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APPENDIX I

This appendix contains some necessary digital communication theory. There are four main topics describing about signals and linear system, baseband digital transmission, digital transmission through bandlimited channels, and digital transmission via carrier modulation.

Fourier Series

The input and output relation of a linear time-invariant system is given by the convolution integral defined by

$$y(t) = x(t) * h(t) \tag{1}$$

$$= \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

In which $h(t)$ denotes the impulse response of the system, $x(t)$ is the input signal, and $y(t)$ is the output signal. If the input $x(t)$ is a complex exponential given by

$$x(t) = Ae^{j2\pi f_0 t} \tag{2}$$

Then the output is

$$y(t) = \int_{-\infty}^{\infty} Ae^{j2\pi f_0 (t-\tau)} h(\tau) d\tau$$

$$= A\left[\int h(\tau) e^{-j2\pi f_0 \tau} d\tau\right] e^{j2\pi f_0 t} \tag{3}$$

In other words, the output is a complex exponential with the same frequency as the input the (complex) amplitude of the output, however, is the (complex) amplitude of the input amplified by

$$\int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f_0 \tau} d\tau$$

Note that the above quantity is a function of the impulse response of the LTI system, $h(t)$, and the frequency of the input signal, f_0. Therefore, computing the response of LTI systems to exponential inputs is quite easy. Consequently, it is natural in linear system analysis to look for methods of expanding signals as the sum of complex exponentials. Fourier series and Fourier transform are techniques for expanding signals in terms of complex exponentials.

Fourier series is the orthogonal expansion of periodic signals with period T_0 when the signal set $\left\{e^{j2\pi t/T_0}\right\}_{n=-\infty}^{\infty}$ is employed as the basis for the expansion. With this basis, any periodic signal $x(t)$ with period T_0 can be expressed as

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{j2\pi nt/T_0} \tag{4}$$

Where the x_n 's are called the Fourier series coefficients of the signal $x(t)$ and are given by

$$x_n = \frac{1}{T_0} \int_{\alpha}^{\alpha+T_0} x(t) e^{-j2\pi nt/T_0} dt \tag{5}$$

Here α is an arbitrary constant chosen in such a way that the computation of the integral is simplified. The frequency $f_0 = 1/T_0$ is called the fundamental frequency of the periodic signal, and the frequency $f_n = nf_0$ is called the $x(t)$ harmonic. In most cases either $\alpha = 0$ or $\alpha = -T_0/2$ is a good choice.

This type of Fourier series is known as the exponential Fourier series and can be applied to both real-valued and complex-valued signals $x(t)$ as long as they are periodic. In general, the Fourier series coefficients $\{x_n\}$ are complex numbers even when $x(t)$ is a real-valued signal.

When $x(t)$ is a real-valued periodic signal, we have

$$
\begin{aligned}
x_{-n} &= \frac{1}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)e^{j2\pi nt/T_0}\,dt \\
&= \frac{1}{T_0}\left[\int_{\alpha}^{\alpha+T_0} x(t)e^{-j2\pi nt/T_0}\,dt\right]^{*} \\
&= x_n^{*}
\end{aligned}
\tag{6}
$$

From this it is obvious that

$$
\begin{cases}
|x_n| = |x_{-n}| \\
\angle x_n = -\angle x_n
\end{cases}
\tag{7}
$$

Thus the Fourier series coefficients of a real-valued signal have Hermitian symmetry. Therefore, their magnitude is even and their phase is odd. In the other words, their real part is even and their imaginary part is odd.

Another form of Fourier series, known as trigonometric Fourier series, can be applied only to real periodic signals and is obtained by defining

$$x_n = \frac{a_n - jb_n}{2} \tag{8}$$

$$x_{-n} = \frac{a_n + jb_n}{2} \tag{9}$$

With the use of Euler's relation

$$e^{-j2\pi nt/T_0} = \cos\left(2\pi t\frac{n}{T_0}\right) - j\sin\left(2\pi t\frac{n}{T_0}\right) \tag{10}$$

We have

$$a_n = \frac{2}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)\cos\left(2\pi t\frac{n}{T_0}\right)dt$$
$$b_n = \frac{2}{T_0}\int_{\alpha}^{\alpha+T_0} x(t)\sin\left(2\pi t\frac{n}{T_0}\right)dt \tag{11}$$

therefore,

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos\left(2\pi t\frac{n}{T_0}\right) + b_n\sin\left(2\pi t\frac{n}{T_0}\right) \tag{12}$$

Periodic Signals and LTI Systems

When a periodic signal $x(t)$ is passed through a linear time-invariant (LTI) system, as shown in figure 1, the output signal $y(t)$ is also periodic, usually with the same period as the input signal and therefore has a Fourier series expansion.

$x(t)$ and $y(t)$ are expanded as

$$x(t) = \sum_{n=-\infty}^{\infty} x_n e^{j2\pi nt/T_0} \tag{13}$$

$$y(t) = \sum_{n=-\infty}^{\infty} y_n e^{j2\pi nt/T_0} \tag{14}$$

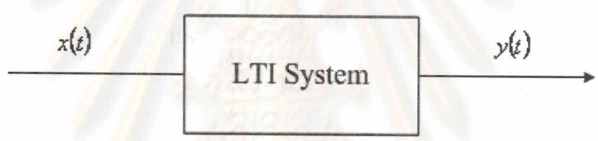

Figure 1 Periodic signals through LTI systems

Then the relation between the Fourier series coefficients of $x(t)$ and $y(t)$ can be obtained by employing the convolution integral

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$$

$$= \int_{-\infty}^{\infty}\sum_{n=-\infty}^{\infty} x_n e^{j2\pi n(t-\tau)/T_0} h(\tau)d\tau$$

$$= \sum_{n=-\infty}^{\infty} x_n \left(\int_{-\infty}^{\infty} h(\tau)e^{-j2\pi n\tau/T_0} d\tau \right) e^{j2\pi nt/T_0}$$

$$= \sum_{n=-\infty}^{\infty} y_n e^{j2\pi nt/T_0} \tag{15}$$

From the preceding relation we have

$$y_n = x_n H\left(\frac{n}{T_0}\right) \quad (16)$$

Where $H(f)$ denotes the transfer function of the LTI system given as the Fourier transform of its impulse response $h(t)$:

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}\,dt \quad (17)$$

Fourier Transforms

The Fourier transform is the extension of the Fourier series to nonperiodic signals. The Fourier transform of a signal $x(t)$ that satisfies certain conditions, known as Dirichlet's conditions [25], is denoted by $X(f)$ or, equivalently, $F[x(t)]$ and is defined by

$$F[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt \quad (18)$$

The inverse Fourier transform of $X(f)$ is $x(t)$, given by

$$F^{-1}[X(f)] = x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df \quad (19)$$

If $x(t)$ is a real signal, then $X(f)$ satisfies the Hermitian symmetry, that is,

$$X(-f) = X^*(f) \quad (20)$$

Sampling Theorem

The sampling theorem is one of the most important results in signal and system analysis. It forms the basis for the relation between continuous-time signals and discrete-time signals. The sampling theorem says that a bandlimited signal, a signal whose Fourier transform vanishes for $|f| > W$ for some W, can be completely described in terms of its sample values taken at intervals T_s as long as $T_s \leq 1/2W$. If the sampling is done at intervals $T_s = 1/2W$, know as the Nyquist interval (or Nyquist rate), the signal $x(t)$ can be reconstructed from the sample values $\{x[n] = x(nT_s)\}_{n=-\infty}^{\infty}$ as

$$x(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\sin(2W(t - nT_s)) \tag{21}$$

This result is based on the fact that the sampled waveform $x_\delta(t)$, defined as

$$x_\delta(t) = \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) \tag{22}$$

Has a Fourier transform given by

$$X_\delta(f) = \frac{1}{T_s}\sum_{n=-\infty}^{\infty} X\left(f - \frac{n}{T_s}\right) \text{ for all } f$$

$$= \frac{1}{T_s}X(f) \text{ for } |f| < W \tag{23}$$

$$= \frac{1}{T_s}X(f) \text{ for } |f| < W \tag{24}$$

So passing it through a lowpass filter with a bandwidth of W and a gain of T_s in the passband will reproduce the original signal.

