



## CHAPTER V

## CALCULATION AND RESULTS

In this chapter we will show how the screening factor  $\$(\vec{r})$  and the function  $h(r)$  can be calculated from McMillan's curve (Fig.5.1) and Puoskari's curve (HNCE scaling in Fig.5.2). The ground state energy and the condensate fraction in liquid  ${}^4\text{He II}$  at 0 K are determined from the expression involving the interatomic potential  $V(\vec{r})$ , the screening factor  $\$(\vec{r})$ , and the function  $h(\vec{r})$ . In the calculation, we have used the computer (number 252558, BASIS(MAD-FLY), 130 columns) for determining  $\$(\vec{r})$ ,  $h(\vec{r})$  and the areas under curve of function  $I_a$  and  $I_b$ . In the last section, we have compared the numerical value of the ground energy and the condensate fraction at 0 K obtained when various interatomic potential of our calculations with the results of others in Table I and II.

5.1 The Screening Factor  $\$(\vec{r})$  and The Function  $h(\vec{r})$ 

The numerical value of the ground state energy and the condensate fraction in liquid  ${}^4\text{He II}$  at 0 K is determined from equations (4-23) and (4-55), when the screening factor  $\$_1(\vec{r})$ ,  $\$_2(\vec{r})$  and the function  $h(\vec{r})$  are known. If we assume that  $\$_1(\vec{r}) = \$_2(\vec{r}) = \$(\vec{r})$  equation (4-23) becomes,

$$m\mu_0 = 4\pi\rho\int I_b d\vec{r} + 4\pi\rho\int I_a d\vec{r} ,$$

where 
$$I_b = V(r) \xi(\vec{r}, T = 0 K) r^2 \quad (5-1)$$

and 
$$I_a = V(r) \xi(\vec{r}, T = 0 K) h(\vec{r}, T = 0 K) r^2 \quad (5-2)$$

$\xi_1(\vec{r})$  at 0 K is known from the works of McMillan(2) and Puoskari et.al(6) while  $\xi_2(\vec{r})$  is unknown, but is presumed to be similar  $\xi_1(\vec{r})$ . McMillan has calculated a quantity which he calls the  $\vec{x}$ -space pairing function

$$m(\vec{x}_N - \vec{x}_{N+1}) = \int \dots \int \psi(\vec{x}_1, \dots, \vec{x}_{N-1}) \psi(\vec{x}_1, \dots, \vec{x}_{N-1}, \vec{x}_N, \vec{x}_{N+1}) d\vec{x}_1, \dots, d\vec{x}_{N-1} \quad (5-3)$$

where  $\psi$  is the ground state (real) wave function.

The  $\vec{x}$ -space pairing function  $m(\vec{x}_N - \vec{x}_{N+1})$  can be expressed in terms of the particle destruction operator  $\psi(\vec{x})$  in the ground state for N and N+2 particles as

$$m(\vec{x}_N - \vec{x}_{N+1}) = \langle N | \psi(\vec{x}_N) \psi(\vec{x}_{N+1}) | N+2 \rangle \quad (5-4)$$

According to the definition of  $\Omega_2$  and the conditions for  $\Omega_2$  as well as to a plausible form for  $\Omega_2$ ,  $\Omega_2$  can be identified, in the limit when the pair  $(\vec{x}', \vec{y}')$  is taken to be far from the pair  $(\vec{x}'', \vec{y}'')$ , as

$$\Omega_2(\vec{x}', \vec{y}'; \vec{x}'', \vec{y}'') \longrightarrow m(\vec{x}' - \vec{y}') m(\vec{x}'' - \vec{y}'') \quad (5-5)$$

$$\vec{x}' \approx \vec{y}', \text{ and } \vec{x}'' \approx \vec{y}''$$

In the bulk system, the equation

$$\Omega_2 \longrightarrow \xi_1^2(\vec{r}) \rho_c^2 \quad (5-6)$$

for  $(\vec{x}', \vec{x}'')$  taken to be far from  $(\vec{y}', \vec{y}'')$ ,  $\vec{x}' \approx \vec{x}''$ ,  $\vec{y}' \approx \vec{y}''$ , and  $|\vec{x}' - \vec{y}'| \approx |\vec{x}'' - \vec{y}''| \approx \vec{r}$

