

การหาตำแหน่งของลักษณะเด่นบนใบหน้าโดยปราศจากการพิจารณาการปรากฏของเนื้อภาพ



นายศุภกานต์ พิมลธเรศ

ศูนย์วิทยทรัพยากร  
จุฬาลงกรณ์มหาวิทยาลัย

วิทยานิพนธ์นี้เป็นส่วนหนึ่งของการศึกษาตามหลักสูตรปริญญาวิทยาศาสตรดุษฎีบัณฑิต

สาขาวิชาวิทยาการคอมพิวเตอร์ ภาควิชาคณิตศาสตร์

คณะวิทยาศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

ปีการศึกษา 2547

ISBN 974-17-6472-3

ลิขสิทธิ์ของจุฬาลงกรณ์มหาวิทยาลัย

FACIAL FEATURE LOCALIZATION WITHOUT CONSIDERING  
THE APPEARANCE OF IMAGE CONTEXT

Mr.Suphakant Phimoltares

ศูนย์วิทยทรัพยากร

A Dissertation Submitted in Partial Fulfillment of the Requirements  
for the Degree of Doctor of Philosophy in Computer Science

Department of Mathematics

Faculty of Science

Chulalongkorn University

Academic year 2004

ISBN 974-17-6472-3

Thesis Title FACIAL FEATURE LOCALIZATION WITHOUT  
CONSIDERING THE APPEARANCE OF IMAGE  
CONTEXT

By Mr. Suphakant Phimoltares


Field of study Computer Science

Thesis Advisor Professor Chidchanok Lursinsap, Ph.D.

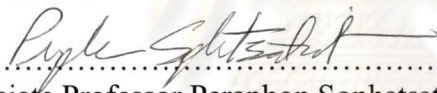
Thesis Co-advisor Associate Professor Kosin Chamnongthai, Ph.D.

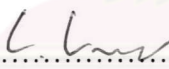
---

Accepted by the Faculty of Science, Chulalongkorn University in Partial  
Fulfillment of the Requirements for the Doctor's Degree


  
..... Deputy Dean for Administrative Affairs  
Acting Dean, The Faculty of Science  
(Associate Professor Tharapong Vitidsant, Ph.D.)


THESIS COMMITTEE


  
..... Chairman  
(Associate Professor Peraphon Sophatsathit, Ph.D.)

  
..... Thesis Advisor  
(Professor Chidchanok Lursinsap, Ph.D.)

  
..... Thesis Co-advisor  
(Associate Professor Kosin Chamnongthai, Ph.D.)

  
..... Member  
(Associate Professor Boonserm Kijisirikul, Ph.D.)

  
..... Member  
(Assistant Professor Rajalida Lipikorn, Ph.D.)

  
..... Member  
(Pirawat Watanapongse, Ph.D.)

นายศุภกานต์ พิมลธเรศ : การหาตำแหน่งของลักษณะเด่นบนใบหน้าโดยปราศจากการพิจารณาการปรากฏของเนื้อหา. (FACIAL FEATURE LOCALIZATION WITHOUT CONSIDERING THE APPEARANCE OF IMAGE CONTEXT) อ. ที่ปรึกษา : ศาสตราจารย์ ดร. ชิดชนก เหลือสินทรัพย์, อ.ที่ปรึกษาร่วม : รองศาสตราจารย์ ดร. โกสินทร์ จ้านงไทย จำนวนหน้า 92 หน้า. ISBN 974-17-6472-3.

