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นายฐานิสร์ สุรภาพวงศ์



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# UNCERTAINTY ESTIMATION OF OIL IN-PLACE CALCULATED FROM MATERIAL BALANCE EQUATION

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A Thesis Submitted in Partial Fulfillment of the Requirements
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ในงานวิจัยนี้ผู้ทำการวิจัยได้ใช้วิธีทางสถิติในการประเมินความไม่แน่นอนของตัวแปรต่างๆในการประยุกต์ใช้สมการสมดุลมวลสารเพื่อเปรียบเทียบผลลัพธ์ที่ได้ของแต่ละวิธี และศึกษาผลกระทบที่เกิดขึ้นของการคำนวณปริมาณน้ำมันในแหล่ง โดยวิธีทางสถิติต่างๆนั้นทางผู้ทำการวิจัยได้นำวิธีทางสถิติต่างๆมาจากบทความตีพิมพ์ที่เกี่ยวข้องและงานวิจัยต่างในเรื่องที่เกี่ยวกับหลักการวิเคราะห์ความไม่แน่นอน โดยขั้นแรกนั้นวิธีการมอนติคาร์โล ได้ถูกนำมาใช้ในการสร้างค่าตัวแปรสุ่มในสมการสมดุลมวลสาร จากนั้นผู้ทำการวิจัยได้นำวิธีการวิเคราะห์ความไม่แน่นอนมาประยุกต์ใช้ เช่น การประมาณค่าอันดับหนึ่งภายใต้อนุกรมเทเลอร์ การใช้สมการพื้นผิว และ การออกแบบการทดลอง โดยค่าผลลัพธ์ทางสถิติของวิธีการพยากรณ์ในแต่ละวิธี ก็จะถูกนำไปเปรียบเทียบกับกรณีตั้งต้น

จากค่าผลลัพธ์ทางสถิติที่ได้ในแต่ละวิธี เราสามารถที่จะตั้งข้อสรุปได้ว่าผลลัพธ์ทางสถิติที่ได้จากวิธีการใช้สมการพื้นผิวอันดับที่สองของออกแบบการทดลองของบอกซ์-เบคเค่น นั้นมีค่าที่ใกล้เคียงอย่างสมเหตุสมผล ส่วนค่าผลลัพธ์ที่ได้จากการพยากรณ์จากสมการพื้นผิวอันดับที่สองของออกแบบการทดลองของบอกซ์-เบคเค่น สามารถที่จะให้ค่าผลลัพธ์ที่ได้จากการพยากรณ์ใกล้เคียงกับกรณีตั้งต้นมากที่สุด ยิ่งกว่านั้นวิธีที่ใช้สมการพื้นผิวอันดับที่สองของออกแบบการทดลองของบอกซ์-เบคเค่นก็ยังสามารถที่จะให้ทั้งค่าผลลัพธ์ทางสถิติและค่าผลลัพธ์ที่ได้จากการพยากรณ์ได้ใกล้เคียงอย่างสมเหตุสมผลที่สุด



ภาควิชา.....วิศวกรรมเหมืองแร่และปิโตรเลียม..... ลายมือชื่อนิสิต.....

สาขาวิชา.....วิศวกรรมปิโตรเลียม..... ลายมือชื่ออาจารย์ที่ปรึกษา.....

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THANIS SURAPAPWONG: UNCERTAINTY ESTIMATION OF OIL IN-PLACE CALCULATED FROM MATERIAL BALANCE EQUATION.
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In this research, the author uses the statistical methods for assessing the various sources of uncertainties in material balance equation application, to compare how well the result of each uncertainty analysis method and to study how they affect the amount of original oil in-place calculation. Following those statistical methods, the author has brought several statistical methods from the relevant papers and researches regarding the uncertainty analysis method. Firstly, the Monte Carlo simulation procedure has been used to generate random variables in the material balance equation. Then the author also applied other uncertainty analysis methods for instance the first-order approximation based on Taylor's series expansion, response surface method and experimental design. The statistical results of traditional Monte Carlo simulation will be set as the reference point in order to compare to other methods. In addition, the predictive result of each method will be also compared to the base case scenario.

From the statistical result of each method, we can conclude that the statistical result of the Box-Behnken experimental design with second-order response surface is reasonably accurate. In addition, the predictive result of the Box-Behnken experimental design with second-order response surface can provide the nearest predictive result to the base case scenario. Moreover the Box-Behnken experimental design with second-order response surface can provide both statistical and predictive result at optimum accuracy.

Department: Mining and Petroleum Engineering Student's signature .....
Field of study: Petroleum Engineering Advisor's signature .....
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# Nomenclatures

| | |
|---|---|
| $B_g$ | gas formation volume factor (rb/scf) |
| $B_{gi}$ | gas formation volume factor at initial condition (rb/scf) |
| $B_o$ | oil formation volume factor (rb/stb) |
| $B_{oi}$ | oil formation volume factor at initial condition (rb/stb) |
| $B_w$ | water formation volume factor (bbl/stb) |
| $c_f$ | pore compressibility factor ($psi^{-1}$) |
| $c_w$ | water compressibility factor ($psi^{-1}$) |
| $f$ | fraction of reservoir encroachment angle |
| $HCPV$ | Hydrocarbon pore volume |
| $h$ | thickness (ft) |
| $m$ | ratio of the initial hydrocarbon pore volume of the gascap to that of the oil |
| $N$ | oil initial in-place (stb) |
| $N_p$ | cumulative oil production (stb) |
| $P$ | reservoir pressure (psi) |
| $P_i$ | reservoir pressure at initial condition (psi) |
| $R_p$ | cumulative gas oil ratio (scf/stb) |
| $R_s$ | solution gas oil ratio (scf/stb) |
| $R_{si}$ | solution gas oil ratio at initial condition (scf/stb) |
| $r_e$ | external boundary radius (ft) |
| $r_o$ | reservoir radius (ft) |
| $S_g$ | gas saturation |
| $S_o$ | oil saturation |
| $S_{wc}$ | connate water saturation |
| $W_e$ | cumulative water influx (stb) |
| $W_p$ | cumulative water produced (stb) |
| $W_i$ | volume of water at initial condition (bbl) |
| $\phi$ | porosity |

# CHAPTER I

# INTRODUCTION

The importance of uncertainty and risk has been well recognized in the petroleum engineering literature, especially in the areas of exploration and reserves estimation. In recent years, petroleum engineers have also been focusing on methods for assessing the uncertainty in forecasts of original oil in-place and corresponding drainage area.

In the area of reservoir engineering, the sources of uncertainty have three major causes: 1) the model, because it is an imperfect representation of reality, 2) geologic parameters, because of a limited samplings and 3) measurement errors in the experiments performed to determine inputs. Thus, a statistical approach that recognizes both the lack of knowledge and the uncertainty of the parameters involved in the forecast of the original oil in-place is desirable.

The stochastic modeling approach Monte Carlo simulation methodology allows a full mapping of the uncertainties in inputs, expressed as probability distributions, into the corresponding uncertainty in model output which is also expressed in terms of a probability distribution. Uncertainties in the model outcome are quantified via multiple model calculations using parameter values drawn randomly from the probability distributions specified for the uncertain inputs.

The Monte Carlo simulation approach offers several advantages for propagating uncertainty in reservoir engineering problems. First advantage is full ranges of each input parameter are sampled and used in producing probabilistic model outcome. The second advantage is the ease of implementation; any input-output model can be utilized in the Monte Carlo process without making any modifications to the original model.

The major disadvantage with the Monte Carlo simulation technique is the need to perform multiple model calculations. In many cases, a limited number of realizations are used for computational expediency, even though there is no assurance that the final results will be statistically robust. A second disadvantage concerns the issue of data availability for defining the range and distributions of the uncertain inputs. In many real-life situations, paucity of data often forces the engineer to make

simplifying assumptions regarding the ranges and shape of the input distributions. Under such circumstances, the justification for using a full-blown Monte Carlo analysis, based on subjective assumptions about data distributions, becomes questionable at best. Though, Monte Carlo simulation methods may not be the most efficient way when the probability associated with only a limited number of model outcomes is desired.

Experiment design and analysis methods have been recently introduced into the oil & gas industry and have been shown to have significant potential in recoverable reserves uncertainty studies, such as sensitivity studies in recoverable reserves, production forecasting and ultimate recovery estimates by representing the numerical reservoir simulation with a surrogate response surface model and development optimization. In such studies, experimental design is generally used in a special purpose manner.

## 1.1 Outline of Methodology

The purpose of this thesis is to study and develop the quantitative estimate uncertainty in original oil in-place prediction based on material balance equation and to make an effective use of the reservoir information, particularly the reservoir description and to demonstrate the efficiency of each approach when compare with traditional Monte Carlo Simulation. The following tasks are to be accomplished:

1) Research and screen the appropriate uncertainty parameters which have an effect on the accuracy of original oil in-place, calculated by using material balance equation.
2) Generate reservoir model based on material balance equation. Set up the general assumption for each reservoir description, producing condition, uncertainty variables and their distribution (possible range of error).
3) Perform Monte Carlo Simulation based on material balance equation and collect the statistical result.
4) Perform first-order analysis to verify the sensitivity coefficient of each variable, variance and the output from first-order term of Taylor series expansion. Response surface method will be applied to Monte Carlo simulation result to find a suit approximation function. All results will be compared to traditional Monte Carlo Simulation.

5) Perform Box-Behnken experimental design and its response surface. Collect the statistical result and compare with the Monte Carlo simulation's result and the others.
6) Analyze overall result and report the recommendation and conclusion.

## 1.2 Thesis Outline

Chapter 2 outlines the list of related work and research in the area of uncertainty assessment using Monte Carlo Simulation and alternatives method. The effects of pressure uncertainty, reservoir drive mechanism on material balance equation and new approach in uncertainty estimation technique are also referred.

Chapter 3 presents the basic comprehension of material balance method, natural water drive mechanism and reservoir parameter consideration.

Chapter 4 describes the Monte Carlo simulation detail study in the uncertainty estimation of original oil in-place assessed by using EXCEL spreadsheet of material balance equation. The relationship between input-output parameters and statistical result will be investigated and analyzed.

Chapter 5 describes the first-order approximation method and its application. The sensitivity coefficient of each variable will be derived. Consequently, the variance and result of original oil in-place can be obtained and compared with reference Monte Carlo simulation result.

Chapter 6 describes the experimental design method and response surface method. The Box-Behnken design is chosen to employ in this study. The response surface will assist in providing the mathematic model for prediction purpose. The statistical result will be compared with traditional Monte Carlo Simulation and other uncertainty methods.

Chapter 7 presents the discussion, conclusion and provides recommendation for future works.

# CHAPTER II

# LITERATURE REVIEW

There are many studies which discuss about the reserve estimation methods to evaluate the uncertainty parameters in material balance equation. The below literatures are the summary of the relevant research regarding the material balance procedure, reservoir drive mechanism effect and uncertainty estimation technique.

B. Wang and R. R. Hwan (1997) investigated the effect pf the pressure data quality and drive mechanisms on the material balance calculation. Result of this research indicate that for a depletion type reservoir, the impact of pressure data error on material balance calculation is minimal, but for a water drive or initial gas-cap reservoir, the impact can be significant, depending on the size of aquifer or gas-cap.

Mike R. Carlson (1997) researched on the number of situation where drastically different interpretations are possible from oil material balances. The conclusion are presented as “tips” and “traps” in many cases, they represent matters of style e.g. to keep some pressure points based on the error analysis. Moreover, the accuracy of each parameter in material balance equation is discussed based on his experience and record.

R.O. Baker, C. Regier, R. Sinclair (2003) emphasized the need to make correction to laboratory data or correction to field data. The result of this study indicate that the impact of PVT error on material balance calculation can be significant if the decrease in reservoir pressure over the production history of the reservoir is quite small, or if the oil is highly volatile. These results are also a good indication of one of the reasons why a reservoir should have a significant amount of production and pressure loss before it becomes a good candidate for analysis using the material balance equation.

J. A. Murtha (1987) researched on the using of Monte Carlo simulation with material balance methods to estimate oil-in-place. The estimating parameters were pointed out only PVT properties for instance solution gas-oil ratio, formation volume factors. The water influx and gas-cap size were not mention in this research. From this study, The Monte Carlo technique can be applied to randomly select value of PVT properties to estimate hydrocarbon in-place.

Mark P. Walsh (1999) investigated the effect of pressure uncertainty and gas-cap size on the reliability of the material-balance method. But their work is limited to an investigation of uncertainty by pressure errors the effect of uncertainty from other variables such as PVT properties and cumulative production measurements is expected to be similar.

C. R. Mc Ewen (1961) presented technique for calculating the original amount of hydrocarbon in place, and for determining the constant characterizing the aquifer performance, based on pressure production data. When water encroachment is occurring, it is desirable to try to infer the behavior of the aquifer. This imposes additional demands on the method of calculation, and uncertainty in the data can result in large uncertainty in the answer. The least-square line lifting can then be applied so as to infer these quantities from observations of pressure and production data.

Srikanta Mishra (1998) researched the alternatives to Monte Carlo simulation for the assessment of uncertainty in reservoir engineering calculations. In this research, he concluded that Monte Carlo simulation is not the most appropriate uncertainty propagation technique when information regarding input distribution is lacking and the probability associated with only a limited number of states is sought. The first order second moment method is efficient alternatives for computing the mean/variance of model output given the mean, variance and correlation matrix of model input.

Chewaroungroaj J. et al. (2000) researched and demonstrated several approaches that qualitatively estimate uncertainty in specific hydrocarbon recovery predictions for instance Monte Carlo simulation, first-order Analysis, Second-order Analysis, response surface and experimental design in order to develop the procedures of hydrocarbon recovery prediction. The conclusion of this research, indicated that the use of experimental design and response surface analysis offer good potential to reduce the effort in uncertainty prediction and maintain the accuracy when compared to the full Monte Carlo simulation.

In this study, several approaches of estimating uncertainty will be applied to compare the behavior of output for each approach and find the way to optimize the simulation case. The expected of outcome from this thesis is to investigate the result of output from each uncertainty analysis techniques.

# CHAPTER III

# MATERIAL BALANCE APPLIED TO OIL RESERVOIR

In this chapter, the general material balance equation will be demonstrated and subsequently applied, using mainly the interpretive technique of Havlena and Odeh, to gain an understanding of reservoir drive mechanisms under primary recovery conditions. The use of basic component in the material balance equation and drive mechanism, are quantitatively discussed. Furthermore the uncertainty in reservoir parameters will be defined in order to assign the appropriate range of observed variables and to be in accordance with relevant research.

## 3.1 General form of the material balance equation for a hydrocarbon reservoir

The general form of material balance equation was first presented by Schilthuis in 1941. The equation is derived as a volume balance which equates the cumulative observed production, expressed as underground withdrawal, to the expansion of the fluids in the reservoir resulting from pressure drop. The situation is depicted in Figure 3.1 in which (a) represents the fluid volume at the initial pressure Pi in a reservoir which has a finite gas cap. The total fluid volume in this diagram is the hydrocarbon pore volume of the reservoir *(HCPV)*. Figure 3.1 (b) illustrates the effect of reducing the pressure by an amount P and allowing the fluid volume to expand, in an artificial sense, in the reservoir. Volume *A* is the increase due to the expansion of the oil plus originally dissolved gas, while volume *B* is due to the expansion of initial gascap gas. The third volume increment *C* is the decrease in *HCPV* due to the combine effects of the expansion of the connate water and reduction in reservoir pore volume.

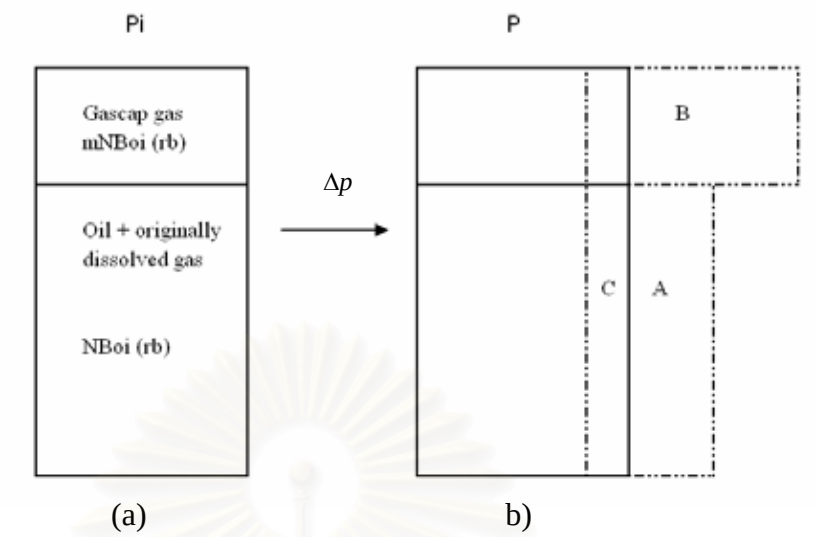



**Figure 3.1: Volume change in the reservoir associated with the finite pressure drop $\Delta p$; (a) volumes at initial pressure, (b) at the reduced pressure.**

If the total observed surface production of oil and gas is expressed in term of an underground withdrawal, evaluate at the lower pressure *p*, (which means effectively taking all the surface production back down to the reservoir at this lower pressure) then it should fit into the volume $A+B+C$ which is the total volume change of the original *HCPV*. Conversely, volume $A+B+C$ results from the expansions which are allowed to artificially occur in the reservoir. In reality, of course, these volume changes correspond to reservoir fluid which would be expelled from the reservoir as production. Thus the volume balance can be evaluated in reservoir barrels as

*Underground withdrawal (rb) = Expansion of oil (rb)*
*+ originally dissolved gas (rb)*
*+ Expansion of gascap gas (rb)*
*+ Reduction inHPVC due to connate water expansion and decrease in pore volume (rb)* (3.1)

Before evaluating the various components in the above equation it is first necessary to define the following parameters;

$N$ is the initial oil in-place in stock tank barrels

$= V\phi(1-S_{wc})/B_{oi}$ stb (3.2)

$m$ is the ratio between initial hydrocarbon volume of the gascap and initial hydrocarbon volume of the oil (and being define under initial condition, is a constant)

$N_p$ is the cumulative oil production in stock tank barrels

$R_p$ is the cumulative gas oil ratio

Then the expansion terms in the material balance equation can be evaluated as follows

*a) Expansion of oil plus original dissolved gas*

There are two components in this term:

Liquid expansion

The $N$ stb will occupy a reservoir volume of $NB_{oi}$ rb, at the initial pressure, while at the lower pressure *p*, the reservoir volume occupied by the $N$ stb will be $NB_{oi}$, where $B_o$ is the oil formation volume factor at the lower pressure. The difference gives the liquid expansion as

$$N(B_o - B_{oi}) \quad \text{(rb)} \tag{3.3}$$

Liberated gas expansion

Since the initial oil is in equilibrium with a gascap, the oil must be at saturation or bubble point pressure. Reducing the pressure below $P_i$ will result in the liberation of solution gas. The total amount of solution gas in the oil is $NR_{si}$ scf. The amount still dissolved in the *N* stb of oil at the reduced pressure is $NR_s$ scf. Therefore, the gas volume liberated during the pressure drop $\Delta p$, expressed in reservoir barrels at the lower pressure, is

$$N(R_{si} - R_s)B_g \quad \text{(rb)} \tag{3.4}$$

*b) Expansion of the gascap gas*

The total volume of gascap gas is $mNB_{oi}$ rb, which in scf may be expressed as

$$G = \frac{mNB_{oi}}{B_{gi}} \quad \text{(scf)} \tag{3.5}$$

This amount of gas, at the reduced pressure *p*, will occupy a reservoir volume

$$mNB_{oi}\frac{B_g}{B_{gi}} \quad \text{(rb)} \tag{3.6}$$

Therefore the expansion of gascap gas is

$$mNB_{oi}\left(\frac{B_g}{B_{gi}}-1\right) \quad \text{(rb)} \tag{3.7}$$

c) *Change in the HCPV due to the connate water expansion and pore volume reduction.*

The total volume change due to these combined effects can be mathematically expressed as

$$d(HCPV) = -dV_w + dV_f \tag{3.8}$$

or, as a reduction in the hydrocarbon pore volume, as

$$d(HCPV) = -(c_w V_w + c_f V_f)\Delta p \tag{3.9}$$

Where $V_f$ is the total pore volume = $HCPV/(1-S_{wc})$

And $V_w$ is the connate water volume = $V_f \times S_{wc} = (HCPV)S_{wc}/(1-S_{wc})$

Since the total *HCPV*, including the gascap, is

$$(1+m)NB_{oi} \quad \text{(rb)} \tag{3.10}$$

Then the *HCPV* reduction can be expressed as

$$-d(HCPV) = (1+m)NB_{oi}\left(\frac{c_w S_{wc} + c_f}{1-S_{wc}}\right)\Delta p \tag{3.11}$$

This reduction in the volume which can be occupied by the hydrocarbon at the lower pressure *p*, must correspond to an equivalent amount of fluid production expelled from the reservoir, and hence should be added to the fluid expansion terms.

d) *Underground withdrawal*

The observed surface production during the pressure drop $\Delta p$ is $N_p$ stb of oil and $N_p R_p$ scf of gas. When these volumes are taken down to the reservoir at the reduced pressure *p*, the volume of oil plus dissolved gas will be $N_p B_o$ rb. All that is known about the total gas production is that, at the lower pressure, $N_p R_s$ scf will be dissolved in the $N_p$ stb of oil. The remaining produced gas, $N_p(R_p - R_s)$ scf is therefore, the total amount of liberated and gascap gas produced during the pressure drop $\Delta p$ and will

occupy the volume $N_p(R_p - R_s)B_g$ rb at the lower pressure. The total underground withdrawal term is therefore

$$N_p\left(B_o + (R_p - R_s)\, B_g\right) \quad \text{(rb)} \tag{3.12}$$

Therefore, equating this withdrawal to the sum of the volume change in the reservoir, equations, give the general expression for the material balance as

$$N_p\left[B_o + (R_p - R_s)B_g\right] = NB_{oi}\left[\frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_{oi}} + m\left(B_g \big/ B_{gi} - 1\right) + (1+m)\frac{(c_w S_w + c_f)}{(1 - S_w)}\Delta p\right] + (W_e - W_p)B_w \tag{3.13}$$

In which the final term $(W_e - W_p)B_w$ is the net water influx into the reservoir. This has been intuitively added to the right hand side of the balance since any such influx must expel an equivalent amount of production from the reservoir thus increasing the left hand side of the equation by the same amount. In this influx term

$W_e$ = cumulative water influx from the aquifer into the reservoir, stb.

$W_p$ = cumulative amount of aquifer water produced, stb.

$B_w$ = water formation volume factor rb/stb.

$B_w$ is generally close to unity since the solubility of gas in water rather small and this condition will be assumed. The following features should be noted in connection with the expanded material balance equation.

## 3.2 Natural water drive

Natural water drive, as distinct from water injection, has already been qualitatively described, in connection with the material balance equation. The same principle applies when including the water influx in the general hydrocarbon reservoir material balance. A drop in the reservoir pressure, due to the production of fluids, causes the aquifer water expands and flow into the reservoir.