From equation (5-5) and (5-6), we get

$$m(\vec{r}) = \rho_c \xi_1(\vec{r}) \quad (5-7)$$

The ground wave function  $\psi$  is approximated as a Jastrow product of the trial function  $f(\vec{x}_i - \vec{x}_j)$ . Equation (5-3) reduces to

$$m(\vec{r}) = \Omega_1(\vec{r}) f(\vec{r}) \quad (5-8)$$

where

$$f(\vec{r}) = \exp\left[-(2.6/r)^5\right] \quad (5-9)$$

and  $r$  is units of Angstrom.

The trial function  $f(\vec{r})$  approaches unity for large  $\vec{r}$ , and equations (5-7) and (5-8) give

$$\xi_1(\vec{r}) = \frac{1}{\rho_c} \Omega_1(\vec{r}) f(\vec{r}) \quad (5-10)$$

In the bulk system,  $\Omega_1(\vec{r})$  can be written as

$$\Omega_1(\vec{r}) = \rho_c + \rho_d h(\vec{r})$$

$$\frac{\Omega_1(\vec{r})}{\rho} = \frac{\rho_c}{\rho} + \frac{\rho_d}{\rho} h(\vec{r})$$

$$\text{thus} \quad h(\vec{r}) = \frac{\rho}{\rho_d} \left\{ \frac{\Omega_1(\vec{r})}{\rho} - \frac{\rho_c}{\rho} \right\} \quad (5-11)$$

At 0 K,  $\Omega_1(\vec{r})$  approaches  $\rho_c$  and  $h(\vec{r})$  approaches zero when  $r \gg 4.5 \text{ \AA}$

## 5.2 Calculation and Results

First, look at McMillan's curve,  $(1/\rho)\Omega_1(\vec{r})$  vs  $r$  (Fig.5.1) and substituting equation (5-9) into equation (5-10) we obtain numerical values of  $\xi(\vec{r})$  and  $h(\vec{r})$  (equations (5-10) and (5-11)). We have plotted the curve of  $\xi(\vec{r}, T = 0 \text{ K})$  vs  $r$  for  $\rho_c/\rho = 0.11$  and  $h(\vec{r}, T = 0 \text{ K})$  vs  $r$  shown in Fig.5.3.

Secondly, from Puoskari's curve,  $(1/\rho)\Omega_1(\vec{r})$  vs  $r$  (Fig.5.2) and substituting equation (5-9) into equation (5-10) also gives numerical values of  $\xi(\vec{r})$  and  $h(\vec{r})$  (equation (5-10) and (5-11)). We have plotted the curve of  $\xi(\vec{r}, T = 0 \text{ K})$  vs  $r$  for  $\rho_c/\rho = 0.14$  and  $h(\vec{r}, T = 0 \text{ K})$  vs  $r$  shown in Fig.5.10.

Next we consider the interatomic potential or the two-body potential interatomic between neutral helium atoms, which have been proposed (7,8,9). Three form are chosen to calculate the numerical value of the ground state energy and the condensate fraction.

The Lennard-Jones 12-6 potential

$$V(r) = 4\varepsilon \left\{ \left( \frac{r_0}{r} \right)^{12} - \left( \frac{r_0}{r} \right)^6 \right\} \quad (5-12)$$

where  $\varepsilon = 10.22 \text{ K}$  and  $r_0 = 2.556 \text{ \AA}$

The Morse- $V_{DD}$  potential

The Morse- $V_{DD}$  potential

$$V(r) = \begin{cases} \epsilon \left[ \exp\{2C(1 - r/r_e)\} - 2\exp\{C(1 - r/r_e)\} \right] & r < 3.6 \text{ \AA} \\ -C_6 r^{-6} - C_8 r^{-8} & r > 3.6 \text{ \AA} \end{cases} \quad (5-13)$$

where  $\epsilon = 9.25 \text{ K}$  ,  $r_e = 2.948 \text{ \AA}$  ,  $C = 6.2059$  ,  $C_6 = 6842 \text{ K.\AA}^6$   
and  $C_8 = 26930 \text{ K.\AA}^8$

