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EFFECTS OF LORENTZ SYMMETRY VIOLATION IN COSMOLOGICAL MODELS

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แม้ว่าสมมาตรลอเรนซ์จะเป็น สมมาตรมูลฐานของธรรมชาติที่มีบทบาทสำคัญทั้งในฟิสิกส์ของอนุภาคในระดับควอนตัม และทฤษฎีโน้มถ่วงแบบฉบับ อย่างไรก็ตาม มีข้อเสนอ ว่าสมมาตรลอเรนซ์อาจจะถูกละเมิดได้ที่ระดับพลังงานของความโน้มถ่วงควอนตัม นอกจากนี้ยังมีความเป็นไปได้ว่าผลกระทบของการละเมิดสมมาตรนี้อาจส่งผลที่สามารถตรวจวัดได้ในทฤษฎีสนามยังผลที่ระดับพลังงานต่ำ การหาผลกระทบนี้ได้ดำเนินการอย่างแพร่หลายในหลากหลายแง่มุม สำหรับผลกระทบในแบบจำลองทางจักรวาลวิทยานั้น การละเมิดนี้จะเป็นผลมาจากการมีอยู่ของสนามอีเทอร์ ในวิทยานิพนธ์ฉบับนี้ ได้แบ่งการศึกษาผลกระทบของสนามอีเทอร์นี้ออกเป็นสองส่วนคือ ส่วนที่มาจากสนามอีเทอร์เสมือนกาลและส่วนที่มาจากสนามอีเทอร์เสมือนอวกาศ สำหรับสนามอีเทอร์เสมือนกาลนั้น เงื่อนไขบังคับของตัวแปรเสริมอีเทอร์จะได้มาจากทั้งผลทดลองและเงื่อนไขจากเสถียรภาพของทฤษฎี โดยผลจากการศึกษาสนามอีเทอร์เสมือนกาลที่เป็นไปตามเงื่อนไขบังคับนี้ในแบบจำลองอินเฟลชันพบว่าพลศาสตร์ของการขยายตัวของเอกภพได้เปลี่ยนไปเล็กน้อย และการรบกวนเริ่มต้นได้เปลี่ยนไปอย่างมีนัยสำคัญ สำหรับสนามอีเทอร์เสมือนอวกาศนั้น เราได้ศึกษาผลของสนามนี้ในแบบจำลองพลังงานมืดแบบแคสิเมียร์ในกาลอวกาศห้ามิติ ทั้งนี้เราพบว่า เมื่อเรายังไม่ได้คิดผลของสนามอีเทอร์ มิติพิเศษจะไม่มีเสถียรภาพได้ถ้าในแบบจำลองแคสิเมียร์นี้รวมผลของสสารเข้าไปด้วย อย่างไรก็ตาม ถ้ารวมผลจากสนามอีเทอร์เข้าไปด้วยแล้วจะพบว่า สนามอีเทอร์จะส่งผลให้แรงยังผลที่กระทำกับสนามเรดิออนมีค่าต่ำลง ซึ่งจะทำให้สนามเรดิออนเคลื่อนที่ช้าลง และสามารถหยุดนิ่งที่จุดต่ำสุดของศักย์ได้และนำไปสู่เสถียรภาพของมิติพิเศษ

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Lorentz symmetry is a fundamental symmetry in nature. It plays an important role in both particle physics in microscopic quantum scale and gravitation theories in macroscopic scale. However, this symmetry is expected to be broken at the Planck scale as the effect suggested from quantum gravity. It is possible that some consequent effects of Lorentz violation can be found in the low-energy effective field theories. Investigations on the Lorentz violation effects gain a lot of attention over the past ten years and many approaches to study these effects have been developed. The Lorentz violation effects in cosmological models are provided by the existence of the æther field. In this thesis, an investigation of the æther field effects is divided into two parts, time-like and space-like æther field. For the time-like æther field, the constraints for the æther parameters are imposed by several experiments. The æther parameters are also constrained by the stability of the models. The effects of the viable time-like æther field in inflationary models are reviewed. The dynamics of inflation is slightly modified and the primordial perturbations are significantly changed. For the effects of space-like æther field on the cosmological models, we consider the Maxwell-like æther field in the Casimir dark energy models in 5-dimensional spacetime. In the Casimir dark energy models, the mechanism for stabilizing the extra dimension are destroyed when non-relativistic matter is taken into account. It is found that the æther field can reduce the effective force acting on the radion field. The radion field will be slowed down before it passes the minimum of the potential. Therefore, the radion field can settle down at the minimum of the potential and the stability is restored.

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CONTENTS

LIST OF TABLES

Table **page**

LIST OF FIGURES

Figure **page**

Chapter I

INTRODUCTION

There are many symmetries found in nature. Lorentz symmetry is a well-known symmetry which characterizes the invariant quantities under rotational and boost transformations. Lorentz symmetry plays a vital part of the special relativity as well as a crucial fundamental symmetry to construct quantum field theory. The standard model is very successful to provide the descriptions of the elementary particles and forces. This model is also based on the quantum field theory and special relativity. Thus one can promote Lorentz symmetry to be the fundamental symmetry of elementary particle system in nature at quantum scale. The successful description of the gravitation is provided by Einstein's general relativity. Lorentz symmetry is also the local symmetry of freely falling frames in Einstein's general relativity. Therefore, it is reasonable to promote that the Lorentz symmetry is the most fundamental symmetry of nature.

Although Lorentz symmetry is a fundamental symmetry of nature there are reasonable motivations for studying the theories that Lorentz symmetry is broken. The key motivations come from the theories of quantum gravity. Lorentz symmetry provides a consistent way to construct both model of particle physics at quantum scale and theory of gravity. However, there is no guarantee that Lorentz symmetry is a symmetry of nature at quantum gravity scale. Furthermore, the candidates of quantum gravity theory such as string theory [1] and loop quantum gravity [2] suggest the possibility of the existence of Lorentz violation effects. Hence, this is one of the theoretical possibilities to probe the physics at quantum gravity scale. Moreover, on the experimental side, the ability to observe physical properties at high energy scale is rapidly improved in several ways including terrestrial, astrophysical and cosmological experiments. This is also the reason why many researchers are interested in the Lorentz violation theories.