Applying the compressibility definition to the aquifer, then

*Water influx = aquifer compressibility x Initial volume of water x Pressure drop*

(3.12)

or

$$W_e = (c_w + c_f)W_i\Delta p \tag{3.13}$$

In which the total aquifer compressibility is the direct sum of the water and pore compressibility since the pore space is entirely saturated with water. The sum of $c_w$ and $c_f$ is usually very small, say $10^{-5}$ psi, therefore, unless the volume of water $W_i$ is very large the influx into the reservoir will be relatively small and its influence as a drive mechanism will be negligible. If the aquifer is large, however, equation will be inadequate to describe the water influx. This is because the equation implied that the pressure drop $\Delta p$, which is in fact the pressure drop at the reservoir boundary, is instantaneously transmitted throughout the aquifer. This will be reasonable assumption only if the dimensions of the aquifer are of the same order of magnitude as the reservoir itself. For a large aquifer there will be a time lag between the pressure change in the reservoir and the full response of the aquifer. In this respect natural water drive is time independent. If the reservoir fluids are produced too quickly, the aquifer will never have a chance to "catch up" and therefore the water influx, and hence the degree of pressure maintenance, will be smaller than if the reservoir were produced at a lower rate. To account for this time dependence in water influx calculations requires knowledge of fluid flow equations

In attempting to use this equation to match the production and pressure history of a reservoir, the greatest uncertainty is always determination of the water influx $W_e$. In fact, in order to calculate the influx the engineer is confronted with what is inherently the greatest uncertainty in the whole subject of reservoir engineering. The reason is that the calculation of $W_e$ requires a mathematical model which itself relies on the knowledge of aquifer properties. These, however, are seldom measured since wells are not deliberately drilled into the aquifer to obtain such information. For instance, suppose the influx could be described using the simple model. Then, if the aquifer shape is radial, the water influx can be calculated as

$$W_e = (c_w + c_f)\pi(r_e^2 - r_o^2) f h \phi \Delta p \qquad (3.14)$$

In which $r_e$ and $r_o$ are the radial of the aquifer and reservoir, respectively, and $f$ is the fractional encroachment angle which is either $\theta/2\pi$ or $\theta/360$, depending on whether $\theta$ is expressed in radians or degrees. It should be realized that the only term in above equation which is known with any degree of certainty is $\pi$. The remaining terms all carry high degree of uncertainty. For instance, what is the correct value of $r_e$? Is the aquifer continuous for 20 kilometers or is it truncated by faulting? What is

the correct value of $h$, the average thickness of aquifer or $\phi$, the porosity? These can only be estimated, based on the values determined in the oil reservoir.

## 3.3 Reservoir parameter consideration

According to the study of parameter's uncertainty in material balance equation, the error in each input parameter will produce the variation of the output which is the original oil in-place. The author has found that it is very difficult to track the uncertainty in all parameters. For this research, mainly, the pressure uncertainty will be brought in consideration because almost all the variables are function of pressure. From Galas's research, the average pressure uncertainty of at least 10 to 50 psi is depended on the method of measurement and reservoir condition. In this study, the 10 psi error in pressure measurement will be assumed. The Standing's correlation has been chosen to transform the pressure related term in material balance equation. It is also necessary to make simplifying assumption regarding PVT properties. Based on the data from numerous studies, black oil PVT data can usually be the reference thus black oil PVT data will be identically used in this research.

**Table 3.1: Reservoir parameter's accuracy (M.R. Carlson and Galas)**

| Observed variables | Possible error |
|---|---|
| Reservoir pressure, $P$ (psi) | $\pm 10$ psi |
| Reservoir initial pressure, $P_i$ (psi) | $\pm 10$ psi |
| Cumulative oil production, $N_p$ (stb) | $\pm 2\%$ |
| Cumulative water production, $W_p$ (stb) | $\pm 2\%$ |
| Connate water saturation, $S_{wc}$ | $\pm 5\%$ |
| Water influx term, $W_e$ (stb) | $\pm 15\%$ |
| Formation compressibility, $C_f$ (1/psi) | $\pm 5\%$ |

The cumulative oil production and cumulative water production is normally obtained from government records of production. Referring Canada measurement is government inspected and accuracy is typical plus or minus 2%. The connate water saturation is typically determined from well log analysis. Log analysis can have significant variation in accuracy. For instance The Energy Resources Conservation

Board (ERCB) normally rounds all saturations to the nearest 5%. All aquifer properties have been group together and considered the data error plus or minus 15% at discrete time in standalone basis. Table 3.1 illustrates the observed variables and their accuracy which will be applied in this research.

# CHAPTER IV

# MONTE CARLO SIMULATION STUDY

This chapter presents the study of Monte Carlo simulation which experienced in investigation of uncertainty estimation in original oil in-place. As demonstrated in previous chapter, the general form of material balance equation has been chosen to study and develop the quantitative estimate uncertainty in original oil in-place prediction.

Moreover, this chapter will contain a description of reservoir, the parameters used to assess original oil in-place uncertainty and describe the Monte Carlo simulation background and its results. In this study, Monte Carlo simulation will be employed as the reference to compare the uncertainty estimations of the method in subsequent chapters.

## 4.1 Uncertainty parameters selection and reservoir model for original oil in-place estimation

The simple original oil in-place of slightly compressible oil was chosen as the study process. This study is to research what are the most significant uncertainties in a material balance equation when estimated the original oil in-place. The uncertainty parameters in this study comprise of cumulative oil production, cumulative water production, initial reservoir pressure, compressibility factor, reservoir pressure, connate water saturation, porosity, formation thickness, reservoir radius, external boundary radius, encroachment angle and pressure drop across aquifer to reservoir boundary. For the remaining parameters, they will be considered as negligible effect. Thus they will be replaced with individual constants.

Totally twelve parameters are considered as the source of uncertainty in material balance equation. To compare the uncertainty estimation with experimental design, the author found that the limited number of uncertainty variables allowed to run via the experimental design software (STATISTICA 6.0) is only seven parameters. As revisit the twelve parameters, the source of uncertainty parameter in

aquifer properties i.e. formation thickness, reservoir radius, external boundary radius, encroachment angle, porosity and pressure drop across aquifer which can be grouped together. Consequently, the uncertainty variables will be reduced to seven parameters i.e. cumulative oil production, cumulative water production , initial reservoir pressure, reservoir pressure, connate water saturation, formation compressibility and water influx term which can be performed via STATISTICA 6.0.

In this study, the reservoir model is assumed to be hypothetical, single tank with natural water drive mechanism as shown in figure 4.1. It was assumed to have no initial gas cap and the pressure drop at the reservoir boundary is assumed to instantaneously transmit throughout the aquifer.

The behavior of a reservoir fluid which is used in this research, have their properties and characteristic close to “Black oil” which is characterized as having initial producing gas-oil ratios of 2000 scf/stb or less. Producing gas oil ratio will increase during production when reservoir pressure falls below the bubble point pressure of the oil. The stock tank oil usually will have gravity below 60 API. Stock tank oil gravity will slightly decrease with time until late in the life of the reservoir when it will increase. Thus the fluid properties which used in this study, will be respected to Black oil fluid model.

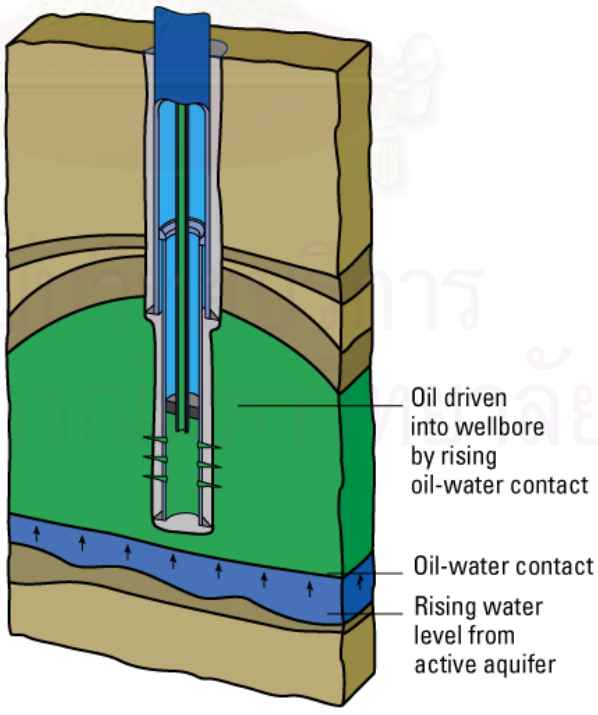



**Figure 4.1: Natural Water Drive Mechanism**

Laboratory analysis will indicate an initial oil formation volume factor of 2.0 rb/stb or less. Oil formation volume factor is the quantity of reservoir liquid in barrels required to produce one stock tank barrel.

The reservoir fluid properties at $25^{th}$ month static time, which employ in this study, has the oil gravity 42.3 API. And the gas oil ratio and oil formation volume factor are 640 scf/stb and 1.489 rb/stb respectively.

The initial reservoir pressure is assumed to be above its bubble point pressure and the producing condition is also assumed to focus at one moment of a static time below the bubble point pressure in order to investigate the PVT properties when a free gas phase exists in the reservoir.

To assess the original oil in-place, the EXCEL spreadsheet of material balance equation has been generated. The seven parameters have been set to have their statistical distribution for instant initial reservoir pressure, reservoir pressure, connate water saturation, formation compressibility, water influx term, cumulative oil production and cumulative water production. All variables are assumed to be independent.

As the triangular distribution is typically used as a subjective description of a population for which there is only limited sample data, and especially in cases where the relationship between variables is known but data is scarce (possibly because of the high cost of collection). Thus the triangle distribution suits oil and gas business and will be chosen as the probability density function of input variables in this study. The input variables in table 4.1, demonstrate the statistical moment and the range of variation which referred to the possible error of each parameter in chapter 3.

**Table 4.1: Range and Statistical moments of input variables for Monte Carlo Simulation**

| Variables | Max | Min | Mean | Var |
|---|---|---|---|---|
| Connate water saturation | 0.37 | 0.33 | 0.35 | 0.00005 |
| Formation compressibility (1/psi)*$10^{-6}$ | 5.25 | 4.75 | 5.00 | 0.00000002 |
| Cumulative oil production (stb) | 112,098 | 107,702 | 109,900 | 861,450 |
| Cumulative water production (stb) | 6,583 | 5,956 | 6,270 | 16,700 |
| Reservoir initial pressure (psi) | 4,020 | 3,980 | 4,000 | 62 |
| Reservoir pressure (psi) | 2,520 | 2,480 | 2,500 | 66 |
| Water influx term (MMstb) | 1,840 | 1,360 | 1,600 | 10,215 |

From table 4.1, the parameters are used as random variables without any modification. The minimum and maximum value of each parameter will be based on the possible range of error as described in chapter 3. The ranges of stochastic variables are chosen to focus the change at static time that may affect the original oil in-place.

Other descriptive parameters necessarily associate to this study are shown in table 4.2. The reservoir is produced naturally with natural water drive mechanism. The production period is chosen to be long enough to experience water production and PVT properties below bubble point pressure.

The base-case scenario in this study is chosen from the combination of the sample mean values of all stochastic variables (table 4.1). The original oil in-place of the base-case scenario is 911,633 stb.

**Table 4.2: List of base-case description parameters**

| Parameters | Value |
|---|---|
| Water formation volume factor, Bw (rb/stb) | 1 |
| Reservoir temperature, (F) | 212 |
| Oil API gravity, (API) | 42.3 |
| Gas gravity | 0.746 |
| Water compressibility, Cw (1/psi) | 0.000003 |
| Oil compressibility, Co (1/psi) | 0.000016 |

As several correlations of the formation volume factor and solution gas oil ratio exist in the oil industry, from the relevant research, the evaluation of empirically derived PVT properties shown that the Standing's correlation have a potential accurate in estimating PVT properties for middle east crude oil and they can be used for estimating the same PVT parameters for all types of oil and gas with properties falling within the range of data used in his study. However, the Standing's correlation is chosen to represent the relationship between formation volume factor and solution gas oil ratio to reservoir pressure since the fluid properties used in this study, are within the range.

Figure 4.2 show the relationship between pressure and compressibility factor. We have fitted up the trend line with field data by using second order polynomial. As the possible range of pressure error is $\pm 10$ psi which has a small effect to compressibility. However, in this study, the compressibility will be converted to pressure dependent term.

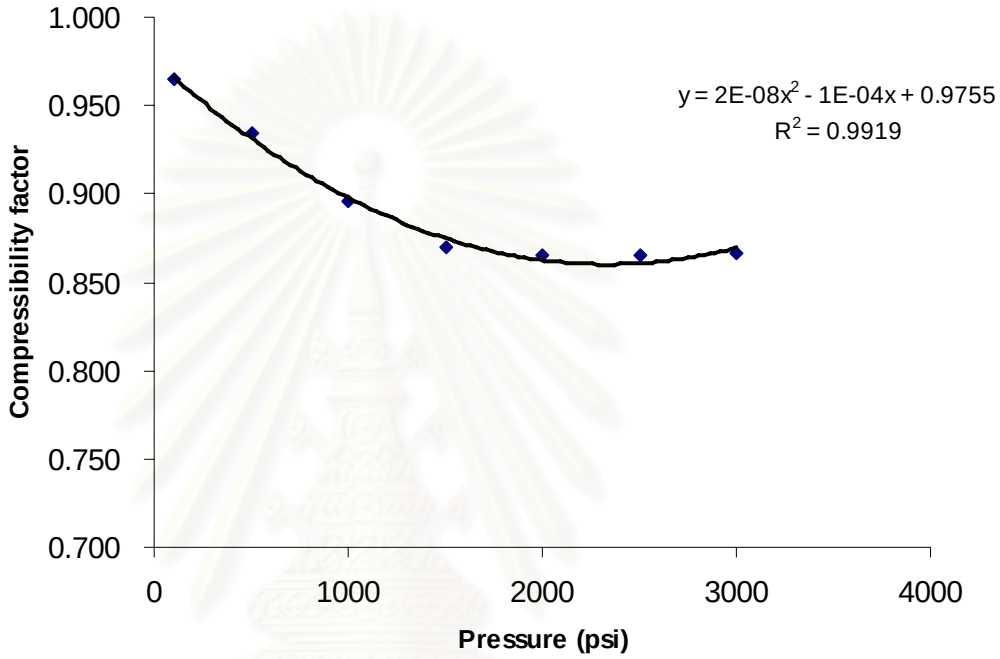



**Figure 4.2: Compressibility factor as function of pressure in base case scenario with second order polynomial trend line**

## 4.2 Monte Carlo Simulation background

Monte Carlo Simulation (MCS) is a computer-based method of analysis developed in the 1940's that uses statistic sampling techniques in obtaining a probabilistic approximation to the solution of a mathematical equation or model. Its methodology allows a full mapping of the uncertainty in model input, expressed as probability distributions, into the corresponding uncertainty in model output which is also expressed in terms of probability distribution.

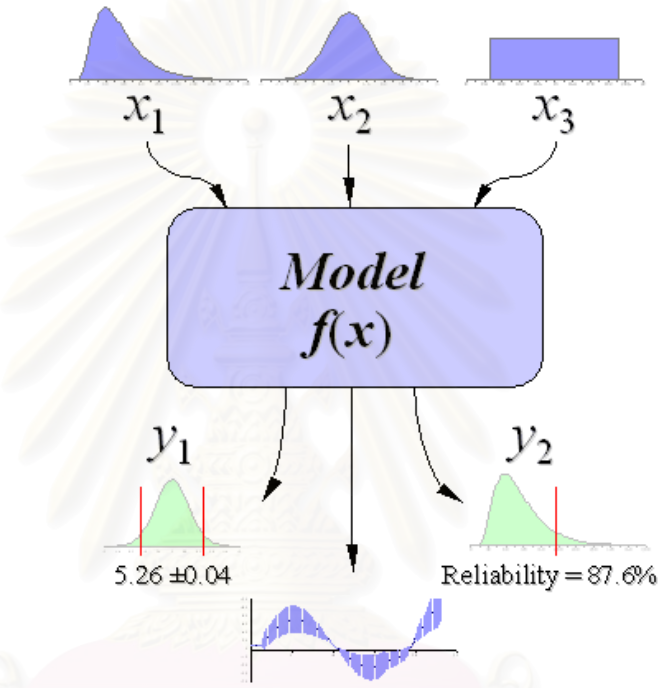



**Figure 4.3: Schematic showing the principal of stochastic uncertainty propagation**

Figure 4.3 demonstrates the Monte Carlo simulation procedure involves the random sampling of each probability distribution within the model to produce hundreds or even thousands of scenarios (also called iterations, realizations or trials) a value is drawn at random from the distribution for each input. Together this set of random values, one of each input, defines a scenario, which is used as input to the model, computing the corresponding output valve. The entire process is repeated *m* times producing *m* independent scenarios with corresponding output values. These *m* output values constitute a random sample from the probability distribution over the output induced by the probability distribution over the inputs. One advantage of this approach is that the precision of the output distribution may be estimated from this

sample of output values using standard statistical techniques. The Monte Carlo Simulation approach offers several advantages for propagating uncertainty in georesources engineering problems. First of these is that full range of each uncertain input parameter is sampled and used in generating the probabilistic model outcome. A second advantage is the ease of implementation – any input – output model can be utilized in the Monte Carlo Simulation process without making any modifications to the original model. Finally, the Monte Carlo Simulation approach is conceptually simple, widely used and easy to explain.

The major disadvantage of with the Monte Carlo Simulation technique is the need to perform multiple model calculations. For large and/or complex models, the computational burden associated with a full Monte Carlo Simulation analysis can be prohibitive. A second disadvantage concerns the issue of data availability for defining the range and distribution of the uncertain inputs. Finally the Monte Carlo Simulation approach may not be the most efficient when the probability associated with only a limited number of model outcomes is desired.

Steps in simulating an iteration of: $x$

1. Generate a vector of statistically independent, uniformly distributed random numbers between 0 and 1, $U_i$.
2. Transform $U_i$ to $x_i$.

**Transformation of single random variables**

Inverse transform method:

$$U = F_x(x) \tag{4.1}$$

$U$ has a uniform distribution between 0 and 1

If we generate an iteration of a uniformly distributed random number, $U_i$, then an iteration of $x$ can be obtain as follows

$$x_i = F_x^{-1}(u_i) \tag{4.2}$$

A common weakness in Monte Carlo studies is that no mention made about the precision of the results. Two items should be always included when reporting Monte Carlo results:

1. The basis for determining the number of simulation used.
2. The precision of the results in the form of confidence bounds.

## 4.3 Monte Carlo Simulation results

We generate 700 trials for all seven variables according to their probability distribution. Each iteration comprises of seven random values of the stochastic variables and other descriptive parameters at their base-case values. The actual statistical moments, means and variance of these inputs are shown in table 4.1. The combinations of the mean values of all variables are considered as the base-case scenario. The 700 trials Monte Carlo Simulation using material balance equation was undertaken to calculate the statistical moment of the original oil in-place at one moment of the time. The original oil in-place in this study is defined as the static measure of hydrocarbon volume at the specific time. This Monte Carlo Simulation results established a reference for comparing other technique employed in this study.

From figure 4.4 shows that the mean value of original oil in-place becomes stabilized after 540 trials. The average value of original oil in-place of all 540 trials and more is around 910,920 – 911,479 stb. The variance of original oil in-place (referred as the uncertainty in the original oil in-place) also shows the same stabilization in figure 4.5. The variance of original oil in-place of all 540 trials and more is around 127,007,420 – 131,583,593 hence, the total number of 700 trials used as the reference statistic should be sufficient. This also confirms the requirement of large number of iteration using Monte Carlo Simulation technique to assess the uncertainty for this study.

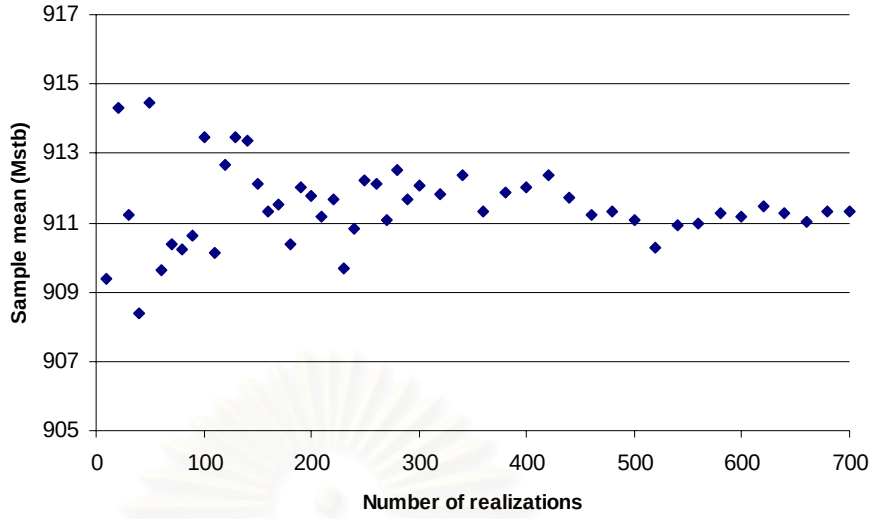



**Figure 4.4: Number of iterations needed to stabilize the sample mean of original oil in-place in the 700 trials Monte Carlo Simulation**

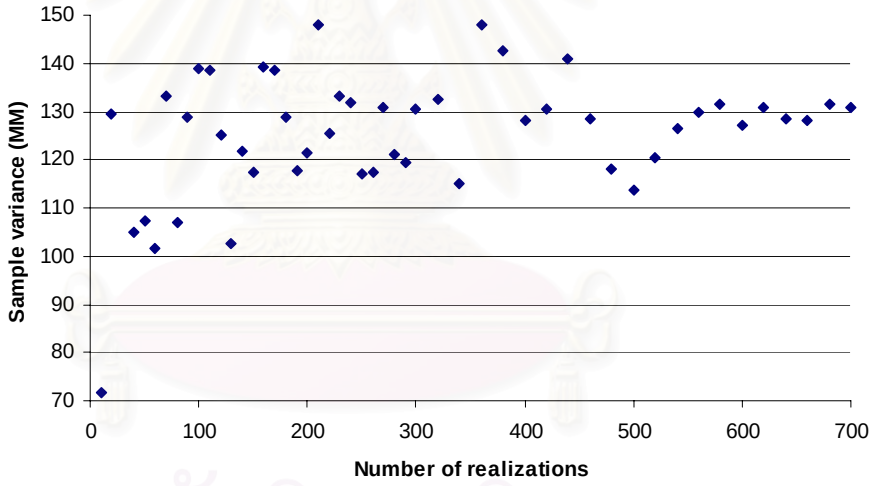



**Figure 4.5: Number of iterations needed to stabilize the sample variance of original oil in-place in the 700 trials Monte Carlo Simulation**

Figure 4.6, 4.7 illustrates the cumulative distribution function (CDF) and probability distribution function (PDF) of the original oil in-place from the 700 trials Monte Carlo Simulation. The calculated cumulative distribution function and probability distribution function follow the procedure described by Jansen et al (1997). The distribution is relatively smooth. Table 4.3 shows statistics summary of the original oil in-place. The Monte Carlo Simulation of the original oil in-place has a mean value of 912,823 stb and a variance of 126,954,400.

