

## CHAPTER 2

### LITERATURE REVIEWS

It is well known that a solid cylinder is the most commonly used specimen in various standard tests in engineering, such as a uniaxial compressive strength test of concrete, a triaxial compressive strength test of soil and a point load strength test of rock. In fact, the stress analysis of an elastic solid cylinder is one of the most fundamental problems in theoretical elasticity and has a rich history in solid mechanics. For example, Lekhniskii (1963) proposed a stress function for solving a transversely isotropic solid under axisymmetric load. Watanabe (1992) presented a solution for axisymmetrically-loaded cylinders by using stress functions in the form of infinite series and presented a solution for stresses, strains and displacements and a comparison with finite element solutions. Wei et al. (1999) used the displacement function approach and presented an analytical solution for the axial point load strength test (PLST) of an isotropic rock. Later, Chau and Wei (2000a, b) obtained analytical solutions for finite elastic isotropic solid cylinders subjected to arbitrary surface loading and presented the stress distributions of the cylinder under the double-punch test. The solution of a transversely isotropic finite cylinder under axisymmetric end loading with stress-free lateral surfaces was given by Vendhan and Archer (1977) by using a displacement potential. Recently, a transversely isotropic rock under a point load strength test was considered by Wei and Chua (2002) by employing the Lekhnitskii's stress functions to uncouple the equations of equilibrium. The stress distributions within a transversely isotropic rock cylinder were presented and then compared to the isotropic solutions (Wei et al. 1999).

All studies mentioned above are concerned with analytical solutions for isotropic and transversely isotropic cylinders. In the context of piezoelectricity, Eringen (1963) derived the variation principle leading to the basic field equations, the boundary conditions and the constitutive equations of an elastic dielectric solid subject to large deformations and polarizations. In the last decade, there are a number of studies involving the development of analytical solutions for boundary value problems related to piezoelectric materials. Wang and Zheng (1995) obtained the

three-dimensional general solutions for a piezoelectric material by employing a set of potential functions and presented a solution for the case of a concentrated shear load on the piezoelectric half-space surface. Dunn and Wienecke (1996) derived a closed-form expression for Green's functions of an infinite piezoelectric medium by using a potential formulation, in which the three displacements and one electric potential are derived from two potential functions. Ding et al. (1996) presented the general solutions for the coupled dynamic equations for transversely isotropic piezoelectric media. Those equations become the general solutions for the corresponding static equilibrium equations on the removal of the terms involving the derivatives of time. For the case of a piezoelectric cylinder, Rajapakse and Zhou (1997) obtained the general solutions of coupling phenomenon between mechanical and electric fields by using Fourier integral transforms and presented solutions of an infinitely long cylinder subjected to a radial ring load and a surface electric charge. To the author's knowledge, the problem of a finite piezoelectric cylinder subjected to surface load and electric fields has never been considered in the past. Such solutions are importance in wide spectrum of applications such as scanning-probe microscopy, fiber optics, telescopes, etc.



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