การหาตำแหน่งของลักษณะเด่นบนใบหน้ามีบทบาทสำคัญในการประยุกต์ใช้งานต่างๆ เช่น การโต้ตอบของคนและคอมพิวเตอร์ ความระวังระไวทางภาพ (video surveillance) การติดตามใบหน้า และ การรู้จำใบหน้า ดังนั้นขั้นตอนวิธีเพื่อการหาตำแหน่งของใบหน้าและลักษณะเด่นบนใบหน้าจึงจำเป็นสำหรับการประยุกต์ใช้งานดังกล่าว วิทยานิพนธ์นี้ได้นำเสนอขั้นตอนวิธีสำหรับภาพใบหน้าทุกชนิดในสภาวะทางภาพที่หลากหลายซึ่งประกอบไปด้วยสองขั้นตอนที่สำคัญ ขั้นตอนหนึ่งเป็นการหาตำแหน่งของหน้าจากภาพต้นฉบับ การหาขอบภาพแค่นี้ (Canny edge detection) ถูกนำมาใช้เพื่อหาขอบของภาพ จากนั้นจะคัดเลือกบริเวณที่น่าจะเป็นใบหน้าจากบริเวณที่มีจำนวนของจุดภาพ (pixel) สอดคล้องกับแผ่นแบบของใบหน้า (face template) หลังจากนั้นใช้ค่าความสอดคล้องที่คำนวณได้ในการหาบริเวณที่เป็นใบหน้าจริง ขั้นตอนที่สองคือการหาตำแหน่งของลักษณะเด่นบนใบหน้าซึ่งได้นำมาใช้กับใบหน้าที่ได้จากขั้นตอนแรกโดยนำเสนอแบบจำลองภาพทางระบบประสาท (neural visual model) ในการรู้จำตำแหน่งของลักษณะเด่นบนใบหน้าที่เป็นไปได้อย่างหมด พารามิเตอร์ที่นำมาใช้กับแบบจำลองหาได้จากลักษณะพิเศษของใบหน้าและตำแหน่งของลักษณะเด่นบนใบหน้าที่เป็นอิสระจากข้อมูลของความเข้มสี หลังจากนั้นได้นำเอาวิธีการประมวลผลภาพที่เรียกว่าการพองภาพ (image dilation) มาใช้เพื่อเอาส่วนที่ไม่เกี่ยวข้องที่หลงเหลือออกเพื่อให้ได้ผลที่ดีขึ้น นอกจากนี้แล้วขั้นตอนวิธีดังกล่าวยังสามารถนำไปใช้กับปัญหาการไม่แปรเปลี่ยนแบบหมุน (rotational invariance) โดยการใช้การแปลงเรดอน (Radon transform) เพื่อหามุมหลักของใบหน้า ขั้นตอนวิธีดังกล่าวได้นำมาใช้ทดสอบกับภาพใบหน้าหลายชนิดกว่า 1,000 ภาพเช่น ใบหน้าสี, ใบหน้าเทา, ใบหน้าขาวดำ, ใบหน้าที่มีการสวมแว่นตากันแดด, ใบหน้าที่มีการสวมผ้าพันคอ, ใบหน้าที่มีการแสดงสีหน้า, ใบหน้าที่มีผลกระทบจากแสง, ใบหน้าพรมัวหรือมีสัญญาณรบกวน, ใบหน้าสีและใบหน้าที่เขียนแบบหวัดจากภาพเปรียบเทียบ (animated cartoon) ด้วยขั้นตอนวิธีดังกล่าว อัตราการค้นหา (detection rate) มีมากกว่าร้อยละ 94 โดยเฉลี่ย

ภาควิชา คณิตศาสตร์  
สาขาวิชา วิทยาการคอมพิวเตอร์  
ปีการศึกษา 2547

ลายมือชื่อนิสิต..... *Suphakant Phimoltaxes*.....  
ลายมือชื่ออาจารย์ที่ปรึกษา..... *F. L. ...*.....  
ลายมือชื่ออาจารย์ที่ปรึกษาร่วม..... *Worin. Chumongthai*.....

## 4373839723 : MAJOR COMPUTER SCIENCE

KEY WORD: FACIAL FEATURE LOCALIZATION, FACE DETECTION, FACE LOCATION, NEURAL VISUAL MODEL, NEURAL NETWORKS, CANNY EDGE DETECTION, IMAGE DILATION, RADON TRANSFORM

SUPHAKANT PHIMOLTARES : FACIAL FEATURE LOCALIZATION WITHOUT CONSIDERING THE APPEARANCE OF IMAGE CONTEXT. THESIS ADVISOR : PROF. CHIDCHANOK LURSINSAP, Ph.D., THESIS CO-ADVISOR : ASSOC. PROF. KOSIN CHAMNONGTHAI, Ph.D., 92 pp. ISBN 974-17-6472-3.

Facial feature detection plays an important role in various applications such as human computer interaction, video surveillance, face tracking, and face recognition. Efficient face and facial feature detection algorithms are required for applying to those tasks. This dissertation presents the algorithms for all types of face images in the presence of several image conditions. There are two main steps corresponding to face and facial feature detection algorithms. First, the faces are detected from an original image. Canny edge detection is applied to find the edge of the image. A candidate face region can be found from the region having the number of pixels corresponding to average face template. Then, the matching value is calculated and applied to find the actual face. Second, facial feature detection is applied to the actual face obtained from the previous step. A proposed neural visual model (NVM) is used to recognize all possibilities of facial feature locations. The input parameters are obtained from the face characteristics and the locations of facial features which are independent of the intensity information. For the better result, an image processing technique called dilation is applied to remove some irrelevant feature regions. In addition, the algorithms can be extended to cover rotational invariance problem by using Radon transform to extract the main angle of the face. With more than 1,000 experimental images, the algorithms are successfully tested on various types of faces with color intensity, gray intensity, binary intensity, object occlusion such as sunglasses, scarf, and hand, facial expression, lighting effect, noise and blurry images, as well as color and sketchy images from animated cartoon. In particular, the method achieves more than 94% detection rate on the average.

