การเปรียบเทียบเชิงเอมพิริคัลของตัวแบบต้นไม้ทวินามสำหรับออปชันดัชนี SET50

นางสาวนวรัตน์ เอกก้านตรง

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรมหาบัณฑิต

สาขาวิชาวิทยาการคณนา ภาควิชาคณิตศาสตร์

คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

ปีการศึกษา 2552

AN EMPIRICAL COMPARISON OF BINOMIAL TREE MODELS FOR SET50 INDEX OPTIONS

Miss Nawarat Ek-karntrong

A Thesis Submitted in Partial Fulfillment of the Requirements
for the Degree of Master of Science Program in Computational Science

Department of Mathematics
Faculty of Science
Chulalongkorn University

Academic Year 2009

Thesis Title	AN EMPIRICAL COMPARISON OF BINOMIAL TREE MODELS FOR SET50 INDEX OPTIONS
By	Miss Nawarat Ek-karntrong
Field of Study	Computational Science
Thesis Advisor	Khamron Mekchay, Ph.D.
Thesis Co-Advisor	Kittipat Wong, Ph.D.

Accepted by the Faculty of Science, Chulalongkorn University in Partial Fulfillment of the Requirements for the Master's Degree

.... Dean of the Faculty of Science

(Professor Supot Hannongbua, Dr.rer.nat.)

THESIS COMMITTEE

.... Chairman

(Associate Professor Paisan Nakmahachalasint, Ph.D.)

.... Thesis Advisor

(Khamron Mekchay, Ph.D.)

.... Thesis Co-Advisor

(Kittipat Wong, Ph.D.)

.... Examiner

(Assistant Professor Nataphan Kitisin, Ph.D.)

.... External Examiner

(Assistant Professor Siripong Atipan, Ph.D.)

นวรัตน์ เอกก้านตรง : การเปรียบเทียบเชิงเอมพิริคัลของตัวแบบต้นไม้ทวินามสำหรับออปชันดัชนี SET50. (AN EMPIRICAL COMPARISON OF BINOMIAL TREE MODELS FOR SET50 INDEX OPTIONS) อ. ที่ปรึกษาวิทยานิพนธ์หลัก : อ. ดร. คำรณ เมฆฉาย, อ. ที่ปรึกษาวิทยานิพนธ์ร่วม : อ. ดร. กิตติพัฒน์ วอง, 84 หน้า.

เมื่อมีการลงทุน สิ่งที่มาควบคู่กับการลงทุน คือ ความเสี่ยง ดังนั้นผู้ลงทุนจึงต้องทำการบริหารความเสี่ยง และ ในปัจจุบันออปชัน (option) เป็นผลิตภัณฑ์ทางการเงินที่ได้รับความสนใจและนิยมนำมาใช้ในการบริหารความเสี่ยงและผลตอบแทนจากการลงทุน ซึ่งการนำออปชันมาประยุกต์ใช้นั้นจะเกิดประโยชน์สูงสุดต่อผู้ลงทุนขึ้นกับการวิเคราะห์, การประเมินราคา และการเลือกใช้กลยุทธ์การลงทุนที่เหมาะสม

ในงานวิจัยชิ้นนี้ได้สนใจในด้านการประเมินราคาของออปชัน โดยนำตัวแบบในลักษณะต้นไม้ทวินาม คือ ตัวแบบ SBT, ตัวแบบ IBT และตัวแบบ GBT ไปประยุกต์ใช้กับออปชันดัชนี SET50 ซึ่งมีการซื้อขายในตลาดอนุพันธ์ของประเทศไทย (Thailand Futures Exchange) จากนั้นวิเคราะห์ผลลัพธ์จากตัวแบบทั้ง 3 กับข้อมูลจริง โดยค่าความคลาดเคลื่อนที่ได้นั้นจะวิเคราะห์ออกมาโดยมี 4 ตัวชี้วัด คือ ความคลาดเคลื่อนเฉลี่ย (Mean error), อัตราความคลาดเคลื่อนเฉลี่ย (Mean percentage error), ความคลาดเคลื่อนกำลังสองเฉลี่ย (Mean square error) และ อัตราความคลาดเคลื่อนสัมบูรณ์เฉลี่ย (Mean absolute percentage error) เพื่อวิเคราะห์ว่าตัวแบบต้นไม้ทวินามแบบใดที่จะเหมาะกับการนำไปทำนายมูลค่าของออปชันดัชนี SET50

ภาควิชา ....คณิตศาสตร์.................... ลายมือชื่อนิสิต..
สาขาวิชา ...วิทยาการคณนา .... ลายมือชื่อ อ.ที่ปรึกษาวิทยานิพนธ์หลัก..................
ปีการศึกษา ......2552 .... ลายมือชื่อ อ.ที่ปรึกษาวิทยานิพนธ์ร่วม ....

5072318923 : MAJOR COMPUTATIONAL SCIENCE
KEYWORDS : OPTION / BINOMIAL TREE MODELS / SET50 INDEX / HEDGING

NAWARAT EK-KARNTRONG : AN EMPIRICAL COMPARISON OF BINOMIAL TREE MODELS FOR SET50 INDEX OPTIONS. THESIS ADVISOR : KHAMRON MEKCHAY, Ph.D., THESIS CO-ADVISOR : KITTIPAT WONG, Ph.D., 84 pp.

An option is a popular security which is used to reduce and manage the investment risks. The best results of applications of option depend on analysis of option, valuation of option, and some suitable investing strategies. In this research we concern about valuation of option using an empirical comparison of binomial tree models. We compare binomial tree models for SET50 index option in terms of pricing and hedging performances. The underlying asset of SET50 index option is the Stock Exchange Thailand index 50 (SET50) which is traded in the Thailand Futures Exchange (TFEX). The sample data are taken from December 28, 2007 through December 29, 2008.

The empirical comparisons are employed among three binomial tree models: the standard binomial tree (SBT), the implied binomial tree (IBT), and the generalized binomial tree (GBT). The performances are measured in terms of the mean error (ME), the mean percentage error (MPE), the mean squared error (MSE), and the mean absolute percentage error (MAPE).

Department : ....Mathematics......................	Student's Signature : นวรัตน์ เอกการณ์ทรง	
Field of Study : ....Computational Science.........	Advisor's Signature :	
Academic Year : ....2009.........................	Co-Advisor's Signature :	

ACKNOWLEDGEMENTS

I am greatly indepted to Dr.Kamron Mekchey, my thesis advisor, and Dr.Kittipat Wong, my thesis co-advisor, for their willingness to sacrifice their time to suggest and advise me in preparing and writing this thesis.

Also I would like to thank Associate Professor Paisan Nakmahachalasint, Ph.D., Associate Professor Nataphan Kitisin, Ph.D., and Assistant Professor Siripong Atipan, Ph.D., my thesis commitee, for their suggestion on this thesis.

I therefore thank all of my teachers for my knowledge and skill.

In particular, thank to my dear friends for giving me good advices for my thesis and experiences at Chulalongkorn university.

Finally, I would like to express my gratitude to my beloved family for their love and encouragement throughout my graduate study.

This research was financially supported by Development and Promotion of Science and Technology Talents Project (DPST).

CONTENTS

Page

ABSTRACT IN THAI iv

ABSTRACT IN ENGLISH v

ACKNOWLEDGEMENTS vi

CONTENTS vii

LIST OF TABLES ix

LIST OF FIGURES x

CHAPTER

I INTRODUCTION 1

II OPTION AND OPTION PRICING 4

2.1 Option 4

2.1.1 Classification of options 4

2.1.2 Contract Specifications 10

2.1.3 Option value 10

2.2 Option pricing 13

2.2.1 Black-Scholes model 14

2.2.2 Standard Binomial Tree (SBT) Model 15

2.2.3 Implied Binomial Tree (IBT) Model 19

2.2.4 Generalized Binomial Tree (GBT) Model 23

2.3 Hedge 24

2.3.1 Types of hedging 27

III DATA AND METHODOLOGY 29

3.1 Data 29

3.1.1 Contract Specifications 30

3.2 Methodology 31

3.2.1 The SBT algorithm 31

3.2.2 The IBT algorithm 33

3.2.3 The GBT algorithm 34

3.2.4 Pricing Errors 35

3.2.5 Hedging errors 35

3.3 Analysis of Empirical Errors 36

IV RESULT 53

4.1 Results of option pricing 53

4.1.1 Pricing performance 54

4.1.2 Hedging performance 54

V DISCUSSIONS AND CONCLUSIONS 73

REFERENCES 75

APPENDICES 76

APPENDIX A 77

APPENDIX B 79

VITAE 84

LIST OF TABLES

3.1 Contract Specification of SET50 Index Options 33

3.2 Ending-nodal risk-neutral probabilities of Longstaff's and Shimko's methods 40

3.3 pricing errors of the IBT for calls. 41

3.4 Estimation results for different weight functions 41

4.1 pricing errors of C1 and C2. 65

4.2 pricing errors of C3 and C4. 66

4.3 pricing errors of P1 and P2 67

4.4 pricing errors of P3 and P4 68

4.5 hedging errors of C1 and C2 69

4.6 hedging errors of C3 and C4 70

4.7 hedging errors of P1 and P2 71

4.8 hedging errors of P3 and P4 72

LIST OF FIGURES

2.1 The 1-step binomial trees of the underlying asset price (left) and the value of call option (right) 16

2.2 The n-step standard binomial tree of the underlying asset price . 17

2.3 The 3-step standard binomial tree of the option price and option price 18

2.4 Risk-Neutral Probability Distribution 21

2.5 An illustration of various weight functions 25

3.1 Flowchart of the SBT model 38

3.2 Flowchart of the SBT model 39

3.3 Flowchart of the IBT model [1] 42

3.4 Flowchart of the IBT model [2] 43

3.5 Flowchart of the Longstaff's method used in the IBT model . . . 44

3.6 Flowchart of the Shimko's method used in the IBT model 45

3.7 Flowchart of the Rubinstein's method used in the IBT model . . 46

3.8 Flowchart of the Jackwerth and Rubinstein's method used in the IBT model 47

3.9 Flowchart of the IBT model [3] 48

3.10 Flowchart of the GBT model 49

3.11 Flowchart of the linear (concave and convex) weight functions used in the GBT model 50

3.12 Flowchart of the quadratic (concave and convex) weight functions used in the GBT model 51

3.13 Flowchart of the S-curve weight functions used in the GBT model 52

4.1 SET50 Index 56

4.2 call price at the strike price 530 of the call option with the SET50 Index as the underlying asset that expired on 31 March, 2008 . . 57
4.3 call price at the strike price 600 of the call option with the SET50 Index as the underlying asset that expired on 31 March, 2008 . . 58
4.4 call price at the strike price 630 of the call option with the SET50 Index as the underlying asset that expired on 31 March, 2008 . . 59
4.5 Put price at the strike price 530 of the put option with the SET50 Index as the underlying asset that expired on 31 March, 2008 . . 60
4.6 Put price at the strike price 600 of the put option with the SET50 Index as the underlying asset that expired on 31 March, 2008 . . 61
4.7 Put price at the strike price 630 of the put option with the SET50 Index as the underlying asset that expired on 31 March, 2008 . . 62
4.8 Call price at the strike price 320 of the call option with the SET50 Index as the underlying asset that expired on 31 September, 2008 63
4.9 Put price at the strike price 310 of the put option with the SET50 Index as the underlying asset that expired on 31 September, 2008 64

5.1 Example of data 78
5.2 Using excel find call premium as determined by the Black-Scholes formula 80
5.3 Using excel find put premium as determined by the Black-Scholes formula 81
5.4 Using excel find cimplied volatility as determined by the Black-Scholes formula 83

CHAPTER I

INTRODUCTION

Derivatives are financial instruments whose prices derived from the value of the underlying. The underlying on which a derivative based on can be the price of an asset (commodities, equities, stock, residential mortgages, commercial real estate, loan, bond), the value of an index (interest rate, exchange rate, stock market index, consumer price index). Derivatives are very important especially in finance and investment. The typical forms of derivatives traded in the exchange traded markets are forwards, futures, options, and swap. Some forms are also traded in over-the-counter markets. Options are derivatives that have gained attention and popularity and often used for hedging

An option contract [1] [2] is a financial asset where the price depends on another financial asset. The word option indicates that the contract has specific choices or alternative built in. Options are classified by the right as *call* or *put* options. In general, a call option is a contract that gives the holder the right, but not obligation, to buy a fixed amount of underlying assets at a specified time in future for an already agreed price (called *strike price* or *exercise price*) from the seller (called *writer*) of the option. Oppositely, a buyer of a put option receives the right to sell a fixed amount of assets to the writer of the option for the strike price at a specified time. Here the writer of the put option is obliged to buy the underlying asset while the holder can decide on selling or not. We also usually distinguish between American options and European options. American options give the holder the right to exercise (sell or buy) the underlying assets during the whole time span of the contract, in contrast to a European option where the holder can only exercise his option at maturity of the contract. The day when option contracts cease to exist is called the *expiration date*. There exist options on several forms of underlying assets, for example, stocks, equities, bonds, goods such as oil, corns, currencies, or even options. They are traded in enormous volumes on stock exchange markets all over the world.

The main reason for buying options is the possibility to hedge against non-favorable price fluctuations of underlying assets. The type of option the holder uses for hedging depends on which risks (price increase or price decrease) he hedge against. With the help of options risks of future cash flow can be eliminated or bounded. The main problem we are facing is reasonable the price of an option the price the buyers pay the sellers for such a contract or the *premium*. Many people have tried and produced some valuation methods or formulas to solve this problem. However, most of these methods or formulas are not complete and cannot be applied since the real world situations are more complicate.

In 1973, the option pricing formula was introduced and so-called the Black-Scholes model [3], has been a great attraction on option pricing and hedging. This model is quite popular and influences investors in valuation of pricing for options, because it is simple and its parameters are mostly visible, except for the volatility.

In 1979, John C. Cox, Stephen A. Ross and M. Rubinstein suggested the discrete-time pricing model called Standard Binomial Tree (SBT) model [4]. The SBT model assumes that the ending-nodal probability distribution is log-normal and the transposition probabilities are the same entirely of the tree. The Implied Binomial Tree (IBT) model (1994, [5]) estimates the risk-neutral ending-nodal probability distribution by using the various computational methods. Jens

Carsten Jackwerth placed the emphasis on the weight function used in the IBT model then he proposed the Generalized Binomial Three (GBT) model (1997, [6]). The GBT model adjusts the linear weight function used in the IBT model by using other weight functions.

This research concentrates on the three binomial tree models; SBT, IBT and GBT, in terms of their pricing and hedging performances. The objective of this study is to compare the performances of these three binomial tree models for the SET50 Index options.

In this work the three binomial tree models are applied to SET50 Index options. The SET50 Index options are traded in the Thailand Futures Exchange (TFEX) which has launched on October 2007 to complement the SET50 Index futures. The SET50 Index options offer the investors an opportunity to both capitalize on anticipated market movements and limit the risks of adverse market direction. Investors can also use the SET50 Index options to protect the value of their equity portfolios. This study is the first to examine the performance of the three binomial tree models for the SET50 index options.

This research is organized as follows. In chapter II, we give some literature reviews and basic knowledges about option pricing using binomial tree models. We explain data and methodology in chapter III, and discuss the results in chapter IV. Finally, the last chapter provides the conclusions and remarks about this work.

CHAPTER II

OPTION AND OPTION PRICING

In this chapter, we describe about properties of option and option pricing and explain about hedging strategy.

2.1 Option

Options are derivative securities derived from underlying assets. Options have been traded for centuries, but it was in this century that they gained economic importance. They have become especially popular since 1973 when they were traded in some organized ways on the Chicago Board Options Exchange. An option is a contract between a buyer and a seller, that gives the buyer (holder) the right, but not obligation, to buy or sell particular assets (the underlying asset) on or before the option expiration time at the agree price (the strike price).

2.1.1 Classification of options

classified by the right

There are two types of options, calls and puts, available in the market when they are classified by the right.

Call option is the contract that gives the buyer (holder) the right to buy underlying assets at a specified time in the future. The buyer of the call option has the right, but not the obligation to buy an agree quantity of underlying asset from the seller of the option at the expiration date for the strike price. The seller is obligated to sell the underlying asset should the buyer so decides. The buyer pays a premium fee for the contract that gives the right.

Call options are most profitable for the buyer when the underlying asset is moving up, making the underlying asset price highter than the exercise price. The buyers of call options usually believe that the underlying asset price will rise by the exercise time. The profit for the buyer can be very large but is limited by how high underlying asset price rises. When the underlying asset price surpasses the exercise price, the option is said to be *in-the-money*. On the other hand, the call writer (seller) usually does not believe that the underlying asset price is likely to rise. The total loss for the writer can be very large, but is only limited by how high the underlying asset price rises.

Example 2.1.1. Trader A (buyer or holder) purchases a call contract to buy 100 shares of X corporation from trader B (writer) at \$50 per share. The current price in the stock market is \$45 a share, and trader A pays a premium of \$500 (\$5 a share). If the stock price rises to \$60 a share right before the expiration, then trader A can exercise the call by buying 100 shares for \$5,000 from trader B and sell them at \$6,000 in the stock market.

Trader A's total earnings (P) can be calculated at \$500. Sale of 100 stock at \$60 =\$6,000 (Q). Amount paid to trader B for the 100 stock bought at strike price of \$50 =\$5,000 (R). Call option premium paid to trader B for buying the contract \$500 (S). Like this equation,

$$P = Q - (R + S) = 6,000 - (5,000 + 500) = 500. \tag{2.1}$$

However, if the price drops to \$40 per share below the strike price, then trader A would not exercise the option. Trader A's option would be worthless and the whole investment would be lost due to the premium for the option contract, \$500 (\$5 a share with 100 shares per contract) . Trader A's total loss is limited to the cast of the call premium.

This example illustrates that a call option has positive monetary value when the underlying asset has underlying asset price (S) above the strike price (K). Since the option will not get exercised unless it is *in-the-money*, the payoff for a call option is

$$(S-K)^{+}=\max[(S-K),0] \tag{2.2}$$

Put option is the contract that gives the buyer (holder) the right to sell underlying assets at the specified time in the future. If the buyer uses the right to sell the underlying assets, the seller (writer) is obliged to buy it at the strike price. In exchange for having this contract, the buyer pays the writer a premium fee.

Put options are most profitable for the buyer when the underlying asset is moving down, making the underlying asset price drops below the exercise price. The writers of put options usually believe that the underlying asset price will rise, not fall. The profit for the holder can be very large but it is limited by how low underlying asset price falls. When the underlying asset price is lower than the exercise price, the option is said to be in-the-money. On the other hand, the call writer (seller) usually does not believe that the underlying asset price is likely to down. The total loss for the writer can be very large, but is only limited by how low the underlying asset price falls.

Example 2.1.2. Trader A (put buyer) purchases a put contract to sell 100 shares of Y corporation to trader B (writer) for \$50 per share. The current price is \$55 a share, and trader A pays a premium of \$5 per share. If the stock falls to \$40 right before expiration, then trader A can exercise the put by buying 100 shares for \$4,000 from the stock market, then selling them to trader B for \$5,000.

Trader A's total earnings (P) can be calculated at \$500. Sale of the 100 shares of stock at strike price of \$50 a share (100 shares) to trader B = \$5,000 (Q). Purchase of 100 shares of stock at \$40 a share = \$4,000 (R). Put option premium paid to trader B for buying the contract = \$500 (S). This implies,

$$P = Q - (R + S) = 5,000 - (4,000 + 500) = 500. \tag{2.3}$$

If, however, the share price never drops below the strike price (in this case \$50), then trader A would not exercise the option. Trader A's option would be worthless and he would lose the whole investment for the premium fee of the option contract, \$500. Trader A's total loss are limited to the cost of the put premium.

A put option is said to have intrinsic value when the underlying asset has a underlying asset price (S) below the option's strike price (K), the put option is said to be in-the-money. Upon exercise, a put option is valued at $K - S$ if it is in-the-money, otherwise zero. Prior to exercise, an option has time value apart from its intrinsic value.

classified by exercise style

The style of an option is a general term denoting the class into which the option falls, usually defined by the dates on which the option may be exercised. The two great families are European and American.

- **European** option is an option that can be exercised by only at the expiration date.

- **American** option is an option that can be exercise at any time before the expiration date.

  For both styles, the pay-off when it occurs is via

  - $\max[(S-K),0]$, for a call option and
  - $\max[(K-S),0]$, for a put option,

  where K is the strike price and S is the underlying asset price of the underlying asset.

The following are examples of other atyles of options besides the two styles above.

- A **bermudan** option is an option where the buyer has the right at a set number of times.

- An **exotic** option is an option which has features making it more complex than commonly traded products.

- A **quanto** option is an option on some underlying in one currency but paid in another currency. The pricing of such options naturally needs to take into account the correlation between the exchange rate of the two currencies involved and the underlying stock price.

- An **asian** option is an option where the payoff is not determined by the underlying price at maturity but by the average underlying asset price over some pre-set period of time.

- A **lookback** option is an option where the option owner has the right to buy (respectively, sell) the underlying asset at its lowest (respectively, highest) price over some preceding period.

- A **russian** option is a lookback option which runs for perpetuity. That is, there is no end to the period into which the owner can look back.

- A **game** option (or **israeli** option) is an option where the writer has the opportunity to cancel the option he has offered, but must pay the payoff at that point plus a penalty fee.

The main reason to buy options is the possibility to hedge against non-favorable price oscillations of underlying asset. Of course, it depends on the type of option against which risk (price increase or price decrease) the holder of the option is hedged. Risks of the future cash flows can be eliminated and bounded by the help of option contracts. A typical application is the following : Assume that a company has to make a payment of 20 million dollars next winter. The company is insured against an increase of the exchange rate for dollars by buying a call option on dollars with a strike price of 1.14 Euro for $1 with maturity next winter. However if the exchange rate would be lower the company could buy the dollars at the market.

Of course, options are also traded by speculators who hope for an over proportional increase of the option value compared to the underlying asset price. For example, it is obvious that the price of an option increases less than one dollar if the underlying asset price increases by one dollar. However, the relative price increase of the option will typically be higher than that of the stock in this case. Moreover, options are attractive for speculators as they are much cheaper than the underlying asset itself, and so with little capital it is possible to make relatively large gains. But it should not be forgotten that with options speculators also suffer big losses; in fact, it is not unusual to lose everything.