The HFDHE-2 potential

$$V(r) = \epsilon \left[ A \exp(-\alpha_1 x) - \{C_6/x^6 + C_8/x^8 + C_{10}/x^{10}\} F(x) \right] \quad (5-14)$$

$$F(x) = \begin{cases} \exp -\{(D/x)-1\}^2 & \text{for } x < D \\ 1 & \text{for } x > D \end{cases}$$

$D = 1.241314$  ,  $x = r/r_m$  ,  $A = 0.5448504(6)$  ,  $\alpha_1 = 13.353384$  ,  
 $C_6 = 1.3732412$  ,  $C_8 = 0.4253785$  ,  $C_{10} = 0.178100$  ,  $\epsilon = 10.8 \text{ K}$   
and  $r_m = 2.9673 \text{ \AA}$

For the other constants, we have used  $m_p = 0.149 \text{ g/cm}^3$   
 $\rho = 2.2 \times 10^{22} \text{ cm}^{-3}$  and the Planck constant  $\hbar = 1.054 \times 10^{-27} \text{ erg-sec}$ .

The quantities  $m\mu_0$  can be calculated from (4-23). We have determined the area under curves of  $I_a$  vs  $r$  and  $I_h$  vs  $r$  (we will be shown in Fig.5.5 for the L-J 12-6 potential) in equations(5-1) and (5-2) in which  $\int I_a dr$  and  $\int I_h dr$  are determined by Simpson's rule method.

All the calculation of  $\int I_a dr$  and  $\int I_b dr$ , we were obtained by numerical method and are given in Table III.

The numerical value of the condensate fraction was calculated from equation (4-55). The values  $\alpha$  and  $\beta$  can be calculated from equations (4-24) and (4-44) respectively.

Our results of the ground state energy and the condensate fraction calculated from the various interatomic potential in equations (5-12), (5-13), and (5-14) are shown in Table I and II, respectively.

#### 5.2.1 From McMillan's curve

In this section, we will show how to calculate the numerical value of the ground state energy and the condensate fraction of liquid helium II at 0 K using the various interatomic potential.

##### 5.2.1.1 The Ground State Energy and The Condensate Fraction

From equation (4-23), we have

$$\mu = \frac{\rho_c}{m} \int V(r) \xi(r) d^3 r + \frac{\rho_d}{m} \int V(r) \xi(r) d^3 r + \frac{\rho_d}{m} \int V(r) \xi(r) h(r) d^3 r$$

$$\mu = 4\pi\rho \int V(r) \xi(r) r^2 dr + 4\pi\rho \cdot \frac{\rho_d}{\rho} \int V(r) \xi(r) h(r) r^2 dr$$

From Table III,

$$\int I_{L-J}^{\mu 1} dr = \int_{L-J} V(r) \xi(r) h(r) r^2 dr = \frac{0.1 \times 677.5628 \times 10^{-24}}{3} = 22.5854 \times 10^{-24}$$

$$\int_{L-J}^{\mu} \Pi dr = \int_{L-J}^{\nu} \xi(r) r^2 dr = \frac{0.1 \times (-1112.78424) \times 10^{-24}}{3} = -37.0928 \times 10^{-24}$$

Therefore, the ground state energy  $m\mu_{L-J}$  due to interatomic L-J potential

$$\begin{aligned} m\mu_{L-J} &= \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} \times (-37.0928 \times 10^{-24}) + \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} \times 0.89 \times 22.5854 \times 10^{-24} \\ &= \frac{4 \times 22 \times 2.2 \times 10^{-2}}{7} (-37.0928 + 0.89 \times 22.5854) \\ &= -4.7 \text{ K/atom} \end{aligned}$$

