APPENDIX I

Using Eq.(5.62), Eq.(5.61) can be written in the form

$$
\begin{aligned}
J(q_{t-\varepsilon},q'_{t-\varepsilon}:q_t,q'_t) \approx \frac{1}{C} exp\frac{i}{\hbar}\Bigg\{ & \frac{1}{2}M\varepsilon\left(\frac{q_{t-\varepsilon}-q_t}{\varepsilon}\right)^2 - \varepsilon V_t\left(\frac{q_{t-\varepsilon}+q_t}{2}\right) \\
& -\frac{1}{2}M\varepsilon\left(\frac{q'_{t-\varepsilon}-q'_t}{\varepsilon}\right)^2 + \varepsilon V_t\left(\frac{q'_{t-\varepsilon}+q'_t}{2}\right) \\
& - M\gamma\Bigg[\varepsilon\left(\frac{q_{t-\varepsilon}+q_t}{2}\right)\left(\frac{q_{t-\varepsilon}-q_t}{\varepsilon}\right) - \varepsilon\left(\frac{q'_{t-\varepsilon}+q'_t}{2}\right)\left(\frac{q'_{t-\varepsilon}-q'_t}{\varepsilon}\right) \\
& + \varepsilon\left(\frac{q_{t-\varepsilon}+q_t}{2}\right)\left(\frac{q'_{t-\varepsilon}-q'_t}{\varepsilon}\right) - \varepsilon\left(\frac{q'_{t-\varepsilon}+q'_t}{2}\right)\left(\frac{q_{t-\varepsilon}-q_t}{\varepsilon}\right)\Bigg]\Bigg\} \\
& \times exp - \frac{2M\gamma kT}{\hbar^2}\varepsilon\left[\frac{q_{t-\varepsilon}+q_t}{2} - \frac{q'_{t-\varepsilon}+q'_t}{2}\right]^2 . \qquad \text{(I - 1)}
\end{aligned}
$$

By using new variables $Q_1 = q_{t-\varepsilon} - q_t$ and $Q_2 = q'_{t-\varepsilon} - q'_t$, one can write Eq.(I-1) in the from

$$
\begin{aligned}
J(q_{t-\varepsilon},q'_{t-\varepsilon}:q_t,q'_t) \approx \frac{1}{C} exp\Bigg\{ & \frac{iM}{2\varepsilon\hbar}Q_1^2 - \frac{i\varepsilon}{\hbar}V_t\left(q-\frac{Q_1}{2}\right) - \frac{iM}{2\varepsilon\hbar}Q_2^2 + \frac{i\varepsilon}{\hbar}V_t\left(q-\frac{Q_2}{2}\right) \\
& - \frac{iM\gamma}{\hbar}\left(q_{t-\varepsilon}-\frac{Q_1}{2}\right)Q_1 + \frac{iM\gamma}{\hbar}\left(q'_{t-\varepsilon}-\frac{Q_2}{2}\right)Q_2 \\
& - \frac{iM\gamma}{\hbar}\left(q_{t-\varepsilon}-\frac{Q_1}{2}\right)Q_2 + \frac{iM\gamma}{\hbar}\left(q'_{t-\varepsilon}-\frac{Q_2}{2}\right)Q_1 \\
& - \frac{2M\gamma kT}{\hbar^2}\varepsilon\left[\left(q_{t-\varepsilon}-\frac{Q_1}{2}\right)-\left(q'_{t-\varepsilon}-\frac{Q_2}{2}\right)\right]^2\Bigg\}, \qquad \text{(I - 2)}
\end{aligned}
$$