2.1.2 Contract Specifications

An option is a contract between the two counter parties with the terms of the option specified in the term sheet. Option contracts usually contain the following specifications :

- whether the option holder has the right to buy (call option) or the right to sell (put option);
- the quantity of the underlying asset(s) (for example, 100 shares of ABC Corporation);
- the strike price (or the exercise price) which is the price at which the underlying transaction will occur upon exercise;
- the expiration date (or expiry) which is the last date the option can be exercised;
- the settlement terms, for instance whether the writer must deliver the actual asset on exercise, or may simply tender the equivalent cash amount;
- the terms by which the option is quoted in the market to convert the quoted price into the actual premium -the total amount paid by the holder to the writer of the option.

2.1.3 Option value

If we buy a put option on a stock. we will not know how much this contract will pay at the exercise time because we do not know with certainty the price of the stock at the exercise time. Thus, we will not know how much should we pay for such a contract, i.e., what should be the price or value of the option.

It has been known for many years that **option value** can be estimated by a formula derived from *Black-Scholes model* or by using empirical technique such as the *Binomial tree models.* In addition, [2], the sensitivity of the option value to the amount of time to expiry is known as the option's *theta.* The option value will never be lower than its intrinsic value.

The value of options depend on two values ;
$OptionValue = IntrinsicValue + TimeValue$

1. Intrinsic value is the greater of zero and the difference between the exercise price of the option (*strike price*), K and the current value of the underlying asset (*underlying asset price*), S depending on the types of options as follows:
   - call value $= Max\,[(S-K),0]$,
   - put value $= Max\,[(K-S),0]$.
2. Time Value is measured by the length of time until expiration. The closer to the expiration is, the smaller time value it has. Time value is the difference between option value and intrinsic value, i.e.

$$TimeValue = OptionValue - IntrinsicValue. \tag{2.4}$$

options classified by moneyness

Moneyness is a measure of the degree to which a derivative is likely to have position monetary value at its expiration, in the risk-neutral measure. It can be measured in percentage probability, or in standard derivations. We can classify the moneyness into these catagories as follows.

- **At-the-money : ATM**

  An option is at-the-money if the strike price is the same as the underlying asset price of the underlying asset on which the option is written. An at-the-money option has no intrinsic value, only time value.

- **In-the-money : ITM**

  An in-the-money option has positive intrinsic value as well as time value. A call option is in-the-money when the strike price is below the underlying asset price. A put option is in-the-money when the strike price is above the underlying asset price.

- **Out-of-the-money : OTM**

  An out-of-the-money option has no intrinsic value, only time value. A call option is out-of-the-money when the strike price is above the underlying asset price of the underlying asset. A put option is out-of-the-money when the strike price is below the underlying asset price.

Option prices are also called premiums. They have two components and are determined by six factors. The price changes are indicated by six factors, called *Greeks*. Note that the interest rate and stock dividend are not generally important to option traders.

Factors that influence the price of options

- The price of the underlying asset. As the price of the underlying asset goes up, the price of the option changes. If the option is a call, the price increases, but decreases for a put. The converse is also true; if the price goes down.

- The strike price. It determines the intrinsic value of the option. For a call option, if the strike price is higher than the stock price, the option is out-of-the-money, and is therefore worthless. If this true for a put option, then it is in-the-money.

- The time value of the option : The life of the option is worth money. The closer to the expiry, the smaller its value.

- Volatility. A volatile stock (asset) is a risky stock. Therefore, the option price for this stock will be higher. If the underlying stock is relatively stable, the price will be relatively low. A highly volatile stock will generally yield better premiums, so is advantageous to the seller in this way.

  - Historical volatility. Historical volatility is the how the underlying asset has changed value in the past.

  - Implied volatility. Implied volatility is obtained by a special mathematical formula to calculate how volatile the market makes that the stock will be in the future.

More specifically, an option's time value reflects the probability the option will gain in intrinsic value or become profitable to exercise before it expires. An important factor is the options *volatility*. Volatile prices of the underlying asset can stimulate option demand, enhancing the value. Numerically, this value depends on the time until the expiration date and the volatility of the underlying asset price. The time value of an option is not negative (because the option value is never lower than the intrinsic value), and converges toward zero with time. At expiration, where the option value is simply its intrinsic value, time value is zero. Prior to expiration, the change in time value with time is non-linear, being a function of the option price.

2.2 Option pricing

In general, we know that the higher the price of the underlying asset (*underlying asset price*) , the greater the value of the option. When the underlying asset price is much greater than the exercise price, the call option is almost surely to be exercised. It goes in the opposite direction for put option.

There are several option pricing models. In this research, we focus only on four option pricing models : Black-Scholes model, Standard binomial tree model, Implied binomial tree model and Generalized binomial tree model.

2.2.1 Black-Scholes model

In 1973, Fischer Black and Myron Scholes suggested the option pricing model, called Black-Scholes model [3]. They derived formula for the value of an option in terms of the price of the stock under *ideal conditions* in the market for the stock and the option, namely with the following assumptions.

1. The short-term interest rate is known and is constant for all time.
2. the stock price follows a random walk in continuous time with a variance rate proportional to the square of the stock price. Thus the distribution of possible stock prices at the end of any finite interval is log-normal. The variance rate of the return on the stock is constant.
3. The stock pays no dividends or other distributions.
4. The option is European, that is, it can only be exercised at the maturity.
5. There are no transaction costs in buying or selling stocks or options.
6. It is possible to borrow any fraction of the price of a security to buy it or to hold it, at the short-term interest rate.
7. There are no penalties to short selling. A seller who does not own a security will simply accept the price of the security from a buyer, and will agree to

settle with the buyer on some future date by paying him an amount equal to the price of the security on that date.

Let $C(S,t)$ be the value of the option written as a function of the stock price S and time t, K be the exercise price, T be the maturity date, r be the risk-free interest rate, and v^2 be the variance rate of the return on the stock. The option pricing formula is given as follows:

$$C(S,t) = S \cdot N(d_1) - Ke^{r(t-T)}N(d_2), \tag{2.5}$$

where

$$\begin{aligned} d_1 &= \frac{ln\frac{S}{K} + \left(r + \frac{1}{2}v^2\right)(T-t)}{v\sqrt{T-t}}, \\ d_2 &= \frac{ln\frac{S}{K} + \left(r - \frac{1}{2}v^2\right)(T-t)}{v\sqrt{T-t}}. \end{aligned} \tag{2.6}$$

Here, $N(\cdot)$ denotes the cumulative normal density function.

2.2.2 Standard Binomial Tree (SBT) Model

SBT model [4] is a discrete-time model that relyes on the assumption that the underlying asset price can changed into two prices in the next time step; it can increase to uS with a probability p or decrease to dS with a probability $1-p$ (see Figure 2.1) where u and d are proportion to increase and decrease, respectively. A 2-step binomial tree can be constructed similarly as in Figure 2.3. Similarly, and also for any n-step binomial trees. (see Figure 2.2).

Let r denote one plus the risk-free interest rate over one period, C be the current price of the call, C_{up} be its price at the end of the period if the underlying

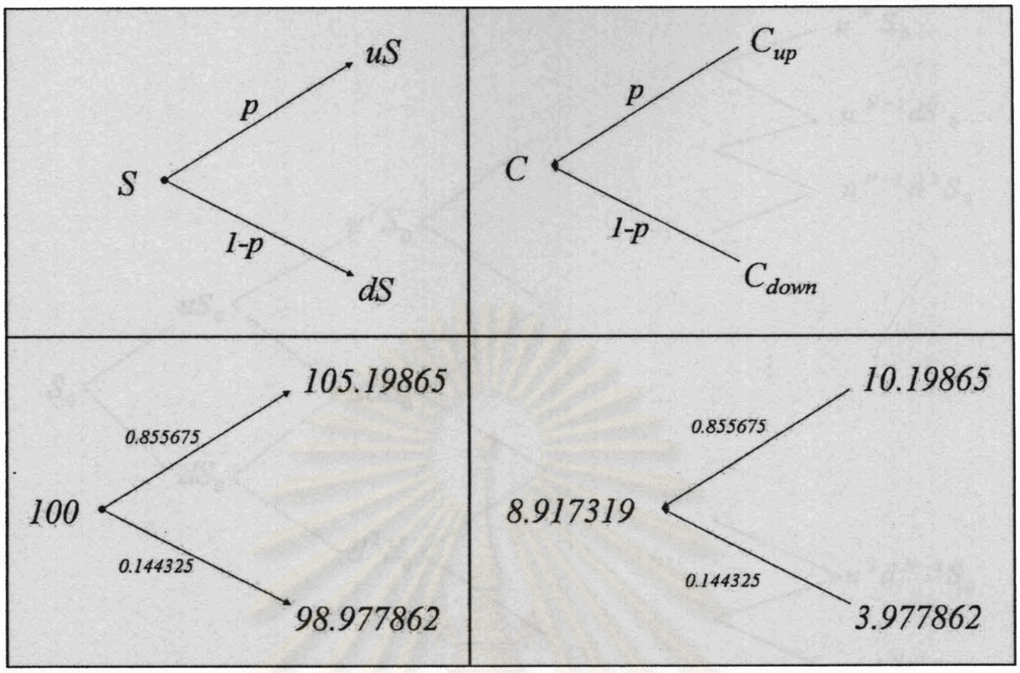

Figure 2.1: The 1-step binomial trees of the underlying asset price (left) and the value of call option (right)

asset price goes to uS, and C_{down} be its price at the end of the period if the underlying asset price goes to dS. Let K be the strike price or the exercise price, σ be a volatility of the underlying asset price, t be time to maturity, and N be the number of steps of the binomial tree. We compute $u = e^{\sigma\sqrt{t/N}}$ and $d = e^{-\sigma\sqrt{t/N}}$ for the proportions to increase and decrease, respectively. Figure 2.1 shows, for example, the two-step tree of the option price. With this idea, we can build the tree of the option price for any n-step, for example, the 2-step tree is shown in Figure 2.3.

This model compute the probability p that a call option price at a underlying asset price is uS from $p = \frac{e^{r(t/N)}-d}{u-d}$. We can calculate option price C when the underlying asset price is S by the equation

$$C = \frac{(pC_{up} + (1-p)C_{down})}{e^{r(t/N)}}. \tag{2.7}$$

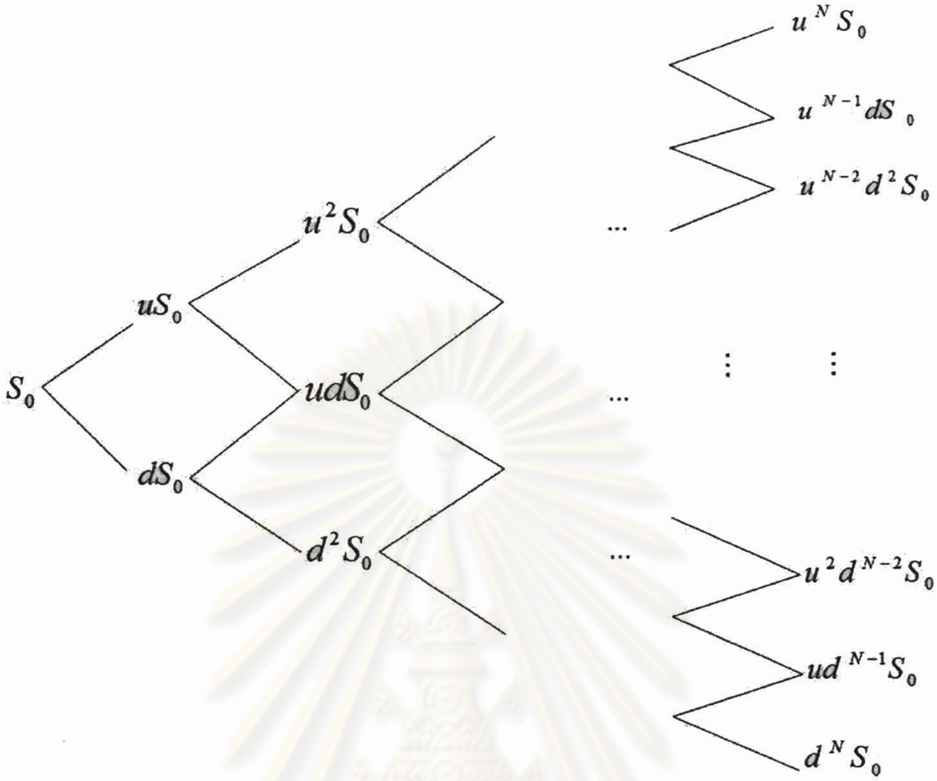

Figure 2.2: The n-step standard binomial tree of the underlying asset price

Repeating this procedure, we can construct the entire tree of the underlying asset prices and the option prices.

Example 2.2.1. Let σ be 0.25, risk-free interest rate be 3% per year, underlying asset price be \$100, and the strike price be \$95. This call option will expire in 30 days and This tree is the 3-step binomial tree.

Solution. With $\sigma = 0.25$, $r = 1 + (0.03 * 30/365)$, $t = 30/365$, $S = 100$, $K = 95$, and $N = 2$. We get $u = 1.042248$, $d = 0.9931749$, $p = 0.719264$. With equation 2.7, we can build the 2-step trees of the underlying asset prices and the option prices shown in figure 2.3.

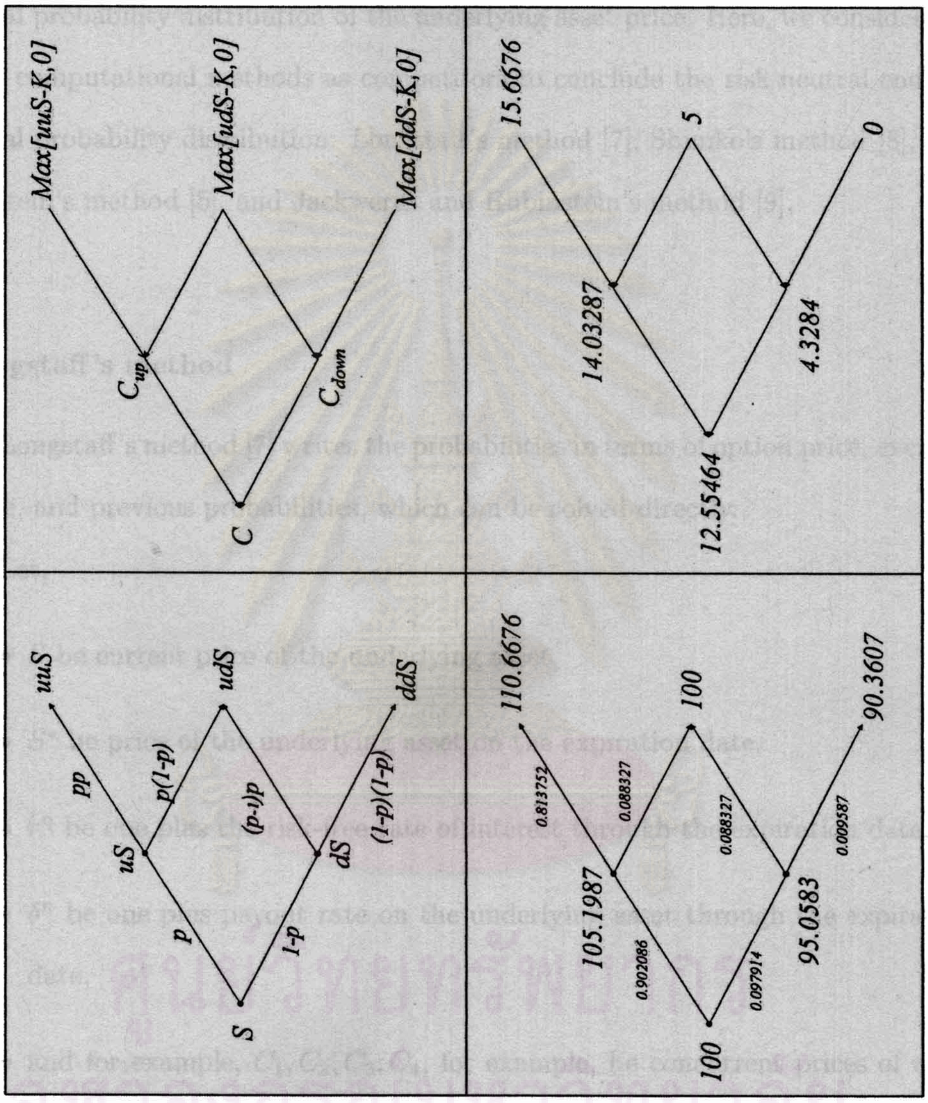

Figure 2.3: The 3-step standard binomial tree of the option price and option price

2.2.3 Implied Binomial Tree (IBT) Model

The basic concept of the IBT model [5] is to build a binomial tree that fits the entire currently traded option price. First, we estimate the risk-neutral ending-nodal probability distribution of the underlying asset price. Here, we consider the four computational methods as competitors to conclude the risk-neutral ending-nodal probability distribution: Longstaff's method [7], Shimko's method [8], Rubinstein's method [5], and Jackwerth and Rubinstein's method [9].

Longstaff's method

Longstaff's method [7] writes the probabilities in terms of option price, exercise price, and previous probabilities, which can be solved directly.

Let,

- S be current price of the underlying asset,
- S^* be price of the underlying asset on the expiration date,
- r^n be one plus the risk-free rate of interest through the expiration date,
- δ^n be one plus payout rate on the underlying asset through the expiration date,
- and for example, C_1, C_2, C_3, C_4, for example, be concurrent prices of associated call options with strike price $K_1 < K_2 < K_3 < K_4$, (in general, we have to consider all strike price $K_1, K_2, ..., K_n$.) all with the same time to expiration.

Assume that, condition on S^* being between adjoining strike prices (including 0), all levels of S^* have equal risk-neutral probabilities. Also assume that there exists a number $K_5 > K_4$ such that the probability that $S^* > K_5$ is zero, and that, conditional on S^* being between K_4 and K_5, all levels of S^* have the same risk-neutral probability. Figure 2.4 depicts this situation.

$$P_1 = 2\left[1 - r^n\left(S\delta^{-n} - C_1\right)K_1^{-1}\right], \tag{2.8}$$

$$P_2 = 2\left[1 - P_1 - r^n\left(C_1 - C_2\right)\left(K_2 - K_1\right)^{-1}\right], \tag{2.9}$$

$$P_3 = 2\left[1 - P_1 - P_2 - r^n\left(C_2 - C_3\right)\left(K_3 - K_2\right)^{-1}\right], \tag{2.10}$$

$$P_4 = 2\left[1 - P_1 - P_2 - P_3 - r^n\left(C_3 - C_4\right)\left(K_4 - K_3\right)^{-1}\right], \tag{2.11}$$

$$P_5 = 1 - P_1 - P_2 - P_3 - P_4, \tag{2.12}$$

$$K_5 = K_4 + \frac{(2r^n C_4)}{P_5}. \tag{2.13}$$

The implied risk-neutral probabilities can be derived by solving the first equation for P_1, using this value for P_1 then solving for P_2, using these value for P_1 and P_2 then solving for P_3, and so on until we get P_5.

Shimko's method

Shimko's idea [8] is to approximate the implied volatilities across strike prices by a quadratic polynomial. By substituting the fitted polynomial for the volatility in the Black-Scholes model, Shimko's method estimates the distribution of the risk-neutral probabilities in term of the second derivative of the call price with respect to exercise price according to the following equation,

$$P_{S_T=X} = e^{rT\frac{\partial^2 C}{\partial X^2}}, \tag{2.14}$$

where

$$\frac{\partial^2 C}{\partial X^2} = SN'(d_1)\left[\frac{1}{X}\left((2aX+b)\sqrt{T} - \frac{\partial d_1}{\partial X}\right) - d_1\frac{\partial d_1}{\partial X}(2aX+b)\sqrt{T} + 2a\sqrt{T}\right],$$

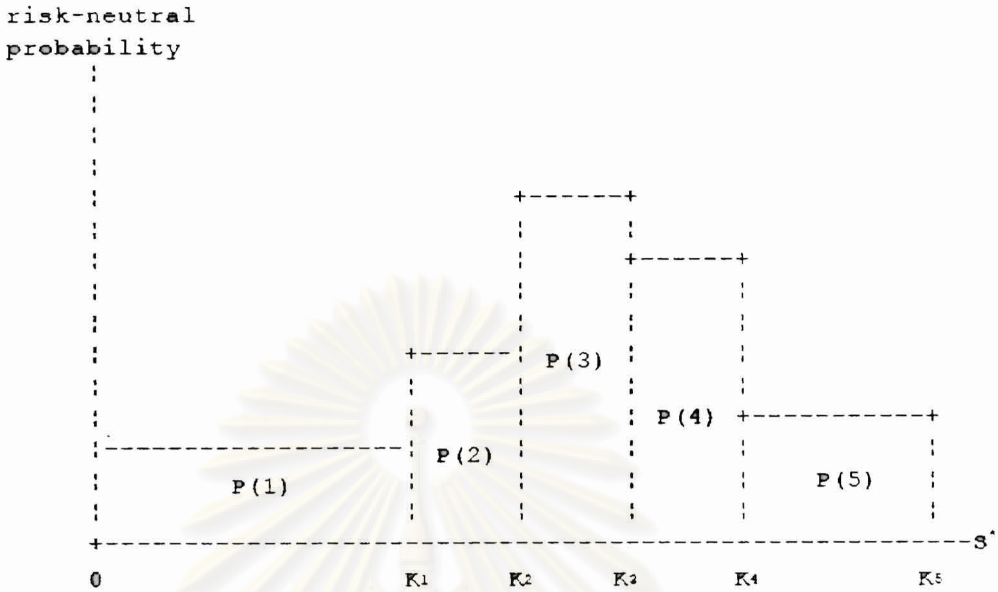

Figure 2.4: Risk-Neutral Probability Distribution

$$\frac{\partial d_1}{\partial X} = \frac{\left[\frac{-1}{X} + (2ax+b)\cdot\sigma(X)\cdot T\right]\left(\sigma(X)\cdot\sqrt{T}\right) - d_1(2aX-b)\cdot\sigma XT}{[\sigma(X)]^2 T},$$

$$d_1 = \left[ln(S/X) + \left(r - q - \frac{[\sigma(X)]^2}{2}\right)T\right] / \left[\sigma(X)\cdot\sqrt{T}\right],$$

$$\sigma(X) = aX^2 + bX + c,$$

$$N'(d_1) = \frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}}. \qquad (2.15)$$

Rubinstein's method

Rubinstein's method [5] minimizes the difference between the implied posterior risk-neutral probabilities $P_{n,j}$ and a prior estimate of risk-neutral probabilities $\hat{P}_{n,j}$, (generated by the SBT), in the square sense under all constrains.