The theoretical models of Lorentz violation have been investigated intensively in various subjects [3, 15]. Since the main goal of this thesis is to investigate the effects of Lorentz violation in cosmological models, it is convenient to consider only a class of the models which is relevant to cosmology. However, to be omniscient, we give a brief review of all interesting models of Lorentz violation in the Chapter II. For our convenience, we classify the Lorentz violation models into two approaches, kinematic and dynamical approaches. For the kinematic approach, we give key ideas and crucial results of two interesting models, i.e., modified dispersion relation and doubly special relativity [12, 13]. For other models of this approach, interested readers can look at [3] and references therein.

For the dynamical approach, we discuss the modification of both the standard model of particle physics and Einstein's general relativity. For the modified standard model or standard model extension, the key concepts and important results are briefly reviewed in the context of effective field theory. However, this model is not directly relevant to cosmology [3, 4, 5]. The direct cosmological applications of Lorentz violation are provided by Einstein-æther models, also known as the æther models [27]. In these models, Einstein's general relativity are modified by including the dynamical vector field with fixed norm. These models are based on the effective field theory with two dynamical fields, the spacetime metric $g_{\mu\nu}$ and the æther field A^{μ}. The general properties of the æther field are characterized in Chapter II.

In the æther models, Lorentz symmetry is spontaneously broken by the vacuum expectation value, *vev*, of the æther field. Due to the fixed norm condition, the effects of the time-like and space-like æther field are significantly different. Thus the study of the æther field can be divided into two parts according to the alignment of the æther field. Generally, there are four covariant scalar terms which are quadratic derivative. These kinetic terms are characterized by 4 æther parameters $\beta_1, \beta_2, \beta_3, \beta_4$. In the final part of Chapter II, we review the constraints of these parameters by using both consistency of the theoretical models and observational data [45].

Since the main goal of this thesis is to investigate the effects of the æther field in cosmological models, it is convenient to discuss the interesting cosmological models. In Chapter III, we provide a brief review of the evolution of the universe and focus on two interesting periods, inflationary and late-time acceleration period. The inflationary models provide a description of the extreme expansion of the universe at the early era. The dynamics of inflation is simply provided by slow-rolling of the scalar field, named *inflaton field*, on the flat potential. The inflationary models satisfy the observational data which suggests the almost perfectly uniform universe. They also provide the primordial perturbations which seed the structure we observe nowadays. We discuss this mechanism in the first part of Chapter III. We devote the later part of Chapter III to dark energy models. The dark energy models provide the late-time acceleration of the universe. These models are classified into three groups, cosmological constant, fluid dark energy models and gravitational dark energy models. The characteristics, the evolution of the universe, the advantages and disadvantages of the basic simple models of dark energy are discussed in detail and are briefly summarized for other complicated models.

The effects of the time-like and space-like æther field are discussed separately. For the time-like case, we expect that the Lorentz violation effects due to the æther field take place near the Planck scale. It is reasonable to investigate the effects of the æther field in the inflationary models since the information of the Planck scale physics can be encoded in cosmological perturbations during the inflationary period. We review these investigations in detail for the time-like æther field in Chapter IV [116, 113, 114, 28].

In the other case, the space-like æther field has not attracted much attention because its existence violates the rotational invariance. The only one investigation is a toy model which the accelerating universe is driven by the cosmological constant and the kinetic terms are only the Maxwell-like term [38]. The results of this investigation show that this model is not viable since it encounters instabilities. Thus we are not interested in this issue.

The interesting application of the space-like æther field is performed in the theories of extra dimensions since the rotational invariance in the three-dimensional space is not violated. This invariance is obtained by phenomenological setting in which the ground state of the æther field aligns in the extra dimensions. In Chapter V, we investigate the effects of the space-like æther field in Casimir dark energy models by using this setting. In this model, Casimir energy of various field fluctuations in extra dimensions plays the role of cosmological constant and drives the late-time accelerated expansion of the universe [107]. These models can solve the cosmological problem and also provide the mechanism to stabilize the extra dimensions. However, the extra dimensions will be destabilized when matter contents are taken into account. In five-dimensional spacetime, we add the æther field into the Casimir dark energy models with the matter contents and show explicitly that the æther field affects these models in such that the stabilization of the extra dimension can be restored [108]. The results and discussions are summarized in Chapter VI. The further interesting investigations are also discussed in that Chapter.

Chapter II

LORENTZ VIOLATION THEORIES

Lorentz symmetry plays an important role in fundamental physics. It is a symmetry of special relativity and the standard model. However, there are suggestions that Lorentz symmetry may be broken, at least spontaneously, at the quantum gravity scale for example in string theory [1] and loop quantum gravity [2]. Since Lorentz violation effects are believed to have the origin from quantum gravity, the study of Lorentz violation models may give some insight to Planck scale physics. Moreover, with rapid improvement of high-energy experiments including terrestrial, astrophysical and cosmological experiments, Lorentz violation theories may provide possible and interesting ways to probe physics at quantum gravity scale experimentally.

In this chapter, we will begin with an overview of Lorentz violation theories. Then we go on to consider a specific class of the models, the so-called æther models as they can provide the Lorentz violation effects which may be observed in cosmological data. The theoretical consistency and observational constraints on æther models are briefly reviewed in the final part of this chapter. Cosmological applications of the æther models will be discussed in the next chapter.

2.1 Overview of Lorentz violation theories

In this section, we review the construction of the Lorentz violation theories and briefly discuss the key ideas of some particular models in each theory. We start by classifying the breaking of Lorentz symmetry into two types. The first type is called the breaking of *particle* Lorentz invariance and the second is the breaking of *observer* Lorentz invariance. Let us explain this issue in more detail.