The discrete Fourier transform (DFT) of the discrete-time sequence $x[n]$ is defined by

$$X_d(f) = \sum_{n=-\infty}^{\infty} x[n]e^{-j2\pi f_n T_s} \tag{25}$$

Comparing equations (25) and (24), we yield

$$X(f) = T_s X_d(f) \tag{26}$$

which gives the relation between the Fourier transform of an analog signal and the discrete Fourier transform of its corresponding sampled signal.

Numerical computation of the discrete Fourier transform is done by the fast Fourier transform (FFT) algorithm. In this algorithm, a sequence of length N of samples of the signals $x(t)$, taken at intervals of T_s, is used as the representation of the signal. The result is a sequence of length N of samples of $X_d(f)$ in the frequency interval $[0, f_s]$, where $f_s = 1/T_s = 2W$ is the Nyquist frequency. When the samples are $\Delta f = f_s / N$ apart, the value of Δf gives the frequency resolution of the resulting Fourier transform. The FFT algorithm is computationally efficient if the length of the input sequence, N, is a power of 2. In many cases if this length is not a power of 2, it is made to be a power of 2 by technique such as zero-padding. Note that since the FFT algorithm essentially gives the DFT of the sampled signal, in order to get the Fourier transform of the analog signal we have to employ equation (2.26). This means that after computing the FFT, we have to multiply it by T_s or, equivalently, divide it by f_s in order to obtain the Fourier transform of the original analog signal.

Frequency-Domain Analysis of LTI System

The output of an LTI system with impulse response $h(t)$ when the input signal is $x(t)$ is given by convolution integral

$$y(t) = x(t) * h(t) \tag{27}$$

Applying the convolution theorem, we obtian

$$Y(f) = X(f)H(f) \tag{28}$$

Where

$$H(f) = F[h(t)] = \int_{-\infty}^{\infty} h(t) e^{-j2\pi ft} dt \tag{29}$$

is the transfer function of the system. Equation (55) can be written in the form

$$\begin{cases} |Y(f)| = |X(f)||H(f)| \\ \angle Y(f) = \angle X(f) + \angle H(f) \end{cases} \tag{30}$$

This shows the relation between the magnitude and phase spectra of the input and the output.

Binary Signal Transmission

In binary communication system, binary data consisting of a sequence of 0's and 1's are transmitted by means of two signal waveforms, presumably $s_0(t)$ and $s_1(t)$. Suppose that the data rate is specified as R bits per second. Then each bit is mapped into a corresponding signal waveform according to the rule

$$\begin{aligned} 0 &\rightarrow s_0(t), \quad 0 \le t \le T_b \\ 1 &\rightarrow s_1(t), \quad 0 \le t \le T_b \end{aligned}$$

Where $T_b = 1/R$ is defined as the bit time interval. We assume that the data bits 0 and 1 are equally probable. In the other words, each occurs with probability $\frac{1}{2}$, and is mutually statistically independent.

The channel through which the signal is transmitted is assumed to corrupt the signal by the addition of noise, denoted as $n(t)$, which is a sample function of a white Gaussian process with power spectrum $N_0/2$ watts/hertz. Such a channel is called an additive white Gaussian noise (AWGN) channel. Consequently, the received signal waveform is given by.

$$r(t) = s_i(t) + n(t), \quad i = 0,1, \quad 0 \le t \le T_b \tag{31}$$

The task of the receiver is to determine whether a 0 or a 1 was transmitted after observing the received signal $r(t)$ in the interval $0 \le t \le T_b$. The receiver is designed to minimize the probability of error. Such a receiver is called the optimum receiver.

Optimum Receiver for the AWGN Channel

In nearly all basic digital communication theory, it is shown that the optimum receiver for the AWGN channel consists of two building blocks. One is either a signal correlator or a matched filter

Signal Correlator

The signal correlator cross-correlates the received signal $r(t)$ with the two possible transmitted signals $s_0(t)$ and $s_1(t)$, as illustrated in figure 2. that is, the signal correlator compute the two outputs

$$
\begin{aligned}
r_0(t) &= \int_0^t r(\tau)s_0(\tau)d\tau \\
r_1(t) &= \int_0^t r(\tau)s_1(\tau)d\tau
\end{aligned}
\tag{32}
$$

In the interval $0 \le t \le T_b$, sample the two outputs at $t = T_b$, and feeds the sampled outputs to the detector.

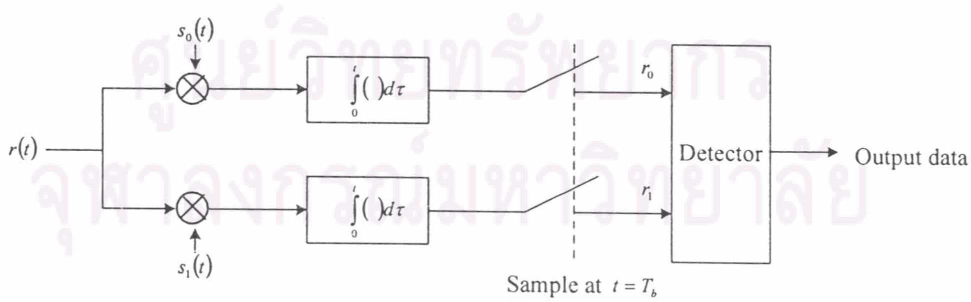

Figure 2. Cross-correlation of the received signal $r(t)$ with the two transmitted signals

Matched Filter

The matched filter provides an alternative to the signal correlator for demodulating the received signal $r(t)$. A filter that is matched to the signal waveform $s(t)$, where $0 \le t \le T_b$, has an impulse response

$$h(t) = s(T_b - t), \quad 0 \le t \le T_b \tag{33}$$

Consequently, the signal waveform say, $y(t)$ at the output of the matched filter, when the input waveform is $s(t)$, is given by the convolution integral

$$y(t) = \int_0^t s(\tau)h(t-\tau)d\tau \tag{34}$$

If we substitute in (34) for $h(t-\tau)$ from (33), we obtain

$$y(t) = \int_0^t s(\tau)s(T_b - t + \tau)d\tau \tag{35}$$

And if we sample $y(t)$ at $t = T_b$, we obtain

$$y(T_b) = \int_0^{T_b} s^2(t)dt = E \tag{36}$$

Where E is the energy of the signal $s(t)$. Therefore, the matched filter output at the sampling instant $t = T_b$ is identical to the output of the signal correlator.

Detector

The detector observes the correlator or matched filter outputs r_0 and r_1 and decides on whether the transmitted signal waveform is $s_0(t)$ or $s_1(t)$, which correspond to the transmission of either a 0 or 1, respective. The optimum detector is defined as the detector that minimizes the probability of error.

Monte Carlo Simulation of a Binary Communication System

Monte Carlo computer simulations are usually performed in practice to estimate the probability of error of a digital communication system, especially in cases where the analysis of the detector performance is difficult to perform. We demonstrate the method for estimating the probability of error for the binary communication system described above.

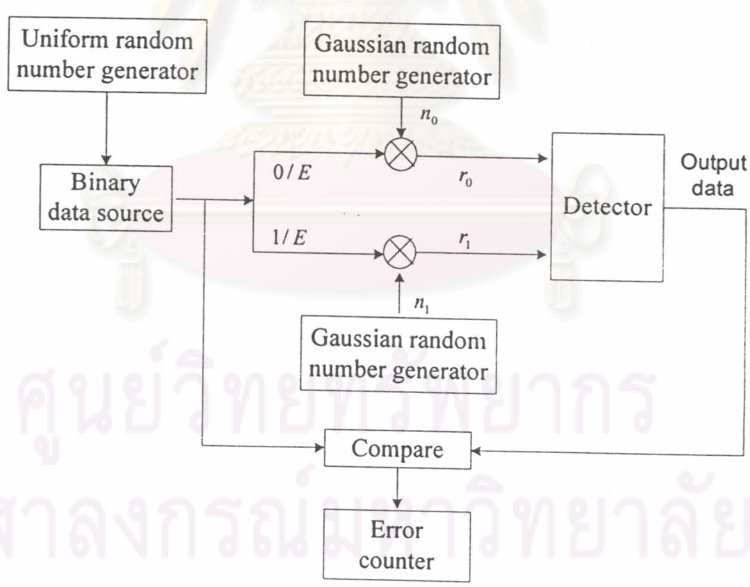

Figure 3 Simulation model for binary communication system

The Power Spectrum of a Digital PAM Signal

A digital PAM signal at the input to a communication channel is generally represented as signals.

$$V(t) = \sum_{n=-\infty}^{\infty} a_n g(t - nT) \quad (37)$$

Where $\{a_n\}$ is the sequence of amplitudes corresponding to the information symbols from the source, $g(t)$ is a pulse waveform, and T is the reciprocal of the symbol rate. T is also called the symbol interval. Each element of the sequence $\{a_n\}$ is selected from one of the possible amplitude values, which are

$$A_m = (2m - M + 1)d, \quad m = 0, 1, \cdots M - 1 \quad (38)$$

Where d is a scale factor that determines the Euclidean distance between any pair of signal amplitudes ($2d$ is the Euclidean distance between any adjacent signal amplitude levels).