Denote the ending-nodal underlying asset price of the tree from the lowest to the highest by $S_{n,j}$ for $j = 0, 1, 2, ..., n$ and . Let S and C_h^b (C_h^a) be the underlying asset price and the option bid (ask) price quotes with strike price $K_h, h = 1, ..., m$ ($m << N$, number of strike price) expiring at the end of the tree.

Then, the implied posterior risk-neutral probabilities $P_{n,j}$ are calculated from the following quadratic minimization problem.

Objective :

$$\min_{P_{n,j}} \sum_j (P_{n,j} - \hat{P}_{n,j})^2. \tag{2.16}$$

Subject to :

$$\sum_j P_{n,j} = 1 \text{ and } P_{n,j} \geq 0 \text{ for } j = 0, ..., N,$$

$$S = \sum_j \frac{P_{n,j} S_{n,j}}{r^N},$$

$$C_h^b \geq C_h \geq C_h^a \text{ where } C_k = (\sum_j P_{n,j} \max[0, S_{n,j} - K_h])/r^n \text{ , for } h = 1, ..., m. \tag{2.17}$$

Here C_h denotes the option price at strike price K_h.

Jackwerth and Rubinstein's method

Jackwerth and Rubinstein's method [9] decides for the implied probability distribution with maximum smoothness. The implied posterior risk-neutral probabilities $P_{n,j}$ are calculated from the following formula,

$$\min_{P_{n,j}} \sum_j (P_{n,j-1} - 2P_{n,j} + P_{n,j+1})^2 \text{ where } P_{n,-1} = P_{n,n+1} = 0, \tag{2.18}$$

with the same constrains as in Rubinstein's method.

With the risk-neutral ending-nodal probabilities $P^{nodal}_{n,j} = P_n, j$ the implied binomial tree of the underlying asset price can be constructed by three steps as follow:

First step.

$$P^{nodal}_{i-1,j-1} := [1 - w(i,j-1)]\, P^{nodal}_{i,j-1} + w(i,j) P^{nodal}_{i,j} \text{ for } i = n, ..., 1, j = 1, ..., i. \tag{2.19}$$

Second step.

$$P_{i-1,j-1} = \frac{w(i,j) P^{nodal}_{i,j}}{P^{nodal}_{i-1,j-1}} \text{ for } i = n, ..., 1, j = 1, ..., i. \tag{2.20}$$

Third step.

$$S_{i-1,j-1} = \left[(1 - P_{i-1,j-1})\, S_{i,j-1} + P_{i-1,j-1} S_{i,j}\right]/r, \tag{2.21}$$

where weight function is linear, defined as $w(i,j) = j/i$. The entire tree of the underlying asset price can be built using three steps above. The tree of the option price can be constructed in the same way as in the SBT model.

2.2.4 Generalized Binomial Tree (GBT) Model

This model generalizes the IBT model by using different weight function $w(i,j)$ in the first and second steps, equation (2.19) and (2.20). We consider five weight functions; linear concave, linear convex, quadratic convex, and S-curve. These functions are defined as follows.

Linear case:

$$w(i,j) = \begin{cases} \left(2\frac{j}{i}\right)\alpha, & \text{for } \frac{j}{i} \in [0, 0.5], \\ 2(1-\alpha)\left(\left(\frac{j}{i}\right) - 0.5\right) + \alpha, & \text{for } \frac{j}{i} \in [0.5, 1], \end{cases} \tag{2.22}$$

$$\text{where} \begin{cases} \alpha \in [0, 0.5) & \text{for convex,} \\ \alpha \in (0.5, 1] & \text{for concave.} \end{cases}$$

Quadratic case:

$$w(i,j) = \alpha \left(\frac{j}{i}\right)^2 + (1-\alpha)\left(\frac{j}{i}\right) \text{ where } \begin{cases} \alpha \in (0, 1] & \text{for convex,} \\ \alpha \in [-1, 0) & \text{for concave,} \end{cases} . \quad (2.23)$$

S-curve case:

$$w(i,j) = \begin{cases} 0, & \text{for} \frac{j}{i} = 0, \\ 1, & \text{for} \frac{j}{i} = 1, \\ CN\left(-5 + 10(\frac{j}{i}), 0, \alpha\right), & \text{for } \frac{j}{i} \in (0,1), \end{cases} \quad (2.24)$$

with $\alpha \in (0, 2.5]$, where $CN(\cdot, 0, \alpha)$ is cumulative normal distribution function with mean of 0 and standard deviation α.

To compute the performances for each weight function [11], we use the root mean square error (RMSE), where the error is the difference between market observed short-term option price and theoretical short-term option price, to find the optimal weight function with the optimal α.

2.3 Hedge

In finance, a hedge is an investment that is taken out specifically to reduce or cancel out the risk in another investment. The term is a shortened form of hedging your bets, a gambling term. Typical hedgers purchase a security that the investor thinks will increase in value, and combine this with a short sell of a related security or securities in case the market as a whole goes down in value.

Example 2.3.1. An investor believes that the company FA is going to do well this month, and wishes to buy some shares of stock so as to profit from their rise

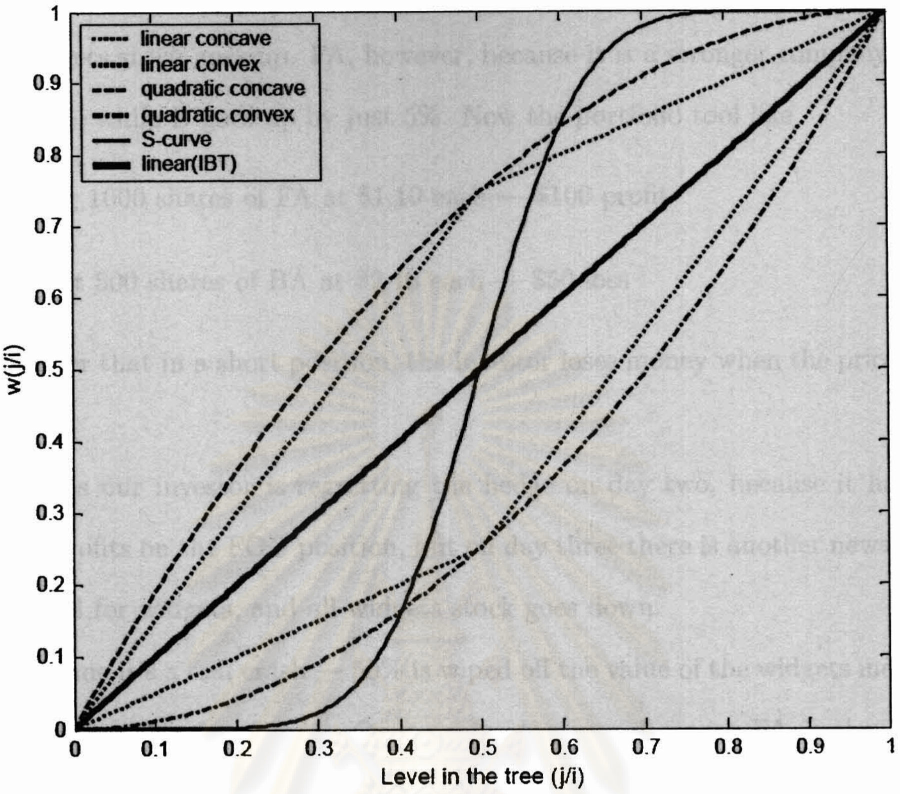

Figure 2.5: An illustration of various weight functions

in value. FA is, however, part of the widgets industry, a sector whose share prices are highly volatile.

Our investor is interested in the company itself, not the vagaries of the industry, and so seeks to hedge out the risk by selling short an equal amount of the shares of FA's direct competitor, BA.

On day one, our investor's portfolio looks like:

- Long 1000 shares of FA at \$1 each
- Short 500 shares of BA at \$2 each

(Notice that the investor has sold short the same value of shares, not the same number).

On day two, there is a big news story about the widgets industry and the value of all widgets stock goes up. FA, however, because it is a stronger company, goes up by 10%, while B goes up by just 5%. Now the portfolio tool like

- Long 1000 shares of FA at $1.10 each — $100 profit
- Short 500 shares of BA at $2.10 each — $50 loss

(Remember that in a short position, the investor loses money when the price goes up)

Perhaps our investor is regretting the hedge on day two, because it has cut into the profits on the FOO position, but on day three there is another news story that is bad for widgets, and all widgets stock goes down.

This time it's a real crash — 50% is wiped off the value of the widgets industry in the course of a few hours. Once again, however, because FA is the better company it suffers less than BA:

Value of long position:

- Day 1 — $1000
- Day 2 — $1100
- Day 3 — $550

Value of short position:

- Day 1 — $1000
- Day 2 — $1050
- Day 3 — $505

Without the hedge, our investor would be looking at a loss of $450. With the hedge, that loss still stands on the long side, but the short side is in profit of $495.

That means our investor in widgets is still $45 in profit on a day when the market suffered a dramatic collapse.

Example 2.3.2. Hedging an agricultural commodity price

A typical hedger might be a commercial farmer. The market values of wheat and other crops fluctuate constantly as supply and demand for them vary, with occasional large moves in either direction. Based on current prices and forecast levels at harvest time, the farmer might decide that planting wheat is a good idea one season, but the forecast prices are only that - forecasts. Once the farmer plants wheat, he is committed to it for an entire growing season. If the actual price of wheat rises a lot between planting and harvest, the farmer stands to make a lot of unexpected money, but if the actual price drops by harvest time, he could be ruined.

If the farmer sells a number of wheat futures contracts equivalent to his crop size at planting time, he effectively locks in the price of wheat at that time - the contract is an agreement to deliver a certain number of bushels of wheat on a certain date in the future for a certain fixed price. He has hedged his exposure to wheat prices; he no longer cares whether the current price rises or falls, because he is guaranteed a price by the contract. He no longer needs to worry about being ruined by a low wheat price at harvest time, but he also gives up the chance at making extra money from a high wheat price at harvest times.

2.3.1 Types of hedging

The stock example above is a classic sort of hedge, known in the industry as a pairs trade due to the trading on a pair of related securities. As investors becomes more sophisticated, along with the mathematical tools used to calculate values, known as models, the types of hedges have increased greatly.

Examples Of Hedging Strategies

This is a list of examples of hedging strategies, for financial derivatives such as call and put options.

- **Risk reversal** Simultaneously buying a call option and selling a put option, This has the effect simulating being long a stock or commodity position.
- **Delta neutral** This is a market neutral position that allows a portfolio to maintain a positive cash flow by dynamically re-hedging to maintain a market neutral position. This is also a type of market neutral strategy.

CHAPTER III

DATA AND METHODOLOGY

We compare the empirical performances of binomial tree models, mentioned in chapter II, for SET50 Index options. In this chapter, we describe the sample data, algorithms for each binomial model, and error estimations.

3.1 Data

In our empirical tests, we use SET50 Index options as data. The SET50 Index options with SET50 index as the underlying assets are traded in the Thailand Futures Exchange(TFEX). The SET50 Index is calculated from the stock prices of the top fifty listed companies on the Stock Exchange Thailand (SET) in terms of large market capitalization, hight liquidity and compliance with requirement regarding the distribution of shares to minor shareholders. The SET50 index option is a European option which can be exercised only at the expiration date. There are four series for both calls and puts; March, June, September and December. In each option contract month, there are at least eleven strike prices, five-OTM, one-ATM, and five-ITM, in each day.

The data sampling period is taken from 28 December 2007 to 27 June 2008, which is also used to estimate parameters for the optimal weight function in the GBT model. The data with best bid and best ask prices at 4.30pm in each trading days and expiration dates are provided from the TFEX. There are 2270 data on over 111 days for empirical testing.

3.1.1 Contract Specifications

The TFEX launched SET50 Index options on October 29, 2007 to complement SET50 Index futures. SET50 Index options offer investors an opportunity to both capitalize on anticipated market movements and limit the risks of adverse market direction. Investors can also use SET50 Index option to protect the value of their equity portfolios.

The TFEX use the monogram of the option contract in terms of "Underlying Symbol + Month + Year + Put/Call Symbol + Strike Price", for example,
"S50H08C500"= SET50 Index call option expires on March 2008 with the strike price 500 point.
"S50M80P530"= SET50 Index put option expires on June 2008 with the strike price 530 point.

- **S50** is SET50 Index
-
  - **H** : the options expire on March
  - **M**: the options expire on June
  - **U** : the options expire on September
  - **Z** : the options expire on December
- 08 is year 2008.
- C is the symbol for call option and P is the symbol of put option.
- 530 is the strike price.

The last trading day of each contract is the day before the last trading day in each expire month.

SET50 Index options are European style options, settled in cash against the value of the SET50 Index on the last trading day. There are SET50 Index put and call option with different exercise prices for various trading strategies. Summary of SET50 Index options contract specifications have shown in Table 3.1.

3.2 Methodology

We build trees of underlying asset prices (SET50 Index) and option prices (SET50 Index option price) by using three models: the SBT, the IBT, and the GBT model. For each contract (strike price) we estimated the option prices for 58 days in sequences. For each binomial tree we set parameters: the number of steps $N = 200$, the risk-free interest rate r calculated by using the 91-day certificate of deposit (CD) rates (provided from Bank of Thailand), the Black-Scholes implied volatility σ calculated with the average of bid and ask quotes of the ATM call options (in this work we calculated in Excel (see Appendix B)), and time to maturity t. For each day we built n-step binomial tree for SET50 Index and used the tree to estimate the SET50 Index option price of the day. Then we compute errors in terms of pricing and hedging performances.

3.2.1 The SBT algorithm

We follow the construction of the n-step binomial tree for SET50 Index $S_{i,j}$ as described in section 2.2 for each contract. From the parameters: N, σ, r, and t we get an up $u = \exp(\sigma\sqrt{(t/N)})$, a down move $d = \exp(-\sigma\sqrt{(t/N)})$, and the transition probability of an up move is $p = \frac{e^{r(t/N)}-d}{u-d}$. The flowchart of the SBT are shown in Figure 3.1. From the binomial tree of SET50 Index we use the ending-nodal price $S_{n,i}(i = 0, 1, ..., n)$ to estimate the SET50 Index option price C_j as described in section 2.2, see Figure 3.2.

Heading	Individual Contract specification
Underlying index	SET50 Index which is compiled, computed and disseminated by the Stock Exchange of Thailand
Contract Multiplier	200 Baht per index point
Contract Months	March, June, September, December up to 4 quarters
Minimum price fluctuations	0.10 index points
Price Limit	+/- 30% of the previous day's SET50 Index
Exercise style	European
Strike price interval	10 points (at least 5 in-the-money strikes, 5 out-of-the-money strikes and 1 at-the-money strike).
Trading Hours	Pre-open: 9.15 - 9.45 hrs. Morning session: 9.45 - 12.30 hrs. Pre-open: 14.00 - 14.30 hrs. Afternoon session: 14.30 - 16.55 hrs.
Speculative Position limit	Net 20,000 delta equivalent SET50 Index Futures contracts on one side of the market in any contract month or all contract months combined.
Final Trading Day	The business day immediately preceding the last business day of the contract month. Time at which trading ceases on Final Trading Day is 16.30 hrs.

Heading	Individual Contract specification
Final Settlement Price	The final settlement price shall be the numerical value of the SET50 Index, rounded down to the nearest two decimal points as determined by the exchange, and shall be the average value of the SET50 Index taken during last 15 minutes plus the closing index value, after deleting the three highest and tree lowest values.
Settlement Procedures	Cash Settlement
Exchange and clearing fee	THB 10 per contract per side
Brokerage commission	Free negotiable

Table 3.1: Contract Specification of SET50 Index Options

3.2.2 The IBT algorithm

This algorithm starts by building the binomial tree of SET50 Index, (the dash border in Figure 3.1). We first estimated the risk-neutral ending-nodal probability distribution of the SET50 Index (see Figure 3.3). We consider the four computational methods as competitors for estimating the probability distribution; Longstaff's method (Figure 3.5), Shimko's method (Figure 3.6), Rubinstein's method (Figure 3.7) and Jackwerth and Rubinstein's method (Figure 3.8). The procedures for these methods are described in section 2.2.3. Once the best result for estimating of the ending-nodal probability distribution is obtained (see the results belows), We then built the binomial tree for the option price, similar to that of SBT method (Figure 3.2).

We find the probabilities of the Longstaff and Shimko's methods are out of range of $[0, 1]$, see Figure 3.2.2. The Jackwerth and Rubinstein's method produce errors larger than the Rubinstein's method, see Figure 3.3 . Then, in our empirical we use the third method (Rubin's) to obtain the implied probability distribution as input for the IBT and GBT models.

3.2.3 The GBT algorithm

The GBT model placed the emphasis on the weight function used in the IBT model. We use five-different weight functions instead of linear weight function in IBT method, the dash border in the algorithm of IBT in figure 3.4. The flowcharts for these weight-functions are displayed as follows: Linear concave , Linear convex in Figure 3.11, quadratic concave, quadratic convex in Figure 3.12 and S-curve in Figure 3.13. Then we obtained the optimal α by using the Root Mean Square Error (**RMSE**) as a measurement

$$\text{RMSE} := \sqrt{\frac{1}{N}\sum_{i}^{N} e_i^2}. \tag{3.1}$$

Table 3.2.3 shows the optimal parameter values and the average values of RMSE (see Equation 3.1) for the five different weight functions in the test using the sample data from 8 December 2007 to 27 June 2008.

As the result from Table 3.2.3 show, we use a S-curve weight function with α of 2.5 to construct the GBT.

In our error estimates, we compare the empirical performances of three models in terms of pricing and hedging errors in moneyness and option types.

3.2.4 Pricing Errors

We examine the option pricing performance in terms of error e_i, by letting e_i be the differences between the option prices C_i, provided by the TFEX (average of best bid and best ask quotes in each day), and the estimated option values from the binomial tree models $\hat{C}_i$, namely

$$e_i = C_i - \hat{C}_i. \tag{3.2}$$

3.2.5 Hedging errors

Hedging performance is significant to investors who often use option as a risk management tool. We explain a hedging strategy as follow.

Consider hedging a long position in a call or a put. Let Δ_i be the number of units of the underlying asset to be sold for an option, and $C_i - \Delta_i \cdot S_i$ be the residual position.

To examine the hedging performance, we

1. construct a hedge portfolio by longing an option and shorting Δ_i units of the underlying asset.

2. borrowing $C_i - \Delta_i \cdot S_i$ in a risk-free rate.

3. compute Δ_i from

$$\Delta_i = \frac{C_{1,1} - C_{1,0}}{S_{1,1} - S_{1,0}} \tag{3.3}$$

where $C_{\cdot,\cdot}$ and $S_{\cdot,\cdot}$ are option values and the underlying asset price at node (i,j) from the binomial tree built on each date.

4. calculate the hedging error from

$$\epsilon_i = \left(\hat{C}_i - C_i\right) - \Delta_i \cdot \left(\hat{S}_i - S_i\right) - (C_i - \Delta_i \cdot S_i) \cdot (r_i \cdot \Delta t) \tag{3.4}$$

where Δt is the length of period between the date when the hedging strategy is performed to the expiration date, r is the risk-free rate and $\hat{S}_i$ is the underlying asset price at $S_{0,0}$ in each binomial tree.

3.3 Analysis of Empirical Errors

We analyze the results from the pricing error and the hedging error based on four error measurements, the mean error (ME), the mean percentage error (MPE), the mean square error (MSE), and the mean absolute percentage error (MAPE).

- **The Mean Error** (ME) is the standard deviation of the differences between the actual values of the dependent variables and the predicted values. This statistic is associated with regression analysis. This measure indicates the direction of error. The formula is

$$\text{ME} := \frac{1}{N}\sum_{i=1}^{N} e_i, \tag{3.5}$$

where N is the number of data.

- **The Mean Percentage Error** (MPE) is the computed average of percentage errors by which estimated forecasts differ from actual values of the quantity being forecast. This measure indicates the direction of errors. The formula is

$$\text{MPE} := \frac{1}{N}\sum_{i=1}^{N} \frac{e_i}{C_i}, \tag{3.6}$$

where N is the number of data.

- **The Mean Square Error** (**MSE**) is a risk function, corresponding to the expected value of the squared error loss or quadratic loss. MSE measures the average of the square of the error. The error is the amount by which the estimator differs from the quantity to be estimated. This measure indicates the volatility of errors. The formula is

$$\text{MSE} := \frac{1}{N}\sum_{i=1}^{N} e_i^2, \tag{3.7}$$

where N is the number of data.

- **The Mean Absolute Percentage Error** (**MAPE**) is commonly used in quantitative forecasting methods because it produces a measure of relative overall fit. The absolute values of all the percentage errors are summed up and the average is computed. The MAPE indicates the magnitude of error. The formula is

$$\text{MAPE} := \frac{1}{N}\sum_{i=1}^{N} \frac{|e_i|}{C_i}, \tag{3.8}$$

where N is the number of data.

