CHAPTER III

ORBITAL FLOW THEORY

As a result of the attempt to understand the circulation of water mass in the Upper Gulf of Thailand, Jesada Jiraporn in 1982[10] accidentally found out that what thought to be wind driven current could not be directly driven by wind. It is actually wave induced current. He developed a new theory called "ORBITAL FLOW THEORY" with the aim to replace Ekman theory on wind drift. The theory is intentionally called "orbital flow" to emphasized the role of wave induced-orbital motion.

Orbital flow theory is based on the second-order wave theory[14] and the Coriolis effect on waves. According to the theory, oceanic surface current is the sum of 1) mass transport current induced by wind waves[15], 2) Coriolis-moment-flow current and 3) Coriolis-shear-flow current.

3.1 Mass Transport Current Induced by Wind Waves

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Free surface water waves make water particles oscillate in both vertical and horizontal directions . For a progressive wave train the phase relationship between the vertical and horizontal oscillations is in such a way that the water particles move circularly in orbits[15] . These orbits translate slowly in the direction of wave propagation resulting in net mass transportation in the direction of wave propagation. According to the second order wave theory, the mass transport velocity distribution is given by [15]

$$v = V_{so}\, e^{-2kz} \tag{3.1}$$

V_{so} is a velocity of the mass transport current at surface which is given by

$$V_{so} = ka^2\sigma \tag{3.2}$$

where a = wave amplitued

k = wave number

σ = wave angular frequency.

For wind waves, it is expected that the mass transport current velocity shall be slightly higher than that for free waves because of the effect of wind stress . Taken this fact in consideration, a factor of (1 + α) is multiplied to the right hand side of Eq. (3.2) to get

$$V_{so} = ka^2\sigma\,(1 + \alpha) \tag{3.3}$$

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3.2 Coriolis-Moment-Flow-Current and Coriolis Shear Flow Current

Coriolis moment flow is a new basic concept firstly introduced in the orbital flow theory[10] . Coriolis moment flow is driven by Coriolis moment which is the result of interaction between the earth rotation and wave-induced orbital motion.

Consider an element of fluid of dimension $\delta x \delta y \delta z$ in deep-water waves propagating in the y-direction, the radius of orbital motion as given by the infinite small amplitude wave theory is

$$r = a e^{-kz} \tag{3.4}$$

And its orbital velocity is

$$q = \sigma a e^{-kz} \tag{3.5}$$

where r = radius of the orbital motion

q = orbital velocity

σ = wave angular frequency.

The horizontal component of the orbital velocity is

$$u = a\sigma e^{-kz} \sin \sigma t \tag{3.6}$$

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The Coriolis force associated with this horizontal velocity component can be determined by the formula

$$F_c = fmu \tag{3.7}$$

where $f = 2\,\Omega \sin\varphi$

$= $ Coriolis parameter.

Substituting mass and velocity from Eq. (3.6) into Eq. (3.7) yields

$$F_c = f(\rho\delta x\delta y\delta z)(a\sigma\, e^{-kz} \sin \sigma t) \tag{3.8}$$

where $\rho =$ water density.

Coriolis moment is defined as the moment of the Coriolis force about the axis passes through the center of orbit and is in the wave propagating direction (Fig. 3). Hence, the time average of the Coriolis moment per unit volume of fluid is

$$M_c = \frac{1}{\delta x\delta y\delta z T}\int_0^T F_c\,(a\, e^{-kz} \sin \sigma t)\, dt \tag{3.9}$$

Substituting Coriolis Moment as the shear stress into Newton's equation for viscosity[15], one obtains

$$\frac{du_m}{dz} = -\frac{1}{K}\left(\frac{\rho f a^2 \sigma\, e^{-2kz}}{2}\right) \tag{3.10}$$

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Assuming that ρ and K are constant, the velocity u_m for any z in an ocean of a finite depth (z = d) can be determined by integrating Eq. (3.10) from any z to z = d.