or in the form

$$
\begin{aligned}
J\left(q_{t+\varepsilon}, q'_{t+\varepsilon}; q_t, q'_t\right) \approx \frac{1}{C} exp\Bigg\{ & \frac{iM}{2\varepsilon\hbar} Q_1^2 - \frac{i\varepsilon}{\hbar} V_r\left(q - \frac{Q_1}{2}\right) - \frac{iM}{2\varepsilon\hbar} Q_2^2 + \frac{i\varepsilon}{\hbar} V_r\left(q - \frac{Q_2}{2}\right) \\
& - \frac{iM\gamma}{\hbar}\left(q_{t+\varepsilon} - \frac{Q_1}{2}\right) Q_1 + \frac{iM\gamma}{\hbar}\left(q'_{t+\varepsilon} - \frac{Q_2}{2}\right) Q_2 \\
& - \frac{iM\gamma}{\hbar}\left(q_{t+\varepsilon} - \frac{Q_1}{2}\right) Q_2 + \frac{iM\gamma}{\hbar}\left(q'_{t+\varepsilon} - \frac{Q_2}{2}\right) Q_1 \\
& - \frac{2M\gamma kT}{\hbar^2} \varepsilon\left(q_{t+\varepsilon} - q'_{t+\varepsilon}\right)^2 + \frac{2M\gamma kT}{\hbar^2} \varepsilon\left(q_{t+\varepsilon} - q'_{t+\varepsilon}\right)\left(Q_1 - Q_2\right) \\
& - \frac{M\gamma kT}{2\hbar^2} \varepsilon\left(Q_1 - Q_2\right)^2 \Bigg\} \qquad (I-3)
\end{aligned}
$$

Inserting Eq.(I-2) into Eq.(5.60) and using $q_{t+\varepsilon} = q$ and $q'_{t+\varepsilon} = q'$, one can get Eq. (5.63).

APPENDIX II

To evaluate Eq.(5.63) in the limit $\varepsilon \to 0$, we must notice that the integral in Eq.(5.63) contains two very fast oscillating terms with exponents proportional to ε^{-1}. It is clear that the main contribution comes from Q_1 and Q_2 very small, otherwise the factor $exp\frac{iM}{2\varepsilon\hbar}\left(Q_1^2 - Q_2^2\right)$ would oscillate wildly giving no finite contribution to Eq. (5.63). To be more specific, we want

$$Q_1 \approx Q_2 \approx \left(\frac{\varepsilon\hbar}{M}\right)^{1/2}, \qquad \text{(II-1)}$$

because in this region the phase of both exponentials will change by an amount of order 1. One might wonder at this point about the contribution of the region $Q_1 \approx Q_2 \approx Q$ with finite Q. In this case the phase of the two exponentials combined would change by an amount of order 1 when

$$\Delta Q \equiv Q_1 - Q_2 \approx \frac{\varepsilon\hbar}{MQ}. \qquad \text{(II-2)}$$

Now , if we define new variables $Q_1' \equiv Q_1 - \gamma(q-q')\varepsilon$ and $Q_2' \equiv Q_2 + \gamma(q-q')\varepsilon$ and then expand the exponentials of Eq.(5.63) in ε we can easily show that in the limit $\varepsilon \to 0$ all terms depending on ΔQ in the integrand will be $O(\varepsilon^2)$. This means that we can safely forget the region $Q_1 \approx Q_2 \approx Q$.

The procedure now is simple. We expand $\tilde{\rho}(q - Q_1, q' - Q_2, t)$ up to the second order for $Q_1 \approx Q_2 \approx 0$, i.e.

$$\tilde{\rho}(q-Q_1,q'-Q_2,t)=\tilde{\rho}(q,q',t)+Q_1\frac{\partial\tilde{\rho}}{\partial q}+Q_2\frac{\partial\tilde{\rho}}{\partial q'}+\frac{1}{2}Q_1^2\frac{\partial^2\tilde{\rho}}{\partial q^2}+Q_1Q_2\frac{\partial^2\tilde{\rho}}{\partial q^2}+\frac{1}{2}Q_2^2\frac{\partial^2\tilde{\rho}}{\partial q'^2}\ .$$