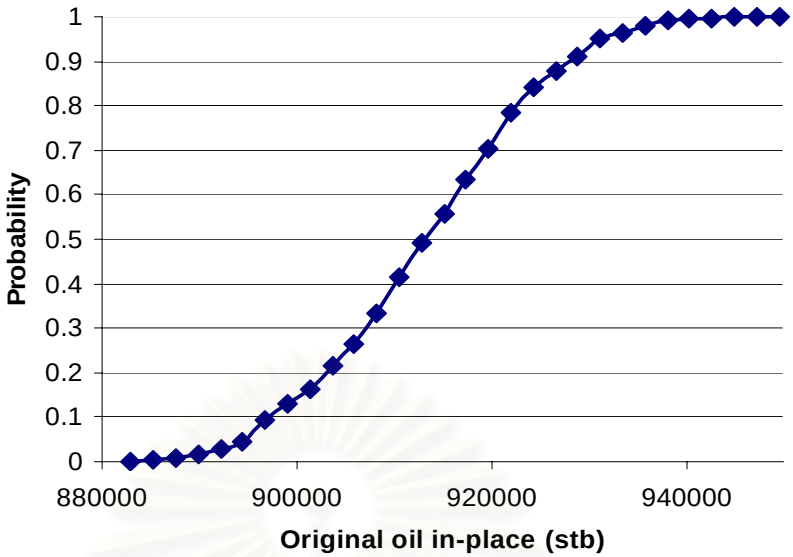



**Figure 4.6: Original oil in-place CDF for the 700 trials Monte Carlo Simulation**

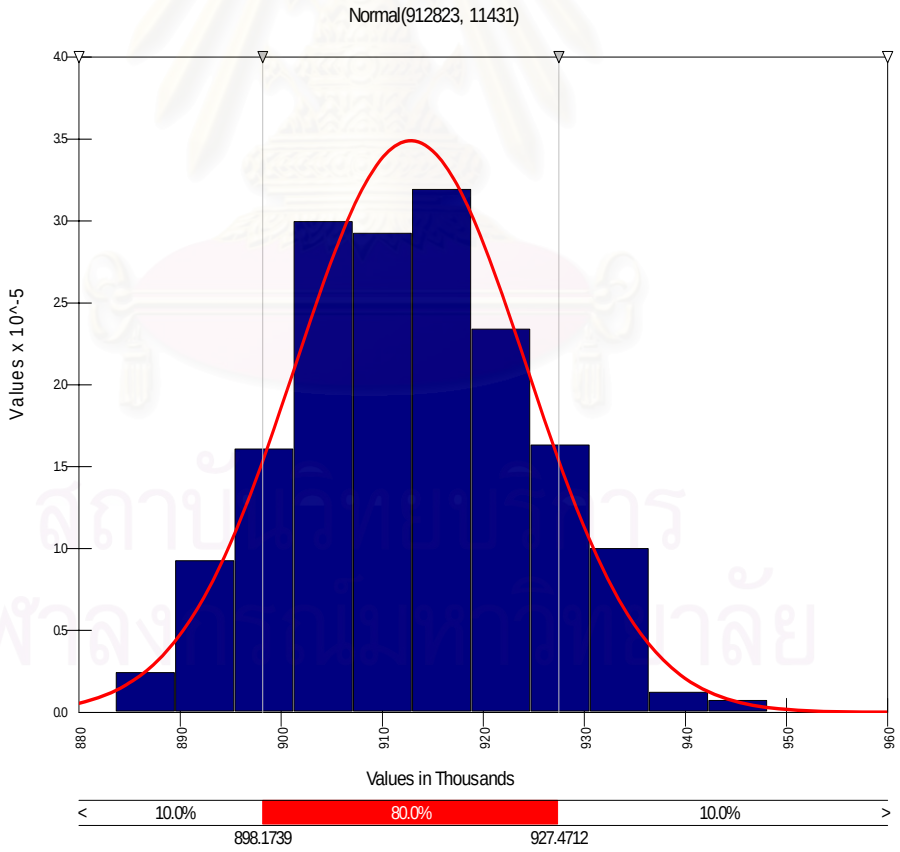



**Figure 4.7: Original oil in-place PDF for the 700 trials Monte Carlo Simulation**

**Table 4.3: Statistical summary of original oil in-place from the 700 trials Monte Carlo simulation**

| Statistics | Sample value |
|---|---|
| Maximum | 938,311 |
| Minimum | 876,969 |
| Mean | 912,823 |
| Variance | 126,954,400 |
| Std Deviation | 11,267 |
| P10 | 898,173 |
| P25 | 903,034 |
| P50 | 912,990 |
| P75 | 918,545 |
| P90 | 927,471 |

From table 4.3, the statistic result from Monte Carlo simulation show the confident bound of P10 at 898,173 stb, P25 at 903,034 stb, P50 at 912,990 stb, P75 at 918,545 stb and P90 at 927,471 stb. These Monte Carlo Simulation results will be a reference for comparing with the other statistical result of each technique employed in this study.

Figure 4.8 thru 4.14 demonstrates the relationship between input variables and original oil in-place from the Monte Carlo simulation. From figure 4.8, We could not establish relationship between the original oil in-place and connate water saturation since it show non-linear relationship and the data are scatted. And also there are no show relationships between other variable for instant reservoir pressure, cumulative water production, formation compressibility, cumulative oil production, initial reservoir pressure and water influx term. The statistical measurement which subject to the data fitting in the simple linear model, is the R square or namely coefficient of determination.

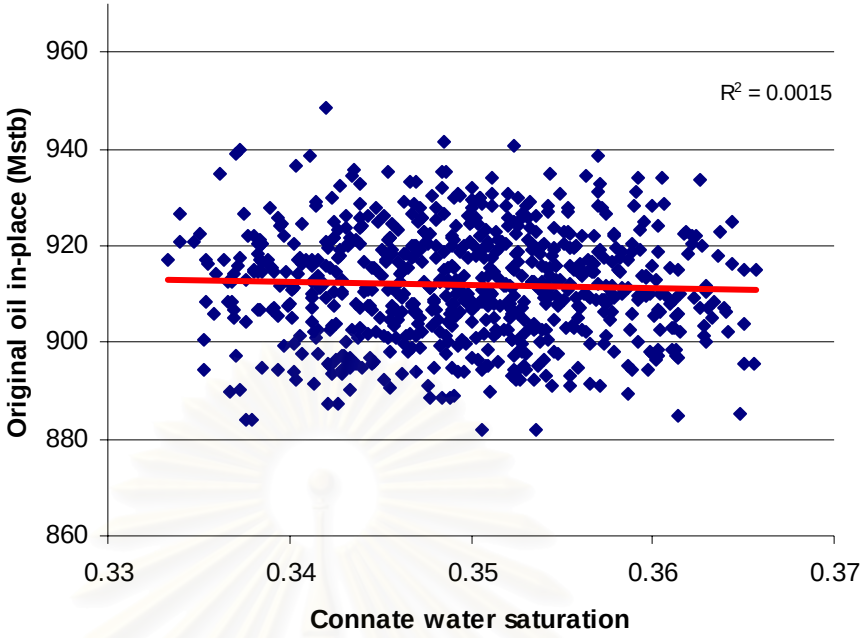



**Figure 4.8: Original oil in-place as function of connate water saturation from the 700 trials Monte Carlo simulation**

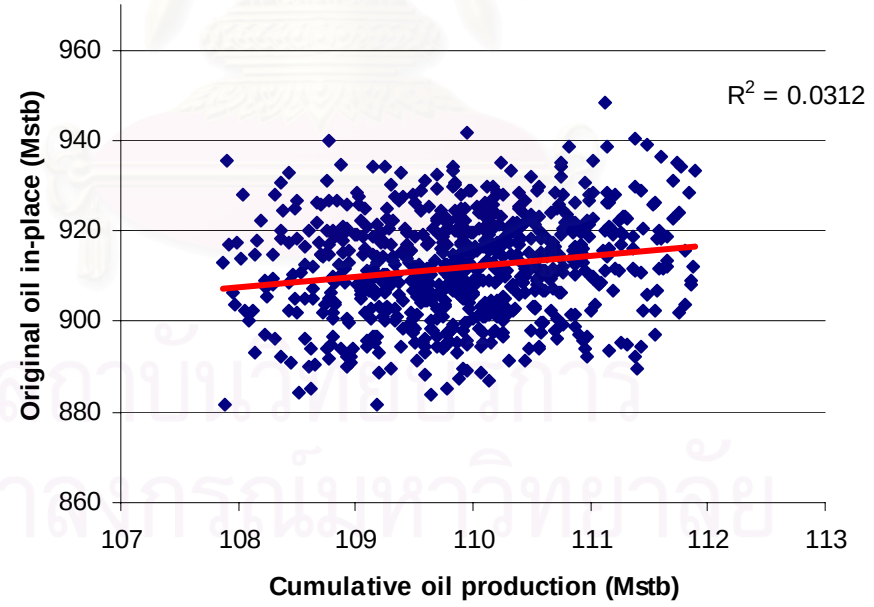



**Figure 4.9: Original oil in-place as function of cumulative oil production from the 700 trials Monte Carlo simulation**

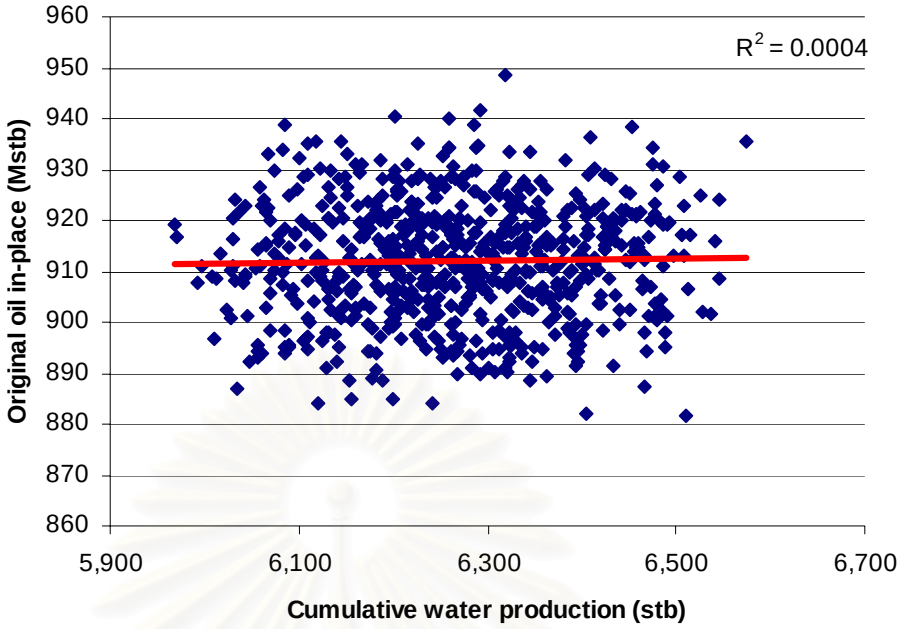



**Figure 4.10: Original oil in-place as function of cumulative water production from the 700 trials Monte Carlo simulation**

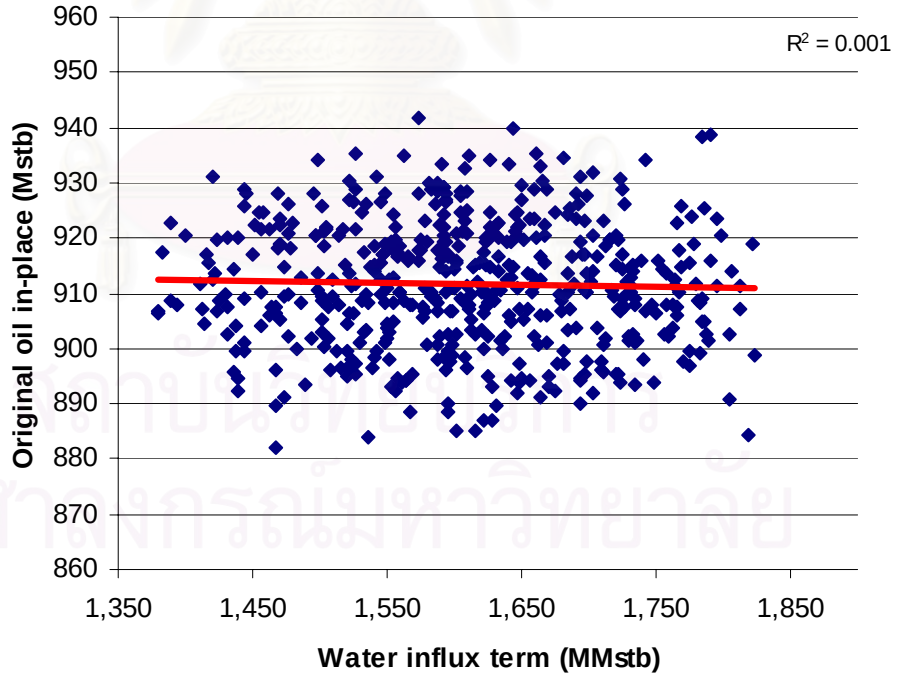



**Figure 4.11: Original oil in-place as function of water influx term from the 700 trials Monte Carlo simulation**

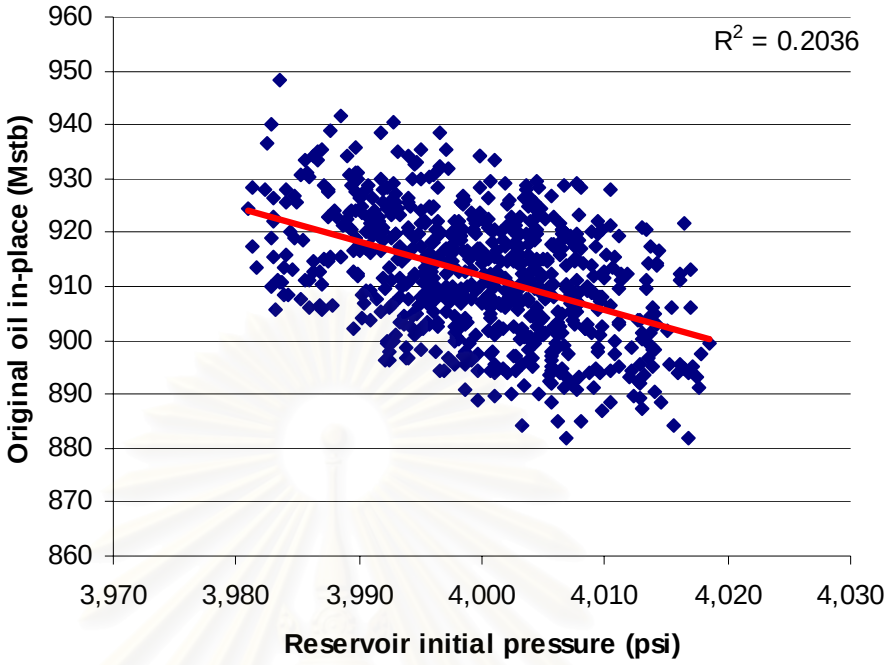



**Figure 4.12: Original oil in-place as function of reservoir initial pressure from the 700 trials Monte Carlo simulation**

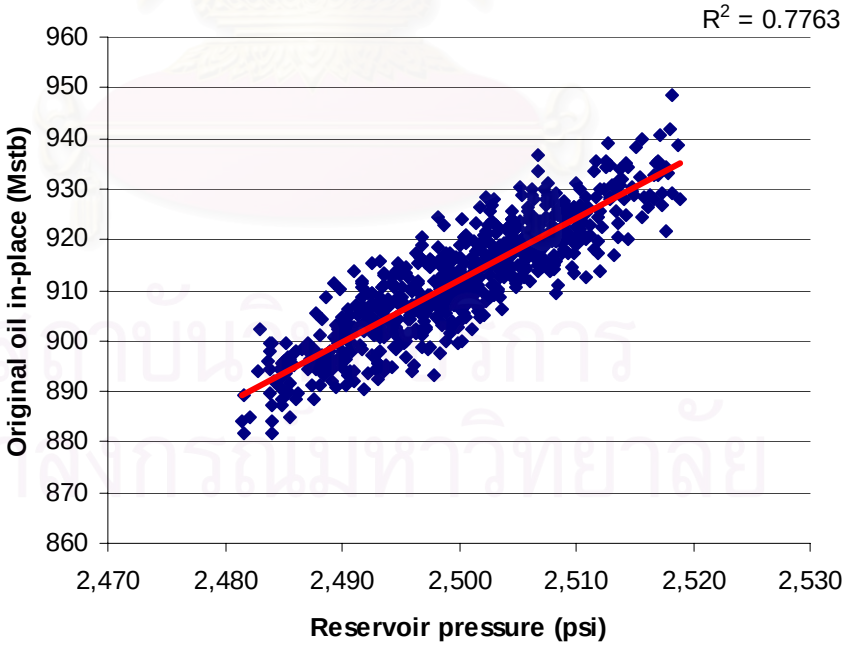



**Figure 4.13: Original oil in-place as function of reservoir pressure from the 700 trials Monte Carlo simulation**

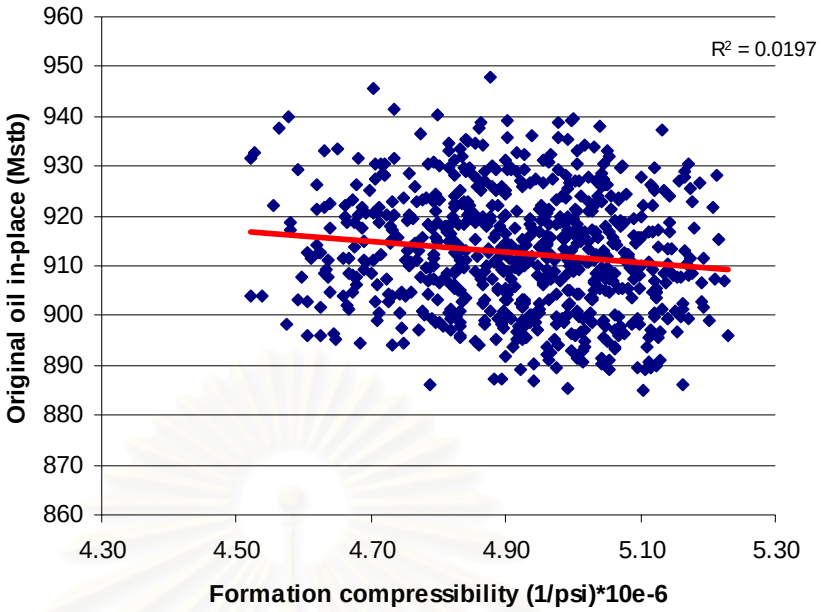



**Figure 4.14: Original oil in-place as function of compressibility factor from the 700 trials Monte Carlo simulation**

In order to investigate the goodness of fit of the model, in regression, the $R^2$ coefficient of determination is a statistical measure of how well the regression line approximates the real data points. From the plots, the $R^2$ coefficients of determination are summarized in table 4.4.

**Table 4.4: Coefficient of determination**

| Variables relationship | R square |
|---|---|
| Original oil in-place vs connate water saturation | 0.0015 |
| Original oil in-place vs cumulative water production | 0.0312 |
| Original oil in-place vs cumulative oil production | 0.0004 |
| Original oil in-place vs water influx | 0.001 |
| Original oil in-place vs reservoir initial pressure | 0.2036 |
| Original oil in-place vs reservoir pressure | 0.7763 |
| Original oil in-place vs formation compressibility | 0.0197 |

More simply, $R^2$ is often interpreted as the proportion of response variation "explained" by the regressors in the model. Thus, $R^2$ = 1 indicates that the fitted model explains all variability in $y$, while $R^2$ = 0 indicates no 'linear' relationship between the response variable and regressors. As concluded in table 4.4, there is only

one parameter which tended to has relationship between original oil in-place and reservoir pressure. For the rest of parameters, the $R^2$ value are almost zero which indicate no linear relationship between the response variables and regressor.

From table 4.4, the coefficient of determination can indicate the degree of strength of linear relationship. The weakest relationship between input and output parameter, is cumulative oil production versus original oil in-place. In the other hand, the strongest relationship is reservoir pressure versus original oil in-place.

For the next chapter, all statistical results from Monte Carlo simulation study will be compared with other uncertainty analysis methods i.e. first-order approximation method, response surface method. Moreover, the result from the base case scenario will be compared with predictive results of the first-order approximation method and the new simplify quadratic equations which obtained from response surface method.

# CHAPTER V
# UNCERTAINTY ANALYSIS

This chapter contains a description of the approximate analytical technique based on Taylor's series expansion of the material balance equation. The collection of mathematical and statistical technique, response surface methodology, that are useful for modeling and analysis of problems are also applied. The result of each uncertainty analysis method will be analyzed and compared with traditional Monte Carlo method. Moreover, the predictive result of first-order approximation and each response surface model will be compared with the base case result.

## 5.1 First-order analysis

The analysis of uncertainty involves measuring the degree to which each input contributes to uncertainty in the output. An input that has a small sensitivity but a large uncertainty may be just as important as an input with a large sensitivity but small uncertainty. One of the simplest approaches to uncertainty analysis, one that explicitly considers the effect of both sensitivity and uncertainty on a variance, is the first-order or Gaussian approximation. This simple method is based on Taylor's series expansion. It provides a way to express the deviation of an output from its base-case value in terms of deviations if its input from their base-case values.

### 5.1.1 First-order approximation

Suppose we have a model of the form

$$Y = f(X1, X2, ..., Xn)$$

In this study, the model, symbolically above as f, will be the Excel spreadsheet of material balance equation, Y is the variable to be predicted (original oil in-place) and $X_i$ is the inputs. A first-order Taylor's series expansion around a base-case value, $X^0$, gives

$$Y - Y_0 \cong \sum_{i=1}^{n} \left[ \frac{\partial Y}{\partial X_i} \right]_{X^0} \left( X_i - X_i^0 \right) \qquad (5.1)$$

And the general first order approximation for the variance of the output is

$$Var(Y) \cong \sum_{i=1}^{n} \left[ \frac{\partial Y}{\partial X_i} \right]_{X^0}^2 Var(X_i) + 2 \sum_{i=1}^{n} \sum_{j=i+1}^{n} Co\,\mathrm{var}(X_i, X_j) \left[ \frac{\partial Y}{\partial X_i} \right]_{X^0} \left[ \frac{\partial Y}{\partial X_j} \right]_{X^0} \quad (5.2)$$

If the inputs are independent, the second terms containing the covariance are zero. The variance of the output is approximately the sum of the product of the squares of the sensitivity of each input and their variance:

$$Var(Y) \cong \sum_{i=1}^{n} \left[ \frac{\partial Y}{\partial X_i} \right]_{X^0}^2 Var(X_i) \quad (5.3)$$

The variability in an output is thus comprised of two components: the sensitivity of the output $Y$ to the input $X_i$ and the variability of the $X_i$ themselves.

To acquire this result, we must calculate the sensitivity coefficient (partial derivatives) for evaluating the variability of an output. The derivative is the ratio of the change in the output to the change in the input. The form of the first-order model is sometime called a main effect model because it includes only the main effects of the variables.

## 5.1.2 Sensitivity coefficient

The sensitivity coefficient or partial derivative of a function of several variables is its derivative with respect to one of those variables with the others held constant. In this research, the material balance equation will be represented as the main function. The sensitivity coefficient or partial derivative of each parameter will be the derivative of the material balance equation with respect to the observing input variable.

In order to obtain the variance of the output, the partial derivative of each parameter in material balance equation has to be determined. The partial derivative of material balance equation with respect to each input variable are summarized.

In this research, there are seven variables to determine sensitivity coefficient i.e. connate water saturation, formation compressibility, cumulative oil production, cumulative water production, initial reservoir pressure, reservoir pressure and water influx term. The details of derived equations are illustrated in appendix A. Totally, the first order approximation result and the output variance are shown in table 5.1.

**Table 5.1: Uncertainty assessment (variance of the original oil in-place) using first-order analysis method.**

| Parameter description | Sensitivity coefficient | Individual variance | $\sum_{i=1}^{n}\left[\frac{\partial Y}{\partial X_i}\right]^2_{X^0} Var(X_i)$ |
|---|---|---|---|
| Connate water saturation | -177,162 | 0.000050 | 1,569,319 |
| Formation compressibility | 7,917,725,060 | $2.2*10^{-14}$ | 1,379,278 |
| Cumulative oil production | 8 | 857,364 | 54,871,296 |
| Cumulative water production | 3 | 16,057 | 144,513 |
| Initial reservoir pressure | 2,770 | 64 | 491,065,600 |
| Reservoir pressure | -216 | 69 | 3,219,264 |
| Water influx term | -3 | 9,213 | 82,917 |
| Variance from First-order approximation method | - | - | 552,332,187 |

## 5.1.3 Uncertainty assessment using first-order method

We assess uncertainty in the original oil in-place by calculating the variance of the original oil in-place using equation 5.3. As the results of calculated variance which demonstrated in table 5.1, the value of calculated output variance (var: 552,332,187) is not close to the reference variance of Monte Carlo simulation (var: 126,954,400) as shown in table 5.3. From this comparison, we can conclude that the uncertainty analysis using the first-order approximation method is overestimate.

We have to note that the first-order approximation method can provide the predictive result equal to base case value when all the input variables are at the base case condition. Hence the expansion term of Taylor's series will be zero and consequently the result will be equal to base case scenario. Thus, in order to get the prediction result, we should have more than one random run to get the average value of the result around the base case.