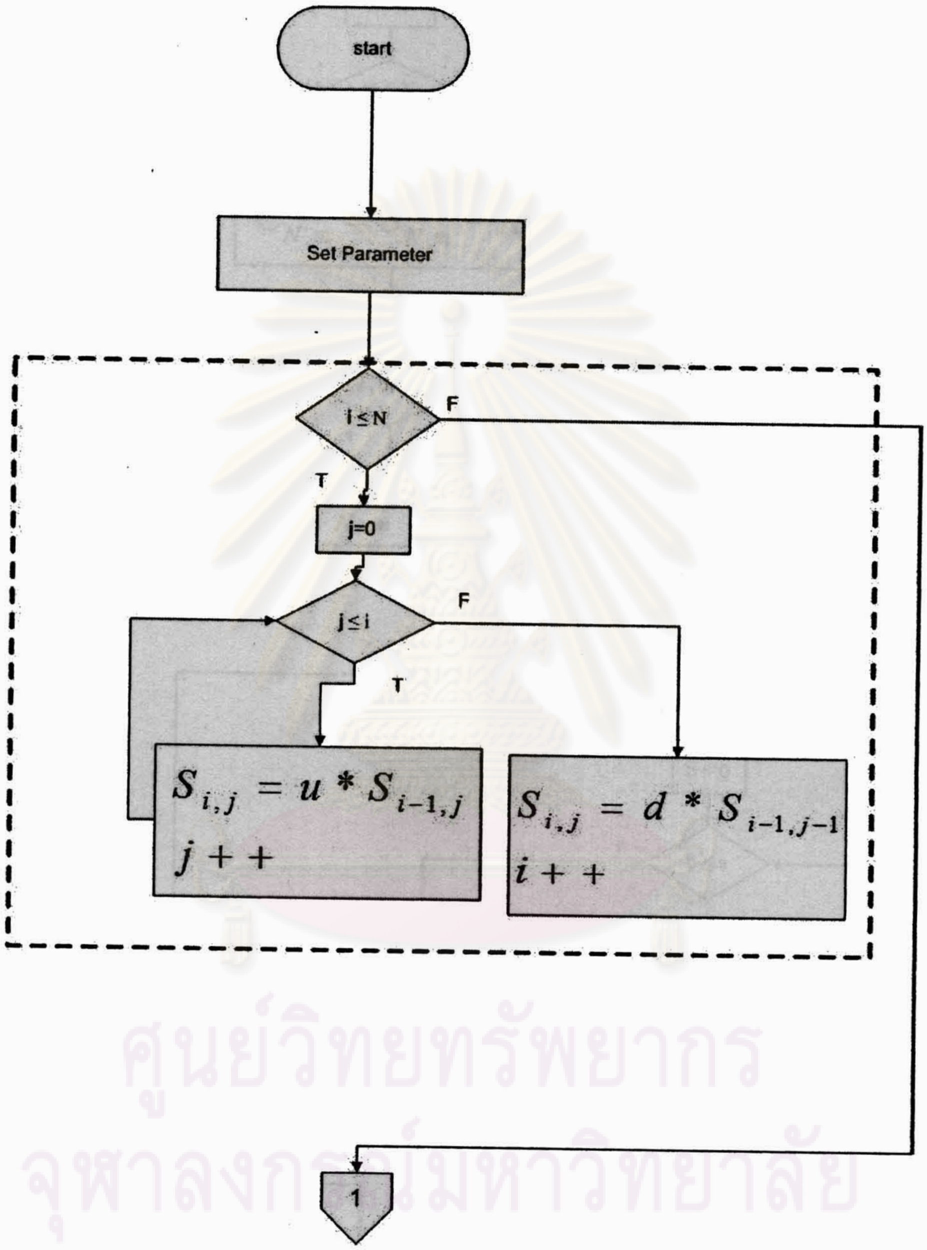

Figure 3.1: Flowchart of the SBT model

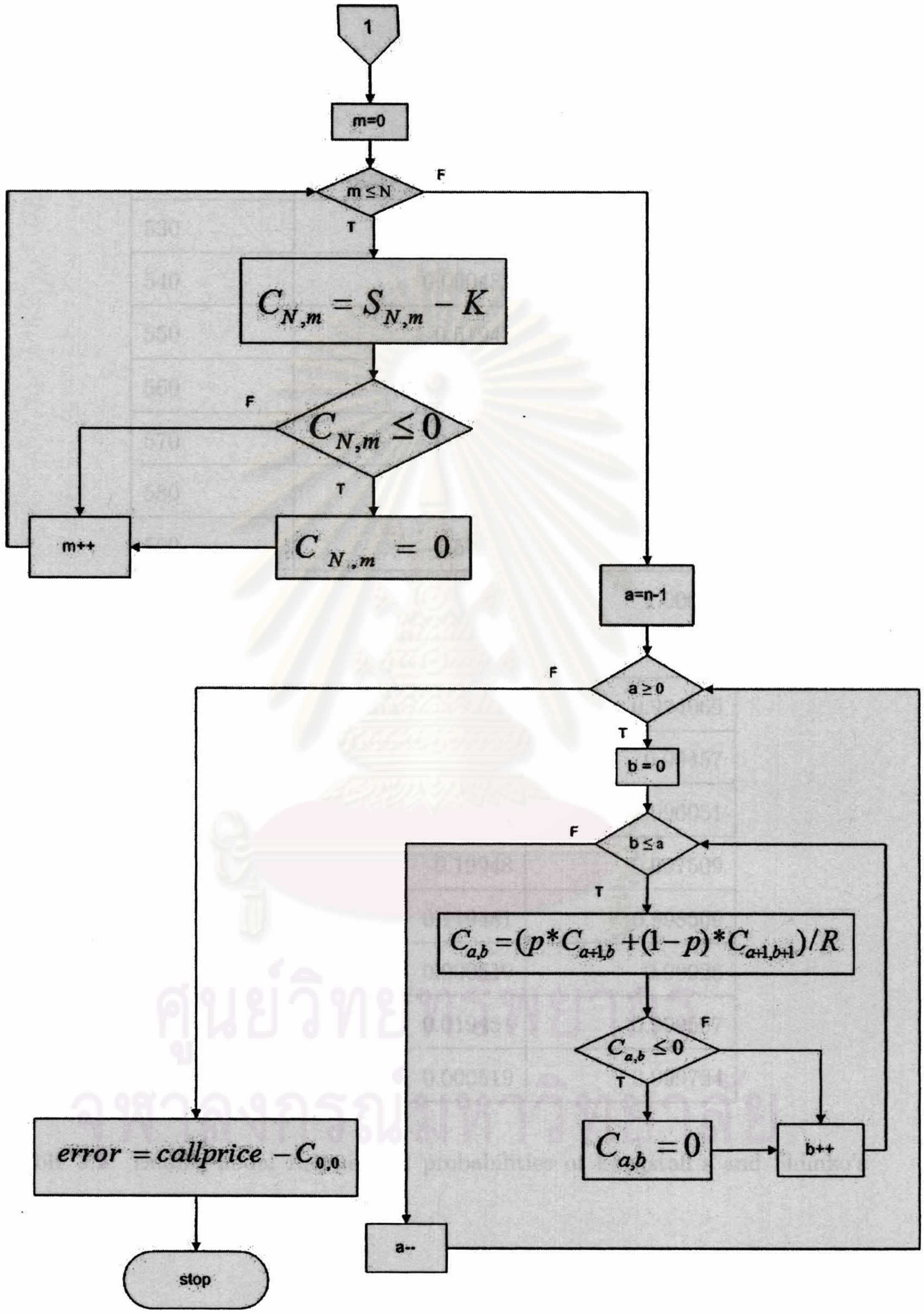

Figure 3.2: Flowchart of the SBT model

strike price	Longstaff (prob.)	Shimko (prob.)
530	0.000255	1.00042
540	0.099482	1.00004
550	-0.51948	1.00042
560	0.839485	1.00212
570	-0.81948	1.00547
580	1.97949	1.00778
590	-1.57949	1.00581
600	0.319482	1.00058
610	0.660523	0.995931
620	-0.06052	0.994069
630	0.10052	0.99457
640	0.039481	0.996051
650	-0.19948	0.997509
660	0.119481	0.998569
670	0.000519	0.99926
680	0.019451	0.999597
690	0.000519	0.999794

Table 3.2: Ending-nodal risk-neutral probabilities of Longstaff's and Shimko's methods

Measures	Methods	C1			
		OTM	NTM	ITM	ALL
ME	method 3	**21.7470**	**7.5315**	**1.2930**	**7.90187**
	method 4	45.8370	20.6662	5.8263	18.6854
MPE	method 3	**0.4503**	**0.3623**	**0.0922**	**0.2395**
	method 4	0.9259	0.9500	0.9895	0.9653
MSE	method 3	**543.1957**	**75.7084**	**6.5713**	**161.4235**
	method 4	2294.0154	495.8334	63.3580	703.4152
MAPE	method 3	**0.4503**	**0.3858**	**0.4921**	**0.4603**
	method 4	0.9630	0.9500	0.9895	0.96598

Table 3.3: pricing errors of the IBT for calls.

This table shows the results of pricing errors of the IBT by using Rubinstein's method (method 3) and Jackwerth and Rubinstein's method (method 4) to estimate the ending-nodal risk-neutral probability distribution for the calls (C1). Out-of-the-money (OTM) options comprise calls with $S/K < 0.97$, Near-the-money (NTM) options comprise calls with $0.97 < S/K < 1.03$, and In-the-money (ITM) options comprise calls with $S/K > 1.03$. ALL includes calls all type. The numbers are based on four measures - mean error (ME), percentage error (MPE), mean square error (MSE) and mean absolute percentage error (MAPE) - calculated from difference between observed market option price and each tree model's theoretical price for the sample period from 8 December 2007 to 27 June 2008 which are used for building the trees.

weight function	weight(α)	RMSE(C1)	weight(α)	RMSE(P1)
Linear concave	0.52	12.2237	0.52	9.19776
Linear convex	0.5	12.7053	0.5	9.46406
Quadratic concave	-0.08	11.947304	-0.09	9.13192
Quadratic convex	0.1	14.84623	0.1	10.4427
S-curve	**2.5**	**8.9968**	**2.5**	**6.67654**

Table 3.4: Estimation results for different weight functions

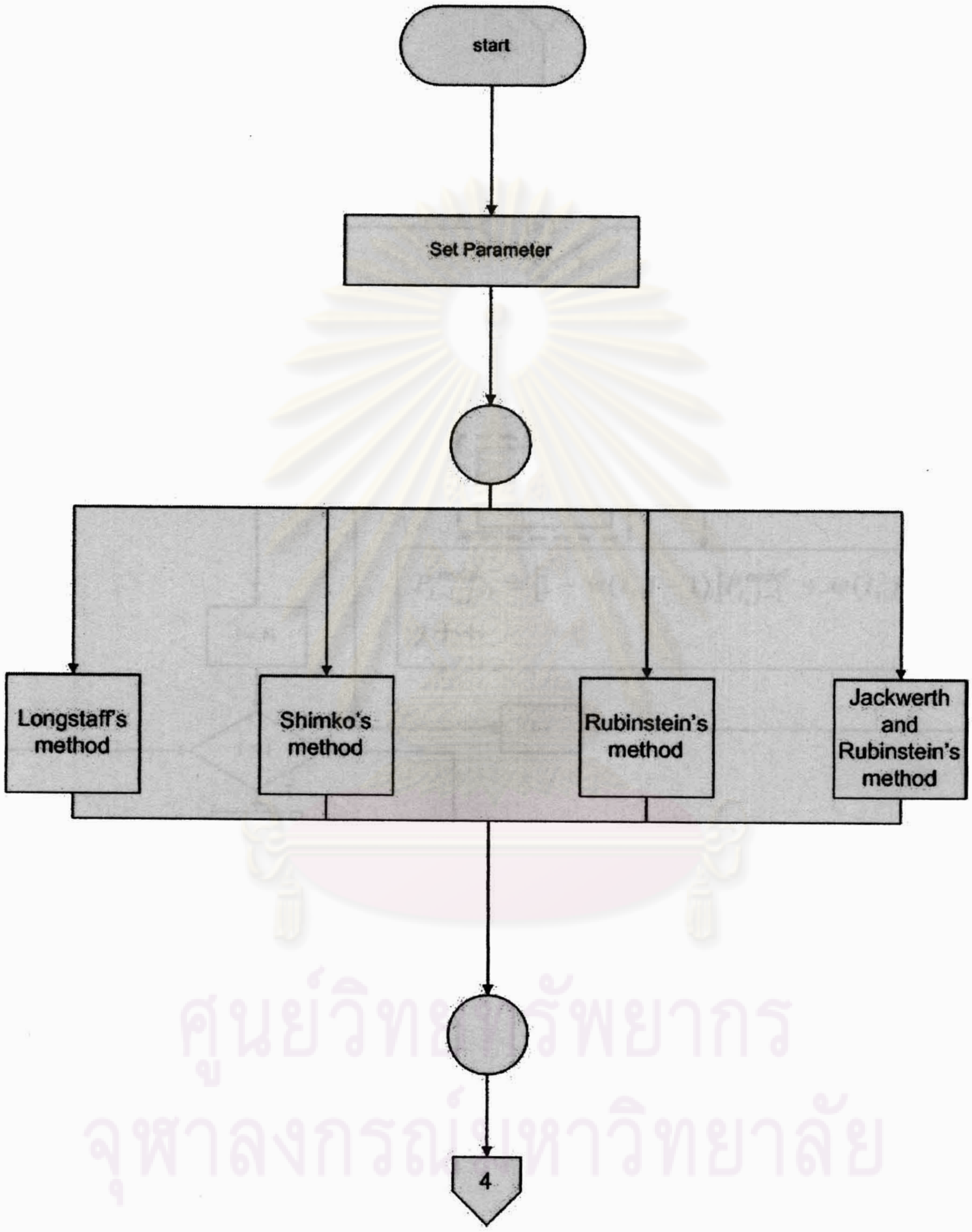

Figure 3.3: Flowchart of the IBT model [1]

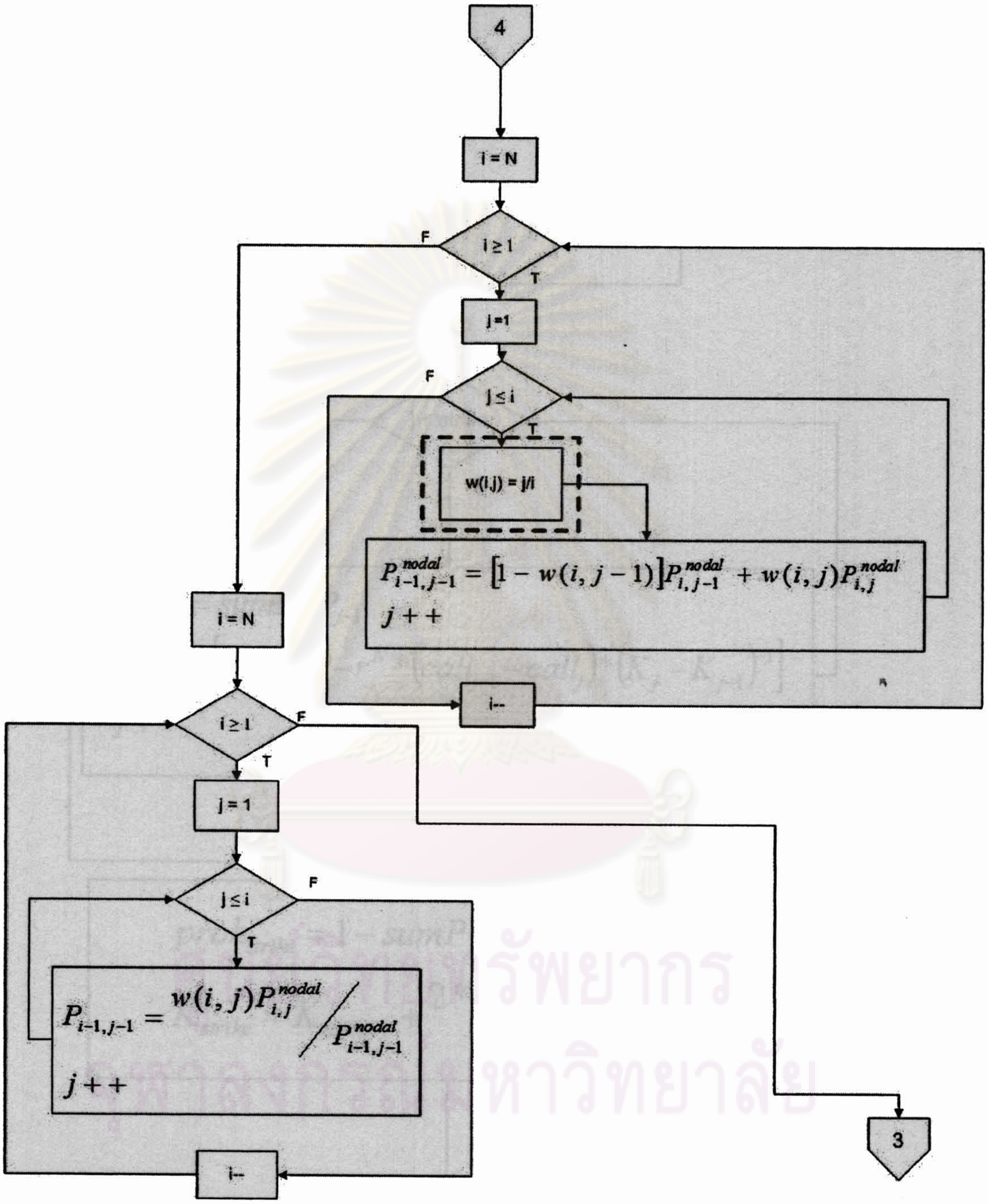

Figure 3.4: Flowchart of the IBT model [2]

