+u3(p-1))+Og_con_b(1,p-
2)+u3(p-2));
      u43(p-1)=eq6g_6*(Og_con_b(1,p-1)+u3(p-1)-Ol_con_b(1,p-1)-v3(p-1));
      u4(p-1)=h*(u41(p-1)+u42(p-1)+u43(p-1));

      u41(p)=eq6g_2*(Og_con_b(1,p)+u3(p)-Og_con_b(1,p-1)-u3(p-1));
      u42(p)=eq6g_5*(Og_con_b(1,p)+u3(p)-2*(Og_con_b(1,p-1)+u3(p-1))+Og_con_b(1,p-
2)+u3(p-2));
      u43(p)=eq6g_6*(Og_con_b(1,p)+u3(p)-Ol_con_b(1,p)-v3(p));
      u4(p)=h*(u41(p)+u42(p)+u43(p));
%connection b liquid
      v41(1)=eq6l_2*(Ol_con_b(1,2)+v3(2)-Ol_con_b(1,1)-v3(1));
      v42(1)=eq6l_5*(Ol_con_b(1,3)+v3(3)-
2*(Ol_con_b(1,2)+v3(2))+Ol_con_b(1,1)+v3(1));
      v43(1)=eq6l_6*(Og_con_b(1,1)+u3(1)-Ol_con_b(1,1)-v3(1));
      v4(1)=h*(v41(1)+v42(1)+v43(1));

      v41(2)=eq6l_3*(Ol_con_b(1,3)+v3(3)-Ol_con_b(1,1)-v3(1));
      v42(2)=eq6l_5*(Ol_con_b(1,3)+v3(3)-
2*(Ol_con_b(1,2)+v3(2))+Ol_con_b(1,1)+v3(1));
      v43(2)=eq6l_6*(Og_con_b(1,2)+u3(2)-Ol_con_b(1,2)-v3(2));
      v4(2)=h*(v41(2)+v42(2)+v43(2));

   for j=3:1:p-2
      v41(j)=eq6l_3*(Ol_con_b(1,j+1)+v3(j+1)-Ol_con_b(1,j-1)-v3(j-1));
      v42(j)=eq6l_5*(Ol_con_b(1,j+1)+v3(j+1)-2*(Ol_con_b(1,j)+v3(j))+Ol_con_b(1,j-
1)+v3(j-1));
      v43(j)=eq6l_6*(Og_con_b(1,j)+u3(j)-Ol_con_b(1,j)-v3(j));
      v4(j)=h*(v41(j)+v42(j)+v43(j));
   end

      v41(p-1)=eq6l_3*(Ol_con_b(1,p)+v3(p)-Ol_con_b(1,p-2)-v3(p-2));
      v42(p-1)=eq6l_5*(Ol_con_b(1,p)+v3(p)-2*(Ol_con_b(1,p-1)+v3(p-1))+Ol_con_b(1,p-
2)+v3(p-2));
      v43(p-1)=eq6l_6*(Og_con_b(1,p-1)+u3(p-1)-Ol_con_b(1,p-1)-v3(p-1));
      v4(p-1)=h*(v41(p-1)+v42(p-1)+v43(p-1));

      v41(p)=eq6l_2*(Ol_con_b(1,p)+v3(p)-Ol_con_b(1,p-1)-v3(p-1));
      v42(p)=eq6l_5*(Ol_con_b(1,p)+v3(p)-2*(Ol_con_b(1,p-1)+v3(p-1))+Ol_con_b(1,p-
2)+v3(p-2));
      v43(p)=eq6l_6*(Og_con_b(1,p)+u3(p)-Ol_con_b(1,p)-v3(p));
      v4(p)=h*(v41(p)+v42(p)+v43(p));

for j=2:1:m
Og_riser(1,j)=Og_riser(1,j)+1/6*(k1(j)+2*k2(j)+2*k3(j)+k4(j));
Ol_riser(1,j)=Ol_riser(1,j)+1/6*(l1(j)+2*l2(j)+2*l3(j)+l4(j));
end

Og_sep1(1)=Og_sep1(1)+1/6*(m1+2*m2+2*m3+m4);
Ol_sep1(1)=Ol_sep1(1)+1/6*(n1+2*n2+2*n3+n4);

for j=2:1:o
Og_con_t(1,j)=Og_con_t(1,j)+1/6*(o1(j)+2*o2(j)+2*o3(j)+o4(j));
Ol_con_t(1,j)=Ol_con_t(1,j)+1/6*(p1(j)+2*p2(j)+2*p3(j)+p4(j));
end

Og_sep2(1)=Og_sep2(1)+1/6*(q1+2*q2+2*q3+q4);
Ol_sep2(1)=Ol_sep2(1)+1/6*(r1+2*r2+2*r3+r4);

for j=2:1:n
Og_downcomer(1,j)=Og_downcomer(1,j)+1/6*(s1(j)+2*s2(j)+2*s3(j)+s4(j));
Ol_downcomer(1,j)=Ol_downcomer(1,j)+1/6*(t1(j)+2*t2(j)+2*t3(j)+t4(j));
end

for j=2:1:p
Og_con_b(1,j)=Og_con_b(1,j)+1/6*(u1(j)+2*u2(j)+2*u3(j)+u4(j));
Ol_con_b(1,j)=Ol_con_b(1,j)+1/6*(v1(j)+2*v2(j)+2*v3(j)+v4(j));
end
%---------------------------------------------------------------------