For the condensate fraction  $\rho_c/\rho$ , we have equations (4-24) and (4-44)

$$\alpha = \frac{4\pi m \rho \int V(r) \xi(r) h(r) r^4 dr}{3h^2}$$

$$\beta = \frac{4\pi m \rho \int V(r) \xi^2(r) h(r) r^4 dr}{3h^2}$$

From Table III

$$\int_{L-J}^{\nu} V(r) \xi(r) h(r) r^4 dr = \frac{0.1 \times 2016.06819 \times 10^{-40}}{3} = 67.20227 \times 10^{-40} \text{ K-cm}^5$$

$$\int_{L-J}^{\nu} V(r) \xi^2(r) h(r) r^4 dr = \frac{0.1 \times (-925.29259) \times 10^{-40}}{3} = -30.84308 \times 10^{-40} \text{ K-cm}^5$$

Therefore

$$\alpha = \frac{4 \times 22 \times 0.149 \times 67.20227 \times 10^{-40}}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = 0.52$$

$$\beta = \frac{4 \times 22 \times 0.149 \times (-30.843 \times 10^{-40})}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = -0.2375$$

From equation (4-55)

$$\frac{\rho_c}{\rho} = \frac{\alpha}{1-\beta} = \frac{0.52}{1-(-0.2375)} = 0.42$$

$$V(r) = \begin{cases} \epsilon \left[ \exp\{2C(1-r/r_e)\} - 2\exp\{C(1-r/r_e)\} \right] & r < 3.6 \\ -C_6 r^{-6} - C_8 r^{-8} & r > 3.6 \end{cases}$$

From Table III

$$\int I_{M-V_{DD}}^{\mu I} dr = \int V(r) \zeta(r) h(r) r^2 dr = \frac{0.1 \times 497.6349 \times 10^{-24}}{3} = 16.5878 \times 10^{-24}$$

$$\int I_{M-V_{DD}}^{\mu II} dr = \int V(r) \zeta(r) r^2 dr = \frac{0.1 \times (-935.4673) \times 10^{-24}}{3} = -31.1822 \times 10^{-24}$$

From equation (4-23)

thus the ground state energy  $\mu_{M-V_{DD}}$  due to interatomic Morse- $V_{DD}$  potential

$$\begin{aligned} \mu_{M-V_{DD}} &= \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} \times (-31.1822 \times 10^{-24}) + \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} \times 0.89 \times 16.5878 \times 10^{-24} \\ &= -4.54 \text{ K/atom} \end{aligned}$$

For the condensate fraction  $\rho_c/\rho$

$$\alpha = \frac{4\pi m \rho \int V(r) \zeta(r) h(r) r^4 dr}{3\hbar^2}$$

$$\beta = \frac{4\pi m \rho \int V(r) \zeta^2(r) h(r) r^4 dr}{3\hbar^2}$$

From Table III

$$\int I_{M-V_{DD}}^{\alpha} dr = \int V(r) \zeta(r) h(r) r^4 dr = \frac{0.1 \times 1525.3768 \times 10^{-40}}{3} = 50.8458 \times 10^{-40} \text{ K-cm}^5$$

$$\int I_{M-V_{DD}}^{\beta} dr = \int V(r) \zeta^2(r) h(r) r^4 dr = \frac{0.1 \times (-645.5889) \times 10^{-40}}{3} = -21.5196 \times 10^{-40} \text{ K-cm}^5$$

$$\alpha = \frac{4 \times 22 \times 0.149 \times 50.8458 \times 10^{-40}}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = 0.394$$

$$\beta = \frac{4 \times 22 \times 0.149 \times (-21.5196 \times 10^{-40})}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = -0.167$$

From equation (4-55)

$$\frac{\rho_c}{\rho} = \frac{\alpha}{1-\beta} = \frac{0.394}{1-(-0.167)} = 0.33$$



$$V(r) = \epsilon \left[ A \exp(-\mu_1 x) - \left\{ C_6/x^6 + C_8/x^8 + C_{10}/x^{10} \right\} F(x) \right]$$