This equation can be written in terms of the new variables, $Q_1'\equiv Q_1-\gamma(q-q')\varepsilon$ and $Q_2'\equiv Q_2+\gamma(q-q')\varepsilon$, as follows (for $\varepsilon\to 0$)

$$\tilde{\rho}(q-Q_1,q'-Q_2,t)=\tilde{\rho}(q,q',t)+Q_1'\frac{\partial\tilde{\rho}}{\partial q}+Q_2'\frac{\partial\tilde{\rho}}{\partial q'}+\frac{1}{2}Q_1'^2\frac{\partial^2\tilde{\rho}}{\partial q^2}+Q_1'Q_2'\frac{\partial^2\tilde{\rho}}{\partial q^2}+\frac{1}{2}Q_2'^2\frac{\partial^2\tilde{\rho}}{\partial q'^2}\ . \tag{II - 3}$$

Using also new variables in the exponent in the integrand of Eq.(5.63), the resulting exponential is (keep up only to the first order in ε and the second order in Q_1' and Q_2')

$$\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp\left[\frac{iM\gamma}{2\hbar}Q_1'^2-\frac{iM\gamma}{2\hbar}Q_2'^2-\frac{i\varepsilon}{\hbar}V_r(q)\,\tilde{\rho}+\frac{i\varepsilon}{\hbar}V_r(q')\,\tilde{\rho}-\frac{2M\gamma kT}{\hbar}\varepsilon(q-q')^2\right]. \tag{II - 4}$$

Notice that we have replaced the potential term $\varepsilon V_r(q-Q_1/2)$ by $\varepsilon V_r(q)$ because the error is of higher order than ε. Expanding the last exponential of (II-4) to first order in ε and second order in Q_1' and Q_2', one obtain

$$\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_1'^2}{2\varepsilon\hbar}\left[1+\left(\frac{iM\gamma}{2\hbar}Q_1'^2-\frac{iM\gamma}{2\hbar}Q_2'^2-\frac{i\varepsilon}{\hbar}V_r(q)\,\tilde{\rho}+\frac{i\varepsilon}{\hbar}V_r(q')\,\tilde{\rho}-\frac{2M\gamma kT}{\hbar}\varepsilon(q-q')^2\right)\right]. \tag{II - 5}$$

$$\tilde{\rho}(q,q',t+\varepsilon) = \iint \frac{dQ_1'dQ_2'}{C^2}\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_1'^2}{2\varepsilon\hbar}\Bigg[\tilde{\rho}(q,q',t) - \frac{\partial\tilde{\rho}}{\partial q}Q_1' - \frac{\partial\tilde{\rho}}{\partial q'}Q_2'$$
$$-\frac{\partial\tilde{\rho}}{\partial q}\gamma(q-q')\varepsilon + \frac{\partial\tilde{\rho}}{\partial q'}\gamma(q-q')\varepsilon + \frac{1}{2}\frac{\partial^2\tilde{\rho}}{\partial q^2}Q_1'^2 + \frac{\partial^2\tilde{\rho}}{\partial q\partial q'}Q_1'Q_2'$$
$$+\frac{1}{2}\frac{\partial^2\tilde{\rho}}{\partial q'^2}Q_2'^2 - \frac{i\varepsilon}{\hbar}V_r(q)\tilde{\rho} + \frac{i\varepsilon}{\hbar}V_r(q')\tilde{\rho} - \frac{2M\gamma kT}{\hbar}\varepsilon(q-q')^2\tilde{\rho}\Bigg]$$

(II - 6)

In oder to evaluate the right-hand side of Eq.(II-6), one shall have to use three integrals

$$\int_{-\infty}^{\infty} e^{ah^2}dh = \left(\frac{\pi}{-a}\right)^{1/2},$$
$$\int_{-\infty}^{\infty} e^{ah^2}h\,dh = 0,$$
$$\int_{-\infty}^{\infty} e^{ah^2}h^2dh = \frac{1}{2}\left(\frac{\pi}{-a}\right)^{3/2}.$$

One can see that

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\tilde{\rho}(q,q',t) = \frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}\tilde{\rho}(q,q',t),$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\frac{\partial\tilde{\rho}}{\partial q}Q_1' = 0,$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\frac{\partial\tilde{\rho}}{\partial q}\gamma(q-q')\varepsilon = \frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}\frac{\partial\tilde{\rho}}{\partial q}\gamma(q-q')\varepsilon,$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\frac{\partial\tilde{\rho}}{\partial q'}\gamma(q-q')\varepsilon = \frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}\frac{\partial\tilde{\rho}}{\partial q'}\gamma(q-q')\varepsilon,$$

$$\int\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\,Q_1'^2\frac{\partial^2\tilde{\rho}}{\partial q^2}=\frac{1}{C^2}\left(\frac{2\pi\varepsilon\hbar}{iM}\right)^{1/2}(2\pi)^{1/2}\left(-\frac{\varepsilon\hbar}{iM}\right)^{3/2}\frac{\partial^2\tilde{\rho}}{\partial q^2}$$
$$=-\frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}\frac{\varepsilon\hbar}{iM}\frac{\partial^2\tilde{\rho}}{\partial q^2},$$

$$\int\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\,Q_2'^2\frac{\partial^2\tilde{\rho}}{\partial q'^2}=\frac{1}{C^2}\left(-\frac{2\pi\varepsilon\hbar}{iM}\right)^{1/2}(2\pi)^{1/2}\left(\frac{\varepsilon\hbar}{iM}\right)^{3/2}\frac{\partial^2\tilde{\rho}}{\partial q'^2}$$
$$=\frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}\frac{\varepsilon\hbar}{iM}\frac{\partial^2\tilde{\rho}}{\partial q'^2}.$$

$$\int\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\frac{\partial^2\tilde{\rho}}{\partial q\partial q'}Q_1'Q_2'=0,$$

$$\int\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\left(\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\right)\frac{i\varepsilon}{\hbar}V_r(q)\,\tilde{\rho}=\frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}\frac{i\varepsilon}{\hbar}V_r(q)\,\tilde{\rho},$$

$$\int\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\left(\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\right)\frac{-i\varepsilon}{\hbar}V_r(q')\,\tilde{\rho}=\frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}\frac{-i\varepsilon}{\hbar}V_r(q')\,\tilde{\rho},$$

and

$$\int\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\left(\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\right)\frac{2M\gamma kT}{\hbar}\varepsilon(q-q')^2\,\tilde{\rho}$$
$$=\frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}\frac{2M\gamma kT}{\hbar}\varepsilon(q-q')^2\,\tilde{\rho}.$$

$$\int\int_{-\infty}^{\infty}\frac{dQ_1'dQ_2'}{C^2}\left(\exp\frac{iMQ_1'^2}{2\varepsilon\hbar}\exp-\frac{iMQ_2'^2}{2\varepsilon\hbar}\right)\frac{2M\gamma kT}{\hbar}\varepsilon(q-q')^2\,\tilde{\rho}$$
$$=\frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}\frac{2M\gamma kT}{\hbar}\varepsilon(q-q')^2\,\tilde{\rho}.$$

The left-hand side of Eq.(II-6) can be expanded up to first order in ε in the form

$$\tilde{\rho}(q,q',t+\varepsilon)=\tilde{\rho}(q,q',t)+\frac{\partial\rho}{\partial t}\varepsilon\ . \tag{II-7}$$

Substituting all integrating results of the right-hand side of Eq.(II-6) and Eq.(II-7) into Eq.(II-6), one can get