Department Mathematics

Student's signature.....*Suphakant Phimoltares*.....

Field of study Computer Science

Advisor's signature.....*L. L. L.*.....

Academic year 2004

Co-advisor's signature.....*Kosin Chamnongthai*.....

## Acknowledgments

I wish to thank the teachers who helped shape my scientific viewpoint and educational style. Two of them stand out. My thesis advisor, Prof. Dr. Chidchanok Lursinsap, is due a special note for his unique way of inspiring students through clarity of thought, enthusiasm, and caring. He is a model for me and many others. He provided a lot of the ideas for the dissertation. I am grateful to my thesis co-advisor, Assoc. Prof. Dr. Kosin Chamnongthai for introducing me to digital image processing and the art of research.

I would like to thank Assoc. Prof. Dr. Petr Musilek for providing some computing facilities and the pleasant atmosphere he created since I visited FACIA (the Facility for Advanced Computational Intelligence and Applications), University of Alberta, Canada.

Also, it is my pleasure to acknowledge the financial support from the Thailand Research Fund through the Royal Golden Jubilee Ph.D. Program.

It has been a pleasure to work with the AVIC members. They provided additional comments that help improve this dissertation and contribute to the environment of the AVIC.

Finally, I am deeply grateful to my parents, grandparents, brother and sister, and Miss Atchara Mahaweerawat for their love, support and patience.

ศูนย์วิจัยทรัพยากร  
จุฬาลงกรณ์มหาวิทยาลัย

# Table of Contents

	page
Thai Abstract .....	iv
English Abstract .....	v
Acknowledgements .....	vi
List of Tables .....	ix
List of Figures .....	x
List of Abbreviations .....	xvi
List of Symbols .....	xvii
Chapter	
1 Introduction .....	1
1.1 Background and Motivation .....	1
1.2 Objective .....	3
1.3 Scope of Work .....	3
1.4 Contributions .....	4
1.5 Dissertation Organization .....	5
2 Related Works .....	6
3 Face Detection .....	10
3.1 Edge Detection .....	10
3.2 Mean Face Template .....	11
3.3 Proposed Algorithms .....	13
4 Facial Feature Detection .....	24
4.1 Neural Visual Model (NVM) .....	24
4.1.1 Facial Parameters .....	25

Chapter	page
4.1.2	The Feedforward Neural Networks . . . . . 29
4.1.3	Implementation of the Neural Visual Model . . . . . 30
4.1.4	Applying Cross Validation to Stopping Criteria . . . . . 34
4.2	Image Dilation . . . . . 38
4.3	Rotational Invariance . . . . . 40
5	Experimental Results . . . . . 44
5.1	Face Database . . . . . 44
5.2	Speed of the Algorithms . . . . . 46
5.3	Combining Overlapping Face Regions . . . . . 46
5.4	Experiments on Real-World Face Images . . . . . 47
6	Conclusion and Future Work . . . . . 62
6.1	Conclusion . . . . . 62
6.2	Future Work . . . . . 63
References	. . . . . 64
Vita	. . . . . 72



## List of Tables

Table	page
5.1 Face and Facial Feature Detection Results of Each Database . . . . .	58
5.2 Face Detection Rate Resulted from Varying Upper Bound of Number of White Pixels in Step 4 of Face Detection Algorithm on 20 Images Chosen at Random from the Databases . . . . .	60
5.3 Comparative Face Detection Performance of the Existing Methods and the Proposed Method on MIT-CMU Testset Containing 130 Images and 507 Faces . . . . .	60