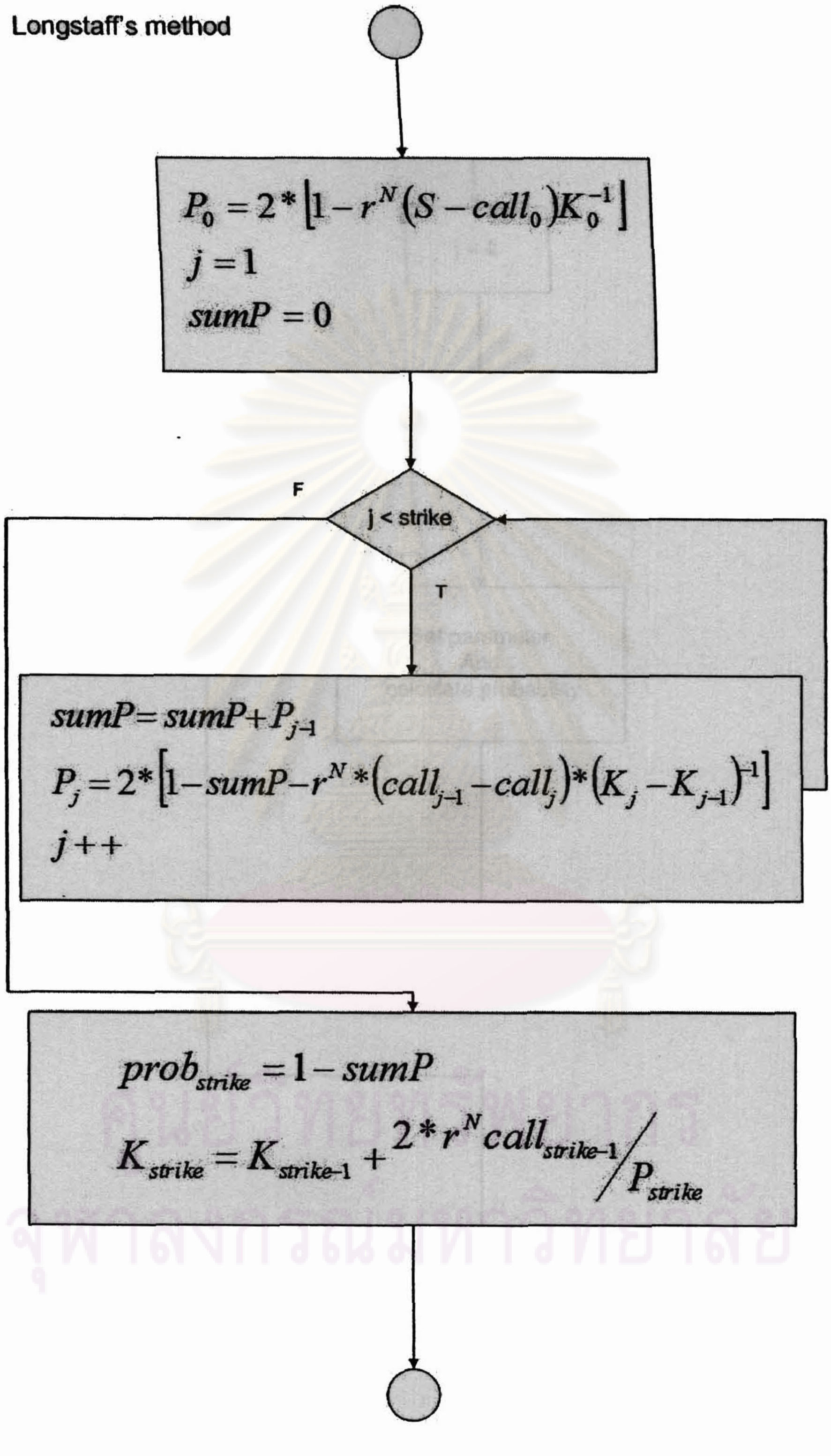

Figure 3.5: Flowchart of the Longstaff's method used in the IBT model

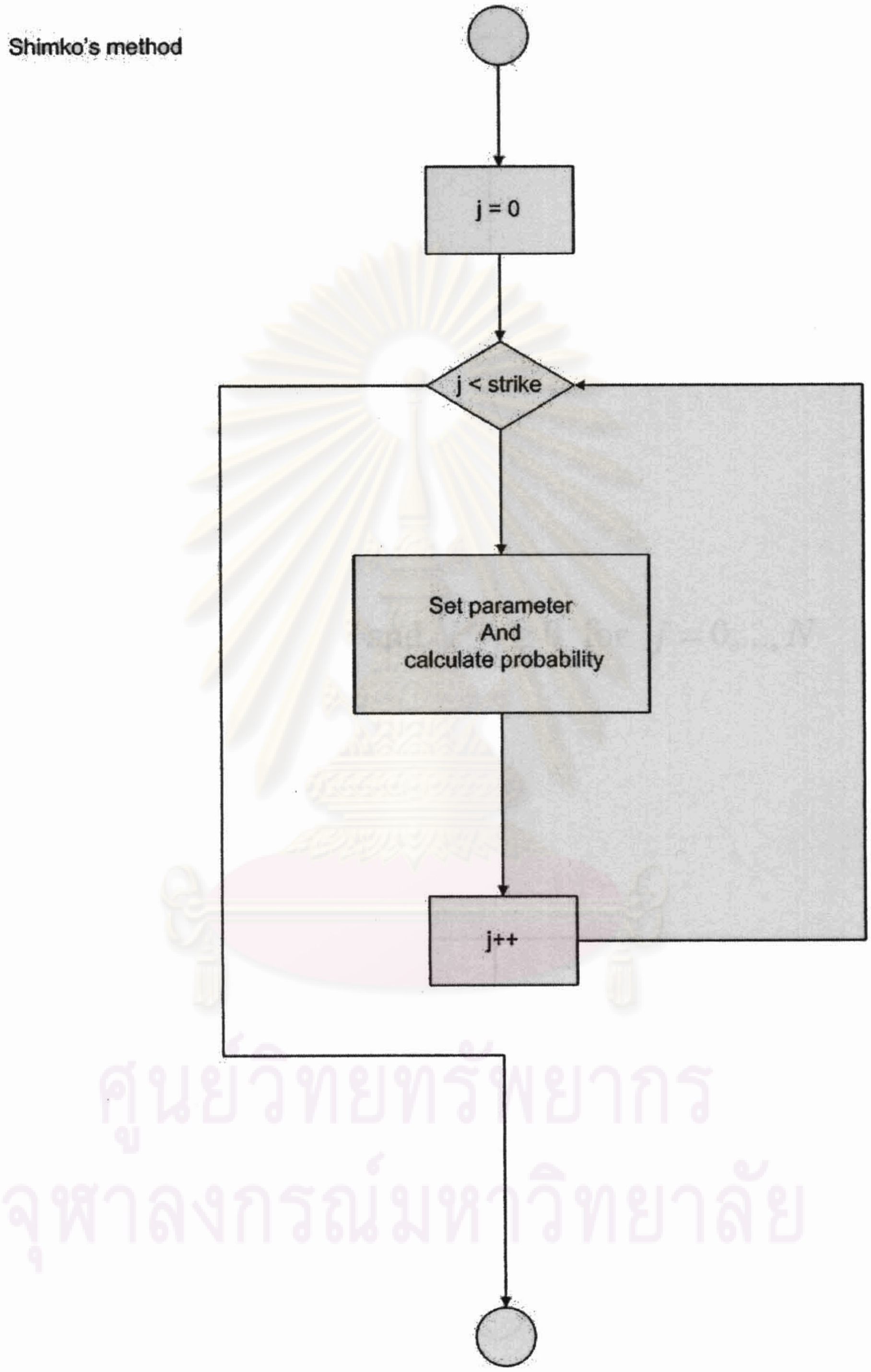

Figure 3.6: Flowchart of the Shimko's method used in the IBT model

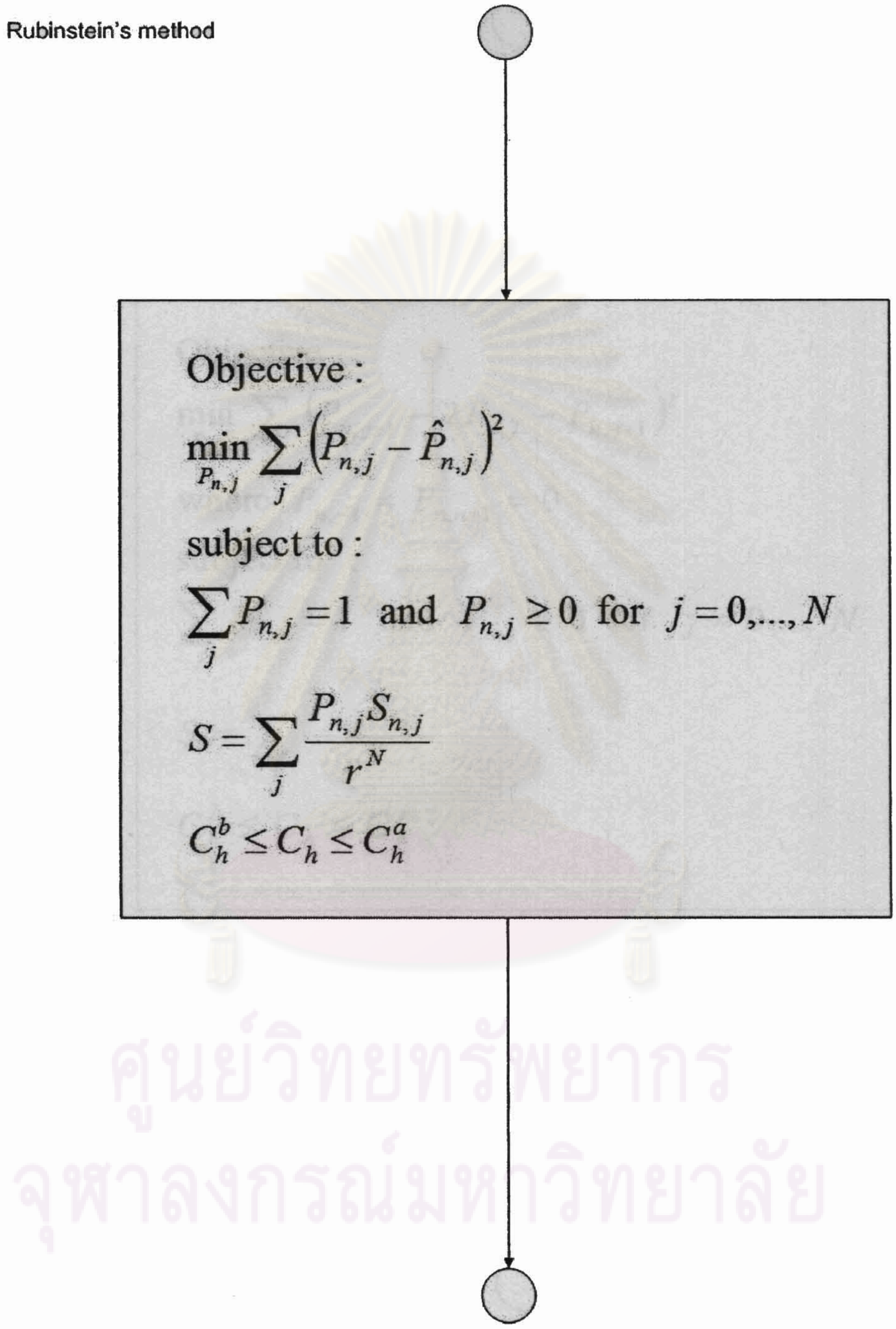

Figure 3.7: Flowchart of the Rubinstein's method used in the IBT model

Jackwert and Rubinstein's method

Objective :

$$\min_{P_{n,j}} \sum_j \left(P_{n,j-1} - 2P_{n,j} - P_{n,j+1}\right)^2$$

where $P_{n,-1} = P_{n,n+1} = 0$

subject to :

$$\sum_j P_{n,j} = 1 \quad \text{and} \quad P_{n,j} \geq 0 \quad \text{for} \quad j = 0, \ldots, N$$

$$S = \sum_j \frac{P_{n,j} S_{n,j}}{r^N}$$

$$C_h^b \leq C_h \leq C_h^a$$

Figure 3.8: Flowchart of the Jackwerth and Rubinstein's method used in the IBT model

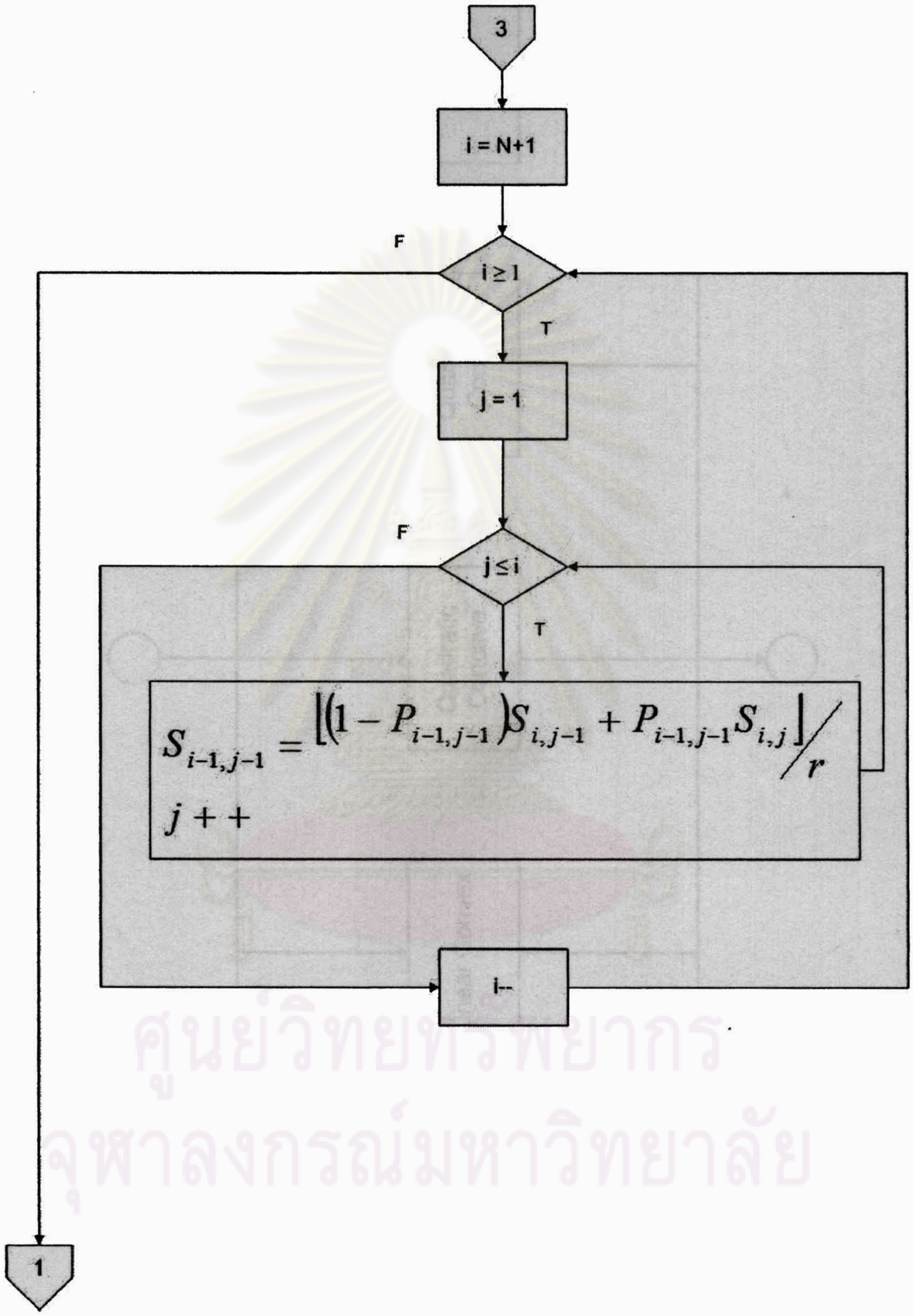

Figure 3.9: Flowchart of the IBT model [3]

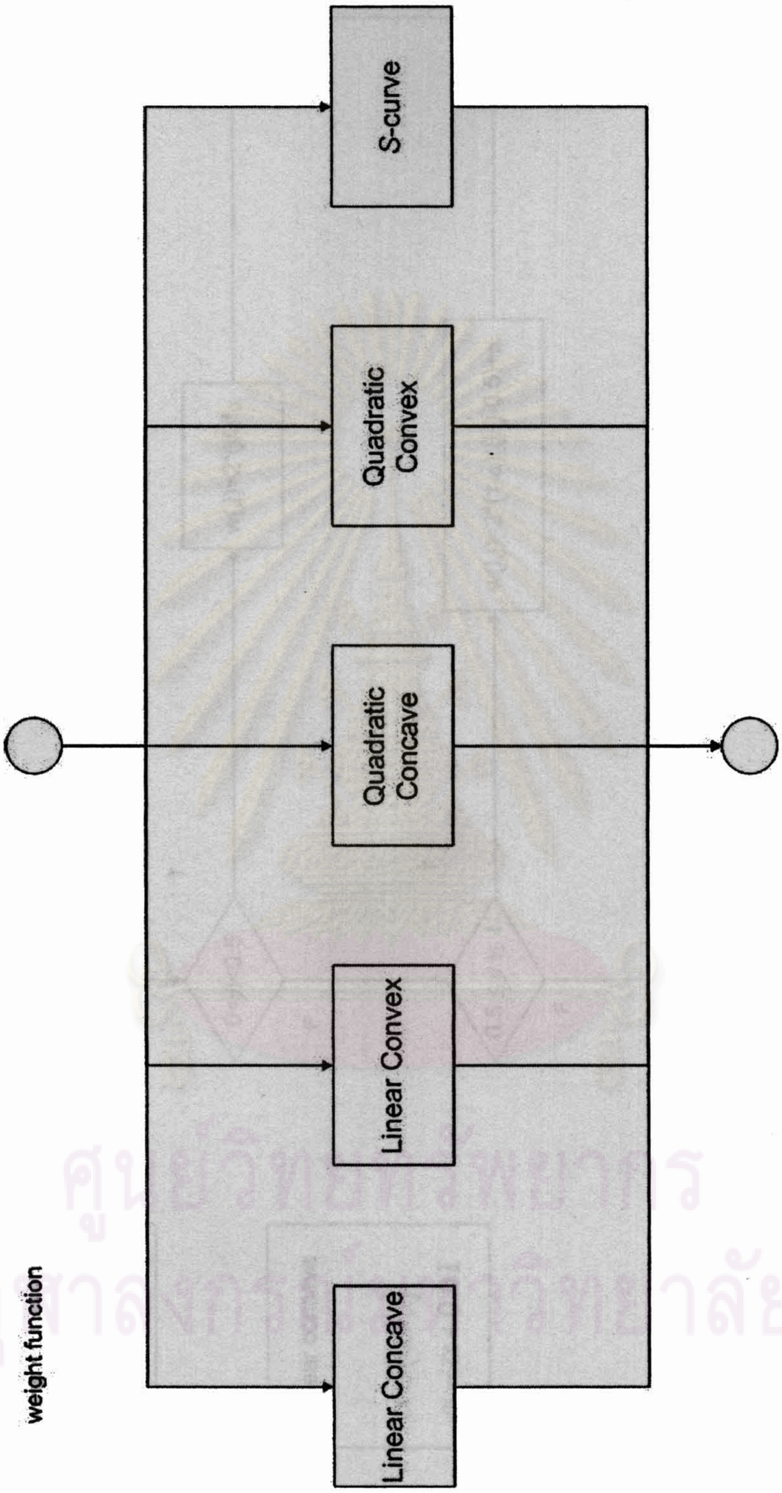

Figure 3.10: Flowchart of the GBT model

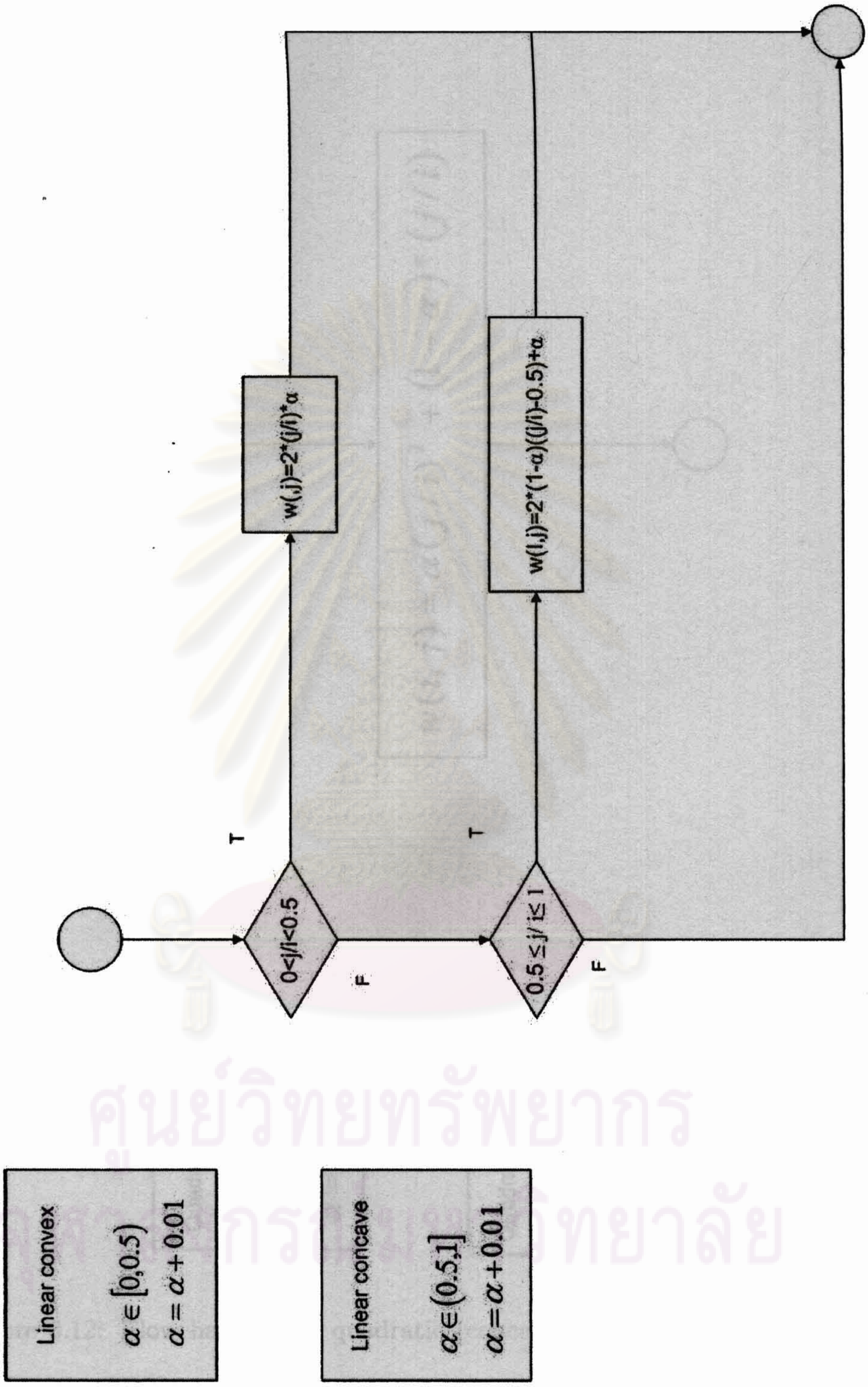

Figure 3.11: Flowchart of the linear (concave and convex) weight functions used in the GBT model

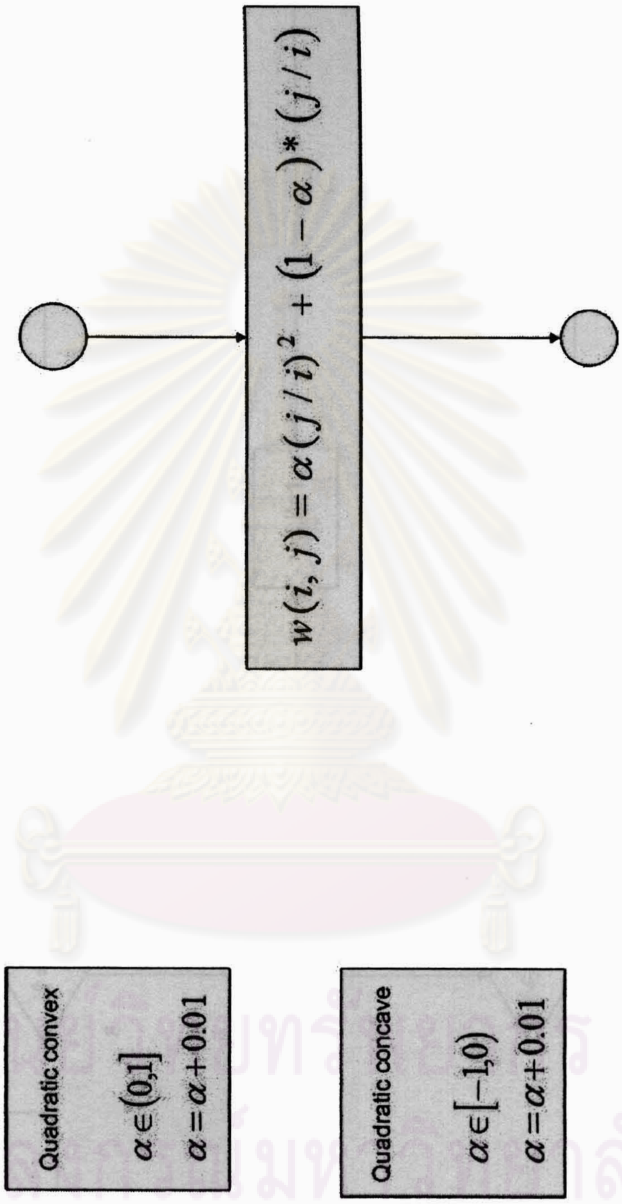

Figure 3.12: Flowchart of the quadratic (concave and convex) weight functions used in the GBT model

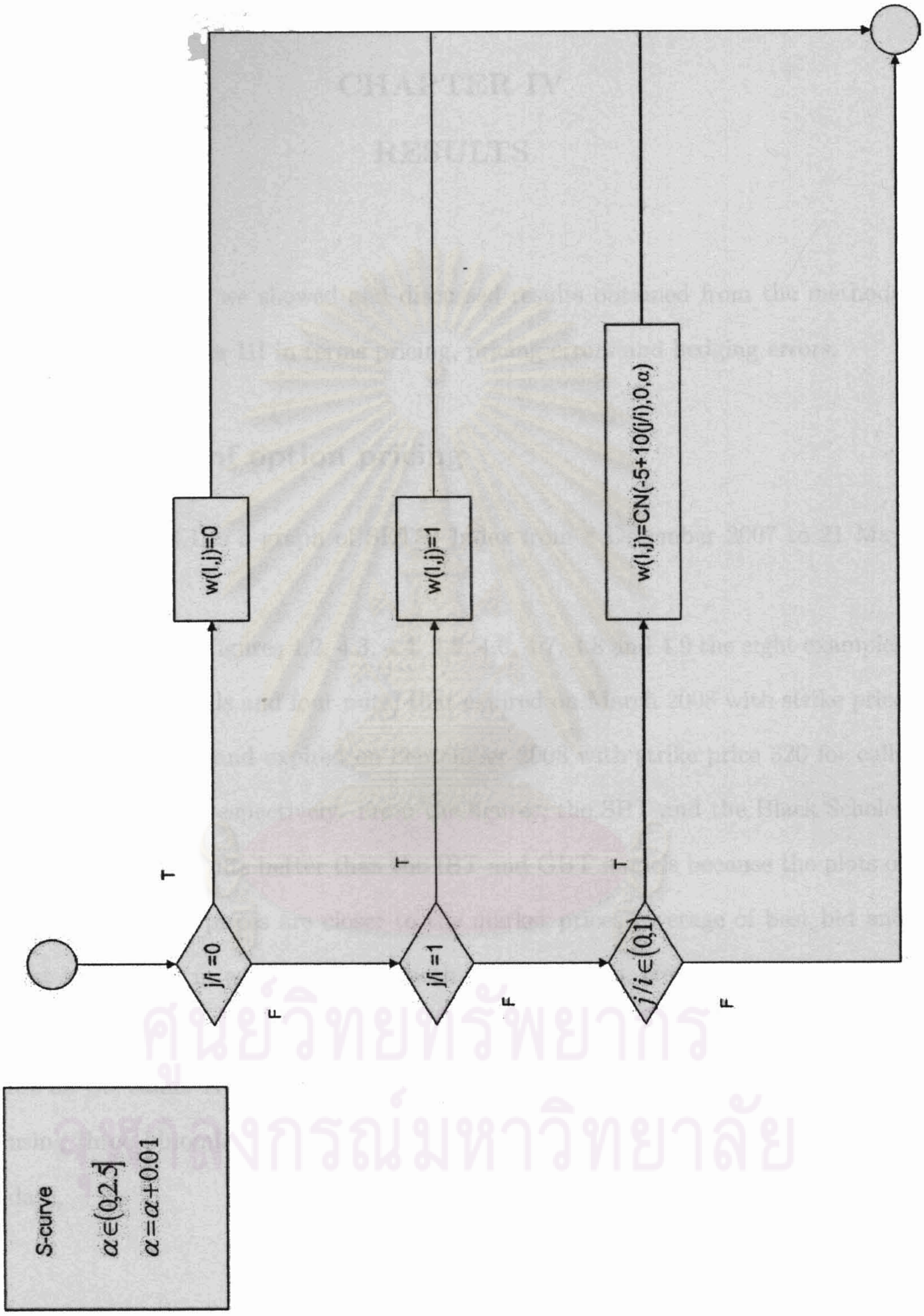

Figure 3.13: Flowchart of the S-curve weight functions used in the GBT model

CHAPTER IV

RESULTS

In this chapter we showed and discussed results obtained from the methods described in chapter III in terms pricing, pricing errors and hedging errors.