Another attempt, we tried on random the input variables into equation 5.1, 32 iterations to see the predictive results of the first-order approximation method. As the result of original oil in-place of first-order approximation method, the minimum and maximum output values are 885,603 stb and 956,179 stb respectively. We also tried on average the output from 36 iterations and compare to the base case. We can get the result close to base case as shown in table 5.2**.**

**Table 5.2: Predictive results of first-order approximation method compared with base case scenario**

| Predictive output | Base-case | First-order approximation method |
|---|---|---|
| Iteration no.1 | 911,633 | 929,338 |
| Iteration no.2 | 911,633 | 941,172 |
| Iteration no.3 | 911,633 | 956,179 |
| Iteration no.4 | 911,633 | 924,576 |
| Iteration no.5 | 911,633 | 915,036 |
| Iteration no.6 | 911,633 | 896,072 |
| Iteration no.7 | 911,633 | 915,337 |
| Iteration no.8 | 911,633 | 885,603 |
| Iteration no.9 | 911,633 | 915,265 |
| Iteration no.10 | 911,633 | 913,196 |
| Iteration no.11 | 911,633 | 915,036 |
| Iteration no.12 | 911,633 | 896,072 |
| Iteration no.13 | 911,633 | 915,337 |
| Iteration no.14 | 911,633 | 885,603 |
| Iteration no.15 | 911,633 | 915,265 |
| Iteration no.16 | 911,633 | 913,196 |
| Iteration no.17 | 911,633 | 924,576 |
| Iteration no.18 | 911,633 | 915,036 |
| Iteration no.19 | 911,633 | 896,072 |
| Iteration no.20 | 911,633 | 915,337 |
| Iteration no.21 | 911,633 | 885,603 |
| Iteration no.22 | 911,633 | 915,265 |
| Iteration no.23 | 911,633 | 913,196 |
| Iteration no.24 | 911,633 | 924,576 |
| Iteration no.25 | 911,633 | 915,036 |
| Iteration no.26 | 911,633 | 896,072 |
| Iteration no.27 | 911,633 | 915,337 |
| Iteration no.28 | 911,633 | 885,603 |
| Iteration no.29 | 911,633 | 915,265 |
| Iteration no.30 | 911,633 | 913,196 |
| Iteration no.31 | 911,633 | 913,196 |
| Iteration no.32 | 911,633 | 913,196 |
| Iteration no.33 | 911,633 | 956,179 |
| Iteration no.34 | 911,633 | 924,576 |
| Iteration no.35 | 911,633 | 885,603 |
| Iteration no.36 | 911,633 | 915,337 |
| Average value | 911,633 | 913,071 |

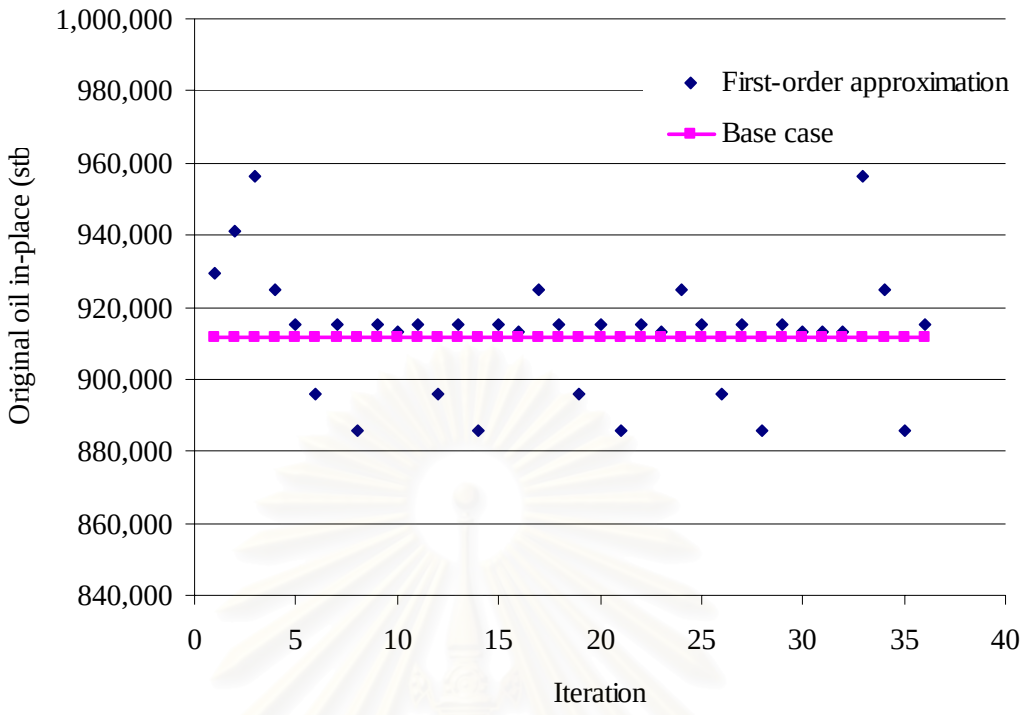



**Figure 5.1: 32 iterations of predictive outputs from first-order approximation method**

**Table 5.3: Statistical result of first-order approximation method compared with Monte Carlo simulation**

| Statistical result | Monte Carlo simulation | First-order approximation method |
|---|---|---|
| Max | 938,311 | 956,179 |
| Min | 876,969 | 885,603 |
| Mean | 912,823 | 913,071 |
| Variance | 126,954,400 | 552,332,187 |
| SD | 11,267 | 23,501 |
| P10 | 898,173 | 885,603 |
| P25 | 903,034 | 904,634 |
| P50 | 912,990 | 915,036 |
| P75 | 918,545 | 915,337 |
| P90 | 927,471 | 924,576 |

In the next section, the predictive value and statistical result of response surface model will be compared with the base case and Monte Carlo simulation in order to determine the benefit of using each method.

## 5.2 Response surface method

### 5.2.1 Background

Response surface methodology is a collection of statistical and mathematical techniques useful for developing, improving, and optimizing processes (Myer and Montgomery, 1995). The most extensive applications of response surface are found where several inputs influence the output of a model. The procedures used in determining a response surface are combination of experimental design, mathematical method, and statistic inference (Box and Wilson, 1951).The response surface is an empirical relationship that satisfies the observed effects of the different factors. Its procedure is not used for understanding the mechanism of the system or process but rather optimizing a process or allow and accurate forecast in a defined region of the total space of the factors or variables (Varela, 1999).

Polynomials are the most common form used for response surface model. The simplest equation describing the relation between a response and variables is the linear model or the first-order model. Each variable independently affects the response. If there exists interactions between variables, quadratic response model, or a second-order model is more suitable.

$$Y = a_0 + \sum_{i=1}^{n} a_i X_i + \varepsilon \qquad \text{(First-order model)} \qquad (5.4)$$

$$Y = a_0 + \sum_{i=1}^{n} a_i X_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} X_i X_j + \varepsilon \qquad \text{(Second-order model)} \qquad (5.5)$$

Where Y is the observed response, and estimates of the coefficients a's, are to be determined by the method of least squares, which minimizes the sum of the squares of the errors, $\varepsilon$. As the number of the variables increase, more coefficients must be estimated, and the number of experimental points must necessarily increase.

There is a close connection between response surface method and linear regression analysis. The coefficients a's are a set of unknown parameters. To estimate the values of these parameters, we must collect data on the system we are studying. Regression analysis is a branch of statistical model building that uses these data to estimate the coefficient a's. It is important to plan the data collection phase of a response surface study carefully. Several designs have been developed to minimize

the number of points required to determine these surface-response equations (Myer and Montgomery, 1995).

### 5.2.2 Uncertainty assessment using response surface method

In general, whenever the first order model does not appear to adequately represent the relationships between variables, then the higher order model approach is appropriate. In this study, we have chosen the second-order model to find a suitable approximation function that allows prediction of original oil in-place and associated uncertainty with fewer simulation runs than the Monte Carlo simulation without losing much accuracy. Because we already have the input and output data from 700 trials Monte Carlo simulation, we use the model fitting to determine the model's coefficients. For the seven variables in this study, the number of samples needed to be estimated in the second-order model is 36. Therefore, we would need at least 36 observation points for the surface response model.

First, we randomly choose 36 different simulations from the 700 trials Monte Carlo simulation and calculate the model's coefficients. Then, we use this derived empirical function to calculate original oil in-place of all 700 trials and estimate their statistics. The procedure is repeated again using another set of 36 simulation runs. For this time, we select the simulation runs by having the input spread out over their ranges (scatter) and at the edge of the range (most of the values are at the upper and lower boundary of their range).

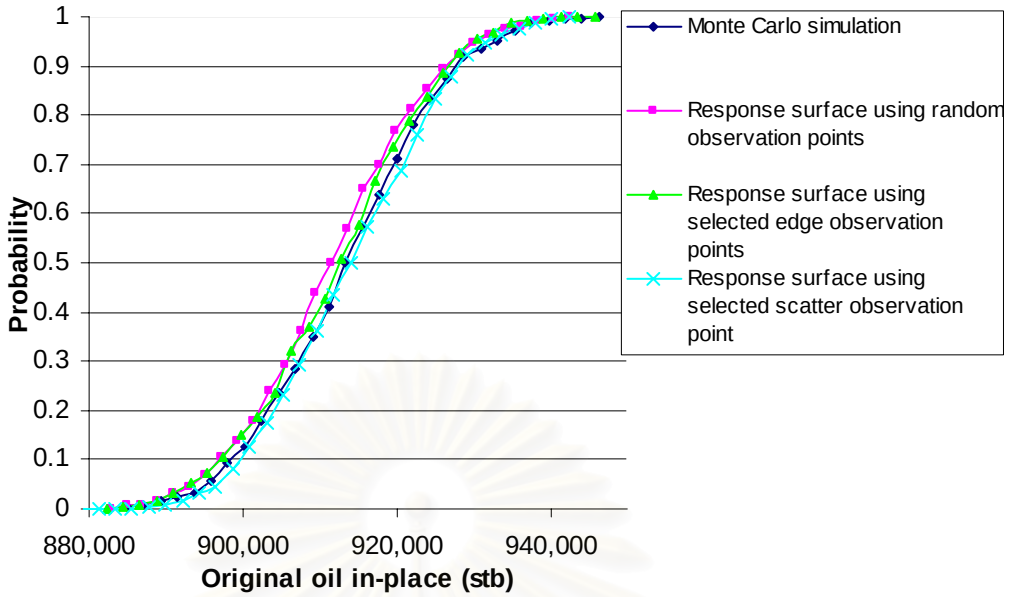



**Figure 5.2: Original oil in-place CDFs comparison of the 700 trials Monte Carlo simulation and the derived response surfaces**

Figure 5.2 illustrates cumulative distribution function of the original oil in-place using derived response surfaces and the reference Monte Carlo simulation. Both derived response surfaces and Monte Carlo simulation produce similar CDFs and the trends are smooth. The derived response surface with selected observation points seems to give a better approximation to the reference Monte Carlo than the random observation one. The derived response surface with random observation point results shifts away from Monte Carlo simulation starting from probability at 0.1 thru 0.9. We found that the maximum difference of the result value between the derivative response surface with random observation points and Monte Carlo simulation occurred at probability (0.9), is approximately 3,924 stb.

Table 5.4, 5.5 and 5.6 show the 36 random simulation results and 36 selected simulation results from 700 trials Monte Carlo simulation which used for fitting the response surface. In the yellow cells, the simulation results were filtered to have only the input at the edge (upper and lower bound) and only the input spread out over both upper and lower range. The response surface equation fitting will be explained the detail in appendix B.

**Table 5.4: Selected input observation points at the edge of boundary from 700 trials Monte Carlo simulation**

| Name | N | Cf | Swc | Np | Wp | Pi | P | We |
|---|---|---|---|---|---|---|---|---|
| Type | Output | Input | Input | Input | Input | Input | Input | Input |
| Iteration 240 | 911,008 | 5.24E-06 | 0.3597 | 109,821 | 6,238 | 3,997 | 2,500 | 1,452 |
| Iteration 311 | 903,462 | 5.23E-06 | 0.3439 | 110,492 | 6,303 | 4,000 | 2,493 | 1,615 |
| Iteration 68 | 905,905 | 5.23E-06 | 0.3356 | 109,549 | 6,285 | 4,010 | 2,502 | 1,756 |
| Iteration 99 | 904,025 | 4.51E-06 | 0.3384 | 109,952 | 6,330 | 4,008 | 2,493 | 1,465 |
| Iteration 452 | 905,258 | 4.52E-06 | 0.3496 | 110,400 | 6,266 | 4,018 | 2,500 | 1,789 |
| Iteration 275 | 904,402 | 5.04E-06 | 0.3669 | 109,544 | 6,173 | 4,005 | 2,499 | 1,545 |
| Iteration 70 | 915,157 | 5.11E-06 | 0.3668 | 109,413 | 6,145 | 3,997 | 2,505 | 1,543 |
| Iteration 579 | 911,692 | 5.14E-06 | 0.3666 | 109,529 | 6,117 | 4,002 | 2,505 | 1,734 |
| Iteration 229 | 894,693 | 4.85E-06 | 0.3327 | 108,772 | 6,398 | 4,002 | 2,487 | 1,679 |
| Iteration 580 | 914,798 | 4.81E-06 | 0.3341 | 108,857 | 6,222 | 4,014 | 2,509 | 1,463 |
| Iteration 611 | 908,143 | 5.18E-06 | 0.3519 | 111,943 | 6,498 | 3,997 | 2,493 | 1,660 |
| Iteration 614 | 921,373 | 4.81E-06 | 0.3428 | 111,933 | 6,340 | 3,995 | 2,500 | 1,643 |
| Iteration 518 | 925,207 | 4.75E-06 | 0.3613 | 111,924 | 6,271 | 3,991 | 2,502 | 1,628 |
| Iteration 450 | 906,401 | 4.88E-06 | 0.3545 | 107,792 | 6,164 | 3,987 | 2,493 | 1,480 |
| Iteration 395 | 910,104 | 4.59E-06 | 0.3614 | 107,826 | 6,518 | 3,998 | 2,499 | 1,535 |
| Iteration 93 | 943,180 | 4.64E-06 | 0.3539 | 111,764 | 6,583 | 3,999 | 2,518 | 1,517 |
| Iteration 536 | 889,146 | 5.20E-06 | 0.3590 | 109,878 | 6,573 | 4,012 | 2,488 | 1,618 |
| Iteration 352 | 895,847 | 4.91E-06 | 0.3616 | 111,100 | 6,571 | 4,011 | 2,489 | 1,601 |
| Iteration 573 | 914,515 | 4.91E-06 | 0.3416 | 109,696 | 5,980 | 4,010 | 2,507 | 1,579 |
| Iteration 263 | 903,452 | 4.74E-06 | 0.3504 | 111,154 | 5,985 | 4,004 | 2,492 | 1,615 |
| Iteration 324 | 899,418 | 5.09E-06 | 0.3495 | 109,729 | 6,168 | 4,019 | 2,500 | 1,578 |
| Iteration 389 | 886,534 | 5.08E-06 | 0.3372 | 110,052 | 6,502 | 4,019 | 2,487 | 1,541 |
| Iteration 632 | 904,670 | 5.06E-06 | 0.3383 | 111,234 | 6,222 | 4,018 | 2,500 | 1,503 |
| Iteration 402 | 924,004 | 5.04E-06 | 0.3510 | 111,334 | 6,270 | 3,981 | 2,499 | 1,640 |
| Iteration 586 | 920,974 | 5.10E-06 | 0.3605 | 110,112 | 6,200 | 3,982 | 2,499 | 1,479 |
| Iteration 213 | 929,485 | 4.91E-06 | 0.3368 | 111,212 | 6,273 | 4,016 | 2,519 | 1,641 |
| Iteration 319 | 933,227 | 4.85E-06 | 0.3617 | 109,852 | 6,079 | 4,000 | 2,518 | 1,581 |
| Iteration 380 | 940,223 | 4.78E-06 | 0.3625 | 111,267 | 6,438 | 3,996 | 2,517 | 1,588 |
| Iteration 211 | 898,369 | 4.89E-06 | 0.3452 | 110,461 | 6,412 | 3,988 | 2,481 | 1,553 |
| Iteration 567 | 902,860 | 4.82E-06 | 0.3433 | 111,731 | 6,323 | 3,987 | 2,481 | 1,507 |
| Iteration 162 | 919,525 | 4.89E-06 | 0.3588 | 108,444 | 6,269 | 3,983 | 2,502 | 1,832 |
| Iteration 648 | 917,875 | 5.14E-06 | 0.3460 | 111,184 | 6,329 | 3,989 | 2,498 | 1,827 |
| Iteration 651 | 904,988 | 4.74E-06 | 0.3433 | 110,081 | 6,331 | 3,999 | 2,492 | 1,825 |
| Iteration 388 | 901,010 | 4.75E-06 | 0.3531 | 108,977 | 6,131 | 3,994 | 2,489 | 1,375 |
| Iteration 597 | 896,789 | 4.59E-06 | 0.3417 | 109,947 | 6,426 | 4,004 | 2,485 | 1,376 |
| Iteration 689 | 898,836 | 4.92E-06 | 0.3516 | 110,153 | 6,293 | 4,010 | 2,493 | 1,380 |

**Table 5.5: Selected input observation points spread out (scatter) over 700 trials Monte Carlo simulation**

| Name | N | Cf | Swc | Np | Wp | Pi | P | We |
|---|---|---|---|---|---|---|---|---|
| Type | Output | Input | Input | Input | Input | Input | Input | Input |
| Iteration 99 | 904,025 | 4.51E-06 | 0.3384 | 109,952 | 6,330 | 4,008 | 2,493 | 1,465 |
| Iteration 509 | 909,094 | 4.75E-06 | 0.3597 | 110,485 | 6,043 | 4,005 | 2,500 | 1,786 |
| Iteration 568 | 925,389 | 4.92E-06 | 0.3534 | 110,230 | 6,181 | 3,994 | 2,508 | 1,754 |
| Iteration 423 | 925,171 | 5.04E-06 | 0.3481 | 109,184 | 6,338 | 4,009 | 2,517 | 1,614 |
| Iteration 240 | 911,008 | 5.24E-06 | 0.3597 | 109,821 | 6,238 | 3,997 | 2,500 | 1,452 |
| Iteration 229 | 894,693 | 4.85E-06 | 0.3327 | 108,772 | 6,398 | 4,002 | 2,487 | 1,679 |
| Iteration 424 | 912,091 | 5.05E-06 | 0.3441 | 109,507 | 6,142 | 3,994 | 2,499 | 1,605 |
| Iteration 601 | 928,483 | 5.06E-06 | 0.3495 | 109,862 | 6,403 | 3,990 | 2,509 | 1,539 |
| Iteration 21 | 903,466 | 4.97E-06 | 0.3560 | 109,934 | 6,031 | 4,008 | 2,498 | 1,601 |
| Iteration 275 | 904,402 | 5.04E-06 | 0.3669 | 109,544 | 6,173 | 4,005 | 2,499 | 1,545 |
| Iteration 450 | 906,401 | 4.88E-06 | 0.3545 | 107,792 | 6,164 | 3,987 | 2,493 | 1,480 |
| Iteration 54 | 915,751 | 4.85E-06 | 0.3353 | 109,158 | 6,444 | 4,001 | 2,503 | 1,638 |
| Iteration 333 | 895,156 | 5.15E-06 | 0.3457 | 109,769 | 6,359 | 4,004 | 2,489 | 1,526 |
| Iteration 452 | 905,258 | 4.52E-06 | 0.3496 | 110,400 | 6,266 | 4,018 | 2,500 | 1,789 |
| Iteration 611 | 908,143 | 5.18E-06 | 0.3519 | 111,943 | 6,498 | 3,997 | 2,493 | 1,660 |
| Iteration 573 | 914,515 | 4.91E-06 | 0.3416 | 109,696 | 5,980 | 4,010 | 2,507 | 1,579 |
| Iteration 330 | 905,393 | 4.94E-06 | 0.3430 | 109,470 | 6,169 | 4,012 | 2,501 | 1,707 |
| Iteration 595 | 896,776 | 4.86E-06 | 0.3508 | 109,984 | 6,269 | 4,000 | 2,487 | 1,649 |
| Iteration 119 | 907,435 | 4.88E-06 | 0.3481 | 108,532 | 6,384 | 4,006 | 2,501 | 1,512 |
| Iteration 93 | 943,180 | 4.64E-06 | 0.3539 | 111,764 | 6,583 | 3,999 | 2,518 | 1,517 |
| Iteration 402 | 924,004 | 5.04E-06 | 0.3510 | 111,334 | 6,270 | 3,981 | 2,499 | 1,640 |
| Iteration 526 | 925,492 | 5.00E-06 | 0.3538 | 109,606 | 6,104 | 3,994 | 2,509 | 1,489 |
| Iteration 141 | 924,302 | 4.94E-06 | 0.3387 | 110,273 | 6,411 | 4,000 | 2,508 | 1,475 |
| Iteration 66 | 895,863 | 4.99E-06 | 0.3544 | 109,438 | 6,256 | 4,006 | 2,491 | 1,712 |
| Iteration 324 | 899,418 | 5.09E-06 | 0.3495 | 109,729 | 6,168 | 4,019 | 2,500 | 1,578 |
| Iteration 211 | 898,369 | 4.89E-06 | 0.3452 | 110,461 | 6,412 | 3,988 | 2,481 | 1,553 |
| Iteration 62 | 903,280 | 5.11E-06 | 0.3365 | 109,128 | 6,313 | 3,997 | 2,493 | 1,526 |
| Iteration 411 | 916,034 | 4.76E-06 | 0.3412 | 108,767 | 6,162 | 3,991 | 2,500 | 1,698 |
| Iteration 23 | 920,048 | 4.89E-06 | 0.3632 | 110,986 | 6,204 | 4,002 | 2,506 | 1,632 |
| Iteration 213 | 929,485 | 4.91E-06 | 0.3368 | 111,212 | 6,273 | 4,016 | 2,519 | 1,641 |
| Iteration 388 | 901,010 | 4.75E-06 | 0.3531 | 108,977 | 6,131 | 3,994 | 2,489 | 1,375 |
| Iteration 542 | 925,862 | 4.92E-06 | 0.3605 | 111,844 | 6,378 | 4,003 | 2,509 | 1,519 |
| Iteration 260 | 897,171 | 4.86E-06 | 0.3425 | 110,459 | 6,487 | 4,000 | 2,485 | 1,591 |
| Iteration 502 | 922,572 | 4.97E-06 | 0.3564 | 109,619 | 6,265 | 3,989 | 2,504 | 1,642 |
| Iteration 284 | 899,151 | 4.90E-06 | 0.3394 | 108,280 | 6,035 | 3,991 | 2,488 | 1,711 |
| Iteration 162 | 919,525 | 4.89E-06 | 0.3588 | 108,444 | 6,269 | 3,983 | 2,502 | 1,832 |