%---------------------------------------------------------------------
Og_riser(1,1)=(e6*A6*vg_6*Og_con_b(1,p)+Qg_in*1)/(e1*A1*vg_1);
%Ol_riser(1,1)=((1-e6)*A6*vl_6*Ol_con_b(1,p))/((1-e1)*A1*vl_1);
Ol_riser(1,1)=Ol_con_b(1,p);

Og_con_t(1,1)=Og_sep1(1);
Ol_con_t(1,1)=Ol_sep1(1);

Og_downcomer(1,1)=Og_sep2(1);
Ol_downcomer(1,1)=Ol_sep2(1);

Og_con_b(1,1)=Og_downcomer(1,n);
Ol_con_b(1,1)=Ol_downcomer(1,n);

z=z+1;
ccc=round/100;
```

```
ddd=round/100;
if floor((z-1)/ccc)==ceil((z-1)/ccc);
   fprintf('z=%f \n',z);
   fprintf('Olr=%f \n',Ol_riser(1,m));
   fprintf('Ogr=%f \n',Og_riser(1,m));
end

   if Ol_riser(1,m)<0.8;
      if floor((z-1)/ddd)==ceil((z-1)/ddd)
         Olrm(k,1)=Ol_riser(1,m);
         Olrm2(k,1)=Ol_riser(1,m-1);
         Oldn(k,1)=Ol_downcomer(1,n);
         Ols1(k,1)=Ol_sep1(1);
         Ols2(k,1)=Ol_sep2(1);
         Olcto(k,1)=Ol_con_t(1,o);
         Olcbp(k,1)=Ol_con_b(1,p);

         Ogrm(k,1)=Og_riser(1,m);
         Ogdn(k,1)=Og_downcomer(1,n);
         Ogs1(k,1)=Og_sep1(1);
         Ogs2(k,1)=Og_sep2(1);
         Ogcto(k,1)=Og_con_t(1,o);
         Ogcbp(k,1)=Og_con_b(1,p);
         k=k+1;
      end
   else
      if floor((z-1)/ddd)==ceil((z-1)/ddd)
         Olrm(k,1)=Ol_riser(1,m);
         Olrm2(k,1)=Ol_riser(1,m-1);
         Oldn(k,1)=Ol_downcomer(1,n);
         Ols1(k,1)=Ol_sep1(1);
         Ols2(k,1)=Ol_sep2(1);
         Olcto(k,1)=Ol_con_t(1,o);
         Olcbp(k,1)=Ol_con_b(1,p);

         Ogrm(k,1)=Og_riser(1,m);
         Ogdn(k,1)=Og_downcomer(1,n);
         Ogs1(k,1)=Og_sep1(1);
         Ogs2(k,1)=Og_sep2(1);
         Ogcto(k,1)=Og_con_t(1,o);
         Ogcbp(k,1)=Og_con_b(1,p);
         k=k+1;
      end
   end
end

t1f=floor((c1-1)/200)*200;
t2i=ceil((c1-1)/500)*500;
t2f=floor((c2-1)/500)*500;

t=[0:200:t1f,t2i:500:t2f-500];

fprintf('--------------------------------------------------- = %0.5f\n',ratio);
fprintf('Ad/Ar = %0.5f\n',ratio);
fprintf('Qg_in = %0.5f\n',Qg_in);
fprintf('usg = %0.5f\n',usg);

fprintf('e riser = %0.5f\n',e1);
fprintf('e downcomer = %0.5f\n',e2);
fprintf('e cont = %0.5f\n',e5);
fprintf('kLa_riser = %0.5f\n',kla1);
fprintf('vg_riser = %0.5f\n',vg_1);
fprintf('vg_con_t = %0.5f\n',vg_5);
fprintf('vg_downcomer = %0.5f\n',vg_2);
fprintf('vl_riser = %0.5f\n',vl_1);
fprintf('vl_con_t = %0.5f\n',vl_5);
fprintf('vl_downcomer = %0.5f\n',vl_2);
%fprintf('v_slip= %0.5f\n',v_slip);
fprintf('t_factor = %0.5f\n',t_factor);
fprintf('z_factor = %0.5f\n',z_factor);
fprintf('c1 = %0.5f\n',c1);
fprintf('c2 = %0.5f\n',c2);
fprintf('del_t1 = %0.5f\n',del_t1);
fprintf('del_t2 = %0.5f\n',del_t2);
```

BIOGRAPHY

Mr. Yuttana Rujiruttanakul was born on 2 nd October, 1982 in Sukhothai. His native home was Sukhothai province. He finished his secondary school from Sawananun Wittayakom School in 2000. He got bachelor degree from Chemical Engineering in Faculty of Engineer at Burapha University in 2004. He continued his further study for master's degree in Chemical Engineering at Chulalongkorn University. He participated in the Biochemical Engineering Research Group and achieved completed his Master's degree in April, 2008.