4.1 Results of option pricing

The figure 4.1 is a graph of SET50 Index from 8 December 2007 to 21 May 2009.

We showed in Figures 4.2, 4.3, 4.4, 4.5, 4.6, 4.7, 4.8 and 4.9 the eight examples of results; (four calls and four puts) that expired on March 2008 with strike price 530, 600 and 630, and expired on September 2008 with strike price 320 for calls and 310 for puts, respectively. From the figures, the SBT and the Black-Scholes model give the results better than the IBT and GBT models because the plots of estimated option prices are closer to the market prices (average of best bid and best ask quotes in each day). For both calls and puts, trends of movements of estimated option prices by using three binomial tree models and the market prices are all the same. At OTM of calls and puts, trends of estimated option prices by using three binomial tree models converge to zero at one day before expiration date.

4.1.1 Pricing performance

We show pricing errors of results in terms of four-measurements; MA, MPE, MSE and MAPE based on moneyness for current month expiration calls (C1 and C3) and subsequent month expiration calls (C2 and C4), and current month expiration puts (P1, P3) and subsequent month expiration puts (P2, P4), see Tables 4.1, 4.2, 4.3 and 4.4.

The Table 4.1 and 4.2 show that the SBT model is superior in terms of the option pricing errors for C1 & C2. Comparing between all four measurements, SBT method seems to have smaller errors than the other methods. This also agree with the results in Tables 4.2 - 4.7. Also comparing between IBT and GBT in terms of four measurements, at ITM of the IBT are smaller than that of the GBT. For OTM that of the GBT is smaller than that of the IBT. These results suggest the SBT is better than The IBT and GBT models in pricing performance for calls.

Not only for calls, the SBT seems to have the errors better than that of IBT and GBT. Comparing between IBT and GBT in terms of four measurements, at OTM and NTM, errors of the GBT are better than that of the IBT. The MSE of these three binomial tree models are very large for C1, C2, C3, C4, P1, P2, P3 and P4, they indicate that these errors are very fluctuating errors.

From the Table 4.1, 4.2, 4.3 and 4.4, they suggest that the SBT is better than the IBT and GBT models.

4.1.2 Hedging performance

We showed in Table 4.5 and 4.6 hedging errors of results in terms of four error measurements; MA, MPE, MSE and MAPE based on moneyness for current month expiration calls (C1, C3) and subsequent month expiration calls (C2, C4); and similarly the results for current month expiration puts (P1 and P3) and subsequent month expiration puts (P2 and P4) are shown in Table 4.7 and 4.8.

The Table 4.5 and 4.6 show that the SBT is superior in terms of the option pricing errors for C1 & C2. Comparing between all four measurements, SBT method seems to have smaller errors than the other methods. Also comparing between IBT and GBT in terms of four measurements, at OTM errors of the GBT is smaller than that of the IBT. At ITM and NTM we can see that the IBT seems to have errors smaller than the GBT. These results suggest the SBT is better than The IBT and GBT models in hedging performance for calls. Not only for C1, C2, C3 and C4, the SBT seems to have the errors better than that of IBT and GBT (see the Table 4.7 and 4.8) for P1, P2, P3 and P4. Comparing between IBT and GBT in terms of four measurements, at OTM and NTM of the GBT are smaller than that of the IBT. The MSE of these three binomial tree models are very large for C1, C2, C3, C4, P1, P2, P3 and P4, these indicate that these errors are very fluctuating errors.

From the Table 4.5, 4.6, 4.7 and 4.8, they suggest that the SBT is better than the IBT and GBT models.

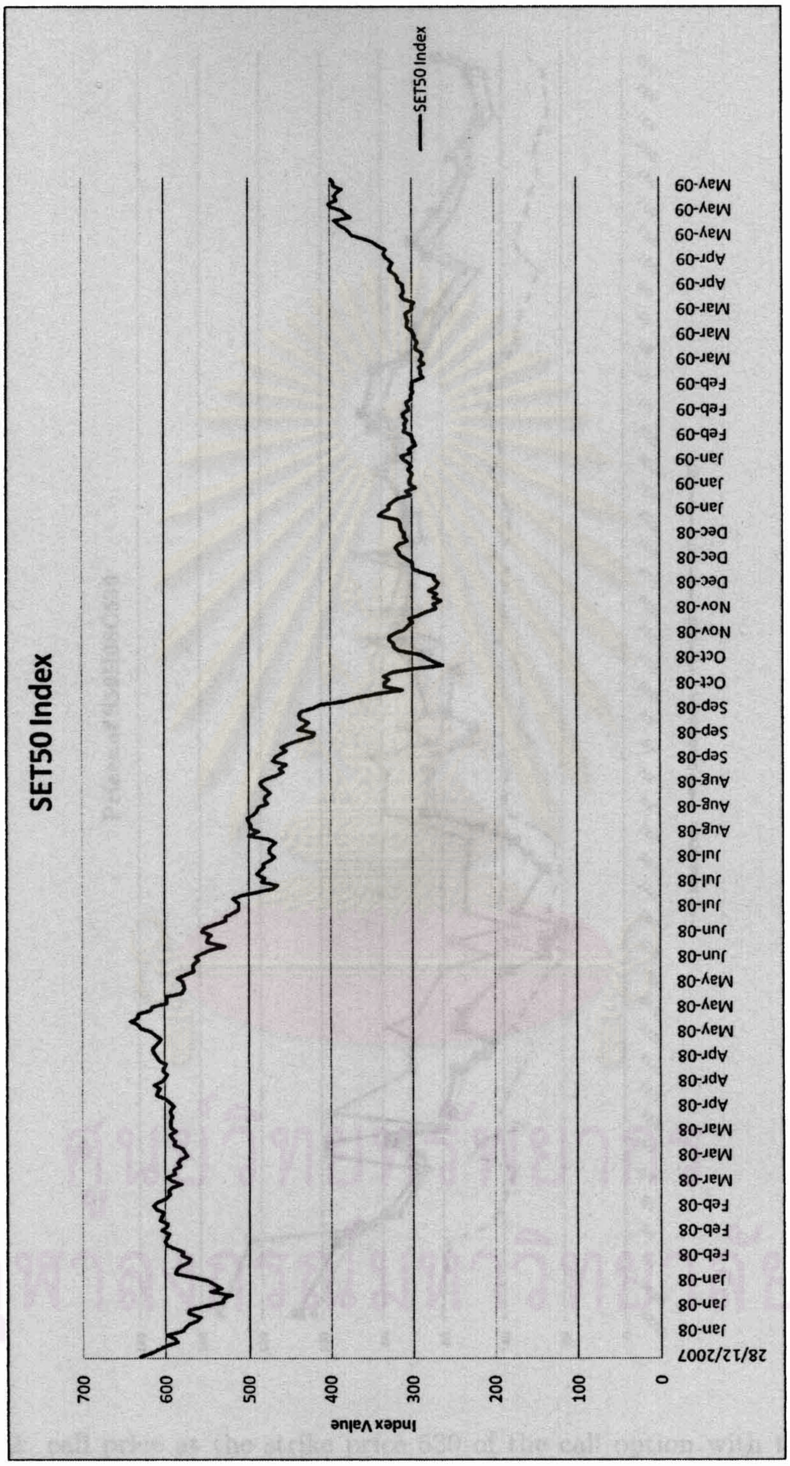

Figure 4.1: SET50 Index

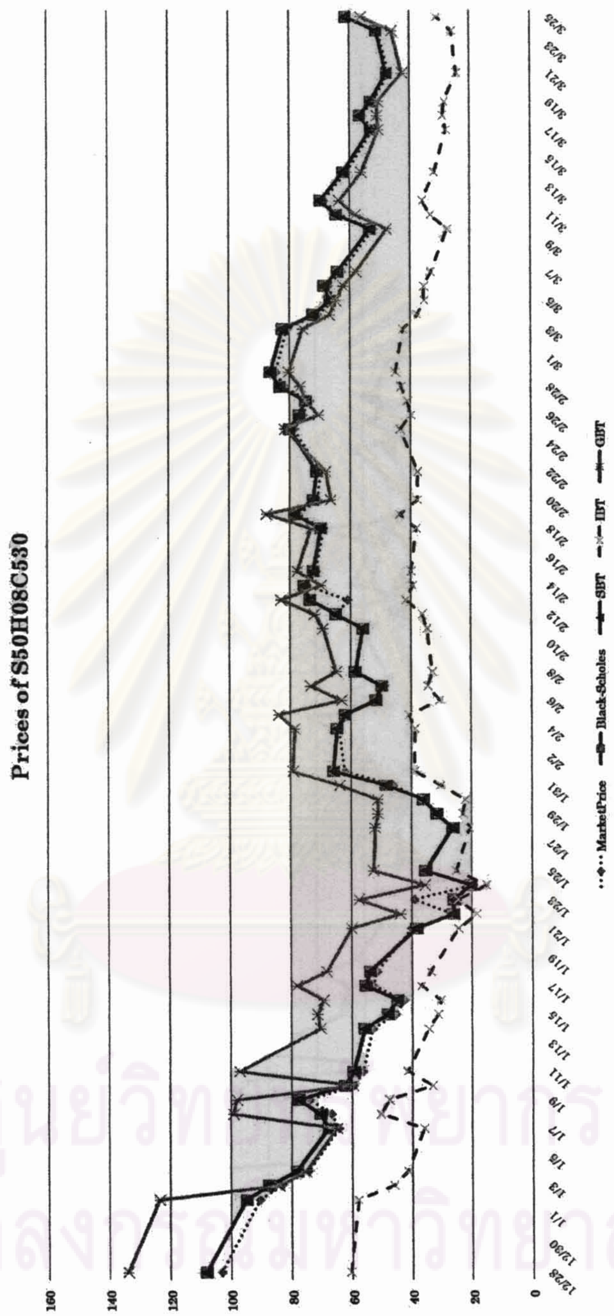

Figure 4.2: call price at the strike price 530 of the call option with the SET50 Index as the underlying asset that expired on 31 March, 2008

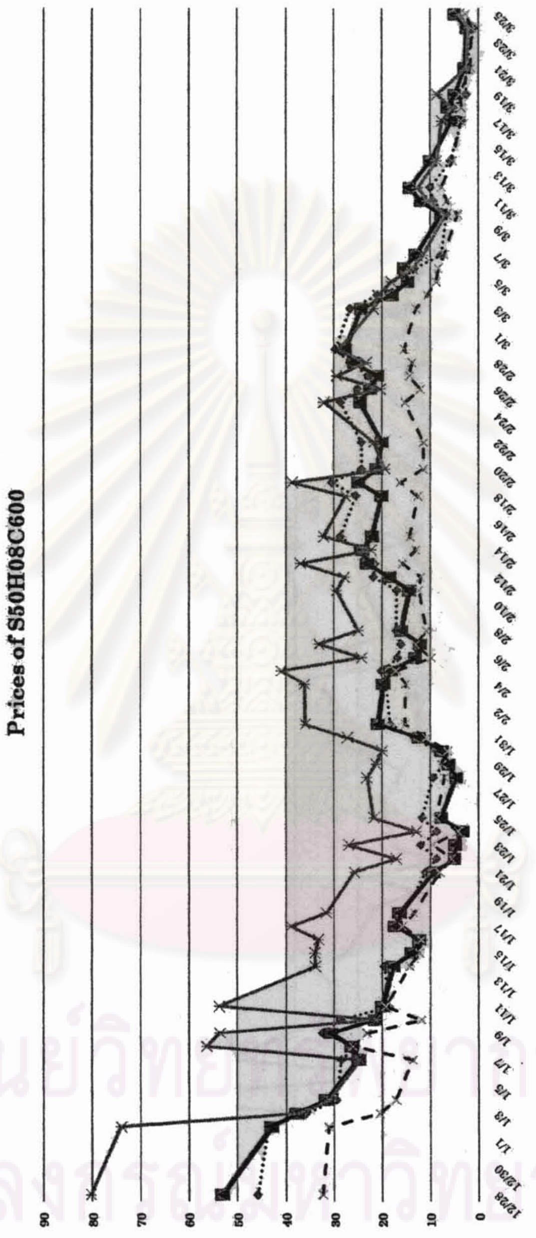

Figure 4.3: call price at the strike price 600 of the call option with the SET50 Index as the underlying asset that expired on 31 March, 2008

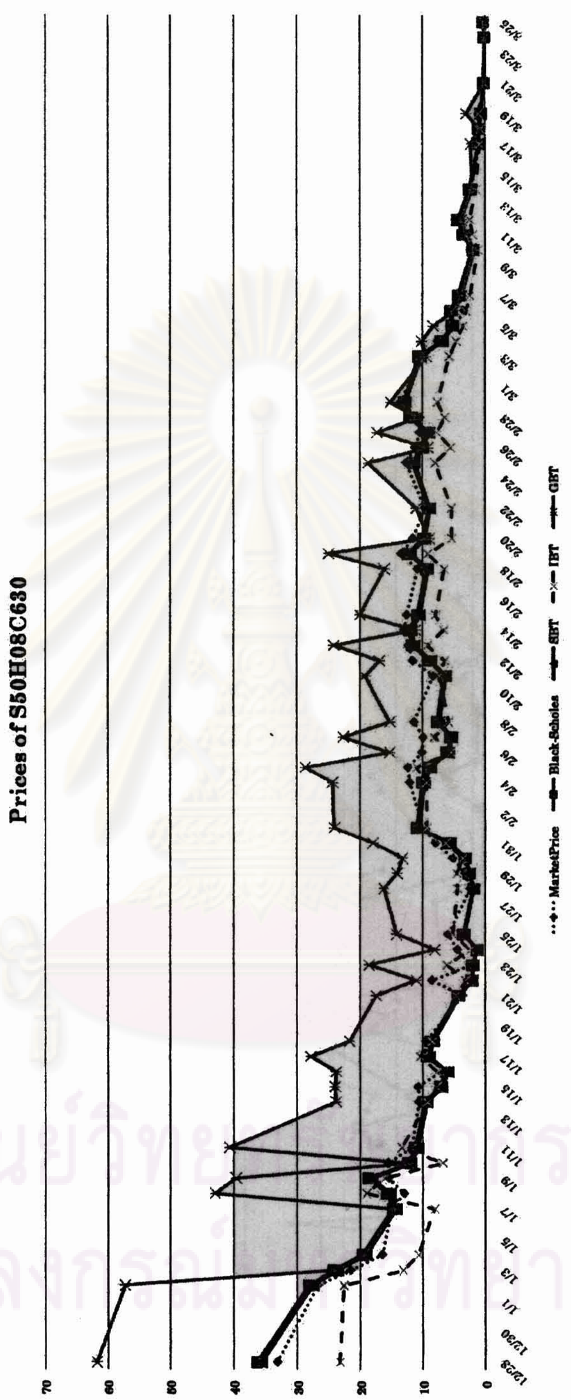

Figure 4.4: call price at the strike price 630 of the call option with the SET50 Index as the underlying asset that expired on 31 March, 2008

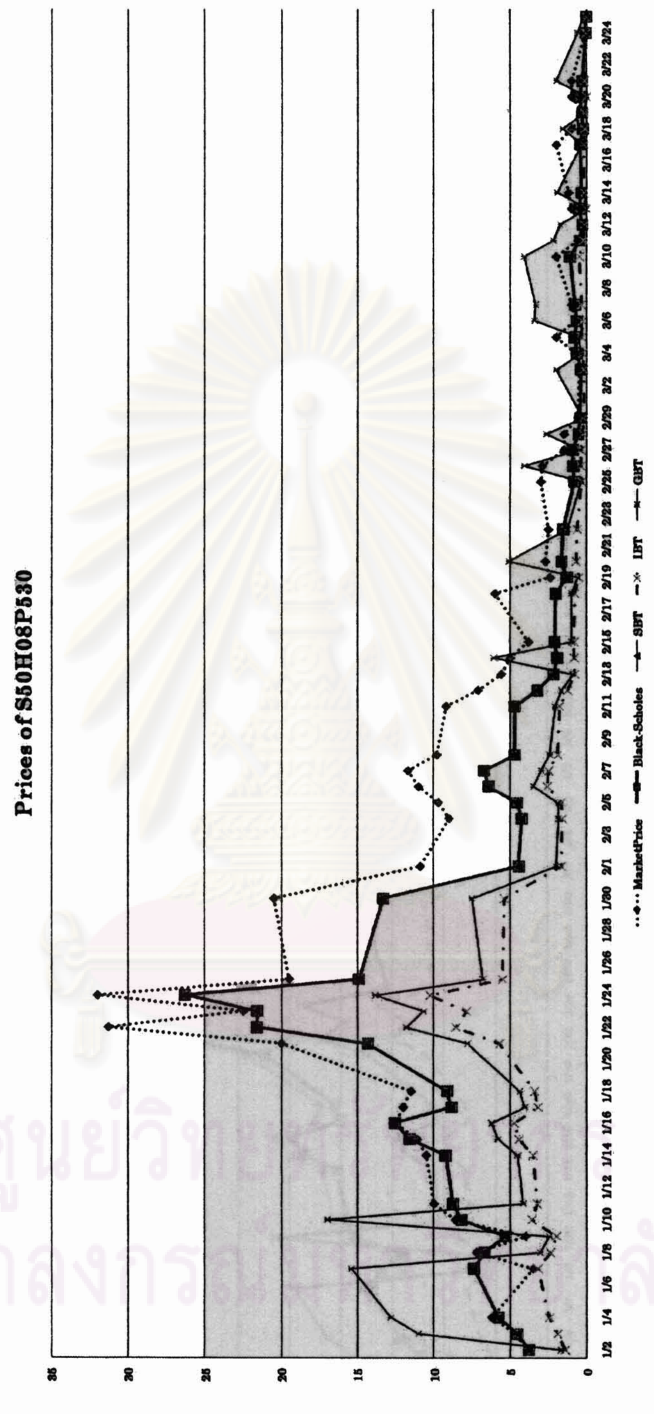

Figure 4.5: Put price at the strike price 530 of the put option with the SET50 Index as the underlying asset that expired on 31 March, 2008

