CHAPTER 2

FLEXURAL THEORY

Basic Assumptions and General Theory of Flexure

Assumptions used in calculation of deflections curvatures and ultimate bending strengths are summarized as follows:

a) Plane sections before bending still remain plane after bending.

b) The stress-strain relationship of steel is known and assumed to be a straight line up to the yield point of the steel.

c) Concrete can take tension until the extreme fiber stress at the critical section is equal to the modulus of rupture which can be approximately obtained from spliting tensile strength. After crack concrete can no longer resist any tensile stress.

d) Bond between concrete and steel is assured, transfering of stress up to the ultimate load.

The above basic assumptions are for conventional reinforced concrete members. The first assumption, Bernoulli's principle, is generally accepted since many tests have provéd its validity. However the strain distribution at the critical sections of the test beams in this study (Fig.4.14 to 4.18) also verified this assumption.

The stress-strain relationship of steel wire with an indefinite yield point is defined its yield strength at 0.2 % offset of the strain

Normally, a bi-linear stress-strain relationship is assumed as shown in Fig. 3.1.

The bond between steel and concrete is true only when no excessive cracks occured. However it is not perfectly correct since local bond failures occur in vicinity of cracks.

Concrete Stress Distribution

Strain profiles and stress distributions due to various stages of loading are shown in Fig. 2.1 to Fig. 2.4.

At early stage since there is no crack in the section, the tensile strength of concrete can take into account for calculating the resisting moment. Both concrete and steel their stress-strain responses are in elastic ranges.

When bottom extreme fiber of the critical sections of test beams start to crack, tension are taken by reinforcement. After that, within elastic range the concrete tensile stress may no longer consider for resisting moment since it is very small when compared to the resisting moment due to concrete compressive stress.

In inelastic range, there are many ways to predict the ultimate bending strength by using appropriate compressive stress distributions such as traperzoidal, parabolic, triangular shape etc. In this study it is assumed that compressive stress distributions are in parabolic, triangular and Nedderman's form, then the results will be compared to that from 1977 ACI Code[1] method.

Internal Force and Moment

Based on the above assumptions, internal forces and moments at various stages can be derived as follows:

a) Before Cracking Stage , $f_t < f_r$ (Fig. 2.1)

$$kd = \frac{bh^2/2+(n-1)dA_s}{bh+(n-1)A_s} \qquad ......(2.1)$$

where b = width of test beam

h = depth of test beam

d = effective depth measured from compression extreme fiber to the centroid of the reinforcement

$n = \frac{E_s}{E_c}$

A_s = area of longitudinal reinforcement

and I_{un} = uncracked moment of inertia

$$= \frac{bh^3}{12} + bh(kd - \frac{h}{2})^2+(n-1)A_s(1-k)^2d^2 \qquad ...(2.2)$$

So the internal resisting moment of critical section is

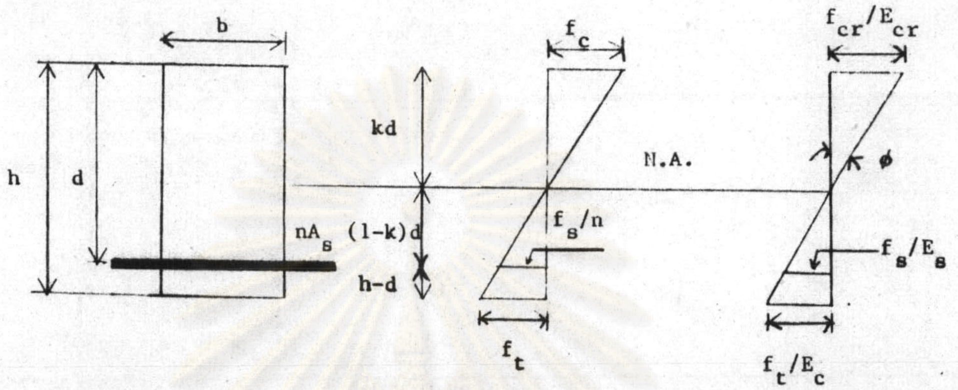
$$M_{un} = \frac{f_t I_{un}}{h-kd} \qquad ......(2.3)$$