From Table III

$$\int I_{H-2}^{\mu_1} dr = \int V(r) \xi(r) h(r) r^2 dr = \frac{0.1 \times 783.4349 \times 10^{-24}}{3} = 26.1145 \times 10^{-24}$$

$$\int I_{H-2}^{\mu_2} dr = \int V(r) \xi(r) r^2 dr = \frac{0.1 \times (-694.9097 \times 10^{-24})}{3} = -23.1636 \times 10^{-24}$$

From equation (4-23)

$$\mu_{H-2} = 4\pi\rho \int V(r) \xi(r) r^2 dr + 4\pi\rho \cdot \frac{\rho_d}{\rho} \int V(r) \xi(r) h(r) r^2 dr$$

thus the ground state energy  $\mu_{H-2}$  due to interatomic HFDHE-2 potential

$$\begin{aligned} \mu_{H-2} &= \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} (-23.1636 \times 10^{-24}) + \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} \times 0.89 \times 26.1145 \times 10^{-24} \\ &= 0.22 \text{ K/atom} \end{aligned}$$

For the condensate fraction  $\rho_c/\rho$

$$\alpha = \frac{4\pi m \rho \int V(r) \xi(r) h(r) r^4 dr}{3\hbar^2}$$

$$\beta = \frac{4\pi m \rho \int V(r) \xi^2(r) h(r) r^4 dr}{3\hbar^2}$$

From Table III

$$\int I_{H-2}^{\alpha} dr = \int V(r) \xi(r) h(r) r^4 dr = \frac{0.1 \times 2687.1594 \times 10^{-40}}{3} = 89.5719 \times 10^{-40} \text{ K-cm}^5$$

$$\int I_{H-2}^{\beta} dr = \int V(r) \xi^2(r) h(r) r^4 dr = \frac{0.1 \times (-509.5625 \times 10^{-40})}{3} = -16.9854 \times 10^{-40} \text{ K-cm}^5$$

$$\alpha = \frac{4 \times 22 \times 0.149 \times 89.5719 \times 10^{-40}}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = 0.69$$

$$\beta = \frac{4 \times 22 \times 0.149 \times (-16.9854 \times 10^{-40})}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = -0.132$$

From equation (4-55)

$$\frac{\rho_c}{\rho} = \frac{\alpha}{1-\beta} = \frac{0.69}{1-(-0.132)} = 0.60$$

### 5.2.2 From the work of Puoskari et.al (HNCE scaling curve)

Now, we will use the results obtained by Puoskari et.al (6) in replace of those obtained from McMillan's curve.

#### 5.2.2.1 The Ground State energy and The Condensate Fraction

From Table III

$$\int_{L-J}^{\mu} 1 dr = \int V(r) \xi(r) h(r) r^2 dr = \frac{0.1 \times 465.601 \times 10^{-24}}{3} = 15.52 \times 10^{-24}$$

$$\int_{L-J}^{\mu} \Pi dr = \int V(r) \xi(r) r^2 dr = \frac{0.1 \times (-1498.3795 \times 10^{-24})}{3} = -49.9459 \times 10^{-24}$$

From equation (4-23)  $m\mu = 4\pi\rho \int V(r) \xi(r) r^2 dr + 4\pi\rho \frac{\rho_d}{\rho} \int V(r) \xi(r) h(r) r^2 dr$

thus the ground state energy  $m\mu_{L-J}$  due to interatomic L-J potential

$$\begin{aligned} m\mu_{L-J} &= \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} (-49.9459 \times 10^{-24}) + \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} \times 0.86 \times 15.52 \times 10^{-24} \\ &= -10.12 \text{ K/atom} \end{aligned}$$