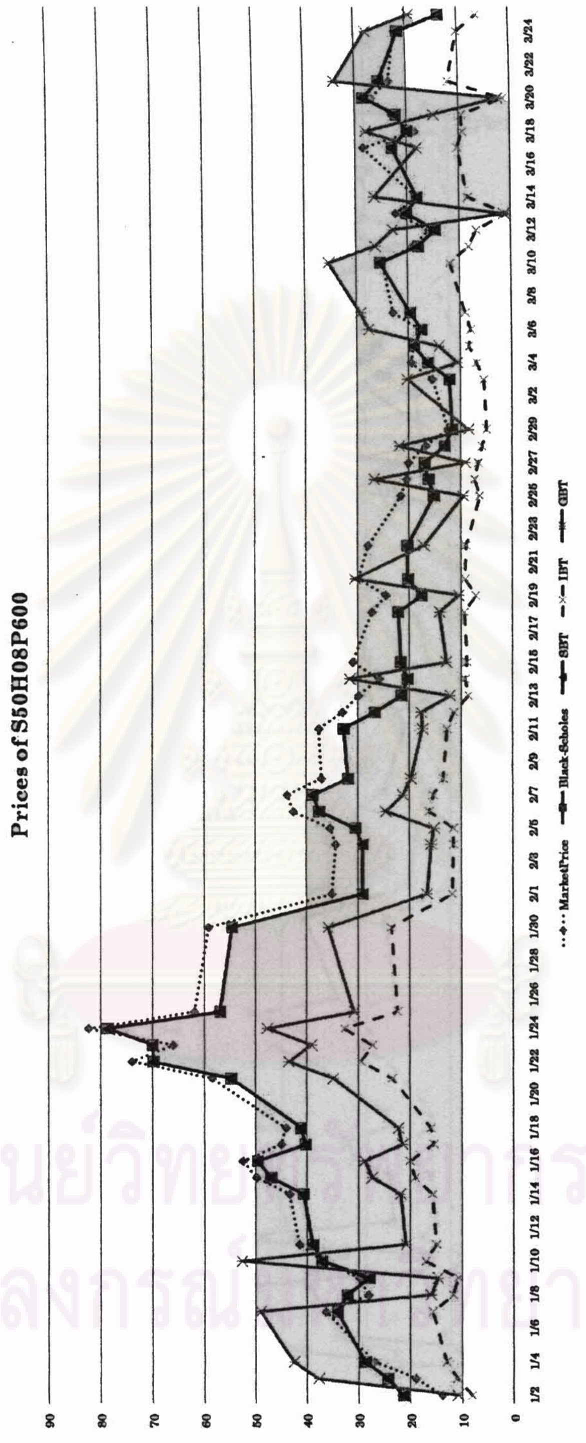

Figure 4.6: Put price at the strike price 600 of the put option with the SET50 Index as the underlying asset that expired on 31 March, 2008

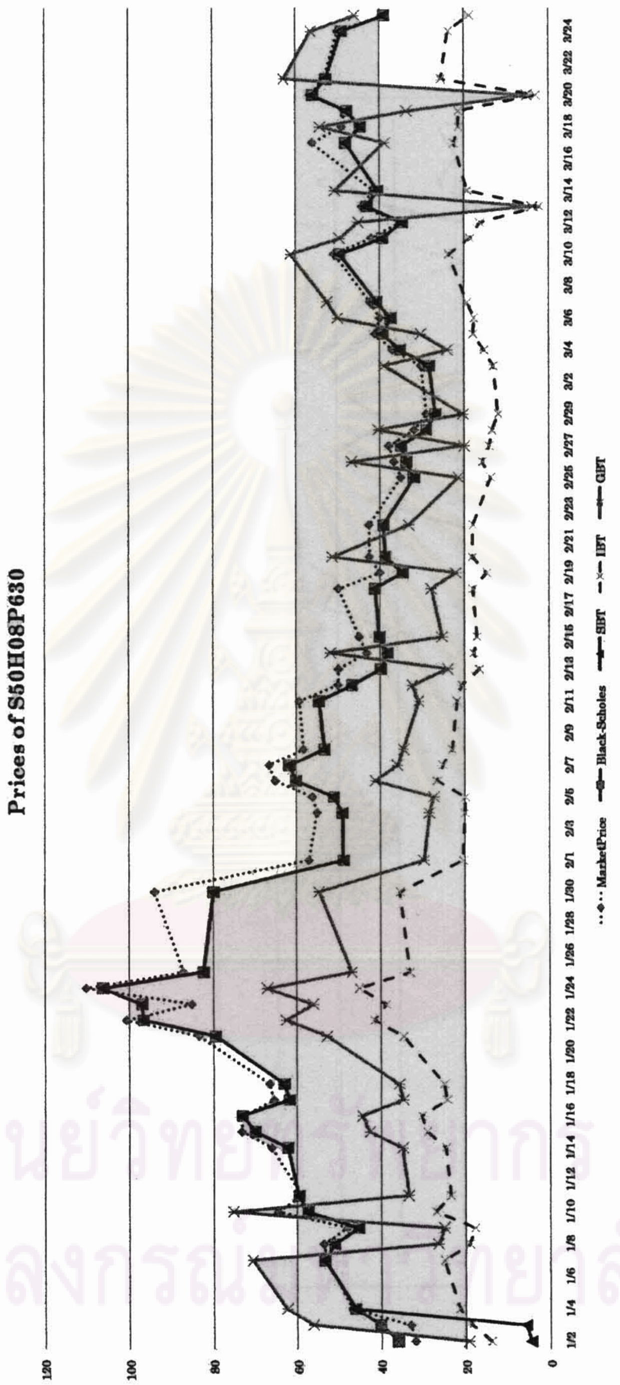

Figure 4.7: Put price at the strike price 630 of the put option with the SET50 Index as the underlying asset that expired on 31 March, 2008

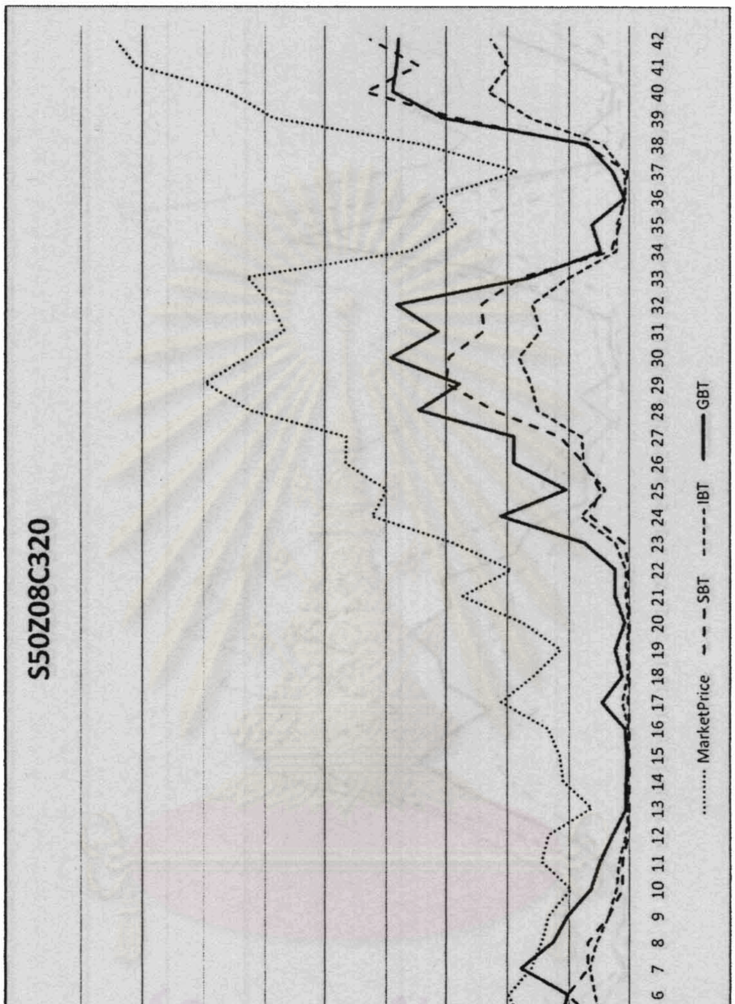

Figure 4.8: Call price at the strike price 320 of the call option with the SET50 Index as the underlying asset that expired on 31 September, 2008

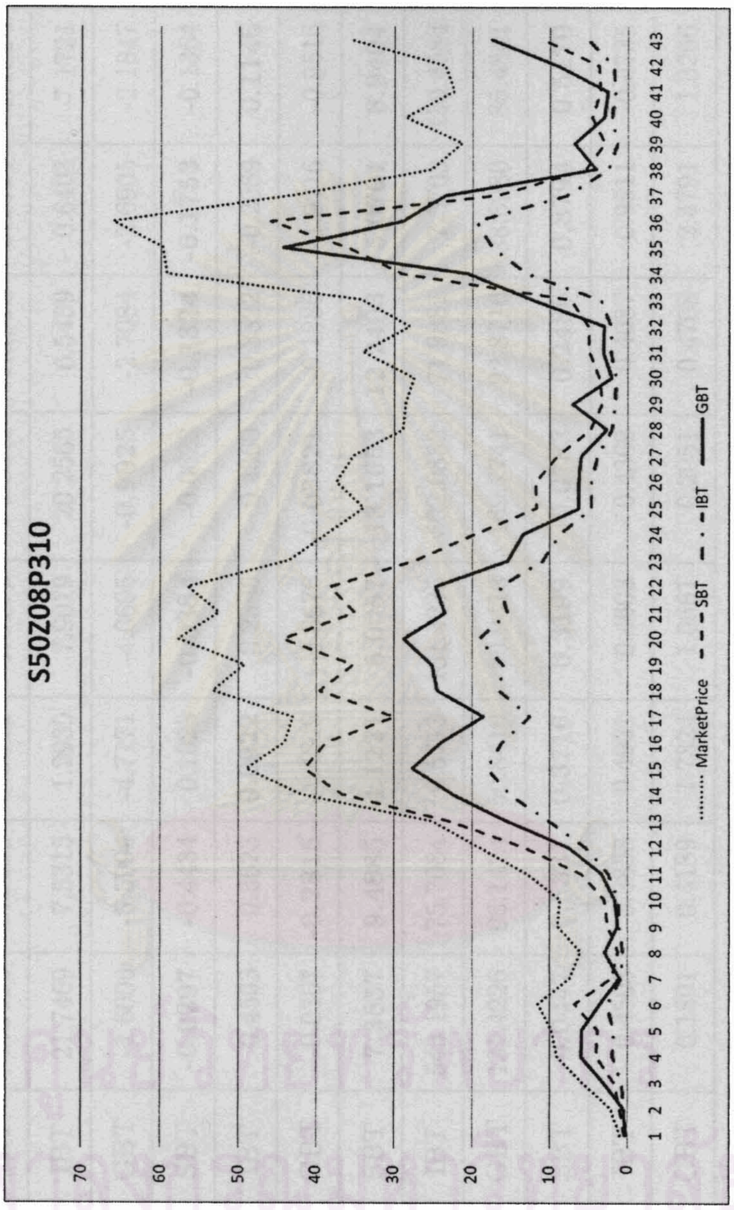

Figure 4.9: Put price at the strike price 310 of the put option with the SET50 Index as the underlying asset that expired on 31 September, 2008

Table 4.1: pricing errors of C1 and C2.

Measures	Models	C1				C2			
		OTM	NTM	ITM	ALL	OTM	NTM	ITM	ALL
ME	SBT	**-1.2416**	**-0.1459**	**0.6612**	**0.0179**	-2.5319	**-1.3009**	**-0.2924**	**-1.1084**
	IBT	21.7469	7.5315	1.2930	7.9019	20.2565	6.5439	0.6403	7.1721
	GBT	-1.6006	-5.5994	-4.7171	-4.0695	**-0.9025**	-2.7084	-2.6905	-2.1847
MPE	SBT	**-0.0297**	-0.4434	0.1023	**-0.0384**	-0.0628	**-0.1324**	**-0.1753**	-0.1364
	IBT	0.4503	0.3623	**0.0922**	0.2395	0.4360	0.3822	-0.2069	**0.1146**
	GBT	-0.0367	**-0.2815**	-1.4849	-0.8676	**-0.03821**	-0.1886	-1.9216	-0.9516
MSE	SBT	**7.3837**	**9.4885**	**4.1221**	**6.0057**	**13.1653**	**12.2403**	**5.0701**	**8.9404**
	IBT	543.1957	75.7084	6.5713	161.4235	502.0833	71.9881	7.5703	150.4584
	GBT	131.4226	96.1424	50.8410	80.9413	169.7741	92.8716	38.6980	85.4841
MAPE	SBT	**0.0447**	**0.5331**	**0.3716**	**0.3199**	**0.0783**	**0.2486**	**0.8794**	**0.5170**
	IBT	0.4503	0.3858	0.4921	0.4603	0.4360	0.4087	0.9811	0.6735
	GBT	0.1801	0.4189	1.7324	1.0661	0.2051	0.4708	2.4791	1.3266

This table shows the results of the pricing performance tests of three binomial tree model for the current month expiration calls (C1) and the subsequent month expiration calls (C2). Out-of-the-money (OTM) options comprise calls with $S/K < 0.97$, Near-the-money (NTM) options comprise calls with $0.97 < S/K < 1.03$, and In-the-money (ITM) options comprise calls with $S/K > 1.03$. ALL includes calls all type. The numbers are based on four measures - mean error (ME), percentage error (MPE), mean square error (MSE) and mean absolute percentage error (MAPE) - calculated from difference between observed market option price and each tree model's theoretical price for the sample period from 8 December 2007 to 29 December 2008 which are used for building the trees.

65

Table 4.2: pricing errors of C3 and C4.

Measures	Models	C3				C4			
		OTM	NTM	ITM	ALL	OTM	NTM	ITM	ALL
ME	SBT	**-0.9571**	**1.8039**	**2.8003**	**1.6789**	**6.9532**	**13.1570**	**8.9009**	9.2400
	IBT	13.4252	6.1388	9.1485	4.3000	23.1701	21.5700	19.2290	9.6677
	GBT	-5.5810	-2.8710	-6.5804	-2.0789	10.8849	14.8033	16.7765	**7.3226**
MPE	SBT	**-0.0246**	**-0.0028**	0.5640	0.2581	**0.1963**	0.5791	**0.9429**	0.7867
	IBT	0.4084	0.3232	2.4446	0.3336	0.6194	0.7702	1.8498	0.6827
	GBT	-0.1583	-0.4129	**0.3544**	**-0.0882**	0.2767	**0.5234**	1.6717	**0.5786**
MSE	SBT	**13.0547**	**12.2066**	**72.7371**	**40.4822**	**116.0390**	**220.6252**	**128.3034**	139.8373
	IBT	196.9085	47.4431	414.8355	76.1630	612.9698	479.4713	253.4402	169.7772
	GBT	75.5394	49.7851	489.1505	62.4643	227.8885	239.8062	189.2650	**97.6344**
MAPE	SBT	**0.0787**	**0.3210**	**0.5549**	0.3755	**0.2625**	0.5958	**0.9429**	0.7983
	IBT	0.4084	0.5922	2.3971	0.3885	0.6206	0.7702	1.8500	0.6828
	GBT	0.2216	0.6896	2.2354	**0.3659**	0.3224	0.5234	1.6716	**0.5823**

This table shows the results of the pricing performance tests of three binomial tree model for the current month expiration calls (C3) and the subsequent month expiration calls (C4). Out-of-the-money (OTM) options comprise calls with $S/K < 0.97$, Near-the-money (NTM) options comprise calls with $0.97 < S/K < 1.03$, and In-the-money (ITM) options comprise calls with $S/K > 1.03$. ALL includes calls all type. The numbers are based on four measures - mean error (ME), percentage error (MPE), mean square error (MSE) and mean absolute percentage error (MAPE) - calculated from difference between observed market option price and each tree model's theoretical price for the sample period from 8 December 2007 to 29 December 2008 which are used for building the trees.

66

Table 4.3: pricing errors of P1 and P2

Measures	Models	P1				P2			
		OTM	NTM	ITM	ALL	OTM	NTM	ITM	ALL
ME	SBT	**3.4886**	**2.8949**	**1.7239**	**2.5373**	**3.0370**	2.3504	0.9936	**1.6601**
	IBT	31.2713	14.2462	5.2369	17.2265	29.9177	9.4087	1.9707	9.4107
	GBT	14.2313	6.3452	2.3371	7.3528	9.9437	**2.0373**	**-0.2102**	2.6057
MPE	SBT	**0.0648**	-0.3014	**0.0754**	**-0.0457**	**0.0660**	0.0994	-0.9200	**-0.0017**
	IBT	0.6247	0.6419	0.6837	0.6566	0.6075	0.5854	**0.2249**	0.4086
	GBT	0.3271	**0.1673**	-0.4618	-0.0766	0.2092	**0.0415**	-2.2878	-1.0209
MSE	SBT	**26.8874**	**21.5724**	**9.5646**	**17.7104**	**31.7914**	**18.7995**	**4.8723**	**13.2541**
	IBT	1105.1587	239.7002	45.1705	356.6261	884.0247	124.0103	11.4117	228.3784
	GBT	586.1844	144.3807	33.8148	199.0287	434.3068	93.2656	15.0704	126.5062
MAPE	SBT	**0.0822**	0.5654	**0.4698**	**0.4020**	**0.0951**	**0.1522**	**0.5381**	**0.3029**
	IBT	0.6247	0.6426	0.7199	0.6730	0.6075	0.5854	0.5893	0.5802
	GBT	0.4348	**0.5535**	1.2437	0.8345	0.3671	0.5343	2.7765	1.5413

This table shows the results of the pricing performance tests of three binomial tree model for the current month expiration puts (P1) and the subsequent month expiration puts (P2). Out-of-the-money (OTM) options comprise calls with $S/K > 1.03$, Near-the-money (NTM) options comprise calls with $0.97 < S/K < 1.03$, and In-the-money (ITM) options comprise calls with $S/K < 0.97$. ALL includes calls all type. The numbers are based on four measures - mean error (ME), mean percentage error (MPE), mean square error (MSE) and mean absolute percentage error (MAPE) - calculated from difference between observed market option price and each tree model's theoretical price for the sample period from 8 December 2007 to 29 December 2008, which are used for building the trees.

67

Table 4.4: pricing errors of P3 and P4

Measures	Models	P3				P4			
		OTM	NTM	ITM	ALL	OTM	NTM	ITM	ALL
ME	SBT	**10.6069**	**8.2351**	6.2460	8.7556	**13.2131**	**18.0792**	**14.7245**	**14.5005**
	IBT	65.3881	13.7411	7.4419	15.4294	298.5508	29.1977	19.9026	25.7271
	GBT	35.5154	8.03596	**5.6782**	**8.7500**	159.6766	23.4865	18.2245	16.4294
MPE	SBT	**0.0985**	**0.3594**	**0.5665**	0.2990	**0.2031**	**0.6640**	0.9184	0.5071
	IBT	1.0471	0.7272	0.8246	0.4527	4.5735	0.8686	0.9629	0.5948
	GBT	0.4940	0.4312	0.6312	**0.2689**	2.5296	0.6993	**0.8855**	**0.4358**
MSE	SBT	**1940.0146**	**95.6241**	50.0842	**906.7513**	**790.2701**	**408.1238**	**283.6320**	563.4924
	IBT	7406.7529	212.2779	64.4164	1381.2179	1420.8050	873.9055	431.3303	1045.0487
	GBT	5760.2978	110.0124	**45.9519**	1063.7081	5297.4282	608.5482	369.0578	**470.8706**
MAPE	SBT	**0.2631**	**0.3871**	**0.5671**	0.3805	4.7284	0.8685	0.9629	0.6040
	IBT	1.0565	0.7272	0.8246	0.4544	4.7284	0.8686	0.9629	0.6040
	GBT	0.6564	0.4639	0.6426	**0.3058**	**2.9444**	**0.6993**	**0.8855**	**0.4604**

This table shows the results of the pricing performance tests of three binomial tree model for the current month expiration puts (P3) and the subsequent month expiration puts (P4). Out-of-the-money (OTM) options comprise calls with $S/K > 1.03$, Near-the-money (NTM) options comprise calls with $0.97 < S/K < 1.03$, and In-the-money (ITM) options comprise calls with $S/K < 0.97$. ALL includes calls all type. The numbers are based on four measures - mean error (ME), mean percentage error (MPE), mean square error (MSE) and mean absolute percentage error (MAPE) - calculated from difference between observed market option price and each tree model's theoretical price for the sample period from 8 December 2007 to 29 December 2008, which are used for building the trees.

68

Table 4.5: hedging errors of C1 and C2

Measures	Models	C1				C2			
		OTM	NTM	ITM	ALL	OTM	NTM	ITM	ALL
ME	SBT	**3.0154**	**1.3895**	**-0.1975**	**0.9327**	**4.8694**	**2.6798**	0.7378	**2.2644**
	IBT	-21.6041	-7.4015	-1.2387	-7.8094	-20.1191	-6.3770	**-0.5712**	-7.0615
	GBT	9.0297	8.9955	5.7687	7.2600	9.5638	6.6940	3.8221	5.94589
MPE	SBT	**0.0657**	0.5089	**-0.0236**	**0.1036**	**0.1165**	**0.2154**	0.3066	0.2359
	IBT	-0.4471	**-0.3540**	-0.0778	-0.2265	-0.4324	-0.3709	**0.2453**	**-0.0929**
	GBT	0.1914	0.4758	1.7735	1.1025	0.2291	0.4371	2.4336	1.3028
MSE	SBT	**17.2942**	**11.4018**	**4.0444**	**8.8644**	**30.6069**	**20.5250**	**6.4427**	**16.0829**
	IBT	537.0018	73.5388	6.3135	159.2280	498.1003	70.3349	7.5149	148.9855
	GBT	213.7224	139.5271	64.28060	118.3836	299.4359	152.1227	54.0354	141.1288
MAPE	SBT	**0.0714**	0.5585	**0.3692**	**0.3304**	**0.1240**	**0.3008**	**0.9674**	**0.5852**
	IBT	0.4471	**0.3793**	0.4887	0.4563	0.4324	0.3998	1.0018	0.6798
	GBT	0.2687	0.5719	1.9536	1.2390	0.3530	0.6498	2.9280	1.6190

This table shows the results of the hedging performance tests of three binomial tree model for the current month expiration calls (C1) and the subsequent month expiration calls (C2). Out-of-the-money (OTM) options comprise calls with $S/K < 0.97$, Near-the-money (NTM) options comprise calls with $0.97 < S/K < 1.03$, and In-the-money (ITM) options comprise calls with $S/K > 1.03$. ALL includes calls all type. The numbers are based on four measures - mean error (ME), mean percentage error (MPE), mean square error (MSE) and mean absolute percentage error (MAPE) - calculated from delta hedge errors for the sample period from 8 December 2007 to 29 December 2008, which are used for building the trees.