where f_t is the tensile stress of concrete

b) At Cracking Stage , $f_t = f_r$

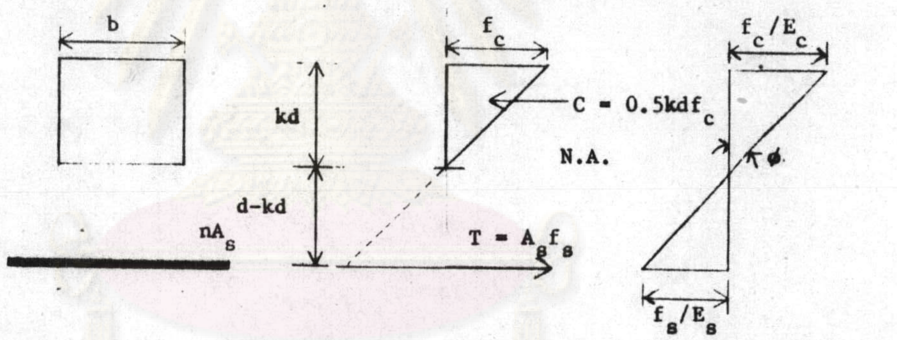
By substituting $f_t = f_r$, modulus of rupture, into equation 2.3 yields.

$$M_{cr} = \frac{f_r . I_{un}}{h-kd} \qquad ......(2.4)$$



a) Transformed area b) Stress distribution c) Strain distribution

Fig. 2.1 Uncracked Transformed Section



a) Transformed area a) Stress distribution c) Strain distribution

Fig. 2.2 Cracked Transformed Section

where f_r is modulus of rupture of concrete approximately obtained from spliting tensile test

c) <u>After Cracking Stage</u> (Fig. 2.2)

$$kd = \sqrt{2pn+(pn)^2}-pn \qquad ......(2.5)$$

where $P = \frac{A_s}{bd}$, and moment of inertia of cracked section can be written as

$$I_{cr} = \frac{bd^3k^3}{3} + nA_s d^2(1-k)^2 \qquad ......(2.6)$$

The internal compression and tension forces can be expressed as

$$C = \frac{1}{2}.f_c bkd \qquad ......(2.7)$$

$$T = A_s.f_s \qquad ......(2.8)$$

The internal resisting moment can be determined from each individual force as

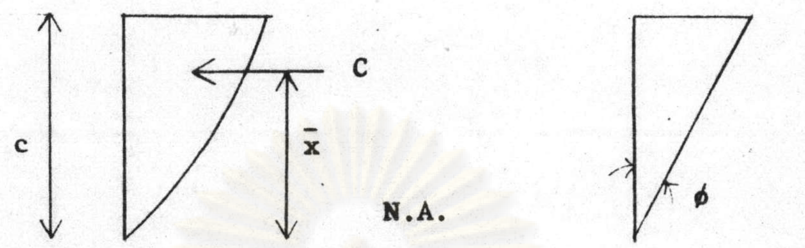
$$M = Td(1-\frac{k}{3}) = Cd(1-\frac{k}{3}) \qquad ......(2.9)$$

The ultimate bending strength by using parabolic compressive stress distribution(17) can be written as: (Fig. 2.3)