**Table 5.6: Random input observation points from 700 trials Monte Carlo simulation**

| Name | N | Cf | Swc | Np | Wp | Pi | P | We |
|---|---|---|---|---|---|---|---|---|
| Type | Output | Input | Input | Input | Input | Input | Input | Input |
| Iteration 1 | 919,825 | 4.80E-06 | 0.344 | 110,328 | 6,172 | 4,003 | 2,505 | 1,479 |
| Iteration 2 | 908,963 | 5.06E-06 | 0.364 | 109,090 | 6,350 | 4,000 | 2,501 | 1,606 |
| Iteration 3 | 909,737 | 4.92E-06 | 0.358 | 109,021 | 6,226 | 3,993 | 2,497 | 1,507 |
| Iteration 4 | 905,510 | 4.82E-06 | 0.352 | 108,950 | 6,402 | 4,018 | 2,504 | 1,604 |
| Iteration 5 | 921,287 | 4.72E-06 | 0.350 | 110,294 | 6,145 | 3,982 | 2,497 | 1,637 |
| Iteration 6 | 916,244 | 4.74E-06 | 0.347 | 109,439 | 6,121 | 4,003 | 2,505 | 1,595 |
| Iteration 7 | 907,551 | 4.71E-06 | 0.349 | 110,103 | 6,254 | 3,996 | 2,492 | 1,692 |
| Iteration 8 | 929,136 | 4.85E-06 | 0.356 | 111,535 | 6,297 | 4,003 | 2,511 | 1,440 |
| Iteration 9 | 906,920 | 4.98E-06 | 0.338 | 109,746 | 6,387 | 4,005 | 2,497 | 1,510 |
| Iteration 10 | 914,932 | 4.98E-06 | 0.341 | 109,728 | 6,234 | 4,002 | 2,503 | 1,496 |
| Iteration 201 | 937,155 | 4.60E-06 | 0.350 | 110,824 | 6,185 | 3,991 | 2,511 | 1,383 |
| Iteration 202 | 908,985 | 4.96E-06 | 0.350 | 110,621 | 6,440 | 3,997 | 2,495 | 1,726 |
| Iteration 203 | 911,933 | 5.04E-06 | 0.341 | 109,918 | 6,310 | 4,005 | 2,502 | 1,592 |
| Iteration 204 | 914,222 | 4.84E-06 | 0.353 | 110,977 | 6,358 | 3,997 | 2,498 | 1,576 |
| Iteration 205 | 931,259 | 4.90E-06 | 0.360 | 109,355 | 6,439 | 3,983 | 2,508 | 1,571 |
| Iteration 206 | 922,711 | 4.87E-06 | 0.343 | 110,374 | 6,144 | 4,001 | 2,507 | 1,491 |
| Iteration 207 | 905,130 | 4.94E-06 | 0.357 | 110,584 | 6,170 | 3,993 | 2,491 | 1,686 |
| Iteration 208 | 911,992 | 5.00E-06 | 0.350 | 110,348 | 6,150 | 4,008 | 2,504 | 1,619 |
| Iteration 209 | 911,240 | 5.06E-06 | 0.351 | 108,828 | 6,378 | 3,992 | 2,498 | 1,701 |
| Iteration 210 | 886,929 | 5.13E-06 | 0.350 | 109,540 | 6,333 | 4,016 | 2,489 | 1,584 |
| Iteration 401 | 912,869 | 4.95E-06 | 0.356 | 111,067 | 6,179 | 4,016 | 2,507 | 1,696 |
| Iteration 402 | 912,851 | 5.16E-06 | 0.360 | 108,856 | 6,211 | 3,996 | 2,503 | 1,490 |
| Iteration 403 | 919,477 | 5.11E-06 | 0.342 | 110,151 | 6,399 | 3,999 | 2,505 | 1,531 |
| Iteration 404 | 902,542 | 4.96E-06 | 0.349 | 109,622 | 6,039 | 4,007 | 2,497 | 1,675 |
| Iteration 405 | 901,162 | 4.98E-06 | 0.361 | 108,964 | 6,336 | 4,009 | 2,498 | 1,603 |
| Iteration 406 | 890,865 | 4.77E-06 | 0.334 | 109,720 | 6,161 | 4,008 | 2,485 | 1,592 |
| Iteration 407 | 896,909 | 4.97E-06 | 0.355 | 110,430 | 6,165 | 4,004 | 2,489 | 1,697 |
| Iteration 408 | 930,150 | 4.69E-06 | 0.333 | 109,769 | 6,219 | 3,997 | 2,510 | 1,467 |
| Iteration 409 | 897,073 | 4.73E-06 | 0.353 | 109,537 | 6,008 | 4,017 | 2,496 | 1,604 |
| Iteration 410 | 895,763 | 4.94E-06 | 0.359 | 110,865 | 6,388 | 3,998 | 2,484 | 1,495 |
| Iteration 601 | 916,152 | 5.02E-06 | 0.366 | 110,409 | 6,357 | 4,003 | 2,505 | 1,632 |
| Iteration 602 | 894,953 | 5.02E-06 | 0.355 | 111,392 | 6,203 | 4,004 | 2,486 | 1,826 |
| Iteration 603 | 908,979 | 4.94E-06 | 0.344 | 110,027 | 6,259 | 4,004 | 2,499 | 1,593 |
| Iteration 604 | 892,246 | 4.99E-06 | 0.348 | 109,678 | 6,203 | 4,000 | 2,484 | 1,544 |
| Iteration 605 | 907,038 | 4.67E-06 | 0.340 | 109,485 | 6,068 | 4,010 | 2,500 | 1,573 |
| Iteration 606 | 896,670 | 4.95E-06 | 0.355 | 109,545 | 6,111 | 4,017 | 2,497 | 1,627 |

From equation 5.6, 5.7 and 5.8, demonstrate the second-order or quadratic models which fitted from 36 random observation points, 36 selected observation points at the edge of the range and 36 selected observation points spread over the range respectively.

$$\begin{aligned}Y = {} & 2{,}530{,}001 + 30{,}830{,}051{,}328X_1 + 482{,}721X_2 - 3X_3 + 13X_4 - 932X_5 - 173X_6 - 12X_7 \\ & - 12{,}697{,}993{,}216X_1X_2 - 42{,}540X_1X_3 + 120{,}697X_1X_4 + 4{,}200{,}648X_1X_5 - 18{,}297{,}976X_1X_6 \\ & - 140{,}928X_1X_7 - 0.4X_2X_3 - 0.7X_2X_4 + 31X_2X_5 - 196X_2X_6 + 3X_2X_7 - X_3X_4 - 0.001X_3X_5 \\ & + 0.004X_3X_6 + 0.00002X_3X_7 - 0.004X_4X_5 + 0.003X_4X_6 - 0.00001X_4X_7 - X_5X_6 + 0.003X_5X_7 \\ & - 0.004X_6X_7 - (4\times10^{13})X_1^{\,2} - 144{,}063X_2^2 + 0.000001X_3^2 + 0.00001X_4^2 + 0.4X_5^2 + X_6^2 + 5X_7^2\end{aligned} \tag{5.6}$$

$$\begin{aligned}Y = {} & 1{,}681{,}644 - 27{,}728{,}052{,}224X_1 + 3{,}792{,}348X_2 - 6X_3 + 104X_4 - 229X_5 - 1072X_6 + 40X_7 \\ & - 69{,}321{,}228{,}288X_1X_2 + 436{,}786X_1X_3 - 2{,}302{,}190X_1X_4 - 12{,}879{,}128X_1X_5 + 19{,}021{,}264X_1X_6 \\ & - 353{,}252X_1X_7 + 7X_2X_3 - 32X_2X_4 - 183X_2X_5 - 1{,}307X_2X_6 + 4X_2X_7 + 0.0004X_3X_4 - 0.004X_3X_5 \\ & - 0.002X_3X_6 - X_3X_7 - 0.006X_4X_5 - 0.04X_4X_6 - 0.002X_4X_7 - X_5X_6 + 0.006X_5X_7 \\ & + 0.01X_6X_7 - (15\times10^{13})X_1^{\,2} - 139{,}636X_2^2 + 0.0000025X_3^2 + 0.00006X_4^2 + 0.4X_5^2 + X_6^2 - 0.002X_7^2\end{aligned} \tag{5.7}$$

$$\begin{aligned}Y = {} & 3{,}810{,}682 - 4{,}096{,}743X_1 - 245{,}012X_2 - 18X_3 + 88X_4 - 3{,}076X_5 + 2{,}365X_6 - 18X_7 \\ & - 1{,}159{,}087X_1X_2 - 22X_1X_3 - 42X_1X_4 + 2{,}500X_1X_5 - 5{,}597X_1X_6 + 25X_1X_7 - X_2X_3 \\ & - 50X_2X_4 + 311X_2X_5 - 614X_2X_6 + 20X_2X_7 - 0.00001X_3X_4 - 0.006X_3X_5 + 0.01X_3X_6 \\ & - 0.0001X_3X_7 - 0.001X_4X_5 + 0.0001X_4X_6 - 0.001X_4X_7 - 2X_5X_6 + 0.008X_5X_7 - 0.008X_6X_7 \\ & - 5{,}396{,}426X_1^{\,2} + 24{,}684X_2^2 - 0.00002X_3^2 - 0.001X_4^2 + 0.8X_5^2 + 2X_6^2 - 0.001X_7^2\end{aligned} \tag{5.8}$$

From the above response surfaces equation 5.6, 5.7 and 5.8, we used the Monte Carlo simulation method to generate the input distribution, the statistical results of original oil in-place for all response surfaces are shown in table 5.7. Furthermore, the predictive results of original oil in-place for all response surfaces can be acquired by plug in the variables at base case condition. All the predictive results are also shown in table 5.8.

**Table 5.7: Comparison of statistical result between Monte Carlo, response surfaces and first-order approximation**

| Statistics | Monte Carlo | RS random | RS selected (edge) | RS selected (scatter) | First-order approximation |
|---|---|---|---|---|---|
| Max | 938,311 | 944,471 | 945,147 | 940,834 | 956,179 |
| Min | 876,969 | 883,521 | 880,589 | 878,930 | 885,603 |
| Mean | 912,823 | 910,693 | 911,925 | 912,704 | 913,071 |
| Variance | 126,954,400 | 126,954,400 | 135,887,300 | 138,227,040 | 552,332,187 |
| SD | 11,267 | 11,267 | 11,657 | 11,757 | 23,501 |
| P10 | 898,173 | 896,155 | 895,406 | 896,489 | 885,603 |
| P25 | 903,034 | 902,094 | 899,571 | 904,048 | 904,634 |
| P50 | 912,990 | 910,534 | 901,800 | 912,556 | 915,036 |
| P75 | 918,545 | 918,394 | 918,903 | 920,860 | 915,337 |
| P90 | 927,471 | 924,757 | 926,801 | 928,190 | 924,576 |

From table 5.7, we can conclude that the response surface method can provide the statistical result close to Monte Carlo simulation. Uses of random sampling points or selected sampling points can reflect the change of response surface model. The response surface model with selected observation points spread over the range can provide a better result than the random observation points and others. Although the selected observation points at the edge of range, have larger variance than the random observation points, but we can ensure that the 36 selected observation points can closely represent the 700 trials of Monte Carlo simulation. Consequently, the cumulative distribution function, P10, P25, P50, P75 and P90 are also close to Monte Carlo simulation.

**Table 5.8: Comparison of predictive result between base case, response surfaces and first-order approximation**

| | Base case | RS random | RS selected (edge) | RS selected (scatter) | First-order approximation |
|---|---|---|---|---|---|
| Predictive output (stb) | 911,633 | 909,778 | 910,171 | 911,556 | 913,071 |

From table 5.8, the predictive result obtained from response surface method, show the good potential of prediction. Especially, if we selected iterations spread over the range, it can provide the simple quadratic equation (surface) with an accurate result. The predictive result of all response surfaces can provide a better result than

the first-order approximation method. The best prediction can be obtained from response surface with the scatter inputs.

As determine the main effect variables of both random observation model and selected observation model, Figure 5.3 and 5.5 illustrated the tornado chart of effect which is the useful plot for identifying the important factors. From the tornado chart of random observation model, the reservoir pressure is the most important factor and secondary the reservoir initial pressure. Identically, figure 5.5, the tornado chart of selected observation model show the same result that the reservoir pressure is the most important factor in the response surface model. The shape of probability distribution function (PDF) for random observation model and selected observation model are also shown in figure 5.4 and 5.6.

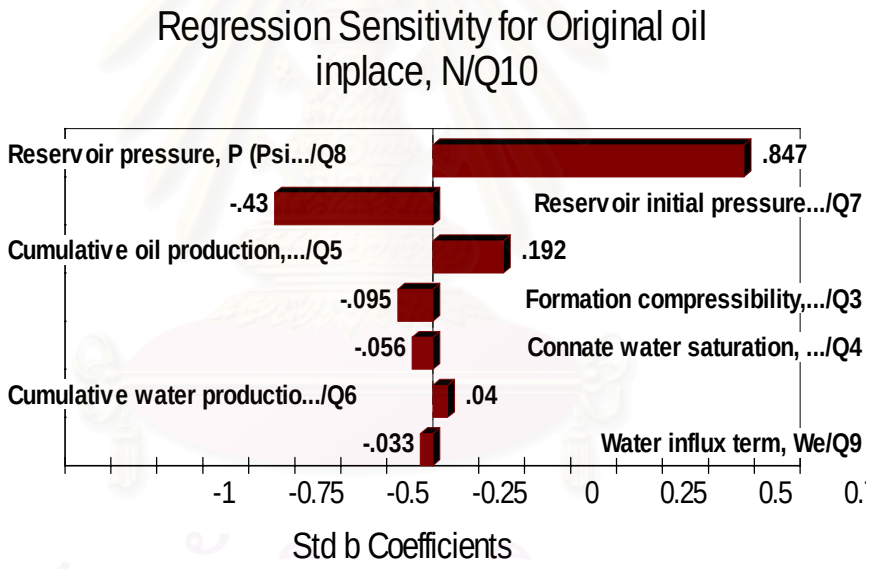



**Figure 5.3: Tornado chart of effect random observation points from 700 trials Monte Carlo simulation**

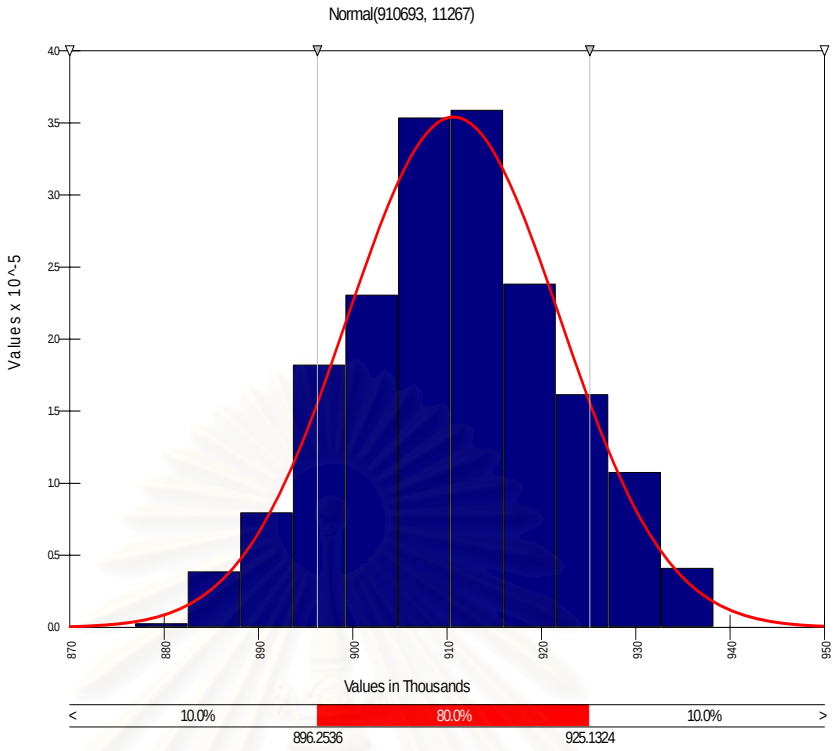



**Figure 5.4: Probability distribution of selected observation points from 700 trials Monte Carlo simulation**

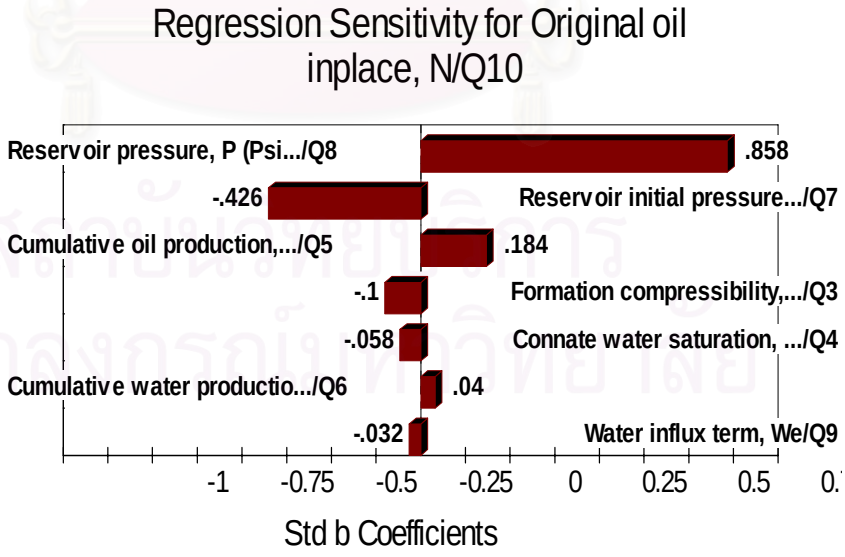



**Figure 5.5: Tornado chart of effect selected observation points from 700 trials Monte Carlo simulation**

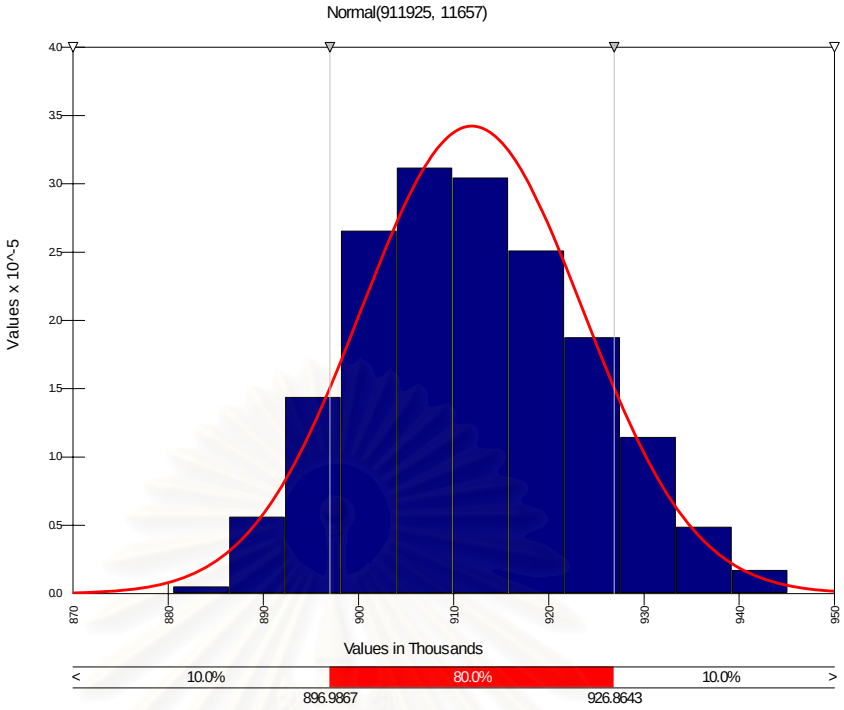



**Figure 5.6: Probability distribution of selected observation points at the edge of range from 700 trials Monte Carlo simulation**

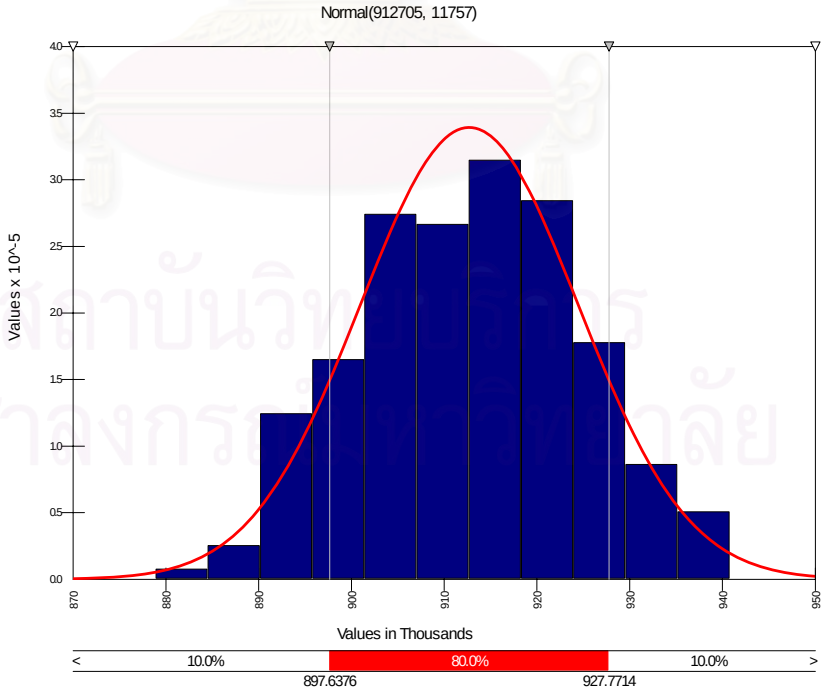



**Figure 5.7: Probability distribution of selected observation points spread over the range from 700 trials Monte Carlo simulation**

The mean value of random observation model and selected observation model are 910,693 stb, 911,925 stb and 912,987 stb respectively. The standard deviation of random observation model and selected observation model is 11,267 and 11,657 and 11,032 respectively. From those results, we can analyze that the statistical output of selected observation models are closer to Monte Carlo simulation method than the random observation model. To select the observation points, we pick the lower and upper value of each parameter or pick the data point spread over the range. Thus it may be possible to have the variance of selected observation points more than the variance of random observation points.

## 5.3 Uncertainty analysis result

From the uncertainty analysis study, the first-order approximation method require only one run at the base case values of all variables and one partial derivative for each variable (sensitivity coefficient). In this research, to evaluate the uncertainty of seven input variables, we must have seven partial derivatives respect to each input variable. But we have to note that the first-order approximation method can provide the predictive result at the mean value when all the input variables are at the base case condition. Hence the expansion term of Taylor's series will be zero and consequently the result will be equal to base case scenario. For the response surface method with second-order model (quadratic equation), the number of simulation runs are $(n+2)!/(2n!)$, where $n$ is the number of variables. Therefore, we must perform 36 different runs for the seven variables study.

The statistical result of the first-order approximation and response surfaces compare to Monte Carlo simulation method, the variance from the first-order approximation method is overestimate. The response surfaces method can give the statistical result close to Monte Carlo simulation method. Especially, if we selected iterations spread over the range, it can provide the simple quadratic equation (surface) which can provide the statistical result close to the Monte Carlo simulation method.

For the predictive purposes, we tried on generate the random input in order to obtain the average output results from the first-order approximation. Then we compare the average output results from first-order approximation with the output result from response surface method. The predictive result which obtained from response surface method, show a good potential of prediction. From this study, we

can conclude that the response surface with selected observation point spread over the range, can give the predictive result close to the base case. From both statistical result and predictive result, we can possibly get better uncertainty estimation by using response surface method with selected observation points spread over the range.

From the uncertainty analysis study, we recognized that the response surface methodology plays an important role in uncertainty estimation and prediction. Thus, the response surface will be utilized again in the next chapter for modeling and analysis of experimental design problem.

# CHAPTER VI
# EXPERIMENTAL DESIGN

This chapter describes about planning and conducting experiments design. The selection of appropriate design, the number of experimental runs and the development of surrogate equation model to be accurately fitted with the experimental response, will be discussed. The strategy of experimentation is also subsequently explained.

There are several strategies that an experimenter could use. In this study, we choose the Box and Behnken design to employ the experiment. The response surface methodology is also applied for modeling and analysis of the problem.

## 6.1 Experimental design background

Experimental design is a well-known technique to maximize the information obtained from a set of experiments (Box et al, 1978). Montgomery (1997) described the design of experiments as a method to select experiments to maximize the information gain from each experiment and to statistically evaluate the significance of the different inputs. In this study, the experiments means the material balance equation and the information obtained from a set of simulations would be the original oil in-place, its statistical values, and relationships between the original oil in-place and stochastic variables.

Experimental design works by measuring the effects that different inputs have on a process. This is done by identifying a prospective set of input variables, varying them over a series of experiments, collecting the data, and analyzing the results. Thus, experimental design is a scientific approach that allows a better understanding of a process and how the inputs affect the output.

Schmidt and Launsby (1989) noted that the engineering usages of experimental designs consist of

1) Efficient methods for gaining an understanding of the relationship between the input variables and the response (output).
2) A means of determining the setting of the input variables which optimize the response.
3) Method for building a mathematical model relating the response to the input variables.

An input variable may be varied over a range of values, for example, two extremes (two-level design) or two extremes and a base-case (three-level design). If we are interested in determining effects of several variables on a certain output, the principle procedure is to design a simple comparative experiment to determine the effect of the first variable while keeping the other variables at their base-case values. Then, the effect of the second variable is determined by a second comparative experiment, and so on. This procedure is commonly known as one-factor-at-a-time experimentation or a one-way sensitivity analysis.

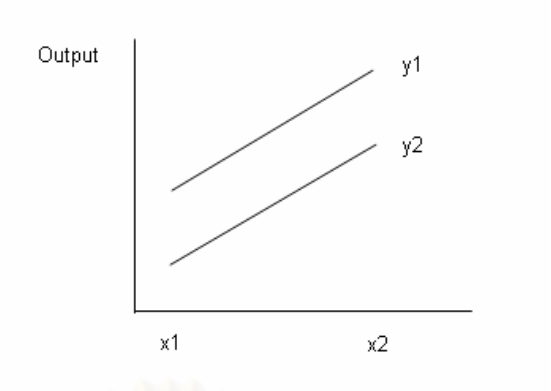
Another experimental arrangement is a factorial arrangement. A factorial design is one in which all levels of a given variable or factors are combined with all levels of every other variable or factor in the experiment (Hicks and Turner Jr., 1999). The value of factorial design is that it looks at several variables simultaneously, which allows us to estimate the various effects and interactions between variables.

An interaction occurs when two or more variables acting together have a different effect on the observed response than the effect of each variable acting individually. The strength of interactions is illustrated in figure 6.1. When the output response increases (or decreases) at the same rate as the value of input *X* changes for different levels of *Y*, i.e. parallel line, this means no interaction between variable *X* and *Y*. when output increases (or decreases) at slightly different rate as *X* varies at different levels of *Y*, there is weak interaction. And when the output changes dramatically different rates for different levels of input *Y*, i.e. the line intersect, this is called a strong interaction between *X* and *Y* inputs and it needs to be considered in the design of experiments.