The particle Lorentz invariance corresponds to the *active* Lorentz transformation which is a transformation of particles or localized field distribution while keeping the spacetime coordinates unchanged. On the other hand, the observer

Lorentz invariance corresponds to *passive* Lorentz transformation which is the transformation of the coordinates, $x'^{\mu} = \Lambda^{\mu}_{\nu}x^{\nu}$, where Λ^{μ}_{ν} is the Lorentz transformation matrix. Note that Λ^{i}_{j} corresponds to the rotational transformations in 3-dimensional space and Λ^{0}_{i} corresponds to the boost transformations along the i direction. In the normal situation, the active and passive Lorentz transformation are equivalent. However, in some particular situation, such as the existence of the preferred reference frame, the particle Lorentz invariance may be broken while the observer Lorentz invariance is still unbroken.

The presence of the Lorentz violation can be induced by the nonzero vacuum expectation value of one or more quantities carrying the Lorentz indices, for example A_{μ}. It is called Lorentz violation coefficient. The active Lorentz transformation leaves the coefficient A_{μ} unaffected. A_{μ} behaves as a set of four scalars under this type of transformation. This means that A_{μ} sets the preferred direction in the spacetime. Thus the *particle* or active Lorentz symmetry is broken. On the other hand, the coefficient A_{μ} is transformed under the observer Lorentz transformation as $A'_{\mu} = (\Lambda^{\nu}_{\mu})^{-1}A_{\nu}$. The coefficient transforms covariantly as a four-vector since the spacetime coordinates act as the basis of the vector. Thus the observer Lorentz symmetry is not broken. The invariance under the passive Lorentz transformation provides the observable physics which does not change when we change the coordinates of the observer. For the rest of this thesis, all Lorentz violation models that we deal with are only the particle Lorentz violation.

Note that the theory of Lorentz violation must be constructed in the way that it does not conflict with the Lorentz invariant theory at the low-energy scale. This means that the Lorentz violation theory must reproduce the standard models and general relativity in some of its limit.

There are some Lorentz violation models that meet these requirements. We will classify all of them into two parts. The first part is kinematic approach. The Lorentz violation model in this approach is not a complete theory by itself. It requires dynamical theory in order to explain all physics at the scale at which the theory takes place. For example, in the modified dispersion relation model, it explains only the physics of free particles but does not for interacting ones. The other part is dynamical approach. It is an approach to the complete theory in the sense that it provides the possibility to describe all physics at low energy scale. Moreover, it should provide the corrections of Lorentz violation effects that are sensible to be observed by experiments.

We will briefly review some interesting models of the Lorentz violation theory in the following subsections. Most of these discussions follow the review paper [3, 15] and references therein.

2.1.1 Kinematic Approach

Generally, Lorentz violation effects can be obtained by modifications of some relations or quantities in Lorentz invariant theory such as dispersion relation. These modifications are basically put in by hand with some phenomenological reason. Most of them have no obvious connection to the underlying fundamental theory. In this subsection, we briefly review the modified dispersion relation and doubly special relativity.

- **Modified dispersion relation**

The dispersion relation of a massive particle can be expressed as $E^2 = m^2 + p^2$, where E is energy, m is mass and p is the momentum of the particle. This relation is Lorentz invariant. Its validity has been confirmed not only by the experiments but also by the consistency of theory. However, Lorentz symmetry might be broken at high-energy scale. We expect a small variation from the above dispersion relation. Our strategy is to rewrite the above dispersion relation in the more general form, $E^2 = F(p, m)$. Of course, this general form should reduce to the Lorentz invariant relation at low energy scale. Thus we need to expand function $F(p, m)$ around the Lorentz invariant relation. Furthermore, we have to specify what the high energy scale we will use to compare is. We expect that this modification is the effect of the quantum gravity with energy scale of Planck energy, E_{pl}. Therefore, the general form of this relation can be expanded in terms of linear momentum, p, as

$$E^2 = m^2 + p^2 + E_{pl} f_i^{(1)} p^i + f_{ij}^{(2)} p^i p^j + f_{ijk}^{(3)} p^i p^j p^k / E_{pl} + ..., \tag{2.1}$$

where $f^{(n)}$ is an arbitrary dimensionless function and the superscript of this function, (n), denotes the order of the expansion. As we have seen in equation (2.1), this relation can break the rotational subgroup of the Lorentz group due to the existence of non-rotational invariant combinations of p^i. Most of these models are constructed in order to avoid this violation because the rotational subgroup is strongly confirmed by many experiments. Moreover, in some criteria, the broken rotational subgroup yields the breaking of boost invariance automatically. Thus more popular version of the modified dispersion relation is expressed as

$$E^2 = m^2 + p^2 + E_{pl} f^{(1)} |p| + f^{(2)} p^2 + f^{(3)} |p|^3 / E_{pl} + ..., \tag{2.2}$$

where $|p|$ is the magnitude of p^i. The third and higher order terms are suppressed by the factor of E_{pl}. Actually, the first order term is more important. We can choose the form of $f^{(1)}$ in such a way that it corresponds to the energy scale we are interested in, for example, $f^{(1)}$ can be chosen as $f^{(1)} = \mu^2/E_{pl}^2$ where μ is the energy scale at which we expect to find the new physical phenomena. As we mentioned before, this is not the complete theory because of the lack of the underlying dynamical mechanism. However, some models in dynamical approach, such as minimal standard model extension [6], will provide similar modification of dispersion relation. A nice review of this issue is [4], and details together with deep concepts are in references therein. Moreover, the modified dispersion relation is often found in other models of Lorentz violation such as Doubly Special Relativity (we will discuss this subject later). Note that modified dispersion relation also provides the variation of equivalence principle [7].