$$\begin{aligned}\tilde{\rho}(q,q',t)+\frac{\partial\rho}{\partial t}\varepsilon &= \frac{1}{C'^2}\frac{2\pi\varepsilon\hbar}{M}\tilde{\rho}(q,q',t)-\frac{1}{C'^2}\frac{2\pi\varepsilon\hbar}{M}\frac{\partial\tilde{\rho}}{\partial q}\gamma(q-q')\varepsilon\\ &+\frac{1}{C'^2}\frac{2\pi\varepsilon\hbar}{M}\frac{\partial\tilde{\rho}}{\partial q'}\gamma(q-q')\varepsilon-\frac{1}{C'^2}\frac{2\pi\varepsilon\hbar}{M}\frac{\varepsilon\hbar}{iM}\frac{\partial^2\tilde{\rho}}{\partial q^2}\\ &+\frac{1}{C'^2}\frac{2\pi\varepsilon\hbar}{M}\frac{\varepsilon\hbar}{iM}\frac{\partial^2\tilde{\rho}}{\partial q'^2}-\frac{1}{C'^2}\frac{2\pi\varepsilon\hbar}{M}\frac{i\varepsilon}{\hbar}V_r(q)\tilde{\rho}\\ &+\frac{1}{C'^2}\frac{2\pi\varepsilon\hbar}{M}\frac{i\varepsilon}{\hbar}V_r(q')\tilde{\rho}-\frac{1}{C'^2}\frac{2\pi\varepsilon\hbar}{M}\frac{2M\gamma kT}{\hbar}\varepsilon(q-q')^2\tilde{\rho}\end{aligned} \tag{II - 6}$$

and equating them to the corresponding ones on the right-hand side one can see that the zero order term in ε gives the result

$$\frac{1}{C^2}\frac{2\pi\varepsilon\hbar}{M}=1, \tag{II-7}$$

which is Eq.(5.64). Using Eq.(II-7) the first order term in ε gives the result (Eq.(5.65))

$$\begin{aligned}\frac{\partial\tilde{\rho}}{\partial t}&=\frac{i\hbar}{2M}\frac{\partial^2\tilde{\rho}}{\partial q^2}-\frac{i\hbar}{2M}\frac{\partial^2\tilde{\rho}}{\partial q'^2}-\gamma(q-q')\frac{\partial\tilde{\rho}}{\partial q}+\gamma(q-q')\frac{\partial\tilde{\rho}}{\partial q'}\\ &+\frac{V_r(q)}{i\hbar}\tilde{\rho}-\frac{V_r(q')}{i\hbar}\tilde{\rho}-\frac{2M\gamma kT}{\hbar^2}(q-q')^2\tilde{\rho}\ .\end{aligned} \tag{II - 8}$$

APPENDIX III

To transform $\tilde{\rho}(q,q',t)$ by using the Wigner transform one must write its position representation in terms of the symmetric, $x=\frac{q+q'}{2}$ and anti- symmetric, $y=q'-q$ coordinates (or $q=x-\frac{y}{2}$, $q'=x+\frac{y}{2}$), $\rho(x,y,t)$. Differential operators appear in Eq.(5.65) can be transformed into these coordinates as follows

$$\frac{\partial}{\partial q}=\frac{\partial y}{\partial \mathrm{q}}\frac{\partial}{\partial \mathrm{y}}+\frac{\partial x}{\partial \mathrm{q}}\frac{\partial}{\partial \mathrm{x}}=-\frac{\partial}{\partial \mathrm{y}}+\frac{1}{2}\frac{\partial}{\partial \mathrm{x}}, \tag{III-1}$$

$$\frac{\partial}{\partial q'}=\frac{\partial y}{\partial \mathrm{q}'}\frac{\partial}{\partial \mathrm{y}}+\frac{\partial x}{\partial \mathrm{q}'}\frac{\partial}{\partial \mathrm{x}}=\frac{\partial}{\partial \mathrm{y}}+\frac{1}{2}\frac{\partial}{\partial \mathrm{x}} \tag{III-2}$$

and the second order are

$$\frac{\partial^2}{\partial q^2}=\frac{\partial^2}{\partial y^2}-\frac{\partial^2}{\partial \mathrm{y}\partial \mathrm{x}}+\frac{1}{4}\frac{\partial^2}{\partial x^2} \tag{III-3}$$

and

$$\frac{\partial^2}{\partial q'^2}=\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial \mathrm{y}\partial \mathrm{x}}+\frac{1}{4}\frac{\partial^2}{\partial x^2}\ . \tag{III-4}$$