Since

$$\alpha = \frac{4\pi\rho \int V(r) \xi(r) h(r) r^4 dr}{3\hbar^2}$$

$$\beta = \frac{4\pi\rho \int V(r) \xi^2(r) h(r) r^4 dr}{3\hbar^2}$$

and since

$$\int_{L-J}^{\alpha} dr = \int V(r) \xi(r) h(r) r^4 dr = \frac{0.1 \times 1336.877 \times 10^{-40}}{3} = 44.5625 \times 10^{-40} \text{ K-cm}^5$$

$$\int_{L-J}^{\beta} dr = \int V(r) \xi^2(r) h(r) r^4 dr = \frac{0.1 \times (-667.488 \times 10^{-40})}{3} = -22.2496 \times 10^{-40} \text{ K-cm}^5$$

we get

$$\alpha = \frac{4 \times 22 \times 0.149 \times 44.5625 \times 10^{-40}}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = 0.345$$

$$\beta = \frac{4 \times 22 \times 0.149 \times (-22.2496 \times 10^{-40}) \times 1.38 \times 10^{-16}}{7 \times 3 \times 0.149 \times (1.054 \times 10^{-27})^2} = -0.17$$

Equation (4-55) yields

$$\frac{\rho_c}{\rho} = \frac{\alpha}{1-\beta} = \frac{0.345}{1-(-0.17)} = 0.29$$

$$V(r) = \epsilon \left[ A \exp(-\frac{\alpha}{1} x) - \{C_6/x^6 + C_8/x^8 + C_{10}/x^{10}\} F(x) \right]$$

From Table III

$$\int I_{H-2}^{\mu 1} dr = \int V(r) \xi(r) h(r) r^2 dr = \frac{0.1 \times 533.799 \times 10^{-24}}{3} = 17.7933 \times 10^{-24}$$

$$\int I_{H-2}^{\mu \Pi} dr = \int V(r) \xi(r) r^2 dr = \frac{0.1 \times (-1208.0418 \times 10^{-24})}{3} = -40.268 \times 10^{-24}$$

From equation (4-23), becomes the ground state energy  $m\mu_{H-2}$  due to interatomic HFDHE-2 potential

$$\begin{aligned} m\mu_{H-2} &= 4 \times \frac{22}{7} \times 2.2 \times 10^{22} \times (-40.268 \times 10^{-24}) + 4 \times \frac{22}{7} \times 2.2 \times 10^{22} \times 0.86 \times 17.7933 \times 10^{-24} \\ &= -6.9 \text{ K/atom} \end{aligned}$$

To obtain the condensate fraction  $\rho_c/\rho$ , we again note that

$$\alpha = \frac{4\pi m \rho \int V(r) \xi(r) h(r) r^4 dr}{3\hbar^2}$$

$$\beta = \frac{4\pi m \rho \int V(r) \xi^2(r) h(r) r^4 dr}{3\hbar^2}$$

From Table III

$$\int I_{H-2}^{\alpha} dr = \int V(r) \xi(r) h(r) r^4 dr = \frac{0.1 \times 1775.4697 \times 10^{-40}}{3} = 59.1823 \times 10^{-40} \text{ K-cm}^5$$

$$\int I_{H-2}^{\beta} dr = \int V(r) \xi^2(r) h(r) r^4 dr = \frac{0.1 \times (-461.4117 \times 10^{-40})}{3} = -15.38 \times 10^{-40} \text{ K-cm}^5$$

we get

$$\alpha = \frac{4 \times 22 \times 0.149 \times 59.1823 \times 10^{-40}}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = 0.467$$

$$\beta = \frac{4 \times 22 \times 0.149 \times (-15.38 \times 10^{-40})}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = -0.12$$

Equation (4-55) now yields

$$\frac{\rho_c}{\rho} = \frac{\alpha}{1-\beta} = \frac{0.467}{1-(-0.12)} = 0.41$$



$$V(r) = \begin{cases} \epsilon \left[ \exp\{2C(1-r/r_e)\} - 2\exp\{C(1-r/r_e)\} \right] & r < 3.6 \\ -C_6 r^{-6} - C_8 r^{-8} & r > 3.6 \end{cases}$$

From Table III

$$\int I_{M-V_{DD}}^{\mu I} dr = \int V(r) \xi(r) h(r) r^2 dr = \frac{0.1 \times 337.6235 \times 10^{-24}}{3} = 11.2541 \times 10^{-24}$$

$$\int I_{M-V_{DD}}^{\mu II} dr = \int V(r) \xi(r) r^2 dr = \frac{0.1 \times (-1226.6324 \times 10^{-24})}{3} = -40.8877 \times 10^{-24}$$