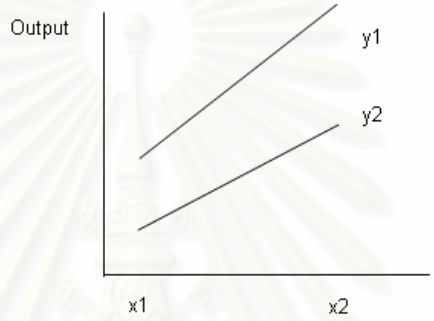
We can do also explain the interaction by considering the first-order Taylor's series expansion of two variables with the cross term included.

$$f(X,Y) = a + bX + cY + dXY \tag{6.1}$$

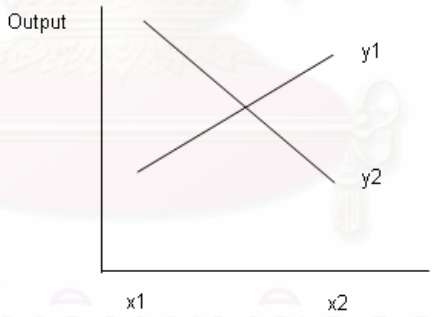
The coefficient d in the cross term indicates the strength of interaction between variables *X* and *Y*. if there is no interaction between two variables, the coefficient *d* become zero. A strong interaction means the negative value of the coefficient *d* while the positive value of *d* represents a weak interaction.




No interaction




Weak interaction




Strong interaction

**Figure 6.1: strength of interactions between input variables.**

To account for all possible main effects, i.e. effects from an individual input, and interaction effects of all variables, we must use the full factorial design. For example, if all seven variables are considered at three levels, a $3^7$ factorial experiment would require 2,187 different experiments. A disadvantage of the factorial design is the number of experiment combinations increase rapidly as the number of variables and/or levels increase. If we are to fit experiment data to a polynomial model, many of these experiments are unnecessary.

One way out of this disadvantage is to consider only a subset of all possible combinations, the so-called fractional factorial or incomplete factorial design. There are many approaches taken to designing fractional factorial experiments that can be determined through the process of assuming certain terms in the experiment are negligible and design the experiments to just estimate the terms of interest. In this study, Box-Behnken design approach will be discussed.

## 6.2 Box-Behnken designs

Box and Behnken (1960) created a series of incomplete three level factorial designs that are useful for estimating the coefficients in a second-order polynomial model. The Box-Behnken designs are constructed by combining two-level factorial designs with incomplete block designs in a particular manner, i.e. combining two-level factorial designs on some of the variables with the center points on the remaining variables. At lease one center points for all variables are added to the designs. The designs are much more efficient and frequently used than three-level full factorial designs (Schmidt and Launsby, 1989). Another advantage of the Box-Behnken design is that as additional variables are added to the model, we are able to use most experience setting from the previous design (design with fewer variables) in the new design.

We can view the Box-Behnken design as a fractional three-level factorial design with only the center point and the edge point of a hypercube being used (figure 6.2). For example, The Box-Behnken design with three variables has only 15 design point compared to 27 design points of the full factorial design with the same number of variables and levels. For three, four, or five-variable designs, two variables are chosen to be at the extreme values, and the remaining variables are fixed at their center values. Each selection of two variables produces four design points, say $\pm 1$ for the two variables and 0 for the remaining variables. Box and Behnken (1960) provided tabulated designs for up to 16 variables. Table 6.1 show the number of design points required for each number of variables. A number in parenthesis means the number of repeated designs at the center point value of variables. The repeated center points are necessary for the detailed analysis of variance.

**Table 6.1: Number of designs in Box-Behnken experimental design**

| Number of variables | Number of experiments |
|---|---|
| 3 | 12+(3) = 15 |
| 4 | 24+(3) = 27 |
| 5 | 40+(6) = 46 |
| 6 | 48+(6) = 54 |
| 7 | 56+(6) = 62 |
| 9 | 120+(10) = 130 |
| 10 | 160+(10) = 170 |
| 11 | 176+(12) = 188 |
| 12 | 192+(12) = 204 |
| 16 | 384+(12) = 396 |

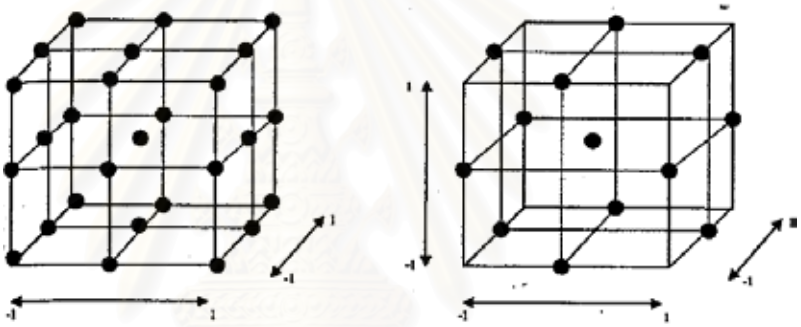


**Figure 6.2: Geometric representations of a $3^3$ full factorial design and a three variable Box-Behnken design.**

## 6.3 Application of Box-Behnken Experimental design

We use seven variables Box-Behnken design to create a set of iterations in simulation study with combine variables. The design table, with coded variables, is shown in table 6.2. The +1 represents the variable at its maximum value, the -1 is for its minimum value and the 0 means the variable is at its base-case value. The $\pm 1$ in the design table means that all combinations of minimum and maximum values are to be run. This design comprises of 62 experiments with 56 different simulation runs because there are 6 repetitions at the base-case values of variables. A set of these 62 experiments are used in the material balance EXCEL spreadsheet to calculate the statistical moments of the original oil in-place.

**Table 6.2: Box-Behnken experimental design matrix with seven variables**

| P | Pi | Np | Wp | Swc | Cf | We |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | ±1 | ±1 | ±1 | 0 |
| ±1 | 0 | 0 | 0 | 0 | ±1 | ±1 |
| 0 | ±1 | 0 | 0 | ±1 | 0 | ±1 |
| ±1 | ±1 | 0 | ±1 | 0 | 0 | 0 |
| 0 | 0 | ±1 | ±1 | 0 | 0 | ±1 |
| ±1 | 0 | ±1 | 0 | ±1 | 0 | 0 |
| 0 | ±1 | ±1 | 0 | 0 | ±1 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |

Where the expansion of the first row is

| 0 | 0 | 0 | -1 | -1 | -1 | 0 |
|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | -1 | -1 | 0 |
| 0 | 0 | 0 | -1 | 1 | -1 | 0 |
| 0 | 0 | 0 | 1 | 1 | -1 | 0 |
| 0 | 0 | 0 | -1 | -1 | 1 | 0 |
| 0 | 0 | 0 | 1 | -1 | 1 | 0 |
| 0 | 0 | 0 | -1 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 |

**Table 6.3: Summary of maximum, minimum and mean**

| Code | P | Pi | Np | Wp | Swc | Cf | We |
|---|---|---|---|---|---|---|---|
| +1 | 2,520 | 4,020 | 112,098 | 6,584 | 0.37 | 5.25E-06 | 1,840 |
| -1 | 2,480 | 3,980 | 107,702 | 5,957 | 0.33 | 4.50E-06 | 1,360 |
| 0 | 2,500 | 4,000 | 109,900 | 6,270 | 0.35 | 5.00E-06 | 1,600 |

Table 6.3 shows the actual value of each parameter which were transformed into the design matrix. Table 6.4 shows the statistical summary of the original oil in-place from the Box-Behnken design. The sample mean value of the original oil in-place is 902,110 stb compared to 912,823 stb from the 700 trials Monte Carlo simulation and the sample variance is 1,029,960,649 compared to 126,954,400 from the reference Monte Carlo simulation. Figure 6.3, shows the CDFs of original oil in-place from Box-Behnken experimental design and Monte Carlo simulation method. From the CDF, we can see that the Box-Behnken design cannot provide the trend close to the Monte Carlo simulation method. The reason is that the Box-Behnken

design matrix practically used two levels and center points to create a set of iterations. Thus there is possibility to have the extreme low-high output results.

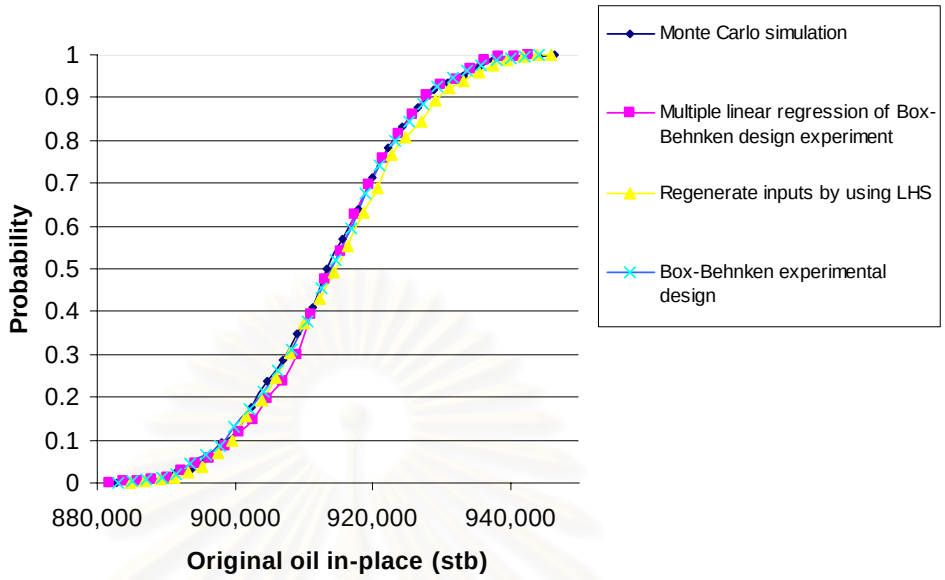



**Figure 6.3: Original oil in-place CDFs comparison of the 700 trials Monte Carlo simulation and the Box-Behnken experimental designs**

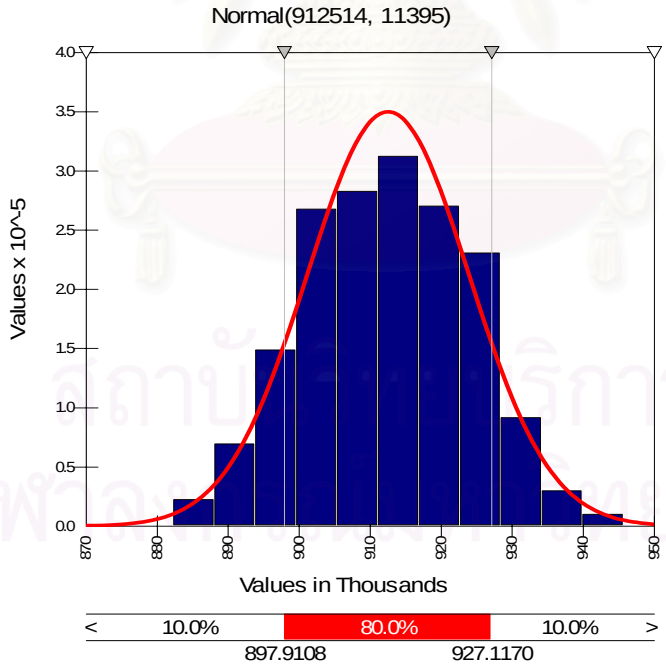



**Figure 6.4: Probability distribution of Box-Behnken experiment design from 700 trials Monte Carlo simulation**

**Table 6.4 Statistical summary of original oil in-place from Box-Behnken experimental design and 700 trials Monte Carlo simulation method**

| Statistical result | Monte Carlo simulation | Box-Behnken experimental design |
|---|---|---|
| Max | 938,311 | 943,207 |
| Min | 876,969 | 881,973 |
| Mean | 912,823 | 912,514 |
| Variance | 126,954,400 | 128,997,352 |
| SD | 11,267 | 11,395 |
| P10 | 898,173 | 897,271 |
| P25 | 903,034 | 904,871 |
| P50 | 912,990 | 912,857 |
| P75 | 918,545 | 920,295 |
| P90 | 927,471 | 927,155 |

**Table 6.5: Comparison of predictive result between base case and Box-Behnken experimental design**

| | Base-case | Box-Behnken |
|---|---|---|
| Predictive output | 911,633 | 911,633 |

$$Y = 2{,}012{,}528 + 20{,}396{,}800{,}000X_1 + 3{,}541{,}795X_2 - 1.6X_3 - 0.57X_4 - 737X_5 - 54X_6 + 0.56X_7$$
$$- 10{,}212{,}340{,}000X_1X_2 - 19{,}657X_1X_3 - 29{,}266X_1X_4 + 4{,}795{,}562X_1X_5 - 16{,}772{,}860X_1X_6$$
$$+ 31{,}139X_1X_7 - 0.2X_2X_3 - 0.3X_2X_4 + 60X_2X_5 - 206X_2X_6 + 0.3X_2X_7 - 2\times10^{-15}X_3X_4 - 0.001X_3X_5$$
$$+ 0.004X_3X_6 + 1\times10^{-15}X_3X_7 - 0.002X_4X_5 + 0.005X_4X_6 + 5\times10^{-16}X_4X_7 - 1.17X_5X_6 + 0.002X_5X_7$$
$$- 0.005X_6X_7 + (6\times10^{13})X_1^2 - 134{,}474X_2^2 - 5\times10^{-8}X_3^2 + 6\times10^{-7}X_4^2 + 0.4X_5^2 + 1.1X_6^2 - 3\times10^{-6}X_7^2 \quad (6.2)$$

From the design matrix in table 6.2, we used 62 design experiments to fit the second-order equation. Thus we get the regression coefficient as shown in equation 6.2. Then we used the Monte Carlo simulation to generate the input sampling into equation 6.2 to get the statistical output result. For the predictive output, we used the input variables at base case condition to fill in equation 6.2.

## 6.4 Response surface with experimental design

As mention in previous chapter, the concept of response surface method and its application were used to determine the uncertainty in original oil in-place.

Response surfaces are used to derive empirical functional dependencies between the response (output) and the input variables (factors) in the system. One application of the experimental design is to build a mathematical model relating the response to the inputs. Therefore, we would be able to derive a response surface using the experimental designs. The design should require as few experimental as possible and provide a good fit to the observed data as much as possible. In addition, as more variables are added into the design, the experimental points in the previously used design should be reusable in the new design (Narayanan, 1999).

The most common form of the response surface model is polynomials. For a linear model, each variable must have at least two levels and the minimum number of design points must be one more than the number of variables. To fit second-order model, there must be at least three levels of each design variable and at least *(n+2)!/(2n!)* distinct design points, where *n* is the number of design variables. For example, a three level of seven variables requires at least 36 different design settings to compute a second-order model. Myer and Montgomery (1995) provided excellent details of different experimental designs for fitting response surfaces.

In this study, we use the Box-Behnken design matrix, as discussed in section 6.2, to fit the second-order model. The design has three level of each variable and their observation points are more than the minimum requirement of 36 distinct design points for the seven variables. We randomly choose 36 different simulations from the 62 experiments from Box-Behnken design matrix and calculate the model coefficients. Unfortunately, the 36 experiments from Box-Behnken cannot be fitted to the second-order (quadratic) equation by any reason. The author surmise that the characteristic of the data and their sensitivity can cause the error when fit the equation.

$$Y = -7{,}388{,}597{,}707X_1 - 79{,}750X_2 + 2.33X_3 + 3.53X_4 - 520X_5 + 1181X_6 - 3.53X_7 - 168{,}029 \qquad (6.3)$$

We used all 62 experiments from Box-Behnken design matrix again. But this time we employed the method of least squares to determine the coefficients in equation 6.3. Then, we calculated the original oil in-place by using these derived empirical functions. The Monte Carlo simulation is also used to generate the random

sampling for each input and estimate the statistical result. The statistical results are also shown in table 6.6.

**Table 6.6 Statistical summary of original oil in-place from Box-Behnken using multiple regression (MLR) and 700 trials Monte Carlo simulation method** (MCS)

| Statistical result | MCS | Box-Behnken (MLR) |
|---|---|---|
| Max | 938,311 | 941,519 |
| Min | 876,969 | 880,669 |
| Mean | 912,823 | 912,908 |
| Variance | 126,954,400 | 116,141,356 |
| SD | 11,267 | 10,776 |
| P10 | 898,173 | 898,358 |
| P25 | 903,034 | 906,470 |
| P50 | 912,990 | 913,414 |
| P75 | 918,545 | 920,305 |
| P90 | 927,471 | 926,366 |

From table 6.6, the multiple linear regression of Box-Behnken design experiment can provide the statistical result accurately when compare to Monte Carlo simulation method.

Figure 6.5 illustrates cumulative distribution function of the original oil in-place using multiple linear regression of Box-Behnken design experiment and the reference Monte Carlo simulation. The multiple linear regression and Monte Carlo simulation produce similar CDFs and the trends are smooth. Using Latin Hypercube Sampling (LHS), it tends to have a bit overestimates of original oil in-place in probability value between 0.1 and 0.9.

From table 6.7, the predictive result obtained from multiple linear regression of Box-Behnken design experiment, show the good potential of prediction. The benefit of multiple linear regressions is the type of equation. According to the accuracy of predictive result, we can reduce the material balance equation to the simple linear equation with an accurate output result.

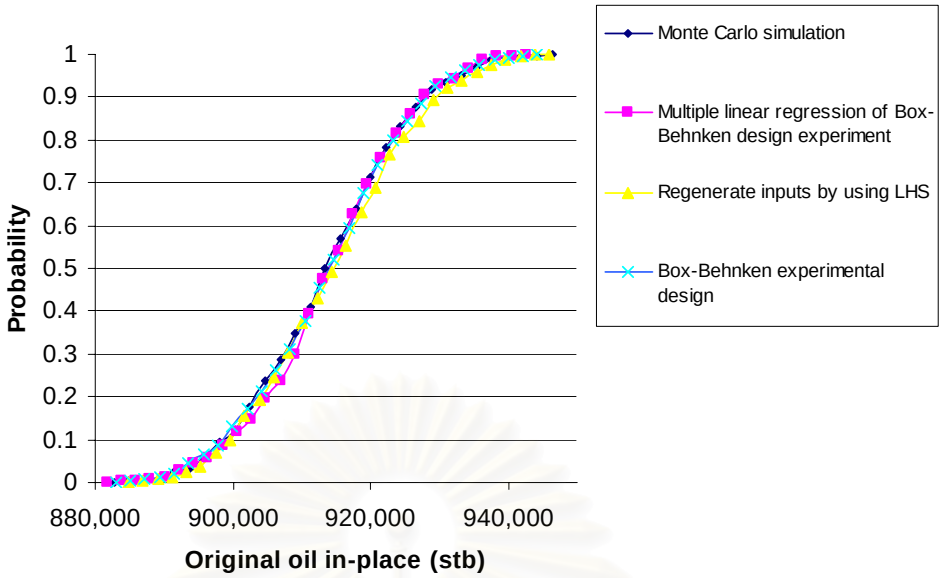



**Figure 6.5: Original oil in-place CDFs comparison of the 700 trials Monte Carlo simulation, multiple linear regression and LHS technique**

**Table 6.7: Comparison of predictive result between base case and Box-Behnken experimental design using multiple linear regression**

| | Base-case | Box-Behnken (MLR) |
|---|---|---|
| Predictive output | 911,633 | 912,161 |

In order to reproduce the new 62 sets of inputs, the author chose the Latin Hypercube Sampling method (LHS) to regenerate the new set of 62 experiments. The basic idea for Latin Hypercube Sampling is to provide a representative set of random variable without wasting iterations. Latin Hypercube Sampling partition a random variable into mutually exclusive bins before sampling. Iteration is then obtained from each bin to provide representatives samples of the random variable. Then we used the new set of 62 observation points to fit the response surface model.

As we experienced in choosing the observation points in chapter 5, the best arrangement is to spread out over the range of observation points. We also repeat that solution again by choosing the new set of 36 experiments to fit the second-order (quadratic) equation.

$$Y = 35{,}356{,}944 - 1{,}515{,}250{,}000{,}000X_1 - 49{,}251{,}278X_2 + 120X_3 + 476X_4 - 8{,}700X_5 + 47{,}520X_6 - 1{,}250X_7$$

$$-43{,}117{,}037{,}056X_1X_2 + 7{,}498{,}281X_1X_3 - 6{,}204{,}738X_1X_4 - 72{,}210{,}980X_1X_5 + 537{,}502{,}708X_1X_6$$
$$-8{,}139{,}331X_1X_7 + 171X_2X_3 + 41X_2X_4 - 509X_2X_5 + 19{,}448X_2X_6 - 89X_2X_7 - 0.006X_3X_4 - 0.006X_3X_5$$
$$-0.16X_3X_6 - 0.004X_3X_7 - 0.058X_4X_5 - 0.001X_4X_6 + 0.059X_4X_7 - X_5X_6 + 0.006X_5X_7$$
$$+0.52X_6X_7 - (3\times10^{16})X_1^{\,2} - 23{,}010{,}554X_2^2 + 0.001X_3^2 + 0.03X_4^2 + 1.5X_5^2 - 6.9X_6^2 - 0.02X_7^2 \quad (6.4)$$

From the above second-order (quadratic) equation 6.4, we used the Monte Carlo simulation method to generate the input distribution, the statistical results of original oil in-place for this response surfaces is shown in table 6.8. The predictive results of original oil in-place for the above response surfaces can be acquired by fill in the variables at base case condition. All the predictive results are also shown in table 6.9.

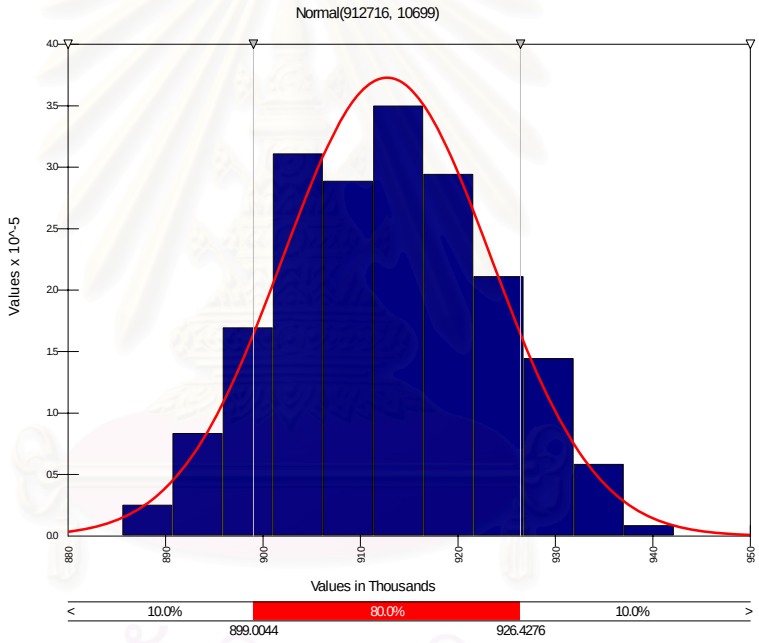



**Figure 6.6: Probability distribution of Box-Behnken using multiple linear regression from 700 trials Monte Carlo simulation**

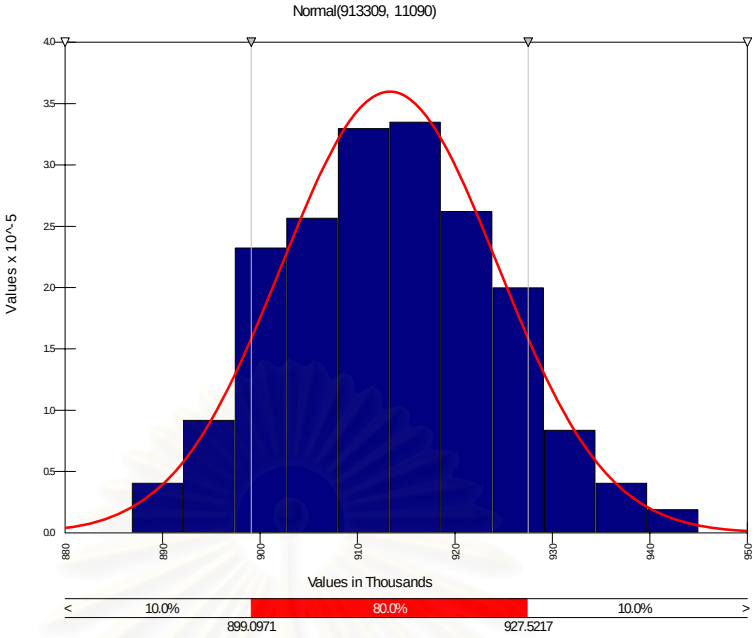



**Figure 6.7: Probability distribution of Box-Behnken using response surface with LHS technique from 700 trials Monte Carlo simulation**

**Table 6.8 Statistical summary of original oil in-place from Box-Behnken using multiple regression, 700 trials Monte Carlo simulation method and response surface using LSH technique**

| Statistical result | MCS | Box-Behnken | Box-Behnken (MLR) | RS (using LHS technique) |
|---|---|---|---|---|
| Max | 938,311 | 943,207 | 941,519 | 974,964 |
| Min | 876,969 | 881,973 | 880,669 | 889,247 |
| Mean | 912,823 | 912,565 | 912,716 | 913,309 |
| Variance | 126,954,400 | 128,997,352 | 114,468,601 | 122,993,584 |
| SD | 11,267 | 11,357 | 10,699 | 11,090 |
| P10 | 898,173 | 897,271 | 899,004 | 898,813 |
| P25 | 903,034 | 904,871 | 906,464 | 905,229 |
| P50 | 912,990 | 912,857 | 913,414 | 913,372 |
| P75 | 918,545 | 920,295 | 920,343 | 920,203 |
| P90 | 927,471 | 927,155 | 926,427 | 928,105 |

**Table 6.9: Comparison of predictive result between base case, response surfaces and first-order approximation**

| | Base-case | Box-Behnken | Box-Behnken (MLR) | RS (using LHS technique) |
|---|---|---|---|---|
| Predictive output | 911,633 | 911,633 | 912,161 | 912,341 |

Table 6.8 provides statistical summary of the original oil in-place using the multiple linear regression of Box-Behnken design experiment and response surface with LHS technique. The sample mean values of the original oil in-place, using the multiple linear regression of Box-Behnken design experiment and response surface with LHS technique, are 912,716 stb and 913,309 stb, respectively, compare to 912,823 stb from 700 trials Monte Carlo simulation. The sample variances, with the multiple linear regression of Box-Behnken design experiment and response surface with LHS technique, are 114,468,601 and 122,993,584, respectively, compare to 126,954,400 from 700 trials Monte Carlo simulation. From figure 6.5, the original oil in-place using the response surface with LHS technique produce the trend line close to the Monte Carlo simulation. Thus we can conclude that the entire range of random samplings generated by LHS technique, will provide the output result close to the Monte Carlo simulation with sufficient number of iterations.