By the reason as above and Eq.(III-1) to Eq.(III-4), Eq.(5.65) can be written in the form

$$\frac{\partial \tilde{\rho}}{\partial t}=-\frac{i\hbar}{M}\frac{\partial^2 \tilde{\rho}}{\partial \mathrm{y}\partial \mathrm{x}}-2\gamma y\frac{\partial \tilde{\rho}}{\partial y}-y\frac{V_r'(x)}{i\hbar}-\frac{2M\gamma kT}{\hbar^2}(-y)^2\,\tilde{\rho}. \tag{III-5}$$

A transformation of the left-hand side of Eq.(III-5) is directly,

$$\frac{1}{2\pi\hbar}\int dy \frac{\partial\tilde{\rho}}{\partial t}\exp\frac{ipy}{\hbar} = \frac{\partial}{\partial t}\left(\frac{1}{2\pi\hbar}\int dy\,\tilde{\rho}\exp\frac{ipy}{\hbar}\right) = \frac{\partial}{\partial t}w(x,p,t) = \frac{\partial w}{\partial t}. \qquad \text{(III-6)}$$

Transformation of the right-hand side can be expressed as follows:

-For the first term, $-\frac{1}{2\pi\hbar}\int dy \frac{i\hbar}{M}\frac{\partial^2\tilde{\rho}}{\partial y\partial x}\exp\frac{ipy}{\hbar}$, by integrating by part one gets

$$\begin{aligned}\frac{i\hbar}{M}\frac{1}{2\pi\hbar}\frac{ip}{\hbar}\int dy\frac{\partial\tilde{\rho}}{\partial x}\exp\frac{ipy}{\hbar} &= -\frac{1}{M}\frac{\partial}{\partial x}\left(p\frac{1}{2\pi\hbar}\int dy\,\tilde{\rho}\exp\frac{ipy}{\hbar}\right)\\ &= -\frac{1}{M}\frac{\partial pw}{\partial x}\end{aligned} \qquad \text{(III - 7)}$$

-The transformation of the second term can be written in the form

$$-2\gamma\ \frac{1}{2\pi\hbar}\int dy\, y\frac{\partial\tilde{\rho}}{\partial y}\exp\frac{ipy}{\hbar} = -2\gamma\ \frac{1}{2\pi\hbar}\int dy\ \frac{\partial\tilde{\rho}}{\partial y}\frac{\hbar}{i}\frac{\partial}{\partial p}\exp\frac{ipy}{\hbar}.$$

Integrating by part and rearranging the result, one gets

$$\begin{aligned}2\gamma\frac{\hbar}{i}\frac{1}{2\pi\hbar}\int dy\,\tilde{\rho}\frac{\partial}{\partial y}\frac{\partial}{\partial p}\exp\frac{ipy}{\hbar} &= 2\gamma\frac{\hbar}{i}\frac{1}{2\pi\hbar}\int dy\,\tilde{\rho}\frac{\partial}{\partial p}\left(\frac{ip}{\hbar}\exp\frac{ipy}{\hbar}\right)\\ &= 2\gamma\frac{\partial}{\partial p}\left(p\frac{1}{2\pi\hbar}\int dy\,\tilde{\rho}\exp\frac{ipy}{\hbar}\right)\\ &= 2\gamma\frac{\partial pw}{\partial p}\end{aligned} \qquad \text{(III - 8)}$$

-A result of the transformation for third term is

$$\begin{aligned}\frac{1}{2\pi\hbar}\int dy\left(-y\frac{V_r'(x)}{i\hbar}\tilde{\rho}\right)\exp\frac{ipy}{\hbar} &= \frac{1}{2\pi\hbar}\frac{V_r'(x)}{i\hbar}\int dy\ y\tilde{\rho}\exp\frac{ipy}{\hbar}\\ &= -\frac{1}{2\pi\hbar}\frac{V_r'(x)}{i\hbar}\int dy\,\tilde{\rho}\frac{\hbar}{i}\frac{\partial}{\partial p}\exp\frac{ipy}{\hbar}\\ &= \frac{\partial}{\partial p}\left(V_r'(x)\frac{1}{2\pi\hbar}\int dy\,\tilde{\rho}\exp\frac{ipy}{\hbar}\right) = \frac{\partial wV_r'(x)}{\partial p}\end{aligned}$$