- **Doubly special relativity (DSR)**

The key idea of DSR is to modify the dispersion relation in which the preferred frame does not exist in the theory. This non-preferred frame effect is the advantage of this model since the observed physical properties are frame-independent and agree with the experiments. The motivation of this model comes from the quantum gravity effect which suggests that there is a fundamental length scale at which quantum theory and general relativity are comparable. This fundamental length scale violates the boost invariance of Lorentz group explicitly. Thus the strategy of this model is to modify action of Lorentz group in which there is an additional invariant quantity other than the speed of light c. The existence of these two invariant quantities in this model is the root of the name, *Doubly Special Relativity.* There are two approaches to this model based on modification of the action of the Lorentz group on physical states, namely DSR1 and DSR2. To see the differences between DSR1 and DSR2, we will consider the boost generators in differential representation. For DSR1, the generator of Lorentz boost along z axis is

$$N_z = p_z\partial_E + (E + \frac{\lambda_{DSR}}{2}|p|^2 - \lambda_{DSR}E^2)\partial_{p_z} + \lambda_{DSR}p_zp_i\partial_{p_i}. \tag{2.3}$$

For DSR2, the generator of Lorentz boost along z axis is

$$N_z = p_z\partial_E + E\partial_{p_z} + \lambda_{DSR}p_z(E\partial_E + p_i\partial_{p_i}), \tag{2.4}$$

where p_i denotes the momentum of the particle along i axis. Mathematically, the modification of both approaches is just the adding non-linear terms to the boost generators in such a way that the new invariant quantity, λ_{DSR} is obtained. One of the modified physical effects for DSR is a modification of dispersion relation which in this case can be expressed as:

$$E^2 = m^2 + p^2 + \lambda_{DSR} E p^2, \tag{2.5}$$

and

$$E^2 = m^2 + p^2 + 2\lambda_{DSR} E (p^2 - E^2), \tag{2.6}$$

for DSR1 and DSR2 respectively. Note that the energy momentum conservation still holds in this theory. The comparisons between these two approaches are investigated in detail in [8]. The original idea of DSR1 and DSR2 is proposed by Amelino-Camelia [9, 10], and Magueijo and Simolin [11] respectively. Although there are many interesting features of this model, it is not a complete theory because there is no dynamical mechanism behind this kinematic approach. However, this theory is in progress on both comparison with observation and consistency of the theory itself. The recent reviews of DSR is [12, 13].

There are other kinematic approaches to Lorentz violation theory including Robertson-Mansouri-Sexl (RMS) model and $TH\epsilon\mu$ model. We will not consider them here. The review of these models is [3] and interested readers can consult the references therein for more detail caculations.

2.1.2 Dynamical Approach

As we have mentioned earlier, the dynamical approach to the Lorentz violation theories is a complete theory in the sense that it includes both interacting and free dynamics of elementary particles. The dynamical approach must cover the results in the kinematic approach such as modified dispersion relation. This subsection is separated into two parts. In the first part, we will discuss the *standard model extension* (SME). This model is the modification of the standard model of particle physics in which the effects of the Lorentz violation are the small corrections of the standard model results. The small value of these corrections comes from the suppression of the energy scale we consider. Thus all of models are based on effective field theory.

In the second part, we discuss a *modified Einstein's general relativity*. Similar to the idea of SME, this model is the extension of Einstein' s general relativity. The effects of the Lorentz violation are provided by introducing new degree of freedom fields such as vector or tensor fields. Some results in this model are similar to vector-tensor theory.

- **The standard model extension**

The standard model is the most successful model describing physics of elementary particles. Most of its prediction are accurately confirmed by experiments. However, there are some phenomena that are not included in the standard model, for example, the existence of neutrino masses. Also, the model itself encounters some crucial problems, for example, the mass hierarchy problem. Thus many models are investigated in order to try to modify the standard model for explaining the experimental data which cannot be described by the standard model. Here, we identify this class of model as the *standard model extension* (SME).

The standard model is the renormalizable field theory. Its Lagrangian has the mass dimension $\leqslant 4$ operators. This model is based on $SU(3) \times SU(2) \times U(1)$ gauge symmetry as well as the Lorentz and CPT symmetries. Strategy of SME in the Lorentz violation theories is slightly different from regular SME. The purpose of regular SME is to explain the experimental data by using the modification of gauge symmetry or adding some mechanisms. However, the strategy of SME in the Lorentz violation theories is to construct the model that predicts the Lorentz violating corrections to the standard model results which will be observed in the next generation experiments. Thus the SME of the Lorentz violation theories can be obtained by adding some renormalizable Lorentz violating terms into the standard model Lagrangian. Let us show how to add the Lorentz violating terms by considering the QED sector of the standard model. These modified terms for electrons are

$$-bA_\mu\overline{\psi}\gamma_5\gamma^\mu\psi + \frac{1}{2}icA_\mu A_\nu\overline{\psi}\gamma^\mu D^\nu\psi + \frac{1}{2}idA_\mu A_\nu\overline{\psi}\gamma_5\gamma^\mu D^\nu\psi, \tag{2.7}$$

and for photons are

$$-\frac{1}{4}k_F A_\rho\eta_{\sigma\mu}A_\nu F^{\rho\sigma}F^{\mu\nu}, \tag{2.8}$$

where b, c, d and k_F are constants represented a strength of the Lorentz violation correction. A^μ is a coefficient characterizing the effect of Lorentz violation, ψ is fermionic field, γ^μ is the gamma matrices and $F^{\mu\nu}$ is field strength tensor of $U(1)$ gauge field. We prefer to break Lorentz symmetry by breaking boost invariance instead of rotational invariance because a rotational symmetry is strongly constrained by observational data. Thus the Lorentz violating coefficient can be normalized to the form $A^\mu = (-1, 0, 0, 0)$. Some effects of these modifications result in the modified dispersion relation expressed as

$$E^2 = m^2 + p^2 - 2bs\ p - (c - ds)\ p^2 \tag{2.9}$$

and

$$E^2 = \left(1 + \frac{1}{2}k_F\right)\ p^2 \tag{2.10}$$

for electrons and photons respectively, where $s = \pm 1$ is the helicity state of electron. The review of the standard model extension is provided in references [3, 4, 5]. However, the first investigation of the SME is discussed in [6].The non-minimal SME which includes the mass dimension > 4 operators is also investigated. The effects of non-minimal SME also result in the modified dispersion relation. In the non-minimal SME, there are infinitely many terms which can include in Lagrangian density. However, in photon sector, by using gauge invariant action, the number of d-dimensional Lorentz violation operator can be counted, for example, 36 terms for $d = 5$ dimensional operator [14]. Recently, the observable effects of Lorentz and CPT violation can be tested by certain experiments. They are summarized in [18]. The progress of Lorentz violation in both experimental and theoretical sides can be monitored by looking at the notes of Kostelecky [15, 16]. More recent issues of Lorentz and CPT violation can be found online in [17]. Note that the CPT violation is implied by Lorentz violation [19]. Therefore, searching for the effects of Lorentz violation is the same as finding the evidence of CPT violation. However, for the recent investigation [134], it is found that CPT violation does not Lead to violation of Lorentz invariance and vice versa.