$$M = C(d-c-\bar{x}) \qquad ......(2.10)$$

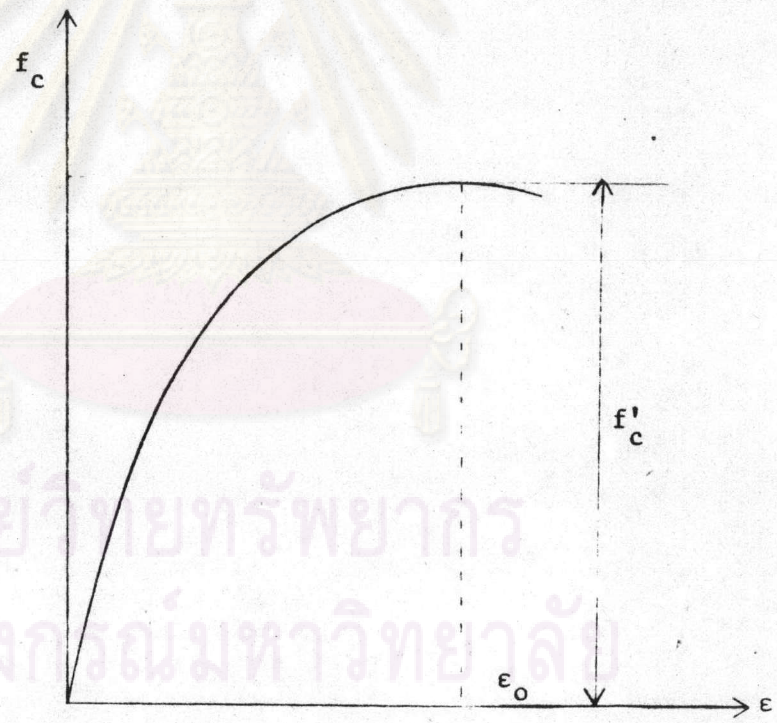
where C = compression force

$$= bf'_c \frac{\phi}{\varepsilon_o} c^2 \left|1-\frac{\phi c}{2\varepsilon_o}\right|$$



a) Parabolic compressive stress-distribution

b) Strain distribution



c) Stress-strain relationship of parabolic shape

Fig. 2.3 Stress and Strain Distribution and its Relationship of Parabolic Shape

c = distance from neutral axis to extreme compression fiber

ε_o = compressive strain at ultimate compressive strength, f'_c

$\bar{x}$ = distance from neutral axis to compression force of concrete

$$= c\left[\frac{8\varepsilon_o - 3\phi c}{12\varepsilon_o - 4\phi c}\right]$$

then
$$M = bf'_c \frac{\phi}{\varepsilon_o} c^2\left[1 - \frac{\phi c}{2\varepsilon_o}\right]\{d - c(\frac{\phi c - 4\varepsilon_o}{12\varepsilon_o - 4\phi c})\} \quad \ldots(2.11)$$

And that by using Nedderman's compressive stress distribution[10] can be written as:

$$M = C(d - k_2 c) \quad \ldots\ldots(2.12)$$

where C = compression force

$= 0.58f'_c bc$

k_2 = the ratio of distance between the extreme fiber and the resultant of compressive stresses to the distance between the extreme fiber and the neutral axis.

$= 0.37$

then
$$M = 0.58f'_c bc(d - 0.37c) \quad \ldots\ldots(2.13)$$

According to 1977 ACI Code(1) the ultimate bending strength can be written as: (Fig. 2.4)

$$M = \phi A_s . f_s (d - \frac{a}{2}) \qquad ......(2.14)$$

where ϕ = strength reduction factor, for fexure = 0.9

A_s = area of tension reinforcement

f_s = stress in reinforcement

a = depth of equivalent rectangular stress block

= $\beta_1 c$

β_1 = 0.85 for concrete strengths f'_c up to and including 280 ksc. For strengths above 280 ksc., β_1 shall be reduced continuously at a rate of 0.05 for each 70 ksc. of strength in excess of 280 ksc., but β_1 shall not be taken less than 0.65

c = distance from extreme compression fiber to neutral axis

Flexural Rigidity

a) Modulus of Elasticity of Concrete

Due to small range in taking tension of concrete into account, it is presumed that modulus of elasticity in tension of concrete is the same as in compression. It is recommended by 1977 1977 ACI Code that

$$E_c = 15,100\sqrt{f'_c} \qquad ......(2.15)$$

where f'_c is cylindrical compressive strength in ksc. and test results[11] shown in Fig. 3.4 have proved its validity.

b) Moment of Inertia

Before cracking stage moment of inertia may be based on the gross concrete section, with generally a small difference arising from whether or not the transformed area of reinforcement is also included. However, as the load increases above the cracking load, the moment of inertia decreases and approaches that of cracked transformed section, although it may be greater between cracks[15].