## 6.5 Experimental design result

From the experimental design study, the Box-Behnken design method requires only 62 experiments at the various levels of each variable. In this research, to evaluate the uncertainty of seven input variables, we tried on using response surface, multiple linear regression, LHS technique and Monte Carlo simulation to assist and acquire the best solution. Firstly, we used 62 experiments from the design matrix to fit the second-order equation. We also tried to reduce the number of runs to 36 experiments to fit the second-order equation. Unfortunately, the 36 experiments from Box-Behnken cannot be fitted to the second-order equation. The author surmise that the characteristic of the data and their sensitivity can cause the error when fit the equation. However, we also tried on using the multiple linear regression. For this time, we intended to use all experiments (62 experiments) in order to have a best fit on the multiple linear regression. Another attempt, we tried on using the LHS to regenerate the inputs. Thus the new sets of inputs are forced to spread out over their range. And then we brought only 36 experiments for fitting the second-order equation.

The statistical result show that the Box-Behnken experimental design using multiple linear regression and quadratic equation provided the overall statistical result close to Monte Carlo simulation. But the variance from multiple linear regression is underestimate.

For the predictive purposes, we summarized the output results from all techniques. The predictive result obtained from those techniques i.e. response surface with second-order equation, multiple linear regression and response surface with LHS technique, show a good potential of accurate prediction. From this study, we can conclude that the Box-Behnken experimental design using second-order equation can give the predictive result closest to the base case.

# CHAPTER VII
# DISCUSSION AND CONCLUSIONS

Several procedures that quantitatively estimate the uncertainty in original oil in-place prediction have been presented. A tradition Monte Carlo simulation is demonstrated in the study of original oil in-place where the principle uncertainties are the reservoir pressure. The Monte Carlo Simulation (MCS) is a typical choice in relating input-output uncertainty in reservoir engineering problems for several reasons. Firstly, the Monte Carlo approach is conceptually simple, easy to explain and widely used. Secondly, it allows a full range of each uncertainty input to be used in generating a probability distribution of the output and the precision of the estimates is independent of the number of inputs. However, the Monte Carlo results are often accepted to be used as the means to verify or validate approximate analytical methods or any other methods as presented in this study.

For the Monte Carlo simulation, the number of simulation runs depends on the relative accuracy required of the output distribution for a given degree of uncertainty. In this study, the number of runs has been investigated as demonstrated in chapter 4, the mean value and variance of outputs become stabilized after 540 trials. In this matter, it can be ensured that the number of realization using in Monte Carlo Simulation technique to assess the uncertainty are adequate in accuracy requirement for this study.

For the first-order analysis, the minimum runs are only one run at the base case values of all variables and one partial derivative for each variable (sensitivity coefficient). In this research, to evaluate the uncertainty of seven input variables, we must have seven partial derivatives with respect to each input variable. But we have to note that the first-order approximation method can provide the predictive result at the mean value when all the input variables are at the base case condition. Hence the expansion term of Taylor's series will be zero and consequently the result will be equal to the base case scenario. For the response surface method (RS) with second-order model (quadratic equation), the number of simulation runs are $(n+2)!/(2n!)$, where n is the number of variables. Therefore, we must perform 36 different runs for the seven variables study.

From the experimental design study, the Box-Behnken design method requires 62 experiments at the various levels of all variables. In this research, to evaluate the uncertainty of seven input variables, we tried on using response surface, multiple linear regression, Latin Hypercube Sampling and Monte Carlo simulation to assist and acquire the best solution. Firstly, we used 62 experiments from the design matrix to fit the second-order equation. We also tried to reduce the number of runs to 36 experiments to fit the second-order equation. Unfortunately, the 36 experiments from Box-Behnken cannot fit to the second-order (quadratic) equation by any reason. The author surmise that the characteristic of the data and their sensitivity can cause the error when fit the equation. However, we also tried on using the multiple linear regression. We intend to use all iterations (62 experiments) in order to have a best fit on the multiple linear regression. Another attempt, we tried on using the Latin Hypercube Sampling to regenerate the inputs. Thus the new inputs are forced to spread out over their ranges. Then we brought only 36 experiments for fitting the second-order equation.

The result of Monte Carlo simulation shows that the mean value of original oil in-place becomes stabilized after 540 trials. The average value of original oil in-place of all 540 trials and more is around 910,920 – 911,479 stb. The variance of original oil in-place (referred as the uncertainty in the original oil in-place) also shows the same stabilization. The variance of original oil in-place of all 540 trials and more is around 127,007,420 – 131,583,593 hence, the total number of 700 trials used as the reference statistic should be sufficient. This also confirms the requirement of large number of iteration using Monte Carlo Simulation technique to assess the uncertainty for this study.

The statistical results of the first-order approximation and response surfaces compare to Monte Carlo simulation method, the variance from the first-order approximation method is overestimate. The response surfaces method can give the statistical result close to Monte Carlo simulation method. Especially, if we selected trials spread over the range, it can provide the simple quadratic equation (surface).

From the statistical result of the multiple linear and response surface using Latin Hypercube Sampling technique, the variance from response surface using Latin Hypercube Sampling technique is also overestimate. The multiple linear regression of Box-Behnken design experiment can give the statistical result close to Monte Carlo simulation method.

From the author's perception, the criteria used to consider the efficiency of each method are the number of run, the accuracy of mean value, the scale or degree of being spread (variance).

From table 7.1, the response surface with scattered observation points can also provide the accurate mean value with the overestimate in variance. The Box-Behnken experimental design fitted by multiple linear regression can provide the accurate mean value and underestimate the variance. Thus we can conclude that the most reasonable accurate approach is Box-Behnken experimental design fitted by second-order (quadratic) equation. Table 7.2 demonstrates the percent deviation of each method compare to the reference Monte Carlo Simulation.

**Table 7.1 Comparison of the statistical results of original oil in-place**

| Statistics | MCS | RS random | RS (edge) | RS (scattered) | First-order | Box-Behnken (quadratic) | Box-Behnken (MLR) | RS (LHS technique) |
|---|---|---|---|---|---|---|---|---|
| Max | 938,311 | 938,311 | 945,147 | 940,834 | 956,179 | 943,207 | 941,519 | 974,964 |
| Min | 876,969 | 876,968 | 880,589 | 878,930 | 885,603 | 881,973 | 880,669 | 889,247 |
| Mean | 912,823 | 910,693 | 911,925 | 912,704 | 919,177 | 912,514 | 912,716 | 913,309 |
| Variance | 126,954,400 | 126,954,400 | 135,887,300 | 138,227,040 | 552,332,187 | 128,997,352 | 114,468,601 | 122,993,584 |
| SD | 11,267 | 11,267 | 11,657 | 11,757 | 23,501 | 11,395 | 10,699 | 11,090 |
| P10 | 898,173 | 895,926 | 897,168 | 896,489 | 886,650 | 897,271 | 899,004 | 898,813 |
| P25 | 903,034 | 898,205 | 899,571 | 904,048 | 908,132 | 904,871 | 906,464 | 905,229 |
| P50 | 912,990 | 910,534 | 901,800 | 912,556 | 914,116 | 912,857 | 913,414 | 913,372 |
| P75 | 918,545 | 900,660 | 901,478 | 920,860 | 915,283 | 920,295 | 920,343 | 920,203 |
| P90 | 927,471 | 903,033 | 903,316 | 928,190 | 915,337 | 927,155 | 926,427 | 928,105 |
| Runs | - | 36 | 36 | 36 | 36 | 62 | 62 | 36 |

**Table 7.2 percent deviation of each method compare to the reference Monte Carlo Simulation**

| Statistics | RS random | RS (edge) | RS (scattered) | First-order | Box-Behnken (quadratic) | Box-Behnken (MLR) | RS (LHS technique) |
|---|---|---|---|---|---|---|---|
| Mean | 0.2 | 0.1 | 0.0 | 0.7 | 0.0 | 0.0 | 0.1 |
| Variance | 0.0 | 7.0 | 8.9 | 335.1 | 1.6 | 9.8 | 3.1 |
| SD | 0.0 | 3.5 | 4.3 | 108.6 | 1.1 | 5.0 | 1.6 |
| P10 | 0.3 | 0.1 | 0.2 | 1.3 | 0.1 | 0.1 | 0.1 |
| P25 | 0.5 | 0.4 | 0.1 | 0.6 | 0.2 | 0.4 | 0.2 |
| P50 | 0.3 | 1.2 | 0.0 | 0.1 | 0.0 | 0.0 | 0.0 |
| P75 | 1.9 | 1.9 | 0.3 | 0.4 | 0.2 | 0.2 | 0.2 |
| P90 | 2.6 | 2.6 | 0.1 | 1.3 | 0.0 | 0.1 | 0.1 |

For the predictive purposes, table 7.3 illustrates that the predictive result obtained from response surface method shows a good potential of accurate prediction.

From this study, the response surface with selected observation point spread over the range can give the predictive result close to the base case. From both statistical result and predictive result, we can possibly get accurate uncertainty estimation using Box-Behnken experimental design with response surface.

Regarding the output results from multiple linear regression and response surface from the design experiment, the predictive result obtained from both techniques show a good potential of accurate prediction. From this study, we can conclude that the Box-Behnken experimental design with response surface technique can provide the closest predictive result to the base case. From both statistical result and predictive result, we recommended that the Box-Behnken experimental design with response surface can possibly provide accurate uncertainty estimation.

**Table 7.3 Comparison of the predictive results of original oil in-place**

| | Base case | RS random | RS (edge) | RS (scattered) | First-order | Box-Behnken (quadratic) | Box-Behnken (MLR) | RS (using LHS technique) |
|---|---|---|---|---|---|---|---|---|
| Predictive output | 911,633 | 909,778 | 910,171 | 911,556 | 919,177 | 911,633 | 912,161 | 912,341 |

Another attempt has been performed to test the effect on the output when there are large variations in the inputs. The new set of PVT data has been used to verify the original oil in-place together with the same uncertainty analysis procedure. For this time, we use the possible range of pressure uncertainty larger than the previous test. The possible range of pressure uncertainty has changed from $\pm 10$ psi to $\pm 30$ psi. The other parameters such as connate water saturation, water influx term and formation compressibility factor have also been changed to have a wider range of uncertainty. On the other hand, we reduce the range of uncertainty in some parameters, i.e. cumulative oil production and cumulative water production in order to be consistent with the field operation.

The same procedure has been used to estimate the uncertainty. But this time, we choose only some methods which show a good potential of accurate prediction, i.e. response surface with scattered observation points, first-order approximation, Box-Behnken experimental design with quadratic surface equation and response surface with Latin Hypercube Sampling technique. For this test, the base case is calculated from the median values of each input parameter.

Table 7.4 demonstrates the statistical output from each method. The result obtained from Box-Behnken experimental design is nearest to the Monte Carlo simulation method. This is to confirm that the Box-Behnken shows a good potential of statistical output estimation. It should be noted that the predictive result of Box-Behnken deviated from the base case value in this set of input parameters because most of the Box-Behnken's experiments are concentrated on the mean value thus the result obtained from those experiments is possibly close to base case which calculated from mean value instead of median value. In addition, the predictive output from the first-order approximation method provides the original oil in-place calculation close to the base case.

**Table 7.4 Comparison of the statistical results of original oil in-place for new PVT data**

| Statistics | MCS | RS (scattered) | First-order | Box-Behnken (quadratic) | RS (LHS technique) |
|---|---|---|---|---|---|
| Max | 1,463,736 | 1,579,981 | 1,466,519 | 1,470,123 | 1,468,510 |
| Min | 1,147,076 | 1,054,765 | 1,318,672 | 1,137,664 | 1,266,122 |
| Mean | 1,291,891 | 1,320,869 | 1,392,833 | 1,300,895 | 1,367,164 |
| Variance | 3,303,181,465 | 6,685,473,334 | 1,906,848,625 | 2,987,005,021 | 1,432,577,164 |
| SD | 57,473 | 81,764 | 43,667 | 54,653 | 37,849 |
| P10 | 1,216,083 | 1,213,533 | 1,350,634 | 1,231,480 | 1,316,759 |
| P25 | 1,250,475 | 1,264,090 | 1,363,689 | 1,265,061 | 1,340,398 |
| P50 | 1,290,026 | 1,327,379 | 1,395,092 | 1,300,103 | 1,367,905 |
| P75 | 1,332,430 | 1,373,642 | 1,417,206 | 1,333,641 | 1,392,838 |
| P90 | 1,369,613 | 1,425,448 | 1,446,082 | 1,371,417 | 1,416,756 |
| Runs | - | 36 | 36 | 62 | 36 |

**Table 7.5 percent deviation of each method compare to the reference Monte Carlo Simulation for new PVT data**

| Statistics | RS (scattered) | First-order | Box-Behnken (quadratic) | RS (LHS technique) |
|---|---|---|---|---|
| Mean | 2.2 | 7.8 | 0.7 | 5.8 |
| Variance | 102.4 | 42.3 | 9.6 | 56.6 |
| SD | 42.3 | 24.0 | 4.9 | 34.1 |
| P10 | 0.2 | 11.1 | 1.3 | 8.3 |
| P25 | 1.1 | 9.1 | 1.2 | 7.2 |
| P50 | 2.9 | 8.1 | 0.8 | 6.0 |
| P75 | 3.1 | 6.4 | 0.1 | 4.5 |
| P90 | 4.1 | 5.6 | 0.1 | 3.4 |

**Table 7.6 Comparison of the predictive results of original oil in-place for new PVT data**

| | Base case | RS (scattered) | First-order | Box-Behnken (quadratic) | RS (using LHS technique) |
|---|---|---|---|---|---|
| Predictive output | 1,396,127 | 1,343,872 | 1,392,833 | 1,299,443 | 1,372,161 |

From the above discussion, the conclusions of this study are following;

1) The reservoir pressure and initial reservoir pressure are an important factor in determination of the original oil in-place of oil reservoir under natural water drive mechanism.
2) There are several uncertainty estimation methods which can be the alternative methods to Monte Carlo simulation and quantitatively estimate uncertainty in original oil in-place with comparable results.
3) The response surface method can be utilized in several manners depend on the pattern of sample arrangement. In order to obtain both statistical and predictive result at optimum accuracy, the Box-Behnken experimental design with response surface method is recommended for fitting the surrogate equation.
4) The experimental designs, especially the Box-Behnken design together with the response surface method, offer good potential to reduce computational efforts in estimating uncertainty in material balance equation while maintaining accuracy comparable to Monte Carlo simulation approach. But it should be noted that the result will be deviated from the base case when the base case is not calculated from the mean value of each parameter.
5) The response surface method provides the advantage of predictive equation since the complex equation can be transformed to the convenience one.

The following are the recommendations for future work. More research is recommended to test these techniques in other drive mechanism or combine drive mechanism reservoir simulation study. The material balance equation is needed to expand to have more complexity and number of uncertainty variables. The detail of each parameter is needed more research i.e. the parameter regarding water influx, gas cap and the time dependent parameter.

Moreover, the author recommends more research to study on other patterns of experimental design method and higher order of surrogate equation for estimating uncertainty in the reservoir engineering study.

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# APPENDICES

# APPENDIX A: Sensitivity coefficients

The sensitivity coefficients (partial derivatives) for each variable are derived as follow:

From the general expression of material balance equation as

$$N_p\left[B_o+\left(R_p-R_s\right)B_g\right]=NB_{oi}\left[\frac{\left(B_o-B_{oi}\right)+\left(R_{si}-R_s\right)B_g}{B_{oi}}+m\left(B_g/B_{gi}-1\right)+\left(1+m\right)\frac{\left(c_wS_w+c_f\right)}{\left(1-S_w\right)}\Delta p\right]+\left(W_e-W_p\right)B_w \tag{3.13}$$

Rearranging, the equation become

$$N=\frac{\left(N_p\left(B_o+\left(R_p-R_s\right)B_g\right)-\left(W_e-W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o-B_{oi}\right)+\left(R_{si}-R_s\right)B_g}{B_{oi}}+\left(\frac{c_wS_{wc}+c_f}{1-S_{wc}}\right)\left(p_i-p\right)\right]} \tag{A.1}$$

To assign $F1$ and $F2$

$$F1=\frac{\left(N_p\left(B_o+\left(R_p-R_s\right)B_g\right)\right)}{B_{oi}\left[\frac{\left(B_o-B_{oi}\right)+\left(R_{si}-R_s\right)B_g}{B_{oi}}+\left(\frac{c_wS_{wc}+c_f}{1-S_{wc}}\right)\left(p_i-p\right)\right]} \tag{A.3}$$

$$F2=\frac{\left(\left(W_e-W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o-B_{oi}\right)+\left(R_{si}-R_s\right)B_g}{B_{oi}}+\left(\frac{c_wS_{wc}+c_f}{1-S_{wc}}\right)\left(p_i-p\right)\right]} \tag{A.4}$$

Then

$$N=F1-F2 \tag{A.5}$$

Where the nomenclature of each parameters are as follow:

| | |
|---|---|
| $R_s$ | solution gas oil ratio (scf/stb) |
| $R_{si}$ | initial solution gas oil ratio (scf/stb) |
| $c_f$ | compressibility factor ($psi^{-1}$) |
| $c_o$ | oil compressibility factor ($psi^{-1}$) |
| $p_i$ | initial reservoir pressure (psi) |
| $p$ | reservoir pressure (psi) |
| $S_{wc}$ | connate water saturation |
| $W_p$ | cumulative produced water (stb) |
| $B_o$ | oil formation volume factor (rb/stb) |
| $B_{oi}$ | initial oil formation volume factor (rb/stb) |

$B_g$ gas formation volume factor (rb/scf)

$R_p$ cumulative gas oil ration (scf/stb)

$N_p$ cumulative oil production (stb)

$W_e$ Water influx term (MMstb)

The derivative of each parameter is shown as follow:

1. Water influx term

From general expression of material balance equation as

$$N = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.1)}$$

$$\frac{\partial N}{\partial W_e} = \frac{\partial F1}{\partial W_e} - \frac{\partial F2}{\partial W_e} \quad \text{(A.6)}$$

$$F1 = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right)\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.3)}$$

$$F2 = \frac{\left(\left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.4)}$$

From constant rule $$\frac{\partial F1}{\partial W_e} = 0 \quad \text{(A.7)}$$

From power rule

$$\frac{\partial F2}{\partial W_e} = \frac{B_w}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.8)}$$

$$\frac{\partial N}{\partial W_e} = \frac{-B_w}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.9)}$$

2. Connate water saturation

From general expression of material balance equation as

$$N = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.1)}$$

Rearranging, the equation become

$$N = \frac{\left[\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)\right]\left(1 - S_{wc}\right)}{B_{oi}\left[\frac{\left(\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g\right)\left(1 - S_{wc}\right)}{B_{oi}} + \left(c_w S_{wc} + c_f\right)\left(p_i - p\right)\right]} \quad \text{(A.10)}$$

From (A.10), we assign

$$A = \left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)\left(1 - S_{wc}\right) \quad \text{(A.11)}$$

$$B = \left(\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g\right)\left(1 - S_{wc}\right) + \left(c_w S_{wc} + c_f\right)B_{oi}\left(p_i - p\right)\right) \quad \text{(A.12)}$$

From quotient rule $$\frac{\partial N}{\partial S_{wc}} = \frac{\left(B\frac{\partial A}{\partial S_{wc}}\right) - \left(A\frac{\partial B}{\partial S_{wc}}\right)}{B^2} \quad \text{(A.13)}$$

From constant rule $$\frac{\partial A}{\partial S_{wc}} = -\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right) \quad \text{(A.14)}$$

From constant rule $$\frac{\partial B}{\partial S_{wc}} = -\left(\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g\right) + \left(c_w B_{oi}\left(p_i - p\right)\right) \quad \text{(A.15)}$$

Then,

$$\frac{\partial N}{\partial S_{wc}} = \frac{\left(\left[\left(\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g\right)\left(1 - S_{wc}\right)\right] + \left(c_w S_{wc} + c_f\right)\left(p_i - p\right)B_{oi}\right)\frac{\partial A}{\partial S_{wc}}}{\left[\left(\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g\right)\left(1 - S_{wc}\right) + \left(c_w S_{wc} + c_f\right)\left(p_i - p\right)B_{oi}\right]^2}$$
$$\frac{-\left(\left[\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)\left(1 - S_{wc}\right)\right]\frac{\partial B}{\partial S_{wc}}\right)}{\left[\left(\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g\right)\left(1 - S_{wc}\right) + \left(c_w S_{wc} + c_f\right)\left(p_i - p\right)B_{oi}\right]^2} \quad \text{(A.16)}$$

3. Formation compressibility

From general expression of material balance equation as

$$N = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.1)}$$

Rearranging, the equation become

$$N = \frac{\left[\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)\right]\left(1 - S_{wc}\right)}{B_{oi}\left[\frac{\left(\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g\right)\left(1 - S_{wc}\right)}{B_{oi}} + \left(c_w S_{wc} + c_f\right)\left(p_i - p\right)\right]} \quad \text{(A.10)}$$

From (A.10), we assign

$$A = \left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)\left(1 - S_{wc}\right) \quad \text{(A.11)}$$

$$B = \left(\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g\right)\left(1 - S_{wc}\right) + \left(c_w S_{wc} + c_f\right)B_{oi}\left(p_i - p\right)\right) \quad \text{(A.12)}$$

From quotient rule $$\frac{\partial N}{\partial c_f} = \frac{\left(B\frac{\partial A}{\partial c_f}\right) - \left(A\frac{\partial B}{\partial c_f}\right)}{B^2} \quad \text{(A.17)}$$