(III - 9)

-The transformation of the last term is

$$\frac{1}{2\pi\hbar}\int dy\left(-\frac{2M\gamma kT}{\hbar^2}\,y^2\,\tilde{\rho}\right)\exp\frac{ipy}{\hbar}=-\frac{2M\gamma kT}{\hbar^2}\frac{1}{2\pi\hbar}\int dy\,\tilde{\rho}\left(\frac{\hbar}{i}\right)^2\frac{\partial^2}{\partial p^2}\exp\frac{ipy}{\hbar}$$

$$=2M\gamma kT\frac{\partial^2}{\partial p^2}\left(\frac{1}{2\pi\hbar}\int dy\,\tilde{\rho}\exp\frac{ipy}{\hbar}\right)$$

$$=D\frac{\partial^2 w}{\partial p^2} \qquad \text{(III - 10)}$$

where $D=2M\gamma kT=\eta kT$.

Finally, substituting all results in Eq.(III-6)-Eq.(III-10) into the Wigner transform of Eq.(III-5) one gets the result in Eq.(5.67).

REFERENCES

[1] Feynman, R. P. Space-Time Approach to Non-Relativistics Quantum Mechanics. **Rev. Mod. Phys.** 20 (April 1948) : 367-387.

[2] Feynman, R. P., and Hibbs, A. R. **Quantum Mechanics and Path Integrals** New York : McGraw-Hill, 1965.

[3] Feynman, R. P. **Statistical Mechanics.** Massachusetts : W. A. Benjamin, 1972.

[4] Feynman, R. P., and Vernon, F. L. Theory of a General System Interacting with a Linear Dissipative System. **Ann. Phys.** 24 (1963) : 118-173.

[5] Caldeira, A. O., and Legett, A. J. Path Integrals Approach to Quantum Brownian Motion. **Physica.** 121A (1983) : 587-616.

[6] Wiegel, F. W. **Some Application of Functional Integration to Classical and Bose Condensation.** Ph.D.'s Thesis Universiteit van Amsterdam, 1973.

[7] Einstein, A. **Investigation on the Theory of the Brownian Movement.** NewYork : Dover Publication, 1956.

[8] Papadopoulos, G. J., and Devreese, J.T. **Path Integral and Their Applications in Quantum Statistical and Solid State Physics.** New York:Plenum, 1977.

[9] Grabert, H., Schramm, P., and Ingold, G. L. Quantum Brownian Motion : The Functional Integral Approach. **Physics Reports** (Review Section of Physics Letters). 168,N3 (1988) 115-207.

[10] Uhlenbeck, G. E., and Ornstein, L. S. On the Theory of the Brownian Motion. **Phys. Rev.** 36 (September 1930) 823-841.

[11] Wang, M. C., and Uhlenbeck, G. E. On the Theory of the Brownian Motion II. **Rev. Mod. Phys.** 17 (April-July 1945) 323-342.

[12] Doob, J. L. The Brownian Movement and Stochastic Equations. **Annals of Mathematics.** 43 (April 1942) 351-369.

[13] Sa-yakanit, V. Functional Integrals in Brownian Motion. **Thai Journal of Physics.** 4 (June 1987) 21-25.

[14] Haken, H. Synergetics. Berlin : Springer-Verlag, 1977.

[15] Reif, F. **Fundamentals of Statistical and Thermal Physics.** New York : McGraw-Hill, 1965.

[16] Papoulis, A. **Probability, Random Variables, and Stochastic Processes.** 2nd ed. New York : McGraw-Hill, 1985.

[17] Goldstein, H. **Classical Mechanics.** 2nd ed. Massachusetts : Addison-Wesley, 1980.

[18] Yourgrau, W., and Mandelstam, S. **Variational Principles in Dynamics and Quantum Theory.** 3rd ed. London : Sir Isaac Pitman & Sons, 1968.

CURRICULUM VITAE

Mr. Porncharoen Palotaidamkerng was born on April 10, 1969 in Bangkok. He received his B.Sc. degree in physics from Srinakarinwirote University in 1992.