The 1971 ACI Code recommended to use effective moment of inertia, as proposed by Branson[15] in 1963, which includes the effect of load level and degree of cracking as:

$$I_{ef} = (M_{cr}/M_{max})^3 I_g + \left[1-(M_{cr}/M_{max})^3\right] I_{cr} \qquad .....(2.16)$$

where M_{cr} = cracking moment

M_{max} = maximum moment for the loading condition under which deflection is computed

I_g = gross moment of inertia (neglecting reinforcement)

I_{cr} = cracked transformed section moment of inertia

Moment-Curvature Relationship

Fig. 2.5 shows an element of a reinforced concrete member with equal end moments. The radius of curvature, R, is measured to the neutral axis. The rotation between the ends of the element is given by

$$\frac{dx}{R} = \varepsilon_c \frac{dx}{kd} \qquad \text{......(2.17)}$$

and the curvature, which is the rotation per unit length of member is

$$\phi = \frac{1}{R} = \frac{\varepsilon_c}{kd} \qquad \text{......(2.18)}$$

$$\phi = \frac{M}{EI}\frac{kd}{kd} = \frac{M}{EI} \qquad \text{......(2.19)}$$

The theoretical moment-curvature relationship may be determined by incrementing concrete strain at the extreme compression fiber and neutral axis depth which satisfied internal force equilibrium is found by adjusting itself until the compression force in concrete is equal to the tension in steel.

The moment and the corresponding curvature can then be determined by using equations 2.2, 2.3, 2.4, 2.6, 2.9, 2.18 and 2.19. Then moment-curvature curve can be plotted within a range of values for compressive strain.

Ductility is defined as the ratio between ultimate and yield curvature. It was proposed by Furlong[16] that