$$\int_z^d du_m = -\frac{\rho f a^2 \sigma}{2K}\int_z^d e^{-2kz}\,dz$$

Because $u_m(d) = 0$, hence

$$-u_m = -\frac{\rho f a^2 \sigma}{2K}\left(\frac{e^{-2kd} - e^{-2kz}}{-2K}\right) \qquad (3.11)$$

Substituting $V_{so} = ka^2\sigma$ from Eq. (3.2) and $A_o = K/\rho$ into Eq. (3.11) yields the velocity distribution

$$u_m = \frac{1}{2}\,\frac{f}{kA_o}\,V_{so}\left(\frac{e^{-2kz} - e^{-2kd}}{2k}\right) \qquad (3.12)$$

The direction of the current in the northern hemisphere is 90 degrees "cum sole" (i.e. to the right) of the wave propagating direction.

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Coriolis-shear-flow current is the result of an interaction between the earth rotation and the wave induced mass transport current. Because of the existence of the net mass transport velocity, the time average of the Coriolis force associated with the horizontal velocity component of the orbital motion is not equal to zero. Its value on fluid can be determined by direct substituting the velocity term from Eq. (3.1) into Eq. (3.7).

$$F_C = f(\rho\delta x\delta y\delta z)\, V_{so}\, e^{-2kz} \tag{3.13}$$

Dividing Eq. (3.13) by $\delta x\delta y\delta z$ and integrating from z = 0 to any z yields shearing stress.

$$\tau(z) = \int_0^z f\rho\, V_{so} e^{-2kz} dz \tag{3.14}$$

This stress produces flow as suggested by Newton's equation of viscosity[16].

$$\frac{du_s}{dz} = -\int_0^z \frac{1}{K}\, f\rho\, V_{so} e^{-2kz} dz \tag{3.15}$$

Assuming that ρ and K are constant, the velocity u_s for any z in an ocean of finite depth (z = d) can be determined by integrating Eq. (3.15) from any z to z = d.

$$\int_z^d du_s = -\frac{1}{K}\left(f\rho\, V_{so} \int_z^d \int_0^z e^{-2kz}\, dz\, dz\right) \tag{3.16}$$

Because $u_s(d) = 0$, hence

$$-u_s = -\frac{f\rho V_{so}}{2kK}\left((d-z) - \frac{e^{-2kd} - e^{-2kz}}{-2k}\right) \tag{3.17}$$

Substituting $A_o = K/\rho$ into Eq. (B.5) yields the velocity distribution of the Coriolis-shear-flow.

$$u_s = \frac{1}{2} \frac{fV_{so}}{kA_o} \left((d - z) - \frac{e^{-2kz} - e^{-2kd}}{2k} \right) \qquad (3.18)$$

The direction of the Coriolis-shear-flow current is also 90 degree "cum sole" to the direction of wave propagation.

3.3 Assumptions and Total Currents

Velocity distributions in Eqs. (3.1), (3.12) and (3.18) are derived under the following assumptions.

1) The ocean is homogeneous.

2) The ocean is unbounded in the horizontal direction.

3) The water depth is greater than 0.5 wavelenght.

4) The Coriolis-moment-flow current (u_m) and the Coriolis-shear-flow current (u_s) are related to Coriolis moment and Coriolis force through the modified Newton's equation for viscosity[16] ($\tau = -K\frac{du}{dz}$, where K is the the apparent viscosity or coefficient of internal friction).

Eq. (3.12) and (3.18) represents currents flowing in the same direction, i.e. "cum sole" to the direction of wave propagation. Hence they can be combined into one equation as

$$u = \frac{1}{2} \frac{fV_{so}}{kAo} (d - z) \qquad (3.19)$$

Eq. (3.1) and (3.19) together represent the total wave induced oceanic surface current which was formerly thought to be directly driven by wind[17,18].

Fig. 4 shows the velocity distribution predicted by the orbital flow theory. The angle between the wind/waves direction and induced current is not fixed to 45 degree as Ekman spiral is.

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