From constant rule $$\frac{\partial A}{\partial c_f} = 0 \quad \text{(A.18)}$$

From constant rule $$\frac{\partial B}{\partial c_f} = \left(B_{oi}\left(p_i - p\right)\right) \quad \text{(A.19)}$$

$$\frac{\partial N}{\partial c_f} = \frac{\left[\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)\left(1 - S_{wc}\right)\right]\left(B_{oi}\left(p_i - p\right)\right)}{\left[\left(\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g\right)\left(1 - S_{wc}\right) + \left(c_w S_{wc} + c_f\right)\left(p_i - p\right)B_{oi}\right]^2} \quad \text{(A.20)}$$

4. Cumulative oil production

From general expression of material balance equation as

$$N = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.1)}$$

$$\frac{\partial N}{\partial N_p} = \frac{\partial F1}{\partial N_p} - \frac{\partial F2}{\partial N_p} \quad \text{(A.21)}$$

$$F1 = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right)\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.3)}$$

$$F2 = \frac{\left(\left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.4)}$$

From power rule $$\frac{\partial F1}{\partial N_p} = \frac{\left(\left(B_o + \left(R_p - R_s\right)B_g\right)\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.22)}$$

From constant rule $$\frac{\partial F2}{\partial N_p} = 0 \quad \text{(A.23)}$$

$$\frac{\partial N}{\partial N_p} = \frac{\left(\left(B_o + \left(R_p - R_s\right)B_g\right)\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.24)}$$

5. Cumulative water production

From general expression of material balance equation as

$$N = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.1)}$$

$$\frac{\partial N}{\partial W_p} = \frac{\partial F1}{\partial W_p} - \frac{\partial F2}{\partial W_p} \tag{A.25}$$

$$F1 = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right)\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \tag{A.3}$$

$$F2 = \frac{\left(\left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \tag{A.4}$$

From constant rule $$\frac{\partial F1}{\partial W_p} = 0 \tag{A.26}$$

From power rule $$\frac{\partial F2}{\partial W_p} = \frac{-B_w}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \tag{A.27}$$

$$\frac{\partial N}{\partial W_p} = \frac{B_w}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \tag{A.28}$$

6. Initial reservoir pressure

From general expression of material balance equation as

$$N = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \tag{A.1}$$

$$\frac{\partial N}{\partial p_i} = \frac{\partial F1}{\partial p_i} - \frac{\partial F2}{\partial p_i} \tag{A.29}$$

$$F1 = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right)\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \tag{A.3}$$

$$F2 = \frac{\left(\left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \tag{A.4}$$

From Standing's correlation

$$R_{si} = \gamma_g\left(\frac{p_i}{18} + 1.4\right)10^E \tag{A.30}$$

and $$E = \left(0.0125API + 0.00091T\right) \tag{A.31}$$

Substitute API = 42.3 and T = 212 F in (A.31)

Then $E = 0.72$

Substitute $E = 0.72$ and $\gamma_g = 0.75$ (A.30)

Then $$R_{si} = 0.21p_i + 5.51 \tag{A.32}$$

From Standing's correlation

$$B_{oi} = 0.97 + 12\times10^{-5}\left(1.1R_{si} + 265\right)^{1.2} \qquad \text{(A.33)}$$

Substitute (A.32) in Equation (A.33)

Then $$B_{oi} = 0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2} \qquad \text{(A.34)}$$

For the common term in (A.3), we assign

$$U = \left[\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right] \qquad \text{(A.35)}$$

Substitute (A.32), (A.34) and (A.35) into (A.3) the equation become

$$F1 = \frac{N_p B_o + N_p R_p B_g - N_p R_s B_g}{B_o - \left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right) + \left(0.21p_i + 5.51\right)B_g - R_s B_g + U\left(p_i - p\right)\left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)} \qquad \text{(A.36)}$$

From (A.36), we assign

$$A_1 = N_p B_o + N_p R_p B_g - N_p R_s B_g \qquad \text{(A.37)}$$

and

$$B_1 = B_o - \left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right) + \left(0.21p_i + 5.51\right)B_g - R_s B_g + U\left(p_i - p\right)\left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right) \qquad \text{(A.38)}$$

Partial derivative of (A.37) and (A.38)

From constant rule $$\frac{\partial A_1}{\partial p_i} = 0 \qquad \text{(A.39)}$$

From power rule

$$\frac{\partial B_1}{\partial p_i} = \left(-3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + 0.21B_g + 0.97U + 12\times10^{-5}U\left[\left(p_i \times 3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + \left(12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)\right] \qquad \text{(A.40)}$$

From quotient rule $$\frac{\partial F1}{\partial p_i} = \frac{B_1\frac{\partial A_1}{\partial p_i} - A\frac{\partial B_1}{\partial p_i}}{B_1^{\,2}} \qquad \text{(A.41)}$$

$$\frac{\partial F1}{\partial p_i} = \frac{-A_1\left[\left(-3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + 0.21B_g + 0.97U + 12\times10^{-5}U\left[\left(p_i \times 3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + \left(12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)\right]\right]}{\left[B_o - \left(0.97 + 12\times10^{-5}\left(0.16p_i + 140\right)^{1.2}\right) + \left(0.18p_i + 3.89\right)B_g - R_s B_g + U\left(p_i - p\right)\left(0.97 + 12\times10^{-5}\left(0.16p_i + 140\right)^{1.2}\right)\right]^2} \qquad \text{(A.42)}$$

$$F2 = \frac{\left(\left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \qquad \text{(A.4)}$$

Substitute (A.32), (A.34) and (A.35) into (A.4) the equation become

$$F2 = \frac{W_e B_w - W_p B_w}{B_o - \left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right) + \left(0.21p_i + 5.51\right)B_g - R_s B_g + U\left(p_i - p\right)\left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)} \tag{A.43}$$

From (A.41), we assign

$$A_2 = W_e B_w - W_p B_w \tag{A.44}$$

and

$$B_2 = B_o - \left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right) + \left(0.21p_i + 5.51\right)B_g - R_s B_g + U\left(p_i - p\right)\left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right) \tag{A.45}$$

Partial derivative of (A.44) and (A.45)

From constant rule $$\frac{\partial A_2}{\partial p_i} = 0 \tag{A.46}$$

From power rule

$$\frac{\partial B_2}{\partial p_i} = \left(-3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + 0.21B_g + 0.97U + 12\times10^{-5}U\left[\left(p_i \times 3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + \left(12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)\right] \tag{A.47}$$

From quotient rule $$\frac{\partial F2}{\partial p_i} = \frac{B_2 \frac{\partial A_2}{\partial p_i} - A_2 \frac{\partial B_2}{\partial p_i}}{{B_2}^2} \tag{A.48}$$

$$\frac{\partial F2}{\partial p_i} = \frac{-A_2\left[\left(-3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + 0.21B_g + 0.97U + 12\times10^{-5}U\left[\left(p_i \times 3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + \left(12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)\right]\right]}{\left[B_o - \left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right) + \left(0.21p_i + 5.51\right)B_g - R_s B_g + U\left(p_i - p\right)\left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)\right]^2} \tag{A.49}$$

$$\frac{\partial N}{\partial p_i} = \frac{-A_1\left[\left(-3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + 0.21B_g + 0.97U + 12\times10^{-5}U\left[\left(p_i \times 3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + \left(12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)\right]\right]}{\left[B_o - \left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right) + \left(0.21p_i + 5.51\right)B_g - R_s B_g + U\left(p_i - p\right)\left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)\right]^2}$$

$$\frac{+A_2\left[\left(-3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + 0.21B_g + 0.97U + 12\times10^{-5}U\left[\left(p_i \times 3\times10^{-5}\left(0.23p_i + 271\right)^{0.2}\right) + \left(12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)\right]\right]}{\left[B_o - \left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right) + \left(0.21p_i + 5.51\right)B_g - R_s B_g + U\left(p_i - p\right)\left(0.97 + 12\times10^{-5}\left(0.23p_i + 271\right)^{1.2}\right)\right]^2} \tag{A.50}$$

7. Reservoir pressure

From general expression of material balance equation as

$$N = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right) - \left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \tag{A.1}$$

$$\frac{\partial N}{\partial p} = \frac{\partial F1}{\partial p} - \frac{\partial F2}{\partial p} \tag{A.51}$$

$$F1 = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right)\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.3)}$$

$$F2 = \frac{\left(\left(W_e - W_p\right)B_w\right)}{B_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + \left(\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right)\left(p_i - p\right)\right]} \quad \text{(A.4)}$$

From Standing's correlation

$$R_s = \gamma_g\left[\left(\frac{p}{18} + 1.4\right)10^E\right] \quad \text{(A.30)}$$

and $$E = \left(0.0125API + 0.00091T\right) \quad \text{(A.31)}$$

Substitute API = 42.3 and T = 212 F @25$^{th}$ month condition in (A.31)

Then $E = 0.72$

Substitute $E = 0.72$ and $\gamma_g = 0.75$ @25$^{th}$ month condition in (A.30)

Then $$R_s = 0.21p + 5.51 \quad \text{(A.52)}$$

From Standing's correlation

$$B_o = 0.97 + 12\times10^{-5}\left(1.1R_s + 265\right)^{1.2} \quad \text{(A.53)}$$

Substitute (A.52) in Equation (A.53)

Then $$B_o = 0.97 + 12\times10^{-5}\left(0.23p + 271\right)^{1.2} \quad \text{(A.54)}$$

From Standing's correlation

$$B_g = \frac{0.00502Z\left(T + 460\right)}{p} \quad \text{(A.55)}$$

Substitute $T$ = 212 F @25$^{th}$ month condition in (A.55)

$$B_g = \frac{3.37Z}{p} \quad \text{(A.56)}$$

From figure 4.8 compressibility factor as function of pressure, substitute in (A.56)

Then $$B_g = 6\times10^{-8}\,p - 3\times10^{-4} + \frac{3.28}{p} \quad \text{(A.57)}$$

For the common term in (A.3), we assign

$$U = \left[\frac{c_w S_{wc} + c_f}{1 - S_{wc}}\right] \quad \text{(A.35)}$$

Substitute (A.52), (A.54), (A.57) and (A.35) into (A.3) the equation become

$$F1 = \frac{\left(N_p\left(B_o + \left(R_p - R_s\right)B_g\right)\right)}{\left[\left(0.97 + 12\times10^{-5}\left(0.23p + 271\right) - B_{oi}\right) + \left(R_{si} - R_s\right)B_g + C\left(p_i - p\right)B_{oi}\right]} \quad \text{(A.58)}$$

From (A.58), we assign

$$A_1 = 12\times10^{-5}(0.23p+271)^{1.2} + 6.74\times10^{-8}\,pR_pN_p - 3.37\times10^{-4}R_pN_p + \frac{3.28R_pN_p}{p} - 1.2\times10^{-9}\,p^2N_p + 6\times10^{-5}\,pN_p + 0.38N_p + \frac{12.75N_p}{p} \quad \text{(A.59)}$$

$$B_1 = 0.97 + 12\times10^{-5}(0.23p+271)^{1.2} - B_{oi} + 6\times10^{-8}\,pR_{si} - 3\times10^{-4}R_{si} + \frac{3.28R_{si}}{p} - 10^{-8}\,p^2 + 5\times10^{-5}\,p - 0.58 + \frac{12.75}{p} \quad \text{(A.60)}$$

Partial derivative of (A.59) and (A.60)

From power rule

$$\frac{\partial A_1}{\partial p} = 3\times10^{-5}(0.23p+271)^{0.2} + 6.74\times10^{-8}R_pN_p - \frac{3.28R_pN_p}{p^2} - 2.4\times10^{-9}\,pN_p + 6\times10^{-5}N_p - \frac{12.75N_p}{p^2} \quad \text{(A.61)}$$

From power rule

$$\frac{\partial B_1}{\partial p} = 3\times10^{-5}(0.23p+271)^{0.2} + 6\times10^{-8}R_{si} - \frac{3.28R_{si}}{p^2} - 2\times10^{-8}\,p + 5\times10^{-5} - \frac{12.75}{p^2} \quad \text{(A.62)}$$

From quotient rule

$$\frac{\partial F1}{\partial p} = \frac{B_1\dfrac{\partial A_1}{\partial p} - A_1\dfrac{\partial B_1}{\partial p}}{B_1^{\,2}} \quad \text{(A.63)}$$

$$\frac{\partial F1}{\partial p} = \frac{B_1\left(3\times10^{-5}(0.23p+271)^{0.2} + 6.74\times10^{-8}R_pN_p - \dfrac{3.28R_pN_p}{p^2} - 2.4\times10^{-9}\,pN_p + 6\times10^{-5}N_p - \dfrac{12.75N_p}{p^2}\right)}{\left(0.97+12\times10^{-5}(0.23p+271)^{1.2} - B_{oi} + 6\times10^{-8}\,pR_{si} - 3\times10^{-4}R_{si} + \dfrac{3.28R_{si}}{p} - 10^{-8}\,p^2 + 5\times10^{-5}\,p - 0.58 + \dfrac{12.75}{p}\right)^2} - \frac{A_1\left(3\times10^{-5}(0.23p+271)^{0.2} + 6\times10^{-8}R_{si} - \dfrac{3.28R_{si}}{p^2} - 2\times10^{-8} + 5\times10^{-5} - \dfrac{12.75}{p^2}\right)}{\left(0.97+12\times10^{-5}(0.23p+271)^{1.2} - B_{oi} + 6\times10^{-8}\,pR_{si} - 3\times10^{-4}R_{si} + \dfrac{3.28R_{si}}{p} - 10^{-8}\,p^2 + 5\times10^{-5}\,p - 0.58 + \dfrac{12.75}{p}\right)^2} \quad \text{(A.64)}$$

Substitute (A.52), (A.54), (A.57) and (A.35) into (A.4) the equation become

$$F2 = \frac{\left((W_e - W_p)B_w\right)}{\left[(0.97 + 12\times10^{-5}(0.23p+271) - B_{oi}) + (R_{si} - R_s)B_g + C(p_i - p)B_{oi}\right]} \quad \text{(A.65)}$$

From (A.65), we assign

$$A_2 = \left((W_e - W_p)B_w\right) \quad \text{(A.66)}$$

$$B_2 = 0.97 + 12\times10^{-5}(0.23p+271)^{1.2} - B_{oi} + 6\times10^{-8}\,pR_{si} - 3\times10^{-4}R_{si} + \frac{3.28R_{si}}{p} - 10^{-8}\,p^2 + 5\times10^{-5}\,p - 0.58 + \frac{12.75}{p} \quad \text{(A.67)}$$

Partial derivative of (A.66) and (A.67)

From constant rule $$\frac{\partial A_2}{\partial p} = 0 \tag{A.68}$$

From power rule

$$\frac{\partial B_2}{\partial p} = 3\times10^{-5}(0.23p+271)^{0.2} + 6\times10^{-8}R_{si} - \frac{3.28R_{si}}{p^2} - 2\times10^{-8}p + 5\times10^{-5} - \frac{12.75}{p^2} \tag{A.69}$$

From quotient rule $$\frac{\partial F2}{\partial p} = \frac{B_2\frac{\partial A_2}{\partial p} - A_2\frac{\partial B_2}{\partial p}}{{B_2}^2} \tag{A.70}$$

$$\frac{\partial F2}{\partial p} = \frac{-A_2\left(3\times10^{-5}(0.23p+271)^{0.2} + 6\times10^{-8}R_{si} - \frac{3.28R_{si}}{p^2} - 2\times10^{-8}p + 5\times10^{-5} - \frac{12.75}{p^2}\right)}{\left(0.97+12\times10^{-5}(0.23p+271)^{1.2} - B_{oi} + 6\times10^{-8}pR_{si} - 3\times10^{-4}R_{si} + \frac{3.28R_{si}}{p} - 10^{-8}p^2 + 5\times10^{-5}p - 0.58 + \frac{12.75}{p}\right)^2} \tag{A.71}$$

$$\frac{\partial N}{\partial p} = \frac{B_1\left(3\times10^{-5}(0.23p+271)^{0.2} + 6.74\times10^{-8}R_pN_p - \frac{3.28R_pN_p}{p^2} - 2.4\times10^{-9}pN_p + 6\times10^{-5}N_p - \frac{12.75N_p}{p^2}\right)}{\left(0.97+12\times10^{-5}(0.23p+271)^{1.2} - B_{oi} + 6\times10^{-8}pR_{si} - 3\times10^{-4}R_{si} + \frac{3.28R_{si}}{p} - 10^{-8}p^2 + 5\times10^{-5}p - 0.58 + \frac{12.75}{p}\right)^2}$$

$$\frac{-A_1\left(3\times10^{-5}(0.23p+271)^{0.2} + 6\times10^{-8}R_{si} - \frac{3.28R_{si}}{p^2} - 2\times10^{-8} + 5\times10^{-5} - \frac{12.75}{p^2}\right)}{\left(0.97+12\times10^{-5}(0.23p+271)^{1.2} - B_{oi} + 6\times10^{-8}pR_{si} - 3\times10^{-4}R_{si} + \frac{3.28R_{si}}{p} - 10^{-8}p^2 + 5\times10^{-5}p - 0.58 + \frac{12.75}{p}\right)^2}$$

$$+\frac{((W_e - W_p)B_w)\left(3\times10^{-5}(0.23p+271)^{0.2} + 6\times10^{-8}R_{si} - \frac{3.28R_{si}}{p^2} - 2\times10^{-8}p + 5\times10^{-5} - \frac{12.75}{p^2}\right)}{\left(0.97+12\times10^{-5}(0.23p+271)^{1.2} - B_{oi} + 6\times10^{-8}pR_{si} - 3\times10^{-4}R_{si} + \frac{3.28R_{si}}{p} - 10^{-8}p^2 + 5\times10^{-5}p - 0.58 + \frac{12.75}{p}\right)^2} \tag{A.72}$$

# APPENDIX B: Response surfaces

The response surface is derived using a second-order model as per below equation

$$Y = a_0 + \sum_{i=1}^{n} a_i X_i + \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij} X_i X_j + \varepsilon \tag{5.5}$$

Where $Y$ is the observed response, i.e. original oil in-place, and the input variables $X_i$ are as follow:

| | |
|---|---|
| $X_1$ | Formation compressibility ($psi^{-1}$) |
| $X_2$ | Connate water saturation |
| $X_3$ | Cumulative oil production (stb) |
| $X_4$ | Cumulative water production (stb) |
| $X_5$ | Reservoir initial pressure (psi) |
| $X_6$ | Reservoir pressure (psi) |
| $X_7$ | Water influx term (stb) |

The $a_0$, $a_1$ and $a_{ij}$ coefficients are determine by the method of least squares, which minimizes the sum of the squares of the error, $\varepsilon$.

From table 5.2, we rearrange the data to matrix form

$$[Y] = [X][\beta] \tag{B.1}$$

Then

$$[\beta] = \left[[X]^t[X]\right]^{-1} \times \left[[X]^t[Y]\right] \tag{B.2}$$

The response surface derived from 36 randomly chosen iterations from the Monte Carlo simulation is,

$$\begin{aligned} Y = {} & 2{,}530{,}001 + 30{,}830{,}051{,}328X_1 + 482{,}721X_2 - 3X_3 + 13X_4 - 932X_5 - 173X_6 - 12X_7 \\ & - 12{,}697{,}993{,}216X_1X_2 - 42{,}540X_1X_3 + 120{,}697X_1X_4 + 4{,}200{,}648X_1X_5 - 18{,}297{,}976X_1X_6 \\ & - 140{,}928X_1X_7 - 0.4X_2X_3 - 0.7X_2X_4 + 31X_2X_5 - 196X_2X_6 + 3X_2X_7 - X_3X_4 - 0.001X_3X_5 \\ & + 0.004X_3X_6 + 0.00002X_3X_7 - 0.004X_4X_5 + 0.003X_4X_6 - 0.00001X_4X_7 - X_5X_6 + 0.003X_5X_7 \\ & - 0.004X_6X_7 - (4\times10^{13})X - 144{,}063X_2^2 + 0.000001X_3^2 + 0.00001X_4^2 + 0.4X_5^2 + X_6^2 + 5X_7^2 \end{aligned} \tag{B.3}$$

The response surface derived from 36 selected chosen iterations from the Monte Carlo simulation, where the input variables are at the edge of the range, is

$$\begin{aligned}Y = {} & 1{,}681{,}644 - 27{,}728{,}052{,}224X_1 + 3{,}792{,}348X_2 - 6X_3 + 104X_4 - 229X_5 - 1072X_6 + 40X_7 \\ & - 69{,}321{,}228{,}288X_1X_2 + 436{,}786X_1X_3 - 2{,}302{,}190X_1X_4 - 12{,}879{,}128X_1X_5 + 19{,}021{,}264X_1X_6 \\ & - 353{,}252X_1X_7 + 7X_2X_3 - 32X_2X_4 - 183X_2X_5 - 1{,}307X_2X_6 + 4X_2X_7 + 0.0004X_3X_4 - 0.004X_3X_5 \\ & - 0.002X_3X_6 - X_3X_7 - 0.006X_4X_5 - 0.04X_4X_6 - 0.002X_4X_7 - X_5X_6 + 0.006X_5X_7 \\ & + 0.01X_6X_7 - (15\times10^{13})X - 139{,}636X_2^2 + 0.0000025X_3^2 + 0.00006X_4^2 + 0.4X_5^2 + X_6^2 - 0.002X_7^2\end{aligned} \tag{B.4}$$

The response surface derived from 36 selected chosen iterations from the Monte Carlo simulation, where the input variables spread out over their entire ranges is,

$$\begin{aligned}Y = {} & 3{,}810{,}682 - 4{,}096{,}743X_1 - 245{,}012X_2 - 18X_3 + 88X_4 - 3{,}076X_5 + 2{,}365X_6 - 18X_7 \\ & - 1{,}159{,}087X_1X_2 - 22X_1X_3 - 42X_1X_4 + 2{,}500X_1X_5 - 5{,}597X_1X_6 + 25X_1X_7 - X_2X_3 \\ & - 50X_2X_4 + 311X_2X_5 - 614X_2X_6 + 20X_2X_7 - 0.00001X_3X_4 - 0.006X_3X_5 + 0.01X_3X_6 \\ & - 0.0001X_3X_7 - 0.001X_4X_5 + 0.0001X_4X_6 - 0.001X_4X_7 - 2X_5X_6 + 0.008X_5X_7 - 0.008X_6X_7 \\ & - 5{,}396{,}426X_1^{\,2} + 24{,}684X_2^2 - 0.00002X_3^2 - 0.001X_4^2 + 0.8X_5^2 + 2X_6^2 - 0.001X_7^2\end{aligned} \tag{B.5}$$

The response surface derived from 36 selected iterations from Box-Behnken using LSH technique, where the input variables spread out over their entire ranges is,

$$\begin{aligned}Y = {} & 35{,}356{,}944 - 1{,}515{,}250{,}000{,}000X_1 - 49{,}251{,}278X_2 + 120X_3 + 476X_4 - 8{,}700X_5 + 47{,}520X_6 - 1{,}250X_7 \\ & - 43{,}117{,}037{,}056X_1X_2 + 7{,}498{,}281X_1X_3 - 6{,}204{,}738X_1X_4 - 72{,}210{,}980X_1X_5 + 537{,}502{,}708X_1X_6 \\ & - 8{,}139{,}331X_1X_7 + 171X_2X_3 + 41X_2X_4 - 509X_2X_5 + 19{,}448X_2X_6 - 89X_2X_7 - 0.006X_3X_4 - 0.006X_3X_5 \\ & - 0.16X_3X_6 - 0.004X_3X_7 - 0.058X_4X_5 - 0.001X_4X_6 + 0.059X_4X_7 - X_5X_6 + 0.006X_5X_7 \\ & + 0.52X_6X_7 - (3\times10^{16})X - 23{,}010{,}554X_2^2 + 0.001X_3^2 + 0.03X_4^2 + 1.5X_5^2 - 6.9X_6^2 - 0.02X_7^2\end{aligned} \tag{B.6}$$

# VITAE

Thanis Surapapwong was born on May 13, 1982 in Bangkok, Thailand. He received his B. Eng. in Mechanical Engineering from the Faculty of Engineering, King Mongkut Institute of Technology Ladkrabang in 2003. After graduating, he worked at Thai Oil Power, Technip Engineering Thailand and PTT Exploration and Producion Ltd. in year 2003, 2004 and 2006 respectively. During his working period at Technip, he continued his study in the Master of Petroleum Engineering program at the Department of Mining and Petroleum Engineering, Faculty of Engineering, Chulalongkorn University.