ศูนย์วิทยทรัพยากร  
จุฬาลงกรณ์มหาวิทยาลัย

## List of Figures

Figure		page
3.1	An edge detection example. (a) Original image. (b) Edge image by Using Canny edge detection. . . . .	11
3.2	Mean face template. (a) Mean frontal-view face template ( $0^\circ$ in yaw) at 96x96 pixels. (b) Black-and-white version of (a). (c) Edge version of (a). (d) Mean left-near-profile-view face template ( $-60^\circ$ in yaw ) at 96x96 pixels. (e) Black-and-white version of (d). (f) Edge version of (d). (g) Mean right-near-profile-view face template ( $60^\circ$ in yaw) at 96x96 pixels. (h) Black-and-white version of (g). (i) Edge version of (g). . . . .	14
3.3	An example of multi-view face image set. The images vary from $-90^\circ$ to $90^\circ$ in yaw and from $-30^\circ$ to $30^\circ$ in tilt at $10^\circ$ intervals. There are three templates: left-near-profile-view face template ( $-60^\circ$ in yaw) for detecting face images of views from $-90^\circ$ to $-30^\circ$ in yaw, frontal-view face template ( $0^\circ$ in yaw) for detecting face images of views from $-30^\circ$ to $30^\circ$ in yaw, and right-near-profile-view face template ( $60^\circ$ in yaw) for detecting face images of views from $30^\circ$ to $90^\circ$ in yaw. . . . .	15
3.4	Multi-view face segmentation. . . . .	15
3.5	A comparison between the edge version of the face template and an actual face. (a) Edge version of the face template. (b) Edge version of an actual face. . . . .	16
3.6	A comparison between the black-and-white version of the face template and the edge version of an actual face. (a) Black-and-white	

Figure	page
version of the face template. (b) Edge version of an actual face.	
(c) Features are apparent in the black holes. . . . .	16
3.7 Black-and-white version of the face template rotated in eight different angles at 64x64 pixels (a) Frontal view. ( b) Left-near-profile view. (c) Right-near-profile view. . . . .	17
3.8 Edge version of the face template rotated in eight different angles at 64x64 pixels (a) Frontal view. (b) Left-near-profile view. (c) Right-near-profile view. . . . .	18
3.9 Face detection process. A large number of non-face regions (negative examples) are discarded by steps 4, 6, and 8 in the algorithm. Subsequently, some mis-detected face regions are eliminated by facial feature detection process. . . . .	21
3.10 Face detection steps. (a) Mean face template. (b) Edge version of template applied to steps 3 and 4. (c) Black-and-white version of template applied to steps 5 and 6. (d) Original image. (e) Edge image. (f) The first candidate face regions by step 4. (white pixel at position $(i, j)$ corresponds to the square candidate region between vertical pixels $i-l+1$ and $i$ , and horizontal pixels $j-l+1$ and $j$ , where the template size is $l \times l$ ). (g) The second candidate face regions by step 6. (h) From step 8, a white pixel at position $(i, j)$ corresponding to a detected region between vertical pixels $i-l+1$ and $i$ , and horizontal pixels $j-l+1$ and $j$ , where the template size is $l \times l$ . (i) Face	

Figure	page
detection result. . . . .	22
3.11 Face detection steps. (a) Mean face template. (b) Edge version of template applied to steps 3 and 4. (c) Black-and-white version of template applied to steps 5 and 6. (d) Original image. (e) Edge image. (f) The first candidate face regions by step 4. (white pixel at position $(i, j)$ corresponds to square candidate region between vertical pixels $i - l + 1$ and $i$ , and horizontal pixels $j - l + 1$ and $j$ , where the template size is $l \times l$ ). (g) The second candidate face regions by step 6. (h) From step 8, a white pixel at position $(i, j)$ corresponding to a detected region between vertical pixels $i - l + 1$ and $i$ , and horizontal pixels $j - l + 1$ and $j$ , where the template size is $l \times l$ . (i) Face detection result. . . . .	23
4.1 Notation used for the facial parameters. $O$ is a face center. Line $EF$ is parallel to the Y-axis. The length of line $EF$ is called face width. Lines $AD$ and $BC$ are perpendicular to each other. The angles $\hat{BOD}$ and $\hat{AOC}$ are divided into two halves by line $EF$ . $A$ , $B$ , $C$ , and $D$ are at four corners of the face. . . . .	25
4.2 Steps for finding face width and face center point. (a) Face detection. (b) Normalized face from (a). (c) Edge version of (b). (d) Dilated boundary from face template. (e) Result of applying $AND$ operation to (c) and (d). (f) Dilated version of (e). (g) Thinned version of (f). (h) Two points at the $64^{th}$ row. The face width is the distance between these points and the face center is at the center of the line between	