Table 4.6: hedging errors of C3 and C4

Measures	Models	C3				C4			
		OTM	NTM	ITM	ALL	OTM	NTM	ITM	ALL
ME	SBT	**1.9968**	**-1.1085**	**-2.5086**	**-1.1158**	**-6.3286**	-12.8132	**-8.8570**	-9.0722
	IBT	-13.2140	-5.9811	-8.9097	4.2112	-23.0950	-21.5200	-19.2458	-9.6625
	GBT	11.8178	6.0454	10.5901	4.1276	-6.6947	**-12.5052**	-16.0507	**-6.5573**
MPE	SBT	**0.0552**	**0.0485**	-0.5330	**-0.2232**	-0.1773	-0.5644	**-0.9415**	-0.7807
	IBT	-0.4017	-0.3097	-2.4099	-0.3267	-0.6169	-0.7684	-1.8526	-0.6832
	GBT	0.3524	0.6718	**0.2312**	0.2237	**-0.1565**	**-0.4415**	-1.6245	**-0.5472**
MSE	SBT	**16.5385**	**10.7713**	**71.5557**	**40.1863**	**107.8741**	209.9181	**126.9213**	136.1724
	IBT	191.2261	45.5426	416.0687	74.9032	610.4447	477.5239	253.7095	169.4803
	GBT	184.9557	86.1459	532.7230	92.0468	176.1530	**173.4664**	179.2869	**82.0020**
MAPE	SBT	**0.0843**	**0.3123**	**0.5253**	**0.3601**	0.2532	0.5813	**0.9412**	0.7937
	IBT	0.4017	0.5837	2.3637	0.3829	0.6184	0.7684	1.8526	0.6833
	GBT	0.3817	0.8283	2.4766	0.4420	**0.2399**	**0.4415**	1.6245	**0.5539**

This table shows the results of the hedging performance tests of three binomial tree model for the current month expiration calls (C3) and the subsequent month expiration calls (C4). Out-of-the-money (OTM) options comprise calls with $S/K < 0.97$, Near-the-money (NTM) options comprise calls with $0.97 < S/K < 1.03$, and In-the-money (ITM) options comprise calls with $S/K > 1.03$. ALL includes calls all type. The numbers are based on four measures - mean error (ME), mean percentage error (MPE), mean square error (MSE) and mean absolute percentage error (MAPE) - calculated from delta hedge errors for the sample period from 8 December 2007 to 29 December 2008, which are used for building the trees.

70

Table 4.7: hedging errors of P1 and P2

Measures	Models	P1				P2			
		OTM	NTM	ITM	ALL	OTM	NTM	ITM	ALL
ME	SBT	**-5.7973**	**-4.1531**	**-2.2788**	**-3.7559**	**-4.7779**	-3.7631	-1.4052	**-2.5588**
	IBT	-31.7052	-14.5047	-5.3291	-14.4519	-27.3629	-9.7095	-2.0550	-9.6340
	GBT	-18.4280	-7.467530	-3.1598	-8.1229	-10.1409	**-3.5015**	**-0.6261**	-3.4777
MPE	SBT	**-0.1094**	**0.2401**	**-0.1478**	**-0.0162**	**-0.1096**	-0.1807	**-0.0708**	**-0.1026**
	IBT	-0.63368	-0.6545	-0.7005	-0.6702	-0.6187	-0.6053	-0.2782	-0.4420
	GBT	-0.3371	-0.24076	0.15871	-0.0847	-0.2208	**-0.1543**	1.1938	0.4696
MSE	SBT	**51.2854**	**32.8265**	**12.3531**	**28.6133**	**53.2433**	**31.2064**	**6.8587**	**21.5321**
	IBT	1134.0215	247.7334	46.5852	366.6064	904.2887	130.5999	12.0580	235.3434
	GBT	555.6000	129.9616	30.6796	185.8527	406.7387	68.5498	8.8405	110.388275
MAPE	SBT	**0.1199**	0.5902	**0.4976**	**0.4314**	**0.1225**	**0.2000**	**0.4982**	**0.3033**
	IBT	0.6337	0.6547	0.7325	0.6846	0.6187	0.6053	0.5792	0.5837
	GBT	0.4078	**0.4767**	0.9717	0.6820	0.3264	0.3981	1.6932	0.9849

this table shows the results of the hedging performance tests of three binomial tree model for the current month expiration puts (P1) and the subsequent month expiration puts (P2). Out-of-the-money (OTM) options comprise calls with $S/K > 1.03$, Near-the-money (NTM) options comprise calls with $0.97 < S/K < 1.03$, and In-the-money (ITM) options comprise calls with $S/K < 0.97$. ALL includes calls all type. The numbers are based on four measures - mean error (ME), mean percentage error (MPE), mean square error (MSE) and mean absolute percentage error (MAPE) - calculated from delta hedge errors for the sample period from 8 December 2007 to 29 December 2008, which are used for building the trees.

71

Table 4.8: hedging errors of P3 and P4

Measures	Models	P3				P4			
		OTM	NTM	ITM	ALL	OTM	NTM	ITM	ALL
ME	SBT	**-11.9108**	**-9.0004**	**-6.6537**	**-9.6645**	**-14.2119**	**-18.5830**	**-14.9054**	**-15.1547**
	IBT	-66.6723	-14.3285	-7.6431	-15.7979	-301.7261	-29.4415	-20.0110	-25.9691
	GBT	-37.3372	-12.2883	-8.1193	-10.2337	-147.5454	-24.3110	-18.6755	-15.9111
MPE	SBT	**-0.1232**	**-0.3933**	**-0.6029**	**-0.3295**	**-0.2198**	**-0.6813**	-0.9274	-0.5214
	IBT	-1.0659	-0.7538	-0.8456	-0.4643	-4.6230	-0.8758	-0.9680	-0.5997
	GBT	-0.5662	-0.6441	-0.9080	-0.3650	-2.3691	-0.7234	**-0.9068**	**-0.4340**
MSE	SBT	**1985.6345**	**110.1375**	**55.9922**	**932.8234**	**819.2208**	**430.4631**	**290.7451**	584.3783
	IBT	7543.9658	231.9079	67.9858	1409.9873	14502.2560	888.4885	436.1279	1065.3256
	GBT	6150.0967	189.7161	85.9941	1154.5879	4742.8300	636.6276	384.3606	**444.8157**
MAPE	SBT	**0.2760**	**0.4166**	**0.6029**	**0.4043**	**0.3062**	**0.6812**	0.9274	0.5656
	IBT	1.0785	0.7538	0.8456	0.4665	4.7770	0.8758	0.9680	0.6089
	GBT	0.7709	0.6647	0.9092	0.4057	2.8510	0.7234	**0.9068**	**0.4626**

this table shows the results of the hedging performance tests of three binomial tree model for the current month expiration puts (P3) and the subsequent month expiration puts (P4). Out-of-the-money (OTM) options comprise calls with $S/K > 1.03$, Near-the-money (NTM) options comprise calls with $0.97 < S/K < 1.03$, and In-the-money (ITM) options comprise calls with $S/K < 0.97$. ALL includes calls all type. The numbers are based on four measures - mean error (ME), mean percentage error (MPE), mean square error (MSE) and mean absolute percentage error (MAPE) - calculated from delta hedge errors for the sample period from 8 December 2007 to 29 December 2008, which are used for building the trees.

72

CHAPTER V

DISCUSSIONS AND CONCLUSIONS

The results for both performances suggest that trends of movement of estimated option prices by using the binomial tree models (the SBT, IBT, and GBT models) and the market prices are all the same. We get estimated option price by using the SBT and the Black-Scholes model are close to the market prices more than the IBT and GBT models from Figures 4.2 - 4.7. The performances, between the IBT and GBT, depend on moneyness and option type. At OTM for all type and all moneyness of pricing and hedging performances of GBT are better than that of the IBT.

Therefore, in our empirical test with SET50 Index option, the SBT shows the better performance in both pricing and hedging performances for data sampling period, from 28 December 2007 to 29 December 2008. It suggests that the SBT which is the discrete version of the Black-Scholes model should work well in emerging market like the Thailand Futures Exchange (TFEX) during this period of the sampling data.

Remarks and Comments

- The method used to estimate the risk-neutral probability distribution is not appropriate for the IBT and GBT. This may be the reason why the pricing errors of these three binomial tree models are very large. However, they still give the movements that are quite reasonable.

- Since the weight function used in the GBT is not the optimal for all moneyness, in our empirical test the GBT method was not better than the IBT. For the better performance of GBT we should use the weight function that is appropriate for each moneyness.

- Since SET50 Index options are new type of derivatives for Thai investors, this market transaction is less active than futures market. The data used in this work is the closing prices (best bid and best ask quotes) of each day. For better estimation, we may use the hour-by-hour transaction price as the data.

- The results from this thesis showed that SBT model is suitable for obtaining SET50 Index option pricing during the period of sampling data. However, it is not clear to conclude that SBT model is better than other model, since this market transaction is less active than the other market.

- Generally, the Black-Scholes model is the popular model which is used to calculate option price to bid and ask the option. It may be the reason why the SBT model is suitable for SET50 Index option during the period of sampling data.

REFERENCES

[1] Ralf Korn and Elke Korn. *Option Pricing and Portfolio Optimization: Modern Methods of Financial Mathematics.* 31. the United States of America. 2001.

[2] Fred Espen Benth. *Option Theory with Stochastic Analysis : An Introduction to mathematical Finance.* Germany:Verlag Berlin Heidelberg, 2004.

[3] Fisher Black and Myron Scholes. "Pricing of options and corporate liabilities". *Journal of Political Economy* 1973 81:637-657.

[4] John C. Cox, Stephen A. Ross and Mark Rubinstein. "Option Pricing : A Simplified Approach". *Journal of Financial Economics* 1979 7:229-263.

[5] Mark Rubinstein. Implied Binomial Trees. *Journal of Finances* 1994 49: 771-818.

[6] Jens Carsten Jackwerth. "Generalized Binomial Trees". *Journal of Derivatives* 1997 5:7-17.

[7] Longstaff F. *Martingale Restriction Tests of Option Pricing Models.* Working paper (University of California, Los Angeles).

[8] Shimko, David. *Beyond inplied volatility: Probability distributions and hedge ratios implied by option price.* Working paper (University of Southern Carifornia).

[9] Jackwerth J. C., Rubinstein M. "Recovering probability distributions and implied binomial trees : A literature review". *Journal of Derivatives* 51:1611-1631.

[10] Gurdip Barshi and Nikunj Kapadia. "Delta-Hedging Gains and the Negative Market Volatility Risk Premium". *The Review of Financial Studies* 2003 vol.16 No.2:527-566.

[11] In Joon Kim and Gun Youb Park. "An empirical comparison of implied tree models for KOSPI 200 index options. *International Review of Economics and Finance* 2006 15:52-71.

APPENDICES

APPENDIX A : DATA

The Figure 5.1 has shown the example of data used in this work.

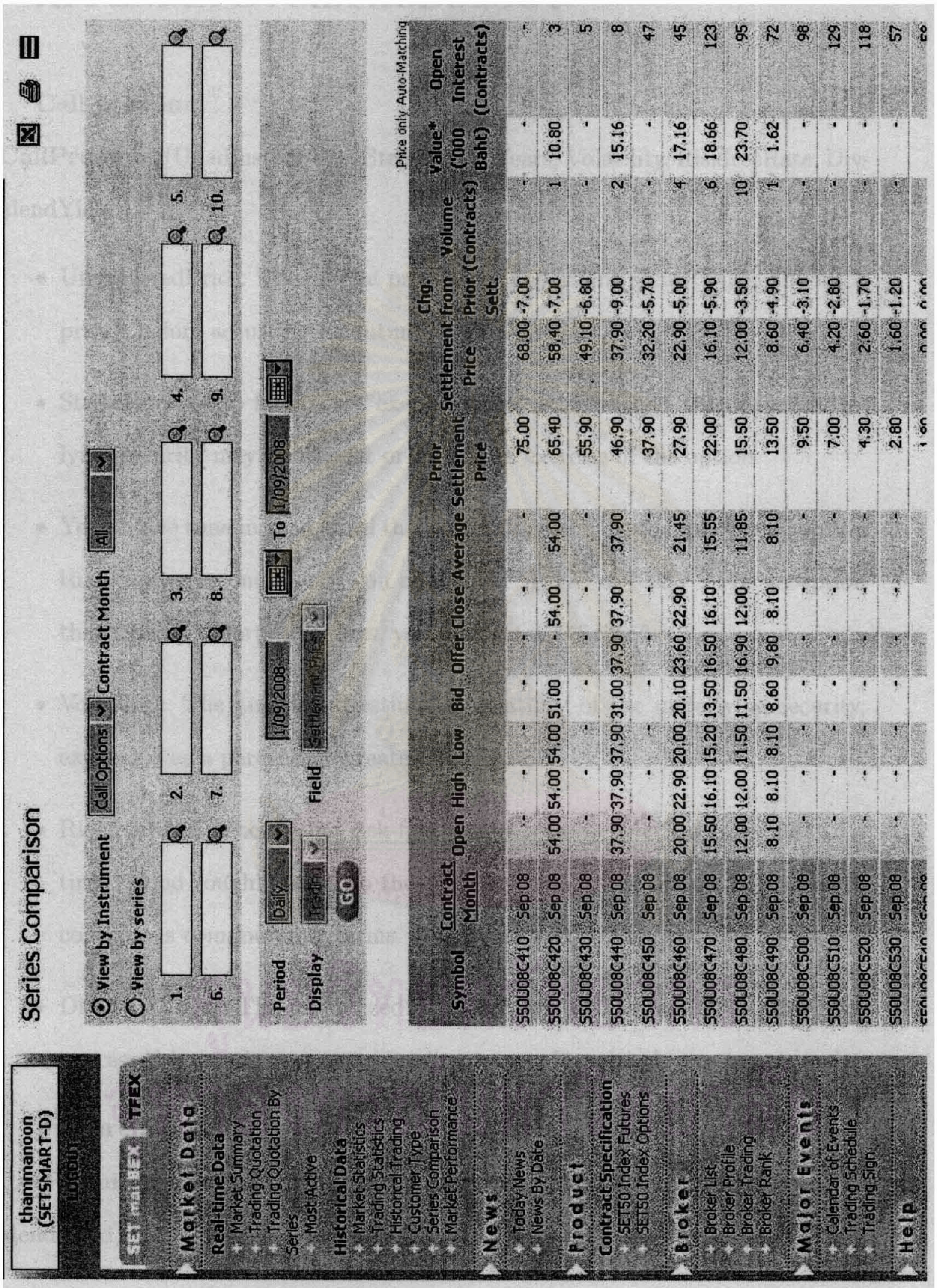

Figure 5.1: Example of data

APPENDIX B : METHODOLOGY

Call premium

CallPremium(UnadjustedPrice, StrikePrice, Years, Volatility, RiskfreeRate, DividendYield)

- UndajustedPrice: The present price of the underlying security (i.e., the stock price), before adjusting for future dividends.
- StrikePrice: Also known as "exercise price", the price at which the underlying security may be bought or sold upon exercise of the option.
- Years: The time in years until the option expires. For example, for an option that expires in one month you may enter "1/12" or ".083". For an option that expires in forty-five days, you may enter "45/365".
- Volatility: The annualized estimated volatility of the underlying security, expressed as a percentage greater than zero.
- RiskfreeRate: The annual risk-free rate of interest which corresponds to a time period roughly equal to the remaining life of the option, expressed in continuous compounding terms. If omitted, zero is assumed.
- DividendYield: The annualized dividend yield of the underlying security, expressed in continuous compounding terms. If omitted, zero is assumed.

Put premium

PutPremium(UnadjustedPrice, StrikePrice, Years, Volatility, RiskfreeRate, DividendYield)

- UndajustedPrice: The present price of the underlying security (i.e., the stock price), before adjusting for future dividends.

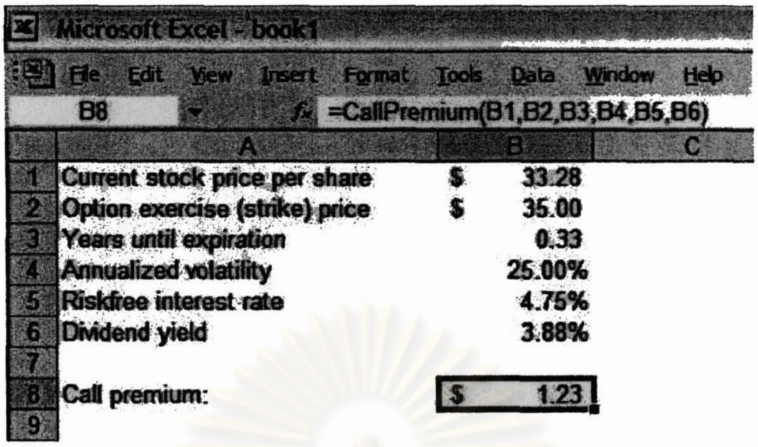

Figure 5.2: Using excel find call premium as determined by the Black-Scholes formula

- StrikePrice: Also known as "exercise price", the price at which the underlying security may be bought or sold upon exercise of the option.
- Years: The time in years until the option expires. For example, for an option that expires in one month you may enter "1/12" or ".083". For an option that expires in forty-five days, you may enter "45/365".
- Volatility: The annualized estimated volatility of the underlying security, expressed as a percentage greater than zero.
- RiskfreeRate: The annual risk-free rate of interest which corresponds to a time period roughly equal to the remaining life of the option, expressed in continuous compounding terms. If omitted, zero is assumed.
- DividendYield: The annualized dividend yield of the underlying security, expressed in continuous compounding terms. If omitted, zero is assumed.

Implied volatility IVol(OptionType, UnadjustedPrice, StrikePrice, StartDate, EndDate, OptionPremium, RiskfreeRate, DividendYield, WeekdaysOnly-

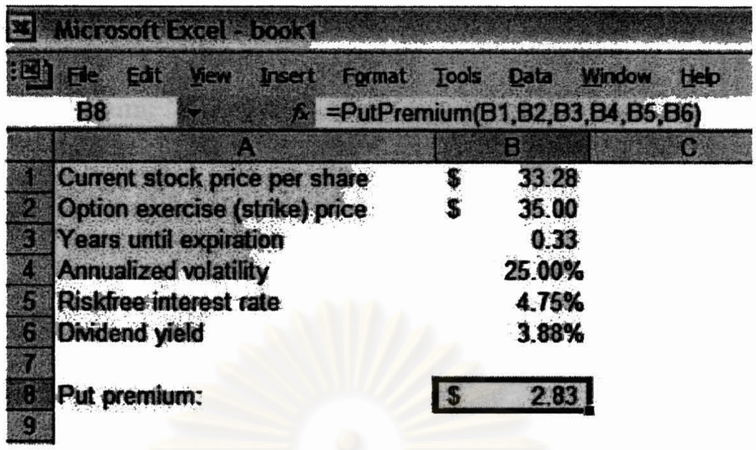

Figure 5.3: Using excel find put premium as determined by the Black-Scholes formula

Mode, Precision)

- OptionType: Enter "C" or "Call" or 0 for call option; "P" or "Put" or 1 for put option. If omitted, the function assumes "Call".

- UndajustedPrice: The present price of the underlying security (i.e., the stock price), before adjusting for future dividends.

- StrikePrice: Also known as "exercise price", the price at which the underlying security may be bought or sold upon exercise of the option.

- StartDate: The beginning day of the period measure. If valuing an option as of today, you may enter "now()" for this argument. If omitted, the function treats this argument as zero. (Using zero as the start date is usually done in conjunction with a fixed time period such as "365," rather than an actual date, as the end date.)

- EndDate: The maturity, expiration, or exercise date of the option. If 0 was entered as the StartDate, then enter as the EndDate the number of days remaining until expiration rather than the absolute date of expiration.

- OptionPremium: The current value or price of the option.

- RiskfreeRate: The annual risk-free rate of interest which corresponds to a time period roughly equal to the remaining life of the option, expressed in continuous compounding terms. (See ContCompRate function.) If omitted, zero is assumed.

- DividendYield: The annualized dividend yield of the underlying security, expressed in continuous compounding terms. (See ContCompRate function). If omitted, zero is assumed.

- WeekdaysOnlyMode: 0 or omitted = OFF; 1 = ON.

- Precision: The acceptable error for the function's result. By default this function will return a correct value ±0.1% (±0.001). Since this function iterates to find the correct answer, setting a higher value may speed execution in a worksheet containing many (¿100) instances of this function. Lower values increase the precision. Example: If the exact implied volatility is 25%, setting "Precision" to .005 will cause the function to return a value between 24.95% and 25.05%.

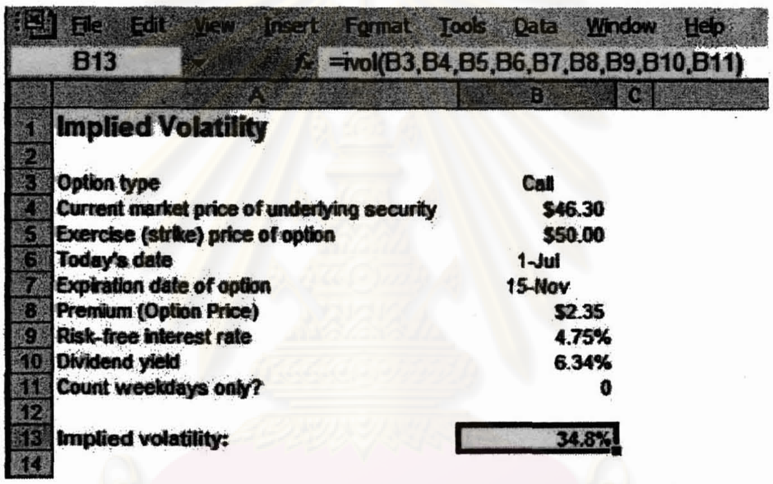

Figure 5.4: Using excel find cimplied volatility as determined by the Black-Scholes formula

VITAE

Miss Nawarat Ek-karntrong was born in December 3, 1984, in Roi-Et. She received a bachelor degree (first-class honor) in Mathematics from Department of Mathematics, Faculty of Science, Khonkaen University, Thailand in 2006. She has been scholared by the Development and Promotion of Science and Technology talents project (DPST) to continue the study from bachelor degree to PhD degree.

PUBLICATION

Nawarat Ek-karntrong, Dr.Khamron Mekchey, and Dr. Kittipat Wong, “An empirical comparison of binomial tree models for SET50 Index Options”, *The 35th Congress on Science and Technology of Thailand (STT35),* October 15-17, 2009.