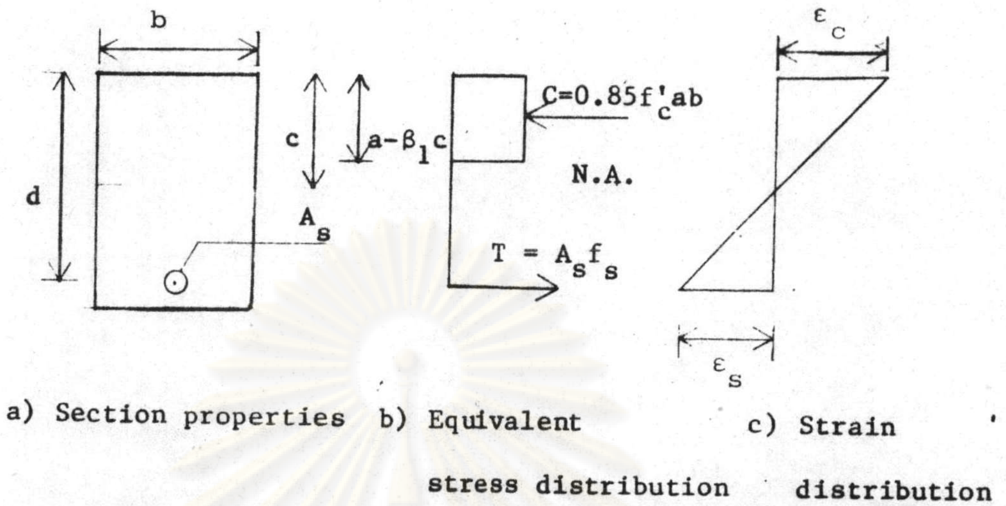


Fig. 2.4 Section Properties and Equivalent Stress Distribution by 1977 ACI Code

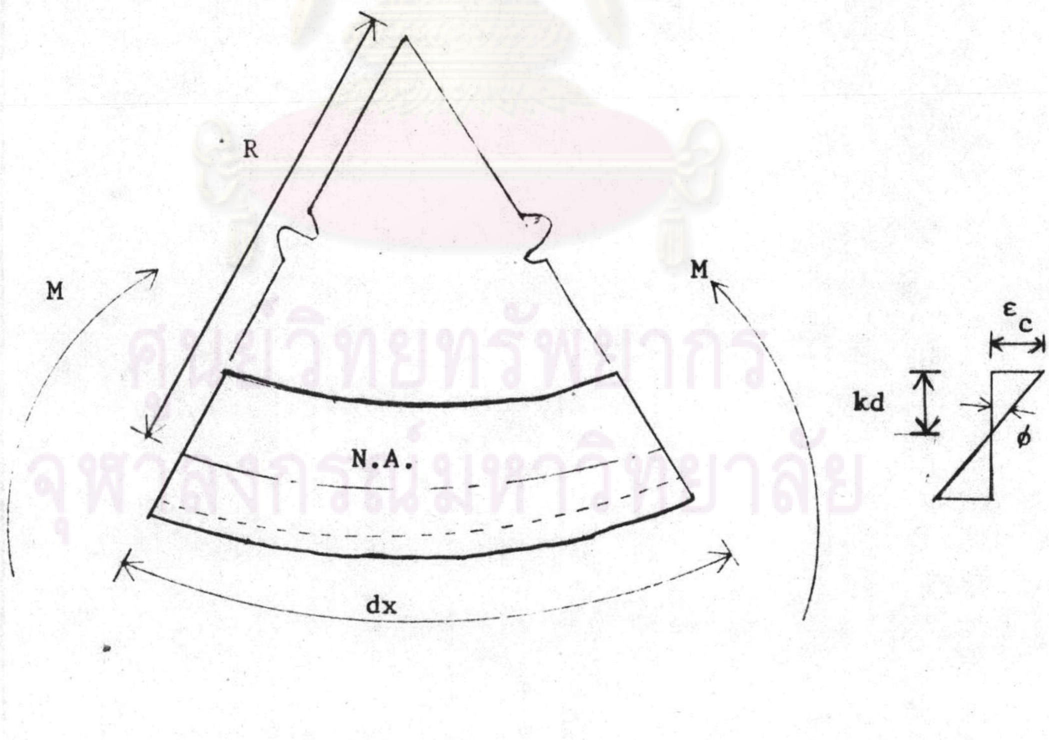


Fig. 2.5 Moment-Curvature

$$U = 1+0.235 \frac{L}{d}$$

where U = ductility index

$$= \frac{\phi_{ul}}{\phi_y}$$

L = span length

d = beam depth

$\frac{L}{d}$ is usually between 15 to 20 hence U will be 4.5 to 5.7

Load-Deflection Relationship

Deflection can be found by solving differential equation of the elastic curve of a beam as

$$\frac{d^2y}{dx^2} = -\frac{M}{EI} \qquad \text{......(2.21)}$$

where y = deflection of a beam

x = horizontal distance along beam length

M = bending moment

E = modulus of elasticity of concrete

I = moment of inertia

For a simply-supported beam, span length L, subjected to any loading shown in Fig. 2.6

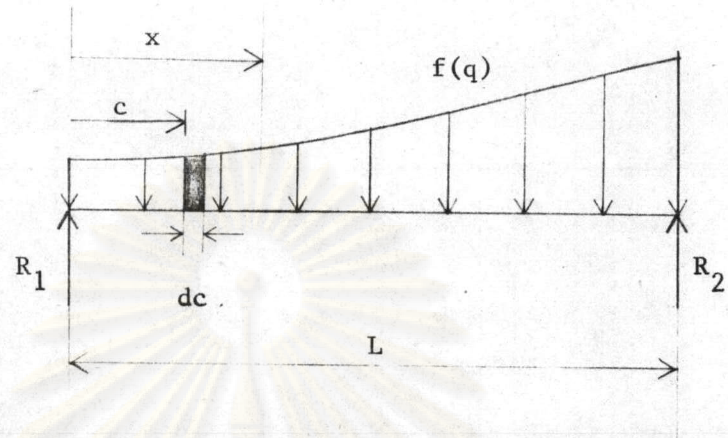


Fig. 2.6 Simply-supported Beam Subjected to any Loading

$$\Sigma M_{R_2} = 0 ; \quad R_1 L = \int_o^L f(q).(L-c)dc$$

$$R_1 = \frac{1}{L}\int_o^L f(q).(L-c)dc \quad .....(2.22)$$

and bending moment at distance x away from R_1 is

$$M_x = R_1.x - \int_o^x f(q).(x-c)dc \quad ......(2.23)$$

Then substitue equation (2.23) into (2.21) and solve for deflection

In this study, test beams are simply-supported beams subjected to two symmetrically concentrated loads at middle-third point of span length shown in Fig. 3.7, the mid span deflection becomes

$$y_{\frac{L}{2}} = \frac{23}{(36)^2}\frac{PL^3}{EI} \quad ......(2.24)$$

$$= \frac{23}{216}\frac{ML^2}{EI} \quad ......(2.25)$$

Due to variation of load and moment of intertia equation (2.25) can be written as

$$\Delta y_{\frac{L}{2}} = \frac{23}{(36)^2}\frac{L^3}{EI}\Delta P - \frac{23}{(36)^2}\frac{PL^3}{EI^2}\Delta I \quad ......(2.26)$$