Figure	page
these points. . . . .	28
4.3 Nonlinear model of a node. . . . .	29
4.4 Architecture of multi-layer perceptron. . . . .	31
4.5 Idealized training and generalization error curves. . . . .	35
4.6 Left-eye region detection steps using image dilation. (a) Face detection. (b) Normalized face from (a). (c) Left-eye region detection from NVM. (d) Black-and-white image obtained from original image with thresh- olding filter (step 1). (e) Dilated image. (f) Left-eye extraction from detected region using the image dilation technique. . . . .	40
4.7 Line expression. . . . .	41
4.8 Detection steps of rotated image. (a) Face detection. (b) Detected face. (c) Edge image. (d) Edge image eliminating face boundary. (e) Radon transform of (d) which the highest intensity is at 45°. (f) Re-rotated im- age obtained by rotating 45° in a counter-clockwise direction. (g) Face and facial feature detection of re-rotated image. (h) Detected image . . .	43
5.1 Example of face images used for generating mean face template and training NVM detector. (a) Left-near-profile-view face images. (b) Frontal-view face images. (c) Right-near-profile-view face images. . . .	45
5.2 Example of combining overlapping face regions. (a) Overlapping face regions. (b) Single detected face. . . . .	47
5.3 The overall process of our proposed face and facial feature detection. .	48
5.4 Detection results of images with any kinds of intensity. (a) and (e) Color images. (b) and (f) Gray images. (c) and (g) Black-and-white	

Figure	page
images. (d) and (h) Edge images. . . . .	49
5.5 Multi-view face detection results. (a) Frontal view. (b) Right-near-profile view. (c) Right-profile view. . . . .	49
5.6 Detection results of face images with some structural components. (a) Face with glasses. (b) Face with beards and mustaches. (c) Face with glasses, beards, and mustaches. . . . .	50
5.7 Detection results of face images with facial expressions. (a) Angry face. (b) Excited face. (c) Very excited face. (d) Happy face. . . . .	51
5.8 Detection results of face images with occlusions. (a) Face with sunglasses. (b) Face with scarf. . . . .	51
5.9 Detection results of rotated face images. (a) 0° in clockwise direction. (b) 30° in clockwise direction. (c) 60° in clockwise direction. (d) 90° in clockwise direction. . . . .	51
5.10 Detection results of face images with lighting effect. (a) Left light source. (b) Front light source. (c) Right light source. . . . .	52
5.11 Poor-quality image detection results. (a) and (d) Blurry images. (b) and (e) Distorted images. (c) Image with 10% uniform noise. (e) Image with 10% Gaussian noise. . . . .	53
5.12 Detection results of unnatural-intensity images. (a) Colored cartoon images. (b) Sketchy cartoon images. (c) Rendered images. (d) Mosaic images. . . . .	53
5.13 More results. (a) Gray-scale cartoon by colored pencil. (b) Rotated face with lighting effect. (c) Color cartoon with rotation. (d) Left-near-	

Figure	page
profile-view cartoon cat face. (e) Color face. (f) Gray face with glasses and facial expression. (g) Inverted black-and-white face like film negative. (h) Color dark-skin face. (i) Sketchy image. (j) Upper-left-near-frontal-view mosaic face. (k) Lower-frontal-view face. (l) Multi-view faces with size variation. ....	54
5.14 More results. (a) Gray image. (b) Right-near-frontal-view face with rotation. (c) Rotated unnatural face. (d) Cartoon and painted face with rotation and facial expression. (e) Multifaces with occlusion and rotation. (f) Gray dark-skin face with dark background. (g) Multifaces including cartoon faces. (h) Multifaces with rotation. (i) Occluded face. (j) Lower-left-profile-view face. (k) Right-near-frontal-view face with beards and poor quality. (l) Left-near-frontal-view face. ....	55
5.15 The comparison of detection results. (a) Detection by our algorithms. The dog's face and facial features are also detected. (b) Detection by Bayesian discriminating feature method [38]. ....	56
5.16 ROC curves for the proposed method on each database. (a) ESSEX-face database. (b) Purdue AR database. (c) MIT-CMU database. (d) Max Planck face database. (e) UMIST face database. ....	57
5.17 Failure cases. (a) Unusual eye locations. (b) Faces without face boundaries. (c) Overlapping boundary regions (upper image) causing a failure detected face (lower image) and partial face(rightmost face). (d) Very small faces with extremely low contrast. (e) Face with incomplete thinned boundary. ....	61