Since the information sequence is random sequence, the sequence $\{a_n\}$ of amplitudes corresponding to the information symbols from the source is also random. Consequently, the PAM signal $v(t)$ is a sample function of a random process $V(t)$. To determine the spectral characteristics of the random process $V(t)$, we must evaluate the power spectrum.

First, we note that the mean value of $V(t)$ is

$$E[V(t)] = \sum_{n=-\infty}^{\infty} E(a_n) g(t - nT) \quad (39)$$

By selecting the signal amplitudes to be symmetric about zero, as given in (38), and equally probable, we have $E(a_n) = 0$ and hence $E[V(t)] = 0$.

The autocorrelation function of $V(t)$ is

$$R_v(t+\tau;t) = E[V(t)V(t+\tau)] \tag{40}$$

It is shown in many digital communications textbooks that the autocorrelation function is a periodic function in the variable t with period T. Random processes that have a periodic mean value and a periodic autocorrelation function are called periodically stationary, or cyclostationary. The time variable t can be eliminated by averaging $R_v(t+\tau;t)$ over a single period, that is,

$$\overline{R}_v(\tau) = \frac{1}{T}\int_{-T/2}^{T/2} R_v(t+\tau;t)dt \tag{41}$$

This average autocorrelation function for the PAM signal can be given by

$$\overline{R}_v(\tau) = \frac{1}{T}\sum_{n=-\infty}^{\infty} R_a(m)R_g(\tau - mT) \tag{42}$$

In which $R_a(m) = E(a_n a_{n+m})$ is the autocorrelation of the sequence $\{a_n\}$ and $R_g(\tau)$ is defined as

$$R_g(\tau) = \int_{-\infty}^{\infty} g(t)g(t+\tau)dt \tag{43}$$

The power spectrum of $V(t)$ is simply the Fourier transform of the average autocorrelation function $\overline{R}_v(\tau)$

$$S_v(\tau) = \int_{-\infty}^{\infty} \overline{R}_v(\tau) e^{-j2\pi f\tau} dt$$

$$= \frac{1}{T} S_a(f)|G(f)|^2 \quad (44)$$

where $S_a(f)$ is the power spectrum of amplitude sequence $\{a_n\}$ and $G(f)$ is the Fourier transform of the pulse $g(t)$. $S_a(f)$ is defined as

$$S_a(f) = \sum_{m=-\infty}^{\infty} R_a(m) e^{-j2\pi fmT} \quad (45)$$

Characterization of Bandlimited Channel and Channel Distortion

Many communication channels, including telephone channels and some radio channels, may be generally characterized as bandlimited linear filters. Consequently, such channels are described by their frequency response $C(f)$, which may be given by

$$C(f) = A(f) e^{j\theta(f)} \quad (46)$$

Where $A(f)$ is called the amplitude response and $\theta(f)$ is called the phase response. Another characteristic that is sometimes used in place of the phase response is the envelop delay, or group delay, which is defined as

$$\tau(f) = -\frac{1}{2\pi} \frac{d\theta(f)}{df} \quad (47)$$

A channel is said to be nondistoring, or ideal, if, within the bandwidth W occupied by the transmitted signal, $A(f)$ = constant and $\theta(f)$ is a linear function of frequency (or the envelope delay $\tau(f)$ = constant). On the other hand, if $A(f)$ and $\tau(f)$ are not constant within the bandwidth occupied by the transmitted signal, the channel distorts the signal. If $A(f)$ is not constant, the distortion is called amplitude distortion, if $\tau(f)$ is not constant, the distortion on the transmitted signal is called delay distortion.

As a result of the amplitude and delay distortion caused by the non-ideal channel frequency response characteristic $C(f)$, a succession of pulses transmitted through the channel at rates comparable to the bandwidth W are smeared to the point that they are no longer distinguishable as well-defined pulses at the receiving terminal. Instead, they overlap, so we have intersymbol interference.

Characterization of Intersymbol Interference

In digital communication system, channel distortion causes intersymbol interference (ISI). In this section, we shall present a model that characterizes ISI. For simplicity, we assume that the transmitted signal is a baseband PAM signal.

The transmitted PAM signal is given by.

$$s(t) = \sum_{n=0}^{\infty} a_n g(t - nT) \tag{48}$$

Where $g(t)$ is the basic pulse that is selected to control the spectral characteristics of the transmitted signal, $\{a_n\}$ is the sequence of transmitted information symbols selected from a signal constellation consisting of M points, and T is the signal interval ($1/T$ is the symbol rate).

The signal $s(t)$ is transmitted over a baseband channel, which may be characterized by a frequency response $C(f)$. Consequently, the received signal can be represented as

$$r(t) = \sum_{n=0}^{\infty} a_n h(t - nT) + w(t) \tag{49}$$

Where $h(t) = g(t) * c(t)$, $c(f)$ is the impulse response of the channel, $*$ denotes convolution, and $w(t)$ represents the additive noise in the channel. To characterize ISI, suppose that the received signal is passed through a receiving filter and then sampled at the rate $1/T$ samples/seconds. In general, the optimum filter at the receiver is matched to the received signal pulse $h(t)$. Hence, the frequency response of this filter is $H^*(f)$. We denote its output as

$$y(t) = \sum_{n=0}^{\infty} a_n x(t - nT) + v(t) \tag{50}$$

Where $x(t)$ is the signal pulse response of the receiving filter, i.e., $X(f) = H(f)H^*(f) = |H(f)|^2$, and $v(t)$ is the response of the receiving filter to the noise $w(t)$. Now, if $y(t)$ is sampled at times $t = kT$, $k = 0, 1, ,2, \ldots, ,$ we have

$$y(kT) = \sum_{n=0}^{\infty} a_n x(kt - nT) + v(kT)$$

$$y_k = \sum_{n=0}^{\infty} a_n x_{k-n} + v_k \tag{51}$$

The sample values $\{y_k\}$ can be expressed as

$$y_k = x_0 \left(a_k + \frac{1}{x_0} \sum_{\substack{n=0 \\ n \neq k}}^{\infty} a_n x_{k-n} \right) + v_k, \qquad k = 0, 1, \ldots \tag{52}$$

The term x_0 is an arbitrary scale factor, which we set equal to unity for convenience. Then

$$y_k = a_k + \frac{1}{x_0}\sum_{\substack{n=0 \\ n\neq k}}^{\infty} a_n x_{k-n} + v_k \tag{53}$$

The term a_k represents the desired information symbol at the k^{th} sampling instant, the term

$$\sum_{\substack{n=0 \\ n\neq k}}^{\infty} a_n x_{k-n} \tag{54}$$

Represents the ISI, and a_k is the additive noise at the k^{th} sampling instant.

Communication System Design for Bandlimited Channels

In this section we consider the design of the transmitter and receiver filters that are suitable for a baseband bandlimited and meet the zeros ISI condition. In this case we assume that the channel is ideal; i.e., $A(f)$ and $\tau(f)$ are constant within the channel bandwidth W. For simplicity, we assume that $A(f)=1$ and $\tau(f)=0$.

Signal Design for Zero ISI

The design of bandlimited signals with zero ISI was a problem considered by Nyquist about 70 years ago. He demonstrated that a necessary and sufficient condition for a signal $x(t)$ to have zero ISI, i.e.,

$$x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \tag{55}$$

Is that its Fourier transform $X(f)$ satisfy

$$\sum_{m=-\infty}^{\infty} X\left(f+\frac{m}{T}\right)=T \tag{56}$$

where $1/T$ is the symbol rate.

In general, there are many signals that can be designed to have this property. One of the most commonly used signals in practice has a raised-cosine frequency response characteristic, which is defined as

$$X_{rc}(f)=\begin{cases} T, & 0<|f|\leq\frac{1-\alpha}{2T} \\ \frac{T}{2}\left[1+\cos\frac{\pi T}{\alpha}\left(|f|-\frac{1-\alpha}{2T}\right)\right], & \frac{1-\alpha}{2T}<|f|\leq\frac{1+\alpha}{2T} \\ 0, & |f|>\frac{1+\alpha}{2T} \end{cases} \tag{57}$$

Where α is called the roll off factor, which takes values in the range $0<\alpha\leq 1$, and $1/T$ is the symbol rate. The frequency response $X_{rc}(f)$ is illustrated in figure 4(a) for $\alpha=0$, $\alpha=\frac{1}{2}$, and $\alpha=1$. Note that when $\alpha=0$, $X_{rc}(f)$ reduces to an ideal "brick wall," physically nonrealizable frequency response with bandwidth occupancy $1/2T$. The frequency $1/2T$ is called the Nyquist frequency. For $\alpha>0$, the bandwidth occupied by the desired signal $X_{rc}(f)$ beyond the Nyquist frequency. For example, when $\alpha=\frac{1}{2}$, the excess bandwidth is 50, and when $\alpha=1$, the excess bandwidth is 100. The signal pulse $x_{rc}(f)$ having the raised-cosine spectrum is

$$x_{rc}(f)=\frac{\sin \pi t/T}{\pi t/T}\frac{\cos(\pi\alpha t/T)}{1-4\alpha^2t^2/T^2} \tag{58}$$

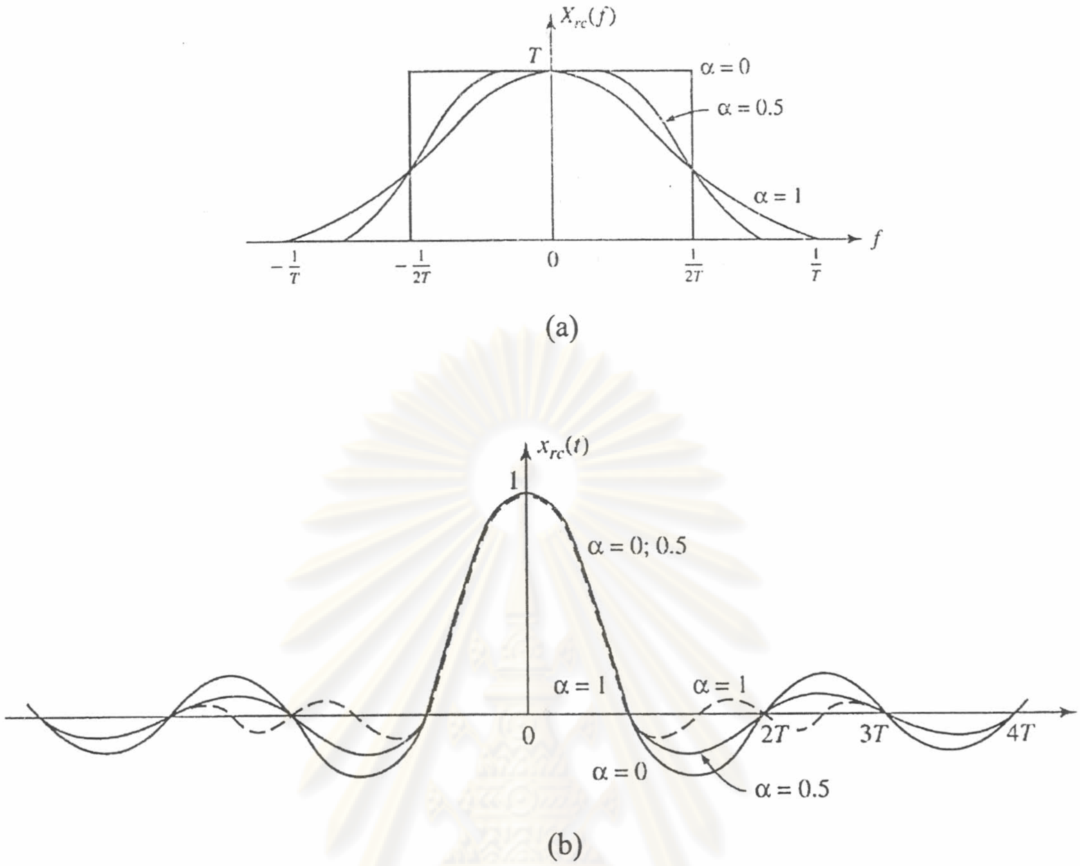

Figure 4 Raised-cosine frequency response and corresponding pulse shape. (a) Raised-cosine frequency response. (b) Pulse shapes for raised-cosine frequency response.

Figure 4(b) illustrates $x_{rc}(f)$ for $\alpha = 0, \frac{1}{2}, 1$. Since $X_{rc}(f)$ satisfies (56), we note that $x_{rc}(f) = 1$ at $t = 0$ and $x_{rc}(f) = 0$ at $t = kT, k = \pm 1, \pm 2, \ldots$. Consequently, at the sampling instants $t = kT, k \neq 0$, there is no ISI from adjacent symbols when there is no channel distortion. However, in the presence of channel distortion, the ISI given by (2.54) is no longer zero, and a channel equalizer is needed to minimize its effect on system performance.

In an ideal channel, the transmitter and receiver filters are jointly designed for zero ISI at the desired sampling instants $t = nT$. Thus, if $G_T(f)$ is the frequency responses of the transmitter filter and $G_R(f)$ is the frequency response of the receiver filter, then the product (cased of the two filters) $G_T(f)\, G_R(f)$ is designed to yield zero ISI. For example, if the product $G_T(f)\, G_R(f)$ is selected as

$$G_T(f)G_R(f) = X_{rc}(f) \tag{59}$$

where $X_{rc}(f)$ the raised-cosine frequency response characteristic, then the ISI is at the sampling times $t = nT$ is zero.

Carrier-Amplitude Modulation

In baseband digital PAM, the signal waveforms have the form

$$s_m(t) = A_m g_T(t) \tag{63}$$

where A_m is the amplitude of the m^{th} waveforms and $g_T(t)$ is a pulse whose shape determines the spectral characteristics of the transmitted signal. The spectrum of the baseband signals is assumed to be contained in the frequency band $|f| \leq W$, where W is the bandwidth of $|G_T(f)|^2$, as illustrated in figure 5. Recall that the signal amplitude takes the discrete values

$$A_m = (2m - 1 - M)d \tag{64}$$

where $2d$ is the Euclidean distance between two adjacent signal points.

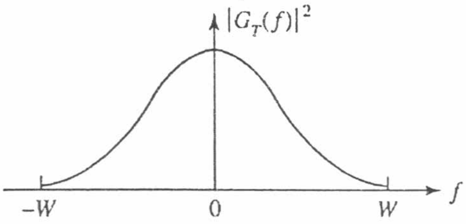

Figure 5 Energy density spectrum of the transmitted signal $g_T(t)$

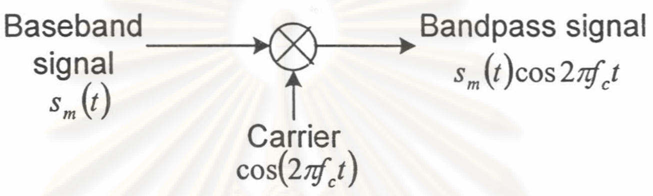

Figure 6 Amplitude modulation of a sinusoidal carrier by the baseband PAM signal

To transmit the digital signal waveforms through a baseband channel, the baseband signal waveforms $s_m(t), \quad m = 1, 2, \ldots M$, are multiplied by a sinusoidal carrier of the form $\cos 2\pi f_c t$, as shown in figure 6, where f_c is the carrier frequency ($f_c > W$) and corresponds to the center frequency in the passband of the channel. Hence, the transmitted signal waveforms are expressed as

$$u_m(t) = A_m g_T(t)\cos 2\pi f_c t, \quad m = 1, 2, \ldots M \tag{65}$$

In the special case when the transmitted pulse shape $g_T(t)$ is rectangular that is

$$g_T(t) = \begin{cases} \sqrt{\dfrac{2}{T}}, & 0 \le t \le T \\ 0, & otherwise \end{cases} \tag{66}$$

The amplitude modulation carrier signal is usually called amplitude shift keying (ASK). In this case the PAM signal is not bandlimited.

Amplitude modulation of the carrier $\cos 2\pi f_c t$ by the baseband signal waveforms $s_m(t)$ shifts the spectrum of the baseband signal by an amount f_c and, thus, places the signal into the passband of the channel. Recall that the Fourier transform of the carrier is $[\delta(f - f_c) + \delta(f + f_c)]/2$. Since multiplication of two signals in the time domain corresponds to the convolution of their spectra in the frequency domain, the spectrum of the amplitude modulated signals is

$$U_m(f) = \frac{A_m}{2}[G_T(f - f_c)G_T(f + f_c)] \tag{67}$$

Thus, the spectrum of the baseband signal $s_m(t) = A_m g_T(t)$ is shifted in frequency by the carrier frequency f_c. The bandpass signal is a double-sideband suppressed-carrier (DSB-SC) AM signal, as illustrated in figure 7.

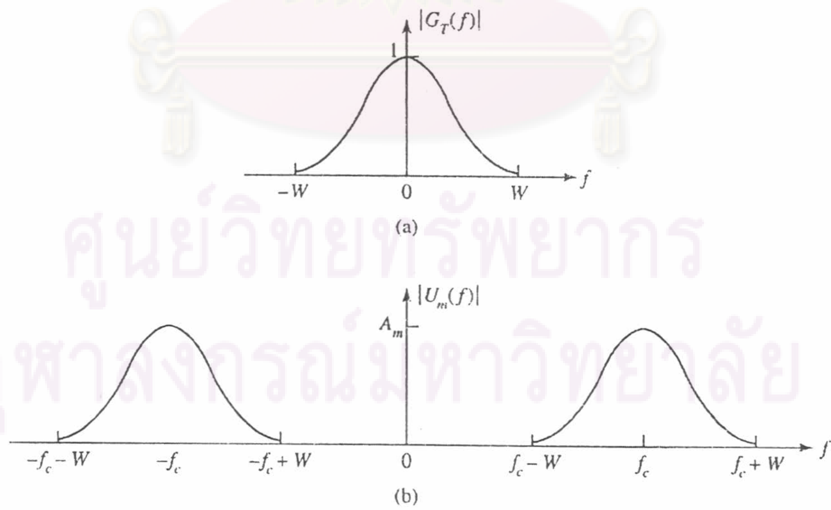

Figure 7 Spectra of (a) baseband and (b) amplitude modulated signals

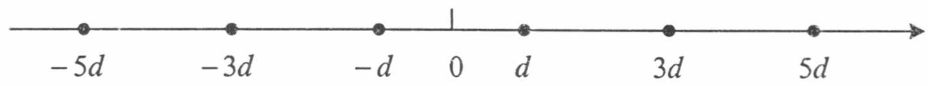

Figure 8 Signal point constellation for PAM signal

We note that impressing the baseband signal $s_m(t)$ onto the amplitude of the carrier signal $\cos 2\pi f_c t$ does not change the basic geometric representation of the digital PAM signal waveforms. The bandpass PAM signal waveforms may be represented in general as

$$u_m(t) = s_m \psi(t) \tag{68}$$

where the signal waveform $\psi(t)$ is defined as

$$\psi(t) = g_T(t)\cos 2\pi f_c t \tag{69}$$

and

$$s_m = A_m, \quad m = 1, 2, \ldots, M \tag{70}$$

denotes the signal points that take the M values on the real line, as shown in figure 8.

The signal waveform $\psi(t)$ is normalized to unit energy; that is,

$$\int_{-\infty}^{\infty} \psi(t)^2 \, dt = 1 \tag{71}$$

Consequently,

$$\int_{-\infty}^{\infty} g_T^2(t)\cos^2 2\pi f_c t dt = \frac{1}{2}\int_{-\infty}^{\infty} g_T^2(t) dt + \frac{1}{2}\int_{-\infty}^{\infty} g_T^2(t)\cos 4\pi f_c t dt \tag{72}$$

But

$$\int_{-\infty}^{\infty} g_T^2(t)\cos 4\pi f_c t \, dt = 0 \qquad (73)$$

Because the bandwidth w of $g_T(t)$ is much smaller than the carrier frequency that is, $f_c << W$. In such a case, $g_T(t)$ is essentially constant within any one cycle of $\cos 4\pi f_c t$; hence, the integral in (2.73) is equal to zero for each cycle of the integrand. In view of (2. 73), it follows that

$$\frac{1}{2}\int_{-\infty}^{\infty} g_T^2 dt = 1 \qquad (74)$$

Therefore, $g_T(t)$ must be appropriately scaled so that (2.71) and (2.74) are satisfied.

Demodulation of PAM Signals

The demodulation of a bandpass digital PAM signal may be accomplished in one of several ways by means of correlation or matched filtering. For illustrative purpose we consider a correlation type demodulator.

The received signal may be expressed as

$$r(t) = A_m g_T(t)\cos 2\pi f_c t + n(t) \qquad (75)$$

where $n(t)$ is a bandpass noise process, which is represented as

$$n(t) = n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \qquad (2.76)$$

And where $n_c(t)$ and $n_s(t)$ are the quadrature components of the noise. By cross-correlating the received signal $r(t)$ with $\psi(t)$ given by (2.69), as shown in figure 2.9, we obtain the output

$$\int_{-\infty}^{\infty} r(t)\psi^{*}(t)dt = A_m + n = s_m + n \tag{77}$$

where n represents the additive noise component at the output of the correlator.

The noise component has a zero mean. Its variance can be expressed as

$$\sigma_n^2 = \int_{-\infty}^{\infty} |\psi(f)|^2 S_n(f)df \tag{78}$$

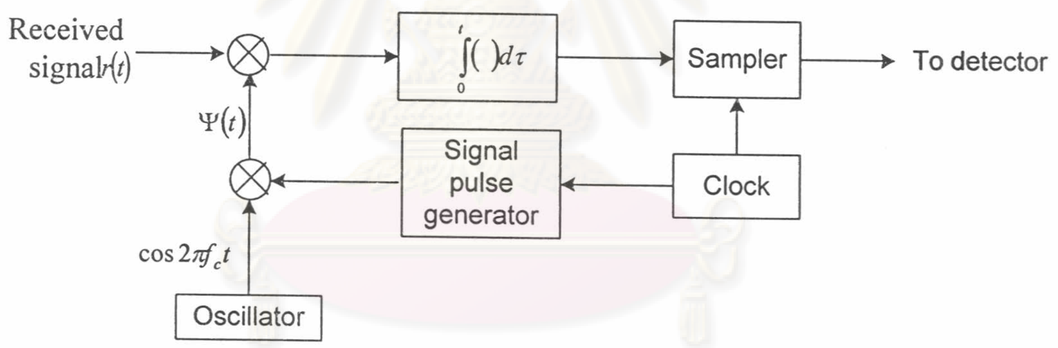

Figure 9 Demodulation of bandpass digital PAM signal

where $\Psi(f)$ is the Fourier transform of $\psi(t)$ and $S_n(f)$ is the power spectral density of the additive noise. The Fourier transform of $\psi(t)$ is

$$\Psi(f) = \frac{1}{2}[G_T(f - f_c)G_T(f + f_c)] \tag{79}$$

And the power spectral density of the bandpass additive noise process is

$$S_n(f) = \begin{cases} \frac{N_0}{2}, & |f - f_c| \le W \\ 0, & otherwise \end{cases} \tag{80}$$

By substitute (79) and (80) into (78) and evaluating the integral, we obtain $\sigma_n^2 = N_0 / 2$.

It is apparent form (77), which is the input to the amplitude detector, that the probability of error of the optimum detector for the carrier modulate PAM signal is identical to that of baseband PAM. That is,

$$P_M \frac{2(M-1)}{M} Q\left(\sqrt{\frac{6(\log_2 M)E_{avb}}{(M^2-1)N_0}}\right) \tag{81}$$

where E_{avb} is the average energy per bit.

Carrier-Phase Modulation

In carrier phase modulation the information that is transmitted over a communication channel is impressed on the phase of the carrier. Since the range of the carrier phase is $0 \le \theta < 2\pi$, the carrier phases used to transmit digital information via digital phase modulation are $\theta_m = 2\pi m / M$, for $m = 0, 1, 2, \ldots M-1$. Thus, for binary phase modulation $(M = 2)$, the two carrier phases are $\theta_0 = 0$ and $\theta_1 = \pi$ rad. For M -ary phase modulation, $M = 2^k$, where k is the number of information bits transmitted symbol.

The general representation of a set of M carrier phase modulated signal waveforms is

$$u_m(t) = Ag_T(t)\cos\left(2\pi f_c t + \frac{2\pi m}{M}\right), \quad m = 0, 1, \ldots M-1 \tag{82}$$

where $g_T(t)$ is the transmitting filter pulse shape, which determines the spectral characteristics of the transmitted signal and A is the signal amplitude. This type of digital phase modulation is called phase shift keying (PSK). We note that PSK signals have equal energy; that is,

$$E_m = \int_{-\infty}^{\infty} u_m^2(t)dt \tag{83}$$

$$= \int_{-\infty}^{\infty} A^2 g_T^2(t)\cos^2\left(2\pi f_c t + \frac{2\pi m}{M}\right)dt$$

$$= \frac{1}{2}\int_{-\infty}^{\infty} A^2 g_T^2(t)dt + \frac{1}{2}\int_{-\infty}^{\infty} A^2 g_T^2(t)\cos^2\left(4\pi f_c t + \frac{4\pi m}{M}\right)dt$$

$$= \frac{A^2}{2}\int_{-\infty}^{\infty} g_T^2(t)dt \tag{84}$$

$$\equiv E_s, \quad for\ all\ m \tag{85}$$

Where E_s denotes the energy per transmitted symbol. The term involving the double frequency component in (83) averages out to zeros when $f_c >> W$, where W is the bandwidth of $g_T(t)$.

When $g_T(t)$ is a rectangular pulse, it is defined as

$$g_T(t) = \sqrt{\frac{2}{T}}, \quad 0 \le t \le T \tag{86}$$

In this case, the transmitted signal waveforms in the symbol interval $0 \le t \le T$ may be expressed as (with $A = \sqrt{E_s}$)

$$u_m(t) = \sqrt{\frac{2E_s}{T}} \cos\left(2\pi f_c t + \frac{2\pi m}{M}\right), \quad m = 0, 1, \ldots M-1 \tag{87}$$

Note that the transmitted signals given by (87) have a constant envelope, and the carrier phase changes abruptly at the beginning of each signal interval. Figure 11 illustrates a four phase $(M = 4)$ PSK signal waveform.

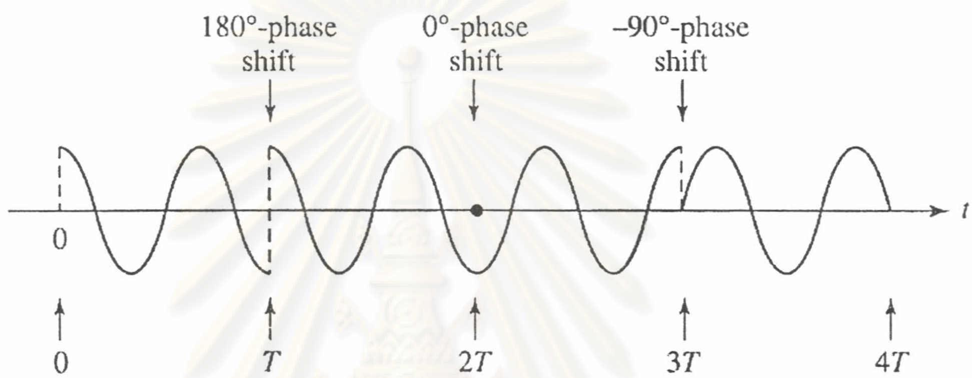

Figure 10 Example of a four phase PSK signal

By viewing the angle of the cosine function in (87) as the sum of two angles, we may express the waveforms in (82) as

$$\begin{aligned} u_m(t) &= \sqrt{E_z}\, g_T(t) \cos\left(\frac{2\pi m}{M}\right) \cos 2\pi f_c t - \sqrt{E_z}\, g_T(t) \sin\left(\frac{2\pi m}{M}\right) \sin 2\pi f_c t \\ &= s_{mc}\psi_1(t) + s_{ms}\psi_2(t) \end{aligned} \tag{88}$$

Where

$$\begin{aligned} s_{mc} &= \sqrt{E_z} \cos\left(\frac{2\pi m}{M}\right) \\ s_{ms} &= \sqrt{E_z} \sin\left(\frac{2\pi m}{M}\right) \end{aligned} \tag{89}$$

And $\psi_1(t)$ and $\psi_2(t)$ are orthogonal basis functions defined as

$$\psi_1(t) = g_T(t)\cos 2\pi f_c t$$
$$\psi_2(t) = -g_T(t)\sin 2\pi f_c t \qquad (90)$$

By appropriately normalizing the pulse shape $g_T(t)$, we can normalize the energy of these two basis functions to unity. Thus, a phase modulated signal may be viewed as two quadrature carriers with amplitudes that depend on the transmitted phase in each signal interval. Hence, digital phase modulated signals are represented geometrically as two-dimensional vectors with components s_{mc} and s_{ms} that is,

$$s_m = \left(\sqrt{E_s}\cos\left(\frac{2\pi m}{M}\right) \quad \sqrt{E_s}\sin\left(\frac{2\pi m}{M}\right)\right) \qquad (91)$$

Signal point constellations for M = 2, 4, and 8 are illustrated in figure 12. We observe that binary phase modulation is identical to binary PAM (binary antipodal signals).

The mapping, or assignment, of k information bits into the $M = 2^k$ possible phases may be done in a number of ways. The preferred assignment is to use Gray encoding, in which adjacent phases differ by one binary digit, as illustrated in figure 12. Consequently, only a single bit error occurs in the k-bit sequence with Gray encoding when noise causes the erroneous selection of an adjacent phase to the transmitted phase.

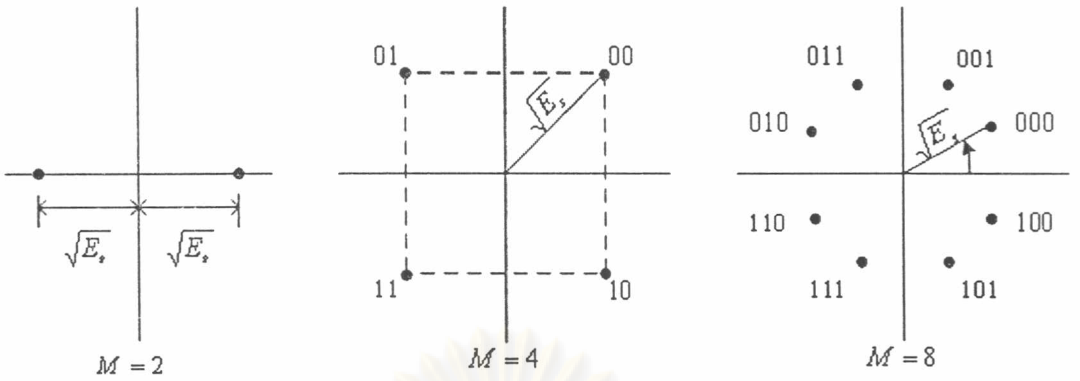

Figure 11 PSK signal constellations

Phase Demodulation and Detection

The received bandpass signal in a signaling interval from an AWGN channel may be expressed as

$$\begin{aligned} r(t) &= u_m(t) + n(t) \\ &= u_m(t) + n_c(t)\cos 2\pi f_c t - n_s(t)\sin 2\pi f_c t \end{aligned} \qquad (92)$$

where $n_c(t)$ and $n_s(t)$ are the two quadrature components of the additive noise.

The received signal may be correlated with $\psi_1(t)$ and $\psi_2(t)$ given by (90). The outputs of the correlators yield the noise corrupted signal components, which may be expressed as

$$\begin{aligned} r &= s_m + n \\ &= \left(\sqrt{E_s}\cos\frac{2\pi m}{M} + n_c \quad \sqrt{E_s}\sin\frac{2\pi m}{M} + n_s \right) \end{aligned} \qquad (93)$$

Where n_c and n_s are defined as

$$n_c = \frac{1}{2}\int_{-\infty}^{\infty} g_T(t) n_c(t) dt$$

$$n_s = \frac{1}{2}\int_{-\infty}^{\infty} g_T(t) n_s(t) dt \qquad (94)$$

The quadrature noise components $n_c(t)$ and $n_s(t)$ are zero-mean Gaussian random processes that are uncorrelated. As a consequence, $E(n_c) = E(n_s) = 0$ and $E(n_c n_s) = 0$. The variance of n_c and n_s is

$$E(n_c^{\,2}) = E(n_s^{\,2}) = \frac{N_0}{2} \qquad (95)$$

The optimum detector projects the received signal vector r onto each of the M possible transmitted signal vectors $\{s_m\}$ and selects the vector corresponding to the largest projection. Thus, we obtain the correlation metrics

$$C(r, s_m) = r \cdot s_m, \quad m = 0, 1, \ldots M - 1 \qquad (96)$$

Because all signals have equal energy, an equivalent detector metric for digital phase modulation is to compute the phase of the received signal vector $r = (r_1, r_2)$ as

$$\theta_r = tah^{-1} \frac{r_2}{r_1} \qquad (97)$$

And select the signal from the set $\{s_m\}$ whose phase is closest to θ_r.

The probability of error at the detector for phase modulation in an AWGN channel may be found in any textbook relevant to digital communications. Since binary phase modulation is identical to binary PAM, the probability of error is

$$P_2 = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{98}$$

where E_b is the energy per bit. Four phase modulations may be viewed as two binary phase modulation systems on quadrature (orthogonal) carriers. Consequently, the probability of a bit error is identical to that for binary phase modulation. For $M > 4$, there is no simple closed form expression for binary phase modulation. A good approximation for P_M is

$$\begin{aligned} P_M &\approx 2Q\left(\sqrt{\frac{2E_s}{N_0}}\sin\frac{\pi}{M}\right) \\ &\approx 2Q\left(\sqrt{\frac{2kE_b}{N_0}}\sin\frac{\pi}{M}\right) \end{aligned} \tag{99}$$

where $k = \log_2 M$ bits per symbol.

The equivalent bit-error probability for M-ary phase modulation is also difficult to derive due to the dependence of the mapping of k-bit symbols into the corresponding signal phases. When a Gray code is used in the mapping, two k-bit symbols corresponding to adjacent signal phases differ in only a single bit. Because the most probable errors due to noise result in the erroneous selection of an adjacent phase to the true phase, most k-bit symbol errors contain only a single bit error. Hence, the equivalent bit-error probability for M-ary phase modulation is well approximated as

$$P_b \approx \frac{1}{k} P_M \tag{100}$$

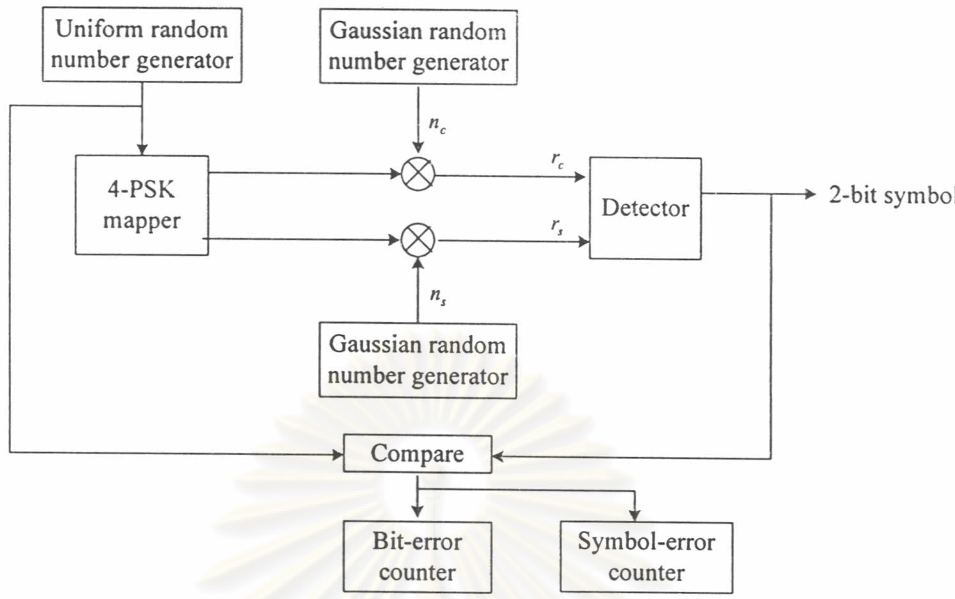

Figure 12 Block diagram of an M = 4 PSK system for a Monte Carlo simulation.

Carrier-Frequency Modulation

We have described methods for transmitting digital information by modulating either the amplitude of the carrier, the phase of the carrier, or the combined amplitude and phase. Digital information can also be transmitted by modulating the frequency of the carrier.

As we will observe from our treatment below, digital transmission by frequency modulation is a modulation method that is appropriate for channels that lack the phase stability that is necessary to perform carrier phase estimation. In contrast, the linear modulation methods that we have introduced namely, PAM, coherent PSK, and QAM require the estimation of the carrier phase to perform phase coherent detection.

Frequency-Shift Keying

The simplest form of frequency modulation is binary frequency shift keying (FSK). In binary FSK we employ two different frequency, say, f_1 and $f_2 = f_1 + \Delta f$, to transmit a binary information sequence. The choice of frequency separation $\Delta f = f_2 - f_1$ is considered later. Thus the two signal waveforms may be expressed as

$$u_1(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_1 t, \quad 0 \le t \le T_b$$

$$u_2(t) = \sqrt{\frac{2E_b}{T_b}} \cos 2\pi f_2 t, \quad 0 \le t \le T_b \qquad (101)$$

where E_b is the signal energy per bit and T_b is the duration of the bit interval.

More generally, M-ary FSK may be used to transmit a block of $k = \log_2 M$ bits per signal waveform. In this case, the M signal waveforms may be expressed as

$$u_m(t) = \sqrt{\frac{2E_s}{T_b}} \cos(2\pi f_c t + 2\pi m \Delta f t), \quad m = 0, 1, \ldots M - 1, \quad 0 \le t \le T_b \qquad (102)$$

where $E_s = kE_b$ is the energy per symbol, $T = kT_b$ is the symbol interval, and Δf is the frequency separation between successive frequencies that is, $\Delta f = f_m - f_{m-1}$ for all $m = 0, 1, \ldots M - 1$, where $f_m = f_c - m\Delta f$.

Demodulation and Detection of FSK Signals

Let us assume that the FSK signals are transmitted through an additive white Gaussian noise channel. Furthermore, we assume that each signal is delayed in the transmission through the channel. Consequently, the filtered received signal at the input to the demodulator may be expressed as

$$r(t)=\sqrt{\frac{2E_s}{T_b}}\cos(2\pi f_c t+2\pi m\Delta f t+\phi_m)+n(t) \tag{103}$$

where ϕ_m denotes the phase shift of the mth signal (due to the transmission delay) and $n(t)$ represents the additive bandpass noise, which may be expressed as

$$n(t)=n_c(t)\cos 2\pi f_c t-n_s(t)\sin 2\pi f_c t \tag{104}$$

The demodulation and detection of the M FSK signals may be accomplished by one of two methods. One approach is to estimate the M carrier phase shifts $\{\phi_m\}$ and perform phase coherent demodulation and detection. As an alternative method, the carrier phases may be ignored in the demodulation and detection.

In phase coherent demodulation, the received signal $r(t)$ is correlated with each of the M possible received signals, $\cos(2\pi f_c t+2\pi m\Delta f t+\hat{\phi}_m)$, for $m=0,1,\ldots M-1$, where $\hat{\phi}_m$ are the carrier phase estimates. A block diagram illustrating this type of demodulation is shown in figure 15. it is interesting to note that when $\hat{\phi}_m\neq\phi_m$ for $m=0,1,\ldots M-1$ (imperfect phase estimates), the frequency separation required for signal orthogonality at the demodulator is $\Delta f=1/T$, which is twice the minimum separation for orthogonality when $\phi=\hat{\phi}$.

The requirement of estimating M carrier phases makes coherent demodulation of FSK signals extremely complex and impractical, especially when the number of signals is large. Therefore, we shall not consider coherent detection of FSK signals.

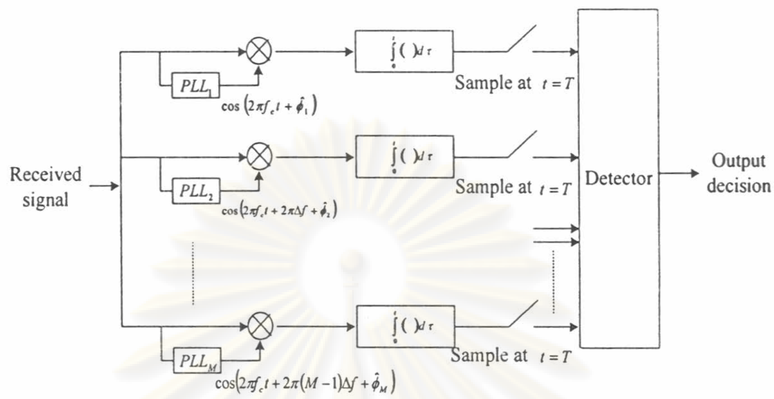

Figure 13 Phase-coherent demodulation of M-ary FSK signals

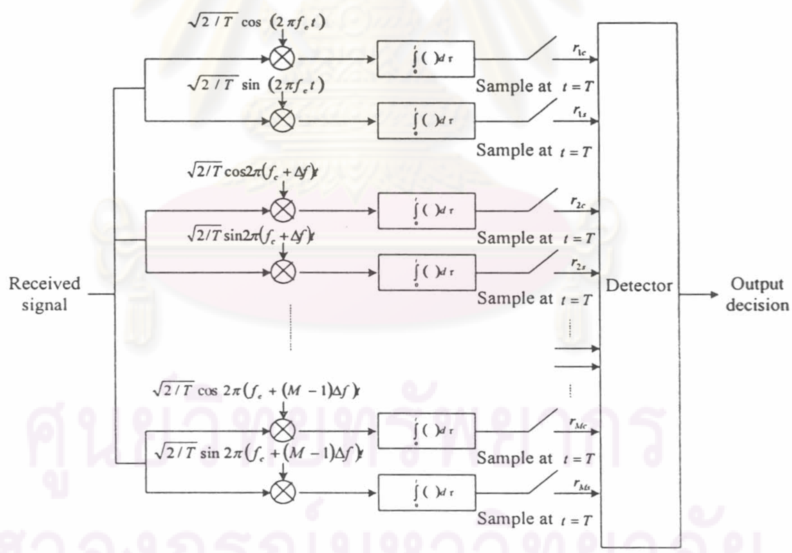

Figure 14 Demodulation of M-ary signals for noncoherent detection

Instead, we consider a method for demodulation and detection that does not require knowledge of the carrier phases. The demodulation may be accomplished as shown in figure 2.14. In this case, there are two correlators per signal waveform, or a total of 2M correlators, in general. The received signal is correlated with the basis functions (quadrature carriers) $\sqrt{2/T}\cos(2\pi f_c t + 2\pi m\Delta f t)$ and $\sqrt{2/T}\sin(2\pi f_c t + 2\pi m\Delta f t)$, for $m = 0, 1, \ldots M-1$. The 2M outputs of the correlators are sampled at the end of the signal interval and are passed to the detector. Thus, if the mth signal is transmitted, the 2M samples at the detector may be expressed as

$$r_{kc} = \sqrt{E_s}\left[\frac{\sin 2\pi(k-m)\Delta f T}{2\pi(k-m)\Delta f T}\cos\phi_m - \frac{\cos 2\pi(k-m)\Delta f T - 1}{2\pi(k-m)\Delta f T}\sin\phi_m\right] + n_{kc}$$

$$r_{ks} = \sqrt{E_s}\left[\frac{\cos 2\pi(k-m)\Delta f T - 1}{2\pi(k-m)\Delta f T}\cos\phi_m - \frac{\sin 2\pi(k-m)\Delta f T}{2\pi(k-m)\Delta f T}\sin\phi_m\right] + n_{ks} \quad (105)$$

Where n_{kc} and n_{ks} denote the Gaussian noise components in the sampled outputs.

We observe that when $k = m$, the sampled values to the detector are

$$r_{mc} = \sqrt{E_s}\cos\phi_m + n_{mc}$$

$$r_{ms} = \sqrt{E_s}\cos\phi_m + n_{ms} \quad (106)$$

Furthermore, we observe that when $k \neq m$, the signal components in the samples r_{kc} and r_{ks} will vanish, independent of the values of the phase shift ϕ_k, provided that the frequency separation between successive frequencies is $\Delta f = 1/T$. In such a case, the other $2(M-1)$ correlator outputs consist of noise only that is,

$$r_{kc} = n_{kc}, \quad r_{ks} = n_{ks}, \quad k \neq m \quad (107)$$

In the following development we assume that $\Delta f = 1/T$, so that the signals are orthogonal.

It can be shown that the 2M noise samples $\{n_{kc}\}$ and $\{n_{ks}\}$ are zero mean, mutually uncorrelated Gaussian random variables with equal variance $\sigma^2 = N_0/2$. Consequently, the joint probability density function for r_{mc} and r_{ms} conditioned on ϕ_m is

$$f_{r_m}\left(r_{mc}, r_{ms} \mid \phi_m\right) = \frac{1}{2\pi\sigma^2} e^{-\left[\left(r_{mc}-\sqrt{E_s}\cos\phi_m\right)^2+\left(r_{ms}-\sqrt{E_s}\sin\phi_m\right)^2\right]/2\sigma^2} \tag{108}$$

And for $m \neq k$, we have

$$f_{r_k}\left(r_{kc}, r_{ks}\right) = \frac{1}{2\pi\sigma^2} e^{-\left[r_{kc}^2+r_{ks}^2\right]/2\sigma^2} \tag{109}$$

Given the 2M observed random variables $\{r_{kc}, r_{ks}\}_{k=0}^{M-1}$, the optimum detector selects the signal that corresponds to the maximum of the posterior probabilities that is,

$$P\left[s_m \; was \; transmitted \mid r\right] \equiv P\left(s_m \mid r\right), \quad m = 0, 1, \ldots, M-1 \tag{110}$$

Where r is the 2M-dimensional vector with elements $\{r_{kc}, r_{ks}\}_{k=0}^{M-1}$. When the signals are equally probable, the optimum detector specified by (115) computes the signal envelopes, defined as

$$r_m = \sqrt{r_{mc}^2 + r_{ms}^2}, \quad m = 0, 1, \ldots, M-1 \tag{111}$$

and selects the signal corresponding to the largest envelope of the set $\{r_m\}$. In this case the optimum detector is called an envelope detector.

An equivalent detector is one that computes the squared envelopes

$$r_m^2 = r_{mc}^2 + r_{ms}^2, \quad m = 0, 1, \ldots, M-1 \tag{112}$$

and selects the signal corresponding to the largest, $\{r_m^2\}$. In this case, the optimum detector is called a square law detector.

BIOGRAPHY

Chaymaly Phakasoum was born in 1982 in Vientiane province, Laos PDR. She received her Dip. Ing. in Electronics Engineering from National University of Laos (NUOL), Laos PDR in 2004 and started studying for the Master's degree in Electrical Engineering Chulalongkorn University, Thailand in the same year.