- **Modified Einstein's general relativity**

Einstein's general relativity provides the description of gravitation at classical level. At quantum gravity scale, we expect that the effects of the Lorentz violation will be explored. It may leave some fingerprints in the low energy effective field theory. In this investigation, we will consider the effective field theory of Einstein's general relativity.

The description of gravitation in Einstein's general relativity is provided by the metric and covariant derivatives that act on vector or tensor representations of $\mathrm{Gl}(4, R)$ under Riemannian spacetime. The description of basic particles and forces in the standard model is provided by spinors and gauge fields based on

$SU(3) \times SU(2) \times U(1)$ gauge group in Minkowski spacetime. From both descriptions of nature, it is not easy to connect them together because there are no spinor representations in Gl$(4, R)$. However, this connection can be obtained by using the fact that there is a local Lorentz frame at every points in spacetime manifold . This local Lorentz frame is a tangent space of a point in spacetime manifold which contains the spinor representations in Minkowski spacetime. The framework that incorporates the quantities between local Lorentz frame and spacetime manifold is vierbein formalism. In this formalism, the gravitational description is provided by vierbein e^a_μ and a spin connection w^{ab}_μ under Riemann-Cartan spacetime where $a, b, c, ...$ are indices describing the local Lorentz frame and $\mu, \nu, \rho, ...$ are the spacetime indices in spacetime manifold. Riemann-Cartan spacetime will be characterized by the curvature tensor $R^\rho_{\sigma\mu\nu}$ and the torsion tensor $S^\lambda_{\mu\nu}$. The vierbein formalism can be reduced to Einstein's general relativity by taking the torsion tensor to be zero. In the other view point, the description of vierbein formalism is similar to the local gauge description of the standard model in which the vierbein acts as the gauge field of translations. The most usefulness of this formalism is that, at each point in spacetime manifold, Lorentz transformations in the local Lorentz frame is independent on general coordinate transformations in spacetime manifold. In other words, physics does not change under coordinate transformation. This provides us the observer Lorentz invariance automatically. Thus it is easy to construct particle Lorentz violation theories while the observer Lorentz symmetry remains unbroken in this formalism.

To construct Lorentz violation theories in the description of gravity, we can follow the Lorentz violation theories in the standard model by introducing the Lorentz violation coefficient, A_a which yields a vector in spacetime coordinates as $A_\mu = e^a_\mu A_a$. For the standard model, the Lorentz violating coefficient A_a can be chosen as a constant time-like vector $A_a = (A, 0, 0, 0)$ in order to ensure that energy and momentum remains conserved. However, in the gravitational description, it leads to non-conservation of energy momentum tensor in spacetime coordinates when the A_a is covariantly constant. The most usefulness of the solutions for this issue is to promote the Lorentz violating coefficient as a dynamical field named *æther field* [20]. The name of æther field comes from the fact that there is a constant vector pointing to a direction in spacetime in the local Lorentz frame every points in spacetime manifold. This means that it has a preferred frame which breaks the local Lorentz invariant for all points in spacetime. For simple model, we can fix the norm of the æther field. In the effective field theory, it can be performed by using constraint of potential term, for example, Lagrange multiplier term. This yields the spontaneous breaking of the local Lorentz symmetry. For the explicit breaking, it is found that the corresponding equation of motion turns out to be inconsistent with Bianchi identity. This implies that explicit Lorentz violation is not compatible with Riemann geometry. However, it is compatible with Cartan-Riemann geometry [21]. Note that the spontaneous breaking of the local Lorentz symmetry is accompanied with spontaneous diffeomorphism violation [22]. Æther theory is the most popular model among all Lorentz violation theories in gravity sector. The aim of this thesis is to investigate the effect of Lorentz violation in cosmological models. Thus it is useful to deal with æther models and we will discuss this topics in detail in the next section.

One of the interesting results of spontaneous local Lorentz violation is the Lorentz violation effect on the standard model. Since the Lorentz symmetry is spontaneously broken, the æther fields can be decomposed into the background solution which is the vacuum solution and its fluctuations. These fluctuations will be represented in the massless Nambu-Goldstone (NG) modes for broken generator [22] and also massive modes [23]. The most interesting result is that some massless modes can be interpreted as photons. This is an alternative way to offer the existence of light while it is the consequence of Lorentz violation instead of U(1) gauge symmetry. The interpretation of graviton is also investigated in the same manner [24, 25].

2.2 Æther theory

As we have mentioned in the previous section, æther theory is the most popular theory among the Lorentz violation theories. It is the dynamical approach to Lorentz violation theory based on the effective field theory. The Lorentz violation of æther theory is provided by a dynamical vector field. It is similar to the vector-tensor gravity theory. The crucial difference is that the æther field is constrained by non-vanishing constant norm. As we have seen in the previous section, the proper æther models must be provided by the spontaneous Lorentz symmetry breaking in order to avoid the inconsistency between the Bianchi identity and the equation of motion in Riemann spacetime. This inconsistency leads to the non-conservation of energy momentum tensor. For the spontaneous breaking, it is provided by the potential term which gives the non-zero vacuum expectation value. Generally, the potential can take the form $V = V(A^\mu A_\mu \pm v^2)$, where A_μ is the æther field and v is vacuum expectation value. In this thesis, we will focus on the Lagrange-multiplier potential since the other forms encounter instabilities [26, 30]. However, we will give a brief review of some interesting results of the other potential forms after the discussion of Lagrange-multiplier potential. Although, in the general framework, one must perform the calculation of this model by using vierbein formalism in Riemann-Cartan spacetime, it is convenient for us to perform calculation in Riemann spacetime by setting the torsion to be zero. The most general action in 4-dimensional space-time of æther theory based on diffeomorphism invariance and quadratic derivative is expressed as

$$S = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g}\Big(R + \mathcal{L}_K + \mathcal{L}_V\Big), \tag{2.11}$$

where

$$\begin{aligned} \mathcal{L}_K &= -\beta_1(\nabla_\mu A_\nu)(\nabla^\mu A^\nu) - \beta_2(\nabla_\mu A^\mu)^2 \\ &\quad -\beta_3(\nabla_\mu A_\nu)(\nabla^\nu A^\mu) - \beta_4 \frac{A^\mu A^\nu}{v^2}(\nabla_\mu A_\rho)(\nabla_\nu A^\rho), \end{aligned} \tag{2.12}$$

and

$$\mathcal{L}_V = \lambda(A_\mu A^\mu \pm v^2). \tag{2.13}$$

Here λ acts as a Lagrange multiplier enforcing the fixed norm condition, M_{pl} is the reduce Planck mass, R is Ricci scalar and A^μ is the æther field. The minus and plus sign in the potential term represent the space-like and time-like vector field respectively.

Conveniently, we take M_{pl}^2 as the overall factor of the action. As a consequence, the æther field is dimensionless. Generally, the kinetic term of the æther field needs to be small because there are no allowed regions of observations in which the magnitude of the æther field is comparable to the metric. With the dimensionless æther field, we can rescale the field to unity by setting its *vev* to be one. This means that the kinetic term is suppressed by the æther parameters, β_i, which is small compared to the Planck mass. The æther parameters are also interpreted as the broken scale of the Lorentz symmetry. However, if we put M_{pl}^2 only in front of R, it will provide the mass dimension to the æther field. For this approach, the kinetic term is suppressed by the ratio of the norm of æther field v to M_{pl}. These two approaches may cause confusion in the later on when we constrain the parameters in the model.

Considering the kinetic terms in Lagrangian (2.12), the last term is quartic in the æther field. It seems like a non-renormalizible term due to the dimension of the coupling, if we consider the case where the æther has a dimension of mass.

However, we are not necessary to consider the renormalizibilty of each term since the theory of gravity is itself not renormalizible. It is interesting to consider this term since it provides the quadratic contribution to the metric and æther perturbations when we expand it around the flat background.

In order to find the equation of motion, we vary action (2.11) with respect to $g^{\mu\nu}$ and A^{μ}. The variation of $g_{\mu\nu}$ and A_{μ} can be written as

$$\delta_g g_{\rho\sigma} = -g_{\rho\mu}g_{\sigma\nu}\delta g^{\mu\nu}, \tag{2.14}$$

$$\delta_g A_{\rho} = -g_{\rho\mu}A_{\nu}\delta g^{\mu\nu}. \tag{2.15}$$

By using the Euler's Lagrange equation, the equations of motion for A^{μ} can be written as

$$\nabla_{\mu}J^{\mu}{}_{\nu} + \frac{\beta_4}{v^2}A^{\mu}(\nabla_{\mu}A^{\rho})(\nabla_{\nu}A_{\rho}) = \lambda A_{\nu}, \tag{2.16}$$

where

$$J^{\mu}{}_{\nu} = -(\beta_1\nabla^{\mu}A_{\nu} + \beta_2\delta^{\mu}{}_{\nu}\nabla_{\rho}A^{\rho} + \beta_3\nabla_{\nu}A^{\mu} + \beta_4\frac{A^{\mu}A^{\rho}}{v^2}\nabla_{\rho}A_{\nu}). \tag{2.17}$$

The equation of motion for $g^{\mu\nu}$ is the Einstein field equation,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}. \tag{2.18}$$

The energy momentum tensor, $T_{\mu\nu}$, is defined as

$$T_{\mu\nu} = -2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} + g_{\mu\nu}\mathcal{L}, \tag{2.19}$$

The first term on the right hand side of the above equation can be expressed as

$$\begin{aligned} -2\frac{\delta\mathcal{L}}{\delta g^{\mu\nu}} &= 2\beta_1(\nabla_{\mu}A^{\rho}\nabla_{\nu}A_{\rho} - \nabla^{\rho}A_{\mu}\nabla_{\rho}A_{\nu}) - 2\beta_4\frac{A^{\rho}A^{\sigma}}{v^2}\nabla_{\rho}A_{\mu}\nabla_{\sigma}A_{\nu} + 2\lambda A_{\mu}A_{\nu} \\ &\quad -2\Big(\nabla_{\rho}(A_{(\mu}J^{\rho}{}_{\nu)}) + \nabla_{\rho}(A^{\rho}J_{(\mu\nu)}) - \nabla_{\rho}(A_{(\mu}J_{\nu)}{}^{\rho})\Big). \end{aligned} \tag{2.20}$$

The equation of motion for Lagrange multiplier is

$$A_{\mu}A^{\mu} = \pm v^2. \tag{2.21}$$

We can eliminate λ in equation (2.17) by contracting A^{ν} with (2.16) and then use (2.21). This gives

$$\lambda = \pm\frac{1}{v^2}\Big(A^{\nu}\nabla_{\mu}J^{\mu}{}_{\nu} + \frac{\beta_4}{v^2}A^{\nu}A^{\mu}(\nabla_{\mu}A^{\rho})(\nabla_{\nu}A_{\rho})\Big). \tag{2.22}$$

The background solutions of the æther field with fixed norm condition also depend on the metric. The dynamics of the isotropic and homogeneous universe is based on the Friedmann Robertson Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t)dx_i dx^i, \tag{2.23}$$

where $a(t)$ is the scale factor. By using this metric, the solution of the equation of motion (2.16) can be chosen as

$$A^\mu = (v, 0, 0, 0), \tag{2.24}$$

for the time-like æther field and

$$A^\mu = (0, v/a(t), 0, 0) \tag{2.25}$$

for the space-like æther field. The difference of the background solutions due to the symmetry of FRW metric results in the different way of analyzing the behavior of the æther field. Thus in order to investigate the effects of the æther field, we have to consider the time-like æther field and space-like æther field separately.

2.3 Theoretical constraints of æther theory

There are many methods for constraining the æther parameters. We divide them into two classes, the theoretical constraints and observational constraints. The theoretical constraints come from the consistency of the theory which needs to be stable and causal. In this section, the stability issue of æther field is reviewed and the causality of the Lorentz violating field is briefly discussed.

2.3.1 Stability

In this subsection, we will focus on the stability of the æther models. It is very subtle to summarize the æther stability due to the non-conclusive status of this argument. There are two main requirements to figure out the stability in effective field theory. The fist requirement for a stable theory is that the perturbation around the background or the vacuum state must converge. Mathematically, the stability of the background solution, X_0, is said to be stable if for any given small neighborhood, U_0, of X_0 always evolves in time to another small neighborhood, U_1 of X_0. According to the fixed norm condition, behavior of the æther field strongly depends on the geometry of spacetime especially for space-like æther model. Therefore, the evolution of the æther perturbations will couple to the metric perturbations. It is obvious that the evaluation of the æther perturbations needs to include the perturbations of the metric. It is convenient to consider this requirement in other view points such as the propagation behavior of the perturbations and the existence of tachyon fields. For the propagation behavior, it requires that the frequency of a perturbation wave must be real. This requirement leads to the constraint on the squared speed as $s^2 \geq 0$. It also leads to the positive squared mass of the massive perturbation modes. In other words, it is not allowed for the existence of tachyon fields. This class of stability is called that the *gradient stability*.

The second requirement for investigating the stability is the absence of ghost field. The ghost field is a field which has the wrong sign of kinetic term. This wrong sign will lead to negative energy of the particles. The problems in this wrong sign kinetic term will not appear if the corresponding particle of this field does not interact to other particles. However, it is not easy to avoid this interaction because all particles will interact at least with a graviton. Thus it is not necessary to ignore the effect of the interaction in this view point. If the fluctuation degrees of freedom of the æther field are ghost fields and couple to the normal fields, the vacuum will decay to the ghost-nonghost states. We cannot use the conservation of energy to impose the limit of this decay due to the negative energy modes of the ghost field. This means that the zero vacuum state can decay into the infinite high-momentum states of ordinary particles. Although we can claim that it is the effective field theory that has the cut-off momentum, it still produces the large number of observable particles from the vacuum state. This phenomenon is not acceptable even though the energy of the system is still conserved. In the quantum level, it encounters the negative probability which is inconsistent to define the observable quantum quantities. One of the convenient ways to deal with the existence of the ghost field is to analyze of the positive Hamiltonian bound. The positive bound from below of the Hamiltonian can infer that there are no ghost fields in the system. The stability analysis in this work is adopted by considering these two requirements.

Mainly, the æther models will be classified into two types, time-like æther model and space-like æther model. Like the Lorentz violation theories, the time-like æther model has received more attention than space-like æther model due to the fact that violation of rotational subgroup of Lorentz group will lead to the violation of boost invariance. Moreover, the rotational invariance is strongly confirmed by many high-precision experiments, for example, the observations of Cosmic Microwave Background (CMB) radiation which suggest that there is the almost perfect rotational invariance. However, there are the space-like æther models that provide a very tiny departure on the smooth CMB. We will discuss this topic in detail later.

Let us start by considering the stability of time-like æther field. For convenience, we will focus on the Minkowski spacetime by following Jacobson and Mattingly [27]. The perturbations of the metric and æther field are expanded around the background $\eta_{\mu\nu}$ and $\bar{A}^{\mu}$ as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2.26}$$

$$A^{\mu} = \bar{A}^{\mu} + \delta A^{\mu} \tag{2.27}$$

where $\eta_{\mu\nu}$ is the Minkowski metric, $h_{\mu\nu}$ is the tensor modes of metric fluctuations and δA^{μ} is the æther fluctuation field. Both $h_{\mu\nu}$ and δA^{μ} are small relative to their background values.

Solving for the perturbations is a very long calculation. Therefore, we skip the detail calculations in this discussion and give only some important steps and the final results. In the first step, the solutions in the equations (2.26) and (2.27) are substituted into the equations of motion, (2.16) and (2.18). Second, the gauge choices are chosen by the symmetry of the action under diffeomorphism transformation. Third, gauge variables will be transformed into Fourier space. This step will provide us the resulting equations for five modes of the perturbations, two modes for spin-2 graviton, two modes for spin-1 æther and one mode for spin-0 æther. Finally, the normal modes of the system of equations are performed. This gives only three propagations of the normal modes whose the squared speeds can be written as

$$s^2_{tt\ metric} = \frac{1}{1-\beta_{13}}, \tag{2.28}$$

$$s^2_{t\ \text{æther}} = \frac{2\beta_1 - \beta_1^2 + \beta_3^2}{2\beta_{14}(1-\beta_{13})}, \tag{2.29}$$

$$s^2_{trace} = \frac{\beta_{123}(2-\beta_{14})}{\beta_{14}(2(1+\beta_2)^2 - \beta_{123}(1+\beta_2+\beta_{123}))}, \tag{2.30}$$

where $s^2 = \omega^2/k^2$ and β_{abc} stands for $\beta_a + \beta_b + \beta_c$. The stability condition requires that the frequency ω must be real. This implies that $s^2 \geq 0$. Some results are recovered and extended to de-Sitter space as well as inflation background by Lim [28] but without β_4 term. The æther perturbations will decay in de-Sitter space and modify the spectrum of the B-mode polarization and also violate the inflationary consistent relation in inflationary universe. The Hamiltonian bound corresponding to the ghost-free condition is also investigated in [28, 29]. In the Maxwell case, corresponding to $\beta_3 = -\beta_1, \beta_2 = \beta_4 = 0$, it is found that there is a positive bound of Hamiltonian. There is another model of æther, the so-called *unleashed æther*, which contains the potential term $V = \lambda(A^\mu A_\mu + v^2)^2$ where λ is a dimensionless parameter [30]. In this model, λ is not the Lagrange mutiplier and then it leads to the weaker condition in which the æther norm does not need to be fixed. However, this model can be viewed as a generalization of the æther models which arising in the limit $\lambda \to \infty$. The results of this class of the æther models show that at least one of the perturbation modes is always a tachyon or a ghost. From the results of above analysis, it follows that the unleashed æther model is not stable.

The more restrictive consideration of stability is investigated by Carroll et al.[31]. Actually, the space-like æther models are also investigated in this paper but we will discuss this topic later. In the point of view of this consideration, it is argued that the stability conditions must hold in all reference frames. This means that the Hamiltonian must be positively bound and the perturbation must converge in all boost frames under the validity of the effective field theory. The result of this consideration shows that only the sigma model, $\beta_3 = \beta_2 = \beta_4 = 0$, satisfies the stability conditions. This means that most of æther models are not stable. The cosmological effects of the sigma model are investigated in [32].

This more restrictive consideration is interpreted as the too much restrictive conditions [33]. The first argument in this issue is that the imaginary of the frequency in the boosted frames, ω', leads to the imaginary wavevector, $k = \gamma(k' - \beta\omega')$ in the rest frame. This imaginary is excluded from the consistent theory even in the Lorentz invariant theory. This argument also results in the Hamiltonian analysis that the positive definiteness of the Hamiltonian is not necessary for the stability in the spontaneous Lorentz violation theory [34]. From this analysis, it is found that the superluminal propagation is also allowed for spontaneous Lorentz violation theory. This superluminal consideration also conflicts with the argument in [31] which uses the stability condition that the propagation speed must not be superluminal.

For the space-like æther field, even though rotational invariance is violated, it is still investigated in order to find the departure of statistical anistropies in the CMB power spectrum [38]. However, it is convenient to consider the space-like æther field in the model of higher dimensions. The higher dimensional æther models can provide the three-dimensional rotational invariance since the norm of the æther is obtained by fixing the direction of the æther to align to the extra dimensions [35, 36]. It has also been investigated phenomenologically from the effect of string theory [37].

In four-dimensional spacetime, the stability of space-like æther model with out β_4 term is investigated in the de-Sitter background [39]. The Hamiltonian bound analysis in the flat space is also considered. It is found that the gradient stability conditions performed in de-Sitter background are expressed as $\beta_{123} \geqslant 0$ and $\beta_1 > 0$. Note that these conditions are obtained by using the limit of a very short wavelength, $k \gg H$, and a very long wavelength, $k \ll H$. The limit of the comparable scale, $k \sim H$ is not considered in this reference. However, in flat space, it implies that only Maxwell model, $\beta_{123} = 0$, is stable. Unfortunately, in the full calculation within de-Sitter background of Maxwell model, it is found that some modes of the perturbations diverge at the horizon crossing, $k \sim H$ [40, 41]. This means that all models of space-like æther are not stable. Similar to the time-like æther model, the more restrictive condition is considered in flat spacetime [31]. This suggests that there are no stable models for space-like æther field in four-dimensional spacetime with or with out including β_4 term. The counter-argument of the over restriction like in the time-like case has not been investigated yet. Moreover, the space-like model including β_4 term in the de-Sitter space has not been considered. We also note that in the case of space-like unleashed æther, it has been investigated and pointed out that there are no stable models as in the time-like case.

2.3.2 Causality

One of the effects of Einstein's general relativity is the possibility to seek the way for violating the causality. It is not clear that this violation of causality can occur in the physical world. However, it is accepted that the well-defined theoretical model must provide the events which is causal. This notion is approved in the low-energy physics and agrees with our intuitive sense. Therefore, in order to construct the well-defined theoretical model, we will avoid violating the causality in Lorentz violation theory. The causality violation have been investigated extensively in the standard model extension. However, it lacks of the investigation of the æther models. Even in the theory of gravitation, the issue of causality is uncertain.

In the point of view of quantum field theory, the causality can be discussed in the term of *microcausality*. The microcausality requires that the commutation relation for any two field operators with space-like separation must vanish, $[\Phi(x), \Phi(x')] = 0$ for $(x - x')^2 > 0$. This ensures that the measurement at x cannot affect any measurements outside the light-cone at x. However, in Lorentz violation theory perspective, specifically SME, it is found that the microcausality will be violated but the two measurements with the space-like separation are still independent [42]. Note that this valids only in the concordant frame, the frame that all Lorentz violating coefficients are small. The causality of the Lorentz violation in QED and also in the modified Maxwell theory with the dimension 5 operator are investigated [43, 44].

In the gravity sector, the causality of the theoretical model can be viewed as the existence of closed time-like curve. For the time-like æther model, the four-vector æther can provide the energy momentum flowing around the closed time-like paths [3]. However, it has not been investigated in the space-like æther model. We note here that some consequences of the theory of quantum gravity also suggest the failure of the causality [3].

2.4 Experimental constraints of æther theory

Among several constructions of the æther models, the observational signals from the theoretical model of the æther field are extensively figured out in order to incorporate the existence of the Lorentz violation effects in the experiments. Some of the theoretical models are ruled out by observational data but some of them are still confront with the more sensitive experiments. In this section, we will briefly review the essential idea of the observational constraints from various experiments. However, some of them are not included here but the references are provided for interested readers. Most of the contents in this section follow the review paper [45].

2.4.1 Parametized-post Newtonian (PPN) parameters

In order to compare the result of æther theory with the Einstein's general relativity, it is convenient to study the model in the static-weak field limit. In this limit, both æther and Einstein's general relativity will reduