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APPENDICES

APPENDIX A

Hilbert Spaces

The Axioms of Hilbert Space

The abstract Hilbert space $\mathcal{H}$ is a collection of objects called vectors, denoted by f, g, ... , which satisfy the following axioms (1) - (4) (Jauch, 1968, pp. 18-19; Sewell, 1986, pp. 35-36).

1. $\mathcal{H}$ is a linear vector space with complex coefficients. This means that to every pair of elements $f, g \in \mathcal{H}$ there is associated a third $(f+g) \in \mathcal{H}$. Furthermore, to every element and every complex number λ there corresponds another element $\lambda f \in \mathcal{H}$. The following rules are postulated :

$$f+g = g+f \; ;$$
$$(f+g) + h = f + (g+h) \; ;$$
$$\lambda\,(f+g) = \lambda f + \lambda g \; ;$$
$$(\lambda+\mu)\, f = \lambda f + \mu f \; ;$$
$$\lambda\,(\mu f) = (\lambda\mu)\, f \; ;$$
$$1 \,.\, f = f.$$

There exists a unique zero vector 0 such that for all f

$$0+f = f \; ;$$
$$0.f = 0.$$

2. There exists a strictly positive scalar product in $\mathcal{K}$. The scalar product (f,g) is a function of pairs of elements $f,g \in \mathcal{K}$ with complex values and satisfying the following conditions :

$$(f,g) = (g,f)^* ;$$
$$(f,g+h) = (f,g) + (f,h) ;$$
$$(f,\lambda g) = \lambda\,(f,g) \quad \text{for all complex } \lambda ;$$
$$(f,f) \geq 0 ;$$

equality in the last formula occurring only if $f = 0$. We define $|f|$, the norm of f, as the non-negative square root of (f,f), i.e.

$$|f| = (f,f)^{1/2} \tag{A.1}$$

If follows from these specifications that the scalar product and norm satisfy the Schwartz inequality :

$$|(f,g)| \leq |f|\,|g| \tag{A.2}$$

and the Minkowski's inequality (also called the triangle inequality) :

$$|f+g| \leq |f| + |g| \tag{A.3}$$

3. The space $\mathcal{K}$ is seperable. This means that there exists a sequence $f_n \in \mathcal{K}$ $(n=1,2, \ldots)$ with the property that it is dense in $\mathcal{K}$ in the following sense : For any $f \in \mathcal{K}$ and any $\epsilon > 0$, there exists at least one element f_n of the sequence such that

$$|f-f_n| < \epsilon ,$$

4. The space $\mathcal{H}$ is complete. A Cauchy sequence, i.e., any sequence $\{f_n\}$ with the property

$$\lim_{n,m\to\infty} |f_n-f_m| = 0$$

defines a unique limit $f \in \mathcal{H}$ such that

$$\lim_{n\to\infty} |f-f_n| = 0$$

Comments on the axioms

The axioms fall into four groups, each of which refers to a different structure property of Hilbert space. Group 1 expresses the fact that $\mathcal{H}$ is a linear vector space over the field of complex numbers. Group 2 defines the scalar product and the metric. Group 3 expresses separability, and group 4, completeness, of the space.

It should be noted that, to some extent at least, quantum mechanics can be formulated also in a Hilbert space over the field of reals or over the skew field of quaternions (Jammer, 1974, p. 358; Jauch, 1968, p. 131), in addition to the field of complex numbers, these being the only finite-dimensional skew fields which according to a theorem by Frobenius contain the reals as a proper subfield. Quantum mechanics in real Hilbert space has been developed by Stueckelberg and he has found that the empirical evidence points towards the existence of a superselection rule, which has the effect that at least for simple systems the proposition system is essentially equivalent to the system of subspaces in a complex Hilbert space. The relation between quaternionic quantum mechanics and the usual complex quantum mechanics has been studied by Emch, who has shown that at least for simple-particle systems relativistic considerations lead to an equivalence between the two formulations.

APPENDIX B

The Formalism of Finite Systems

The quantum theory of finite systems may be summarized as follows (Jammer, 1974, p. 5; Sewell, 1986, p. 9). The primitive (undefined) notions are system, observable (or "physical quantity" in the terminology of von Neumann), and state. To every observable A corresponds uniquely a bounded self-adjoint operator A acting in a certain Hilbert space, $\mathcal{K}$. The pure states of the system correspond to the normalized vectors in $\mathcal{K}$, in such a way that the expectation value of an observable, A, for the state represented by the vector Ψ, is $(\Psi, A\Psi)$. The dynamics is given, in the Schrödinger picture, by the transformations $\Psi \rightarrow \Psi_t \equiv e^{-iHt/\hbar}\Psi$ of the states, where H is the Hamiltonian operator, representing the energy observable of the system. Thus, the expectation value of the observable A at time t, for an evolution from an initial state Ψ, is $(\Psi_t, A\Psi_t)$. The model is therefore determined by the Hilbert space representation of its states and observables, and by the specification of its Hamiltonian.

We remark here that von Neumann's original assumption that observables and bounded self-adjoint operators stand in a one-to-one correspondence and that all nonzero vectors of the Hilbert space are state vectors had to be abandoned in view of the existence of superselection rules, discovered in 1952 by G. C. Wick, A. S. Wightman and E. P. Wigner (1952).

APPENDIX C

The Einstein-Podolsky-Rosen Paradox

C.1 Introduction

This is the remarkable quantum phenomenon discovered by Einstein, Podolsky and Rosen (EPR) in 1935 and called by Einstein himself "spooky action-at-a-distance." It is known today as "quantum non-locality" or "the EPR paradox" (Einstein et al., 1935; Mattuck, 1982, a, b) (see also 2.3.2.4, 2.3.2.5, 2.3.2.6, 4.4.2 and 4.4.3).

The phenomenon is outlined here. Suppose that two particles, 1 and 2, have interacted with each other in the past, but are now so far from each other that further interaction between them is impossible. Then quantum mechanics predicts that a measurement carried out on particle 1 will change the state of particle 2, no matter how far away particle 2 is. The change is instantaneous. Moreover, this "spooky" effect has been confirmed experimentally.

C.2 The EPR Paradox

It's not so surprising that a measuring apparatus changes the state of an atom with which it is physically interacting (although it must be emphasized that no one has succeeded in showing how these physical interaction produce the random collapse of the state vector -- this is the "quantum mechanical measurement problem" (see also

2.5.4)). The surprise first comes when we discover that a measuring apparatus is able to change the state of a distant atom (which may be hundreds of kilometres away) with which it is not physically interacting. This is the EPR paradox (Einstein et al., 1935) which we will present in the form invented by Bohm (1951, pp. 661-623).

Suppose we have a molecule composed by two identical spin - 1/2 atoms, in the singlet state, i.e. the $S^2 = 0$ eigenstate of the total spin operator $S^2 = (S_1+S_2)^2$. Then, neglecting spin-orbit coupling, the spin part of its state vector is

$$|\Psi\rangle = (1/\sqrt{2})\ (|1\overset{a}{+}\rangle|2\overset{a}{-}\rangle - |1\overset{a}{+}\rangle|2\overset{a}{-}\rangle) \qquad \text{(C.1)}$$

where $|1\overset{a}{\pm}\rangle$, $|2\overset{a}{\pm}\rangle$ are the eigenstates of the single-particle spin operators $\sigma_{1\mathbf{a}}$, $\sigma_{2\mathbf{a}}$. Note that **a** is a unit vector in any direction. Observe that the spin state of each atom is indefinite.

The molecule explodes, and the spins move off to distant Stern-Gerlachs A, B, with axes in the direction **a**. If a measurement by A now yields, say $|1\overset{a}{+}\rangle$, then $|\Psi\rangle$ has collapsed to $|1\overset{a}{+}\rangle|2\overset{a}{-}\rangle$. Thus, A's measurement of 1 has instantaneously changed the state of distant atom 2 (which may be 1000 km away) despite the fact that there is no physical interaction between 1 and 2, or A and 2 !

C.3 <u>The Two-Particle Correlation Function</u>

The quantity measured in EPR experiments using two spin - 1/2 particles is the "correlation function", E(**a**,**b**), where **a**, **b** are the orientations of the Stern-Gerlachs A, B, respectively (in the photon case, **a**, **b** are the corresponding optical polarizer orientations). E(**a**,**b**) is just the weighted average of the product of A's result for σ_{1a} and B's result for σ_{2b}.

The quantum mechanical prediction for E(**a**,**b**) is

$$E_{QM}(\mathbf{a},\mathbf{b}) = \sum_{mn} mn\, P_{mn} \qquad m,n = \pm 1$$

$$P_{mn} = |\langle m,n|\Psi\rangle|^2 \qquad \text{(C.2)}$$

where P_{mn} is the probability of observing $|1^{a}_{m}\rangle|2^{b}_{n}\rangle$. Expanding $|2^{a}_{+}\rangle$, $|2^{a}_{-}\rangle$ in (C.1) in terms of $|2^{b}_{+}\rangle|2^{b}_{-}\rangle$ and putting the result for $|\Psi\rangle$ into (C.2) yields

$$E_{QM}(\mathbf{a},\mathbf{b}) = -\,\mathbf{a}\,.\,\mathbf{b} = -\cos\phi \qquad \text{(C.3)}$$

where ø is the angle between the two Stern-Gerlachs.

If both A and B are in the same direction **a**, then $\phi = 0$ and

$$E_{QM}(\mathbf{a},\mathbf{a}) = -1$$

which says that each time A measures spin up (σ_{1a} = +), B measures spin down (σ_{2a} = -1), and vice versa, i.e., perfect correlation.

C.4 Attempt to Resolve the EPR Paradox, by Means of "Local Hidden Variables"

The EPR paradox seems to cry out for an explanation in terms of "hidden variables" (see also 2.5). That is, suppose that in addition to the state vector $|\Psi\rangle$, there is a set of hidden parameters λ ($\equiv \lambda_1, \lambda_2, \ldots, \lambda_n$) associated with the molecule, a different set for each molecule. And suppose that the value of λ, together with $|\Psi\rangle$, is sufficient to determine the result of measurement of σ_{1a} and σ_{2a} e. g. if $a = b = z$, then A's measurement of σ_{1z} to be + 1 would not cause σ_{2z} to become- 1 in some mysterious way ; rather, the value of λ would be the cause of both $\sigma_{1z} = +1$ and $\sigma_{2z} = -1$. In this way we avoid "spooky-action-at-a-distance."

Now, we expect that the result of measuring σ_{1a} should depend on λ and the orientation **a** of apparatus A, but it should not depend on **b**, the setting of the distant apparatus B. This is the very plausible "locality" assumption, and hidden variable theories of this type are called "local hidden variable theories." Thus we may write

$$A(\mathbf{a},\lambda) = \text{result of measuring } \sigma_{1a}$$
$$B(\mathbf{b},\lambda) = \text{result of measuring } \sigma_{2b} \qquad (C.4)$$

C.5 Bell's Inequality

Is it possible to construct a local hidden variable theory which gives agreement with quantum mechanics ? Look at the correlation function $E(\mathbf{a},\mathbf{b})$. This will be just the weighted average of the product $A(\mathbf{a},\lambda)\ B(\mathbf{b},\lambda)$ (analogous to (C.2))

$$E(\mathbf{a},\mathbf{b}) = \int d\lambda\ \rho(\lambda)\ A(\mathbf{a},\lambda)\ B(\mathbf{b},\lambda) \qquad \text{(C.5)}$$

where $\rho(\lambda)\ d\lambda$ is the probability of the hidden variable having a value between λ and $\lambda + d\lambda$.

Without a specific model giving $A(\mathbf{a},\lambda)$, $B(\mathbf{b},\lambda)$ explicitly, we cannot compare E directly with the quantum result (C.3). However, Bell (1964) proved the remarkable result that, for any local hidden variable model where $E(\mathbf{a},\mathbf{b})$ has the form (C.5), E must obey the following inequality

$$|\ - E(\mathbf{a},\mathbf{b}) + E(\mathbf{a},\mathbf{b}') + E(\mathbf{a}',\mathbf{b}) + E(\mathbf{a}',\mathbf{b}')\ | \leq 2 \qquad \text{(C.6)}$$

where **a**, **a'** are two different settings of apparatus A and **b**,**b'** are two different settings of apparatus B. The essential thing in Bell's proof is that A is independent of **b**, and B is independent of **a**. Bell's inequality restricts E so severely that it is easy to show that, for certain pairs of orientations, the inequality is violated by the quantum mechanical correlation function E_{QM}.

Thus, the quantum correlation function violates Bell's inequality. But all local hidden variable correlation functions must satisfy Bell's inequality. Hence no local hidden variable model with $E(\mathbf{a},\mathbf{b})$ of the form (C.5) can give agreement with quantum mechanics.

C.6 Comparision of Local Hidden Variable Theory with Experiment

Since long-distance correlations of the EPR type had never been systematically investigated prior to Bell's inequality, Clauser and Shimony (1978) proposed that the question be examined experimentally. Three types of experiments have been performed.

1. Proton pairs in a singlet state (1 experiment). Low-energy protons were scattered by a hydrogen target, and spin correlations between the incident and recoil protons were measured. The singlet part of the correlation function, E(**a**,**b**) was in good agreement with the quantum prediction, and violated Bell's inequality (Lamehi-Rachti and Mittig, 1976).

2. Low-energy photon pairs emitted in atomic cascade transitions (6 experiments). The polarization of photon pairs were measured by making the isotopes of Calcium (or Mercury) into the exited state by laser absorption, then going back to the ground state in 2 steps. There was a photon released in each step which travelled in opposite directions and had an opposite polarization. That is, the polarization of each pair in the same direction must have a negative correlation.

Five out of six cascade experiments agreed with quantum mechanics (Freedman and Clauser, 1972; Clauser, 1976; Fry and Thompson, 1976; Aspect, Grangier and Roger, 1981, 1982), one by Holt and Pipkin (1974) agreed with local hidden variable theory, but

Clauser (1976) repeated the experiment and the results agreed with quantum mechanics.

The experiment by Aspect, Grangier and Roger (1982) used two-channel analyser for the first time.

3. High-energy photon pair from positron-electron annihilation (4 experiments). This kind of experiments measured polarization of gamma rays (high-energy photons) from positron-electron annihilation. Photon pairs would go in opposite directions having opposite polarization.

In three out of four cases, the results were in good agreement with quantum mechanics (Kasday, Ullman and Wu, 1970, 1975; Wilson et al., 1976; Bruno et al., 1977). The one that disagreed was by Faraci et al. (1974).

C.7 Einstein-Separable Hidden Variable Models

The locality condition which states that the results obtained from A should not depend on the setting of B, though reasonable, has not been stated in any laws of physics. So Aspect (1976) proposed the Einstein-separability condition which states that the results obtained from A should not depend on B provided the two pieces of apparatus are divided by space-like interval according to special relativity.

Thus models that respect the locality condition will always obey the Einstein-separability condition as well.

Aspect, Dalibard and Roger (1982) carried out this kind of experiment by setting the polarizer positions after the cascade photons had emerged from the source, and were in flight. The results were in agreement with quantum mechanics.

C.8 Non-Local Hidden Variable Theories

Many people seem to be under the impression that the disagreement between local and Einstein-separable hidden variable theory and experiment rules out all hidden variable theories. This is not true ! There are still the non-local hidden variable theories left. These have the incredible property that $A = A(\lambda,\mathbf{a},\mathbf{b})$, $B = B(\lambda,\mathbf{a},\mathbf{b})$, i.e., the result of a measurement at B depends on the setting of distant apparatus A, and vice versa. The Bohm and Bub (1966) theory is one such theory (see 2.5.4).

Non-local hidden variable theories obviously cannot cure non-locality since they have the disease themselves. But they provide a model for the collapse of the wavefunction in a measurement, thus solving the quantum measurement problem (see section C.2). Moreover, they yield agreement with both quantum mechanics and experiment.

Thus, instead of a non-local effect from the measurement event at A and B, non-locality now pops up as a non-local effect from the setting-event A to apparatus B. That is, nature is fundamentally non-local, and there is no way around it.

Bohm and Hiley (1981) have proposed a new type of local hidden variable model with non-local distribution function. That is, Bell's locality condition (C.4) holds, but the hidden variable distribution function, ρ, depends on **a** and **b**, as well as on λ. Hence we find for the correlation function

$$E_{\text{Bohm-Hiley}}(\mathbf{a},\mathbf{b}) = \int d\lambda\ \rho(\lambda,\mathbf{a},\mathbf{b})\ A(\mathbf{a},\lambda)\ B(\mathbf{b},\lambda) \qquad (C.7)$$

instead of (C.5). Bell's inequality no longer follows, when (C.7) is used, and it is possible to select $\rho(\lambda,\mathbf{a},\mathbf{b})$ to get agreement with quantum mechanics.

Bohm and Hiley's model appears to be, as yet, purely qualitative, so it is difficult to assess it as present.

C.9 <u>Conclusion</u>

No matter how we try to "resolve" the EPR paradox, it seems that we come up with another paradox. The experimental results up to now seem to be in favour of quantum mechanics. But what is wrong with local hidden variable theories ? If we can prove that the fundamental ideas of local hidden variable theories are locality and realism (as some physicists suspect) then we must choose between abandoning one or the other, either of which is radical and would entirely change our picture of the world.

APPENDIX D

Relations and Orders

Definition of a Relation

A binary relation R on the set A is a subset of the Cartesian product A×A, that is, a set of ordered pairs (a,b) such that a and b are in A (Roberts , 1979, pp. 13-15).

a binary relation (A,R) is :	provided that :
reflexive	$a\,R\,a$, all $a \in A$
nonreflexive	it is not reflexive
irreflexive	$\sim a\,R\,a$, all $a \in A$
symmetric	$a\,R\,b \Rightarrow b\,R\,a$, all $a,b \in A$
nonsymmetric	it is not symmetric
asymmetric	$a\,R\,b \Rightarrow \sim b\,R\,a$, all $a,b \in A$
antisymmetric	$a\,R\,b\ \&\ b\,R\,a \Rightarrow a = b$, all $a,b \in A$
transitive	$a\,R\,b\ \&\ b\,R\,c \Rightarrow a\,R\,c$, all $a,b,c \in A$
nontransitive	it is not transitive
negatively transitive	$\sim a\,R\,b\ \&\ \sim b\,R\,c \Rightarrow \sim a\,R\,c$, all $a,b,c \in A$; equivalently : $x\,R\,y \Rightarrow x\,R\,z$ or $z\,R\,y$, all $x,y,z \in A$
strongly complete	for all $a,b \in A$, $a\,R\,b$ or $b\,R\,a$
complete	for all $a \neq b \in A$, $a\,R\,b$ or $b\,R\,a$
equivalence relation	it is reflexive, symmetric and transitive.

Table D.1 Properties of relations

Note that :

every binary relation which is	is also
irreflexive	nonreflexive
asymmetric	antisymmetric
asymmetric	nonsymmetric
antisymmetric	nonsymmetric
transitive and complete	negatively transitive
strongly complete	complete
strongly complete	reflexive

Table D.2 Implied properties of relations

property	quasi order	weak order	simple order	strict simple order	strict weak order	partial order	strict partial order
reflexive	/					/	
symmetric							
transitive	/	/	/	/		/	/
asymmetric				/	/		/
antisymmetric			/			/	
negatively transitive					/		
strongly complete		/	/				
complete				/			

Table D.3 Order relations

(Only the defining properties are indicated)

Note that :

Quasi order is also called pre-order.

Simple order " linear or total order.

a binary relation which is a	is also a
weak order	quasi order
simple order	weak order
simple order	partial order
quasi order and antisymmetric	partial order
strict weak order	strict partial order
strict simple order	strict weak order

Table D.4 Implied order relations

APPENDIX E′

Lattice Theory

Lattice theory (Encyclopaedia Britannica, 1978, Macropaedia, V.1, p. 519; Jammer, 1974, Appendix) is the branch of mathematics that deals in precise mathematical language with the relation of different parts of the same whole to each other. The fundamental concept is that one part x may include (or contain) another part y, a relation written symbolically as $x \geq y$. In general, in lattice theory the concepts of order and structure are analysed from a mathematical standpoint, much as group theory analyses the concept of symmetry. Below is a short summary of definitions and theorems. (For proofs, see, for example, Birkhoff, 1967 or Grätzer, 1978)

<u>Definition 1</u> A set $S = \{a,b,c,\ldots\}$ is <u>partially ordered</u> if in S a reflexive, transitive, and antisymmetric binary relation $a \leq b$ (read "a is smaller than, or equal to, b" or "a is contained in b," or "b contains a") is defined, that is, for any a,b,c of S $a \leq a$; $a \leq b$ and $b \leq c$ imply $a \leq c$; and $a \leq b$ and $b \leq a$ imply $a = b$. $a < b$ means $a \leq b$ and $a \neq b$ (read "a is properly contained in b" or "b properly contains a"). $a \prec b$ (read "b covers a") means $a < b$ and no c exists such that $a < c < b$.

In the sequel S denotes a partially ordered set and T one of its subsets.

Definition 2 If an element of T is contained in every element of T it is a least element of T. If an element of T contains every element of T it is a greatest element of T. The least element of S, if it exists, is defined by 0 ("zero"); the greatest element of S, if it exists, is denoted by 1 ("unity").

Theorem 1 A least element of T, if it exists, is unique. A greatest element of T, if it exists, is unique.

Definition 3 If an element is contained in every element of T it is a lower bound of T. If an element contains every element of T, it is an upper bound of T. The least element of the set of all upper bounds of T, if it exists, is the least upper bound or supremum of T, sup T, or $\vee$ T. The greatest element of the set of all lower bounds of T, if it exists, is the greatest lower bound or infimum of T, inf T or $\wedge$ T.

Theorem 2 sup T, if it exists, is unique; inf T, if it exists, is unique.

Theorem 3 sup {a,sup{b,c}} = sup {a,b,c};

inf {a,inf{b,c}} = inf {a,b,c}.

Definition 4 A lattice is a partially ordered set (with zero and unity) in which every pair of elements has a supremum and infimum. A lattice is σ-complete if every nonempty denumerable subset of it has a supremum and infimum. A lattice is complete if every nonempty subset of it has a supremum and infimum. A lattice is finite if the number of its elements is finite.

<u>Theorem 4</u> A finite lattice is complete.

In the sequel $a \cup b$ [read "The union (or join, or disjunction) of a and b"] denotes $\bigvee \{a,b\}$; $a \cap b$ [read "the intersection (or meet, or conjunction) of a and b"] denotes $\bigwedge \{a,b\}$; and L denotes a lattice.

Lattices may be graphically represented (Hasse diagrams) as follows : if $a < b$, a is plotted lower than b and a segment is drawn from a to b. A lattice of five elements, for instance, is represented by one of the diagrams in Fig E.1 :

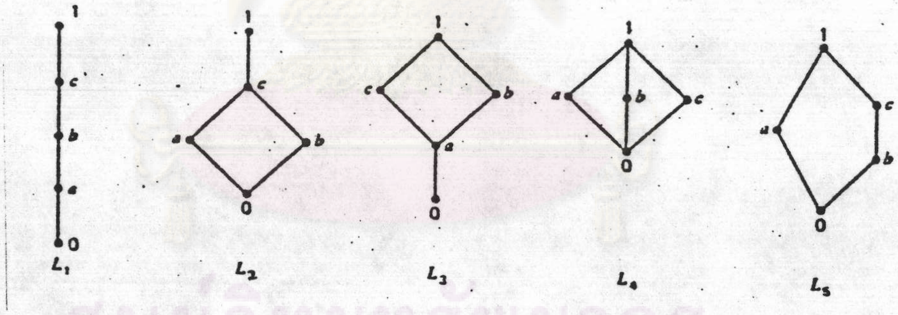

Fig E.1 Hasse diagrams of lattices of five elements

<u>Definition 5</u> An element a of L is an <u>atom</u> if $0 \prec a$. L is <u>atomic</u> if every nonzero element of L contains an atom.

<u>Theorem 5</u> If a,b,c are elements of L then

1. $a \cup a = a$ $\quad a \cap a = a$ $\quad$ (idempotency)

2. $a \cup b = b \cup a$ $\quad a \cap b = b \cap a$ $\quad$ (commutativity)

3. $a \cup (b \cup c) = (a \cup b) \cup c$ $\quad a \cap (b \cap c) = (a \cap b) \cap c$ $\quad$ (associativity)

4. $a \cup (a \cap b) = a$ $\quad a \cap (a \cup b) = a$ $\quad$ (absorptivity)

5. $a \leq b$, $a \cup b = b$, and $a \cap b = a$ imply each other

6. $a \leq b$ implies $a \cup c \leq b \cup c$ and $a \cap c \leq b \cap c$.

<u>Theorem 6</u> A set in which two binary operations $\cup$ and $\cap$ are defined which satisfy the preceding conditions 2,3, and 4 is a lattice with respect to the partial order $a \leq b$ defined by $a \cup b = b$.

<u>Definition 6</u> b is a <u>complement</u> of a if $a \cap b = o$ and $a \cup b = 1$. If every element of L has at least one complement L is <u>complemented</u>. If every element of L has exactly one complement L is <u>uniquely complemented</u>.

<u>Theorem 7</u> In any L, $(a \cap b) \cup (a \cap c) \leq a \cap (b \cup c)$ and $a \cup (b \cap c) \leq (a \cup b) \cap (a \cup c)$.

<u>Definition 7</u> L is <u>distributive</u> if $(a \cap b) \cup (a \cap c) = a \cap (b \cup c)$ and $a \cup (b \cap c) = (a \cup b) \cap (a \cup c)$ for all a,b,c of L. L is <u>Boolean</u> if it is complemented and distributive.

<u>Theorem 8</u> In a distributive lattice $a \cup b = a \cup c$ and $a \cap b = a \cap c$ imply $b = c$ (cancellation rule). A Boolean lattice is uniquely complemented.

<u>Theorem 9</u> In a Boolean lattice (a' denotes the complement of a) :

1. $0' = 1$, $1' = 0$
2. $(a')' = a$
3. $a = b$ if and only if $a' = b'$
4. $(a \cup b)' = a' \cap b'$, $(a \cap b)' = a' \cup b'$
5. $a \leq b$ if and only if $b' \leq a'$
6. $a \leq b$, $a \cap b' = 0$, and $a' \cup b = 1$ imply each other.

<u>Theorem 10</u> In any lattice $a \leq c$ implies $a \cup (b \cap c) \leq (a \cup b) \cap c$ for all a,b,c of L.

<u>Definition 8</u> (b,c) is a <u>modular pair</u> or (b,c)M if for every $a \leq c$ $a \cup (b \cap c) = (a \cup b) \cap c$. A lattice is <u>modular</u> if every two elements of it are a modular pair, that is, if $a \leq c$ implies $a \cup (b \cap c) = (a \cup b) \cap c$ for all a,b,c of the lattice.

<u>Theorem 11</u> Every distributive lattice is modular.

<u>Definition 9</u> A <u>homomorphism</u> is a mapping $h : L_1 \to L_2$ of a lattice L_1 into a lattice L_2 such that $h(a \cup b) = h(a) \cup h(b)$ and $h(a \cap b) = h(a) \cap h(b)$ for all a,b of L_1. An <u>isomorphism</u> is a one-to-one homomorphism. An <u>automorphism</u> is an isomorphism of a lattice with itself. A <u>dual-isomorphism</u> is a one-to-one mapping $d : L_1 \to L_2$ such that $a \leq b$ implies $d(b) \leq d(a)$ for all a,b of L_1. A <u>dual-automorphism</u> is a dual-isomorphism of a lattice with itself. An <u>involutive dual-automorphism</u> of L is a dual-automorphism d such that $d(d(a)) = a$ for all a of L.

<u>Theorem 12</u> An involutive dual-automorphism of L satisfies $d(a \cup b) =$

$d(a) \cap d(b)$ and $d(a \cap b) = d(a) \cup d(b)$ for all a,b of L.

Theorem 13 For an involutive dual-automorphism of L the following three statements imply each other : (1) $a \leq d(a)$ implies $a = 0$, (2) for all a of L $a \cap d(a) = 0$, (3) for all a of L $a \cup d(a) = 1$.

Definition 10 An involutive dual-automorphism of L which satisfies one of the preceding three conditions is an orthocomplementation, L is orthocomplemented and d(a) is the orthocomplement $a^{\perp}$ of a. Being a complement $a^{\perp}$ will be denoted simply by a'.$a \perp b$ means $a \leq b'$.

Theorem 14 L is orthocomplemented if and only if there exists a mapping $a \rightarrow a'$ of L onto itself such that $(a')' = a$, $a \leq b$ implies $b' \leq a'$, $a \cap a' = 0$, and $a \cup a' = 1$ for all a,b of L.

Theorem 15 $a \perp b$ implies $b \perp a$ (a and b are orthogonal or disjoint).

Definition 11 A lattice is weakly modular if it is orthocomplemented and $a \leq b$ implies $b = a \cup (a' \cap b)$. It is quasi-modular if it is orthocomplemented and $a \leq b \leq c'$ implies $a = (a \cup c) \cap b$. It is orthomodular if it is orthocomplemented and $a \perp b$ implies (a,b)M.

Theorem 16 An orthocomplemented lattice is weakly modular if and only if $a \leq b$ implies $a = b \cap (a \cup b')$.

Theorem 17 Each of the three lattice properties ,weakly modularity, quasi-modularity, and orthomodularity, implies the other two.

<u>Theorem 18</u> An orthocomplemented modular lattice is orthomodular. In the sequel, L_o denotes an orthocomplemented lattice and L_{OM} an orthomodular lattice.

<u>Theorem 19</u> In L_o $(a \cap b) \cup (a \cap b') \leq a$ for all a,b of L_o.

<u>Definition 12</u> In L_o a is <u>compatible</u> with b or $a \leftrightarrow b$ if $(a \cap b) \cup (a \cap b') = a$; a is <u>commensurable</u> with b or $a \doteqdot b$ if $a \cup (a' \cap b) = b \cup (b' \cap a)$; a is <u>commeasurable</u> with b or $a \sim b$ if there exists in L_o three mutually orthogonal elements a_1, b_1 and c such that $a = a_1 \cup c$ and $b = b_1 \cup c$.

<u>Theorem 20</u> In L_o $a \leftrightarrow b$ implies $a \leftrightarrow b'$.

<u>Theorem 21</u> In L_{OM} $a \leftrightarrow b$ implies $b \leftrightarrow a$, and $a \doteqdot b$ implies $a \leftrightarrow b$.

<u>Theorem 22</u> An L_o in which $a \doteqdot b$ implies $b \leftrightarrow a$ is an L_{OM}.

<u>Theorem 23</u> In L_o $a \doteqdot b$ implies $b \doteqdot a$.

<u>Theorem 24</u> In L_{OM} each of the three lattice-element relations, compatibility, commensurability, and commeasurability, implies the other two.

<u>Definition 13</u> A subset of a lattice L is a <u>sublattice</u> of L if it is itself a lattice with respect to the lattice operations of L.

Theorem 25 The (set-theoretical) intersection of a family of sublattices of L is a sublattice of L.

Definition 14 If T is a subset of L the lattice generated by T is the intersection of all sublattices of L which contain T.

Theorem 26 In L_o a is commeasurable with b if and only if the lattice generated by a, a', b, and b' is Boolean.

Definition 15 In L_o an element is central if it is commeasurable with all elements of L_o; the set of all central elements is the centre of L_o; L_o is irreducible if its centre is $\{0, 1\}$.

APPENDIX F

Measurement of the First, Second and Third Kind

Wolfgang Pauli (1958) introduced the following definitions :

M1. A measurement is said to be of the first kind if it leaves the measured state in an eigenstate of the measured eigenvalue.

M2. A measurement is said to be of the second kind if it leaves the measured state in some other eigenstate.

Recently, Nick Herbert (1982) introduced a new kind of measurement defined as follows :

M3. A measurement of the third kind is one that duplicates the measured state exactly.

If the measured state is a photon state, it is possible to perform measurements of the third kind using the process of stimulated emission of light in which the emitted photon is "identical" to the impinging one (for details see Garuccio, 1985).

Curriculum Vitae

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