From equation (4-23), we get the ground state energy  $\mu_{M-V_{DD}}$  due to interatomic potential as

$$\begin{aligned} \mu_{M-V_{DD}} &= \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} \times (-40.8877 \times 10^{-24}) + \frac{4 \times 22 \times 2.2 \times 10^{22}}{7} \times 0.86 \times 11.2541 \times 10^{-24} \\ &= -8.63 \text{ K/atom} \end{aligned}$$

The condensate fraction  $\rho_c/\rho$  is obtained by again noting that

$$\alpha = \frac{4\pi m_0 \int V(r) \xi(r) h(r) r^4 dr}{3\hbar^2}$$

$$\beta = \frac{4\pi m_0 \int V(r) \xi^2(r) h(r) r^4 dr}{3\hbar^2}$$

From Table III

$$\int I_{M-V_{DD}}^{\alpha} dr = \int V(r) \xi(r) h(r) r^4 dr = \frac{0.1 \times 989.8217 \times 10^{-40}}{3} = 32.994 \times 10^{-40} \text{ K-cm}^5$$

$$\int I_{M-V_{DD}}^{\beta} dr = \int V(r) \xi^2(r) h(r) r^4 dr = \frac{0.1 \times (-498.6733 \times 10^{-40})}{3} = -16.6224 \times 10^{-40} \text{ K-cm}^5$$

we have

$$\alpha = \frac{4 \times 22 \times 0.149 \times 32.994 \times 10^{-40}}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = 0.256$$

$$\beta = \frac{4 \times 22 \times 0.149 \times (-16.6224 \times 10^{-40})}{7 \times 3 \times (1.054 \times 10^{-27})^2} \times 1.38 \times 10^{-16} = -0.129$$

and so

$$\frac{\rho_c}{\rho} = \frac{\alpha}{1-\beta} = \frac{0.256}{1-(-0.129)} = 0.20$$

Table I The ground state energy of liquid  $^4\text{He II}$ 

	Reading curve	Interatomic potential	Ground state energy $\mu_0$ ( K per atom )	
Our results	McMillan curve ( Fig I )	L-J 12-6	-4.7	
		Morse- $V_{DD}$	-4.54	
		HFDHE-2	0.22	
	HNCE scaling ( Fig II )	L-J 12-6	-10.12	
		Morse- $V_{DD}$	-8.63	
		HFDHE-2	-6.90	
Other theoretical result due to				
McMillan (2)		L-J 12-6	-5.65±0.65	
Cummings (14)		L-J 12-6	-7.1	
Ground state energy $\mu_0$				
		Variational Method	Perturbation Theory	GFMC Method
Kalos et al (13)	L-J 12-6	-5.68	-	-6.85
	HFDHE-2	-5.87	-6.98	-7.12
Masserini et.al (33)	L-J 12-6	-6.10	-	-5.76
Experimental value from Ref(2)			-7.16	

Table II The condensate fraction of liquid  $^4\text{He}$  II at 0 K

	Reading curve	Interatomic potential	Condensate fraction $\rho_c/\rho$ $= \left[ \alpha/(1-\beta) \right] \rho_s/\rho$
Our results	McMillan curve ( Fig I )	L-J 12-6	0.42
		Morse- $V_{DD}$	0.33
		HFDHE-2	0.63
	HNCE scaling ( Fig II )	L-J 12-6	0.29
		Morse- $V_{DD}$	0.20
		HFDHE-2	0.41
Other theoretical results due to			
McMillan (2)			0.11
Lam and Ristig (11)			0.113
Penrose and Onsager (19)			0.08
Schiff and Verlet (1)			0.105
Francis et al (31)			0.25
Kalos et al (13)			0.090
Whitlock (32)			0.115
Visoottiviseth (25)			0.21±0.05
Puoskari et.al (6)			0.14
Experimental result due to			
V.F Sears (26)			0.139±0.023