## List of Abbreviations

- NVM Neural Visual Model  
MLP Multilayer Perceptron  
BP Back-Propagation



ศูนย์วิทยทรัพยากร  
จุฬาลงกรณ์มหาวิทยาลัย



## List of Symbols

$I_p$	image $p$
$\tilde{I}$	average image
$I_q^r(i, j)$	red(r) component of image $I_q$ at position $(i, j)$
$I_q^g(i, j)$	green(g) component of image $I_q$ at position $(i, j)$
$I_q^b(i, j)$	blue(b) component of image $I_q$ at position $(i, j)$
$\tilde{I}^r(i, j)$	red(r) component of average image $\tilde{I}$ at position $(i, j)$
$\tilde{I}^g(i, j)$	green(g) component of average image $\tilde{I}$ at position $(i, j)$
$\tilde{I}^b(i, j)$	blue(b) component of average image $\tilde{I}$ at position $(i, j)$
$m$	image vertical length
$n$	image horizontal length
$q$	image index
$Q$	the number of images in the databases
$\gamma$	the number of white pixels in the edge face template
$\phi_a$	the number of white pixels of region <b>a</b>
$\epsilon_a$	the number of feature pixels appearing in the black holes of the black-and-white face template from candidate region <b>a</b>
$l \times l$	mean face template size
$\phi_{i,j}$	the number of white pixels in the region bounded by rows $i - l + 1$ to $i$ and columns $j - l + 1$ to $j$
$\epsilon_{i,j}$	the number of feature pixels in the region bounded by rows $i - l + 1$ to $i$ and columns $j - l + 1$ to $j$
$\hat{\phi}_{i,j}$	normalized $\phi_{i,j}$
$\hat{\epsilon}_{i,j}$	normalized $\epsilon_{i,j}$
$\lambda_{i,j}$	matching value in the region bounded by rows $i - l + 1$ to $i$

	and columns $j - l + 1$ to $j$
$width_{max} \times length_{max}$	maximum size of detected faces
$(x_o, y_o)$	coordinate of the center point $O$
$(r, \theta)$	polar coordinate at any point $(x, y)$ with respect to the center point
$L_w$	face width
$L_A$	distance between face center and far upper left corner of face
$L_B$	distance between face center and far upper right corner of face
$L_C$	distance between face center and far lower left corner of face
$L_D$	distance between face center and far lower right corner of face
$(x_l, y_l)$	left boundary point of face
$(x_r, y_r)$	right boundary point of face
$x_1, x_2, \dots, x_m$	input signals
$w_{k1}, w_{k2}, \dots, w_{km}$	synaptic weights connecting all input nodes to node $k$
$u_k$	linear combiner output of node $k$ due to the input signals
$b_k$	bias of node $k$
$\psi(\cdot)$	activation function
$y_k$	output signal of node $k$
$input_j$	input signal of node $j$
$out_i$	output signal of node $i$

$w_{ji}$	synaptic weights connecting node $i$ to node $j$
$\eta$	learning rate parameter
$\delta_k$	rate of change of error with respect to the input of node $k$
$d_k$	desired output of node $k$
$E_{msq}$	mean squared error
$p$	index of input pattern
$P$	the number of all input patterns
$E$	objective function (error function) of the training algorithm
$E_{tr}(t)$	average error per example over the training set, measured after epoch $t$
$E_{va}(t)$	average error per example over the validation set, measured after epoch $t$
$E_{te}(t)$	average error per example over the test set, measured after epoch $t$
$E_{opt}(t)$	the lowest validation set error obtained in epochs up to $t$
$GL(t)$	generalization loss at epoch $t$
$GL_{\varsigma}$	generalization loss exceeding a stopping threshold $\varsigma$
$\varsigma$	stopping threshold
$k$	length of training strip
$TP_k$	training progress (in per thousand) measured after $k$ epochs
$PQ_{\varsigma}$	quotient of generalization loss and training progress exceeding a stopping threshold $\varsigma$
$UP_s$	notion of stopping when generalization errors increase in successive steps of strip $s$
$s$	strip index

$X$	set of Euclidean coordinates corresponding to the input binary image
$K$	set of coordinates for the kernel
$Kx$	translation of $K$ so that its origin is at $x$
$(x, y)$	position of pixel in rectangular coordinate
$I_{BW}(x, y)$	black-and-white face image
$I_{Dilated}(x, y)$	dilated image
$I_{NVM}(x, y)$	detected image from NVM
$I_{Detected}(x, y)$	detected image after eliminating the irrelevant regions
$\rho$	smallest distance between a line and the center of image
$\theta$	angle of a line
$I_{edge}(x, y)$	edge image
$\alpha$	radius parameter for rotational invariant facial feature detection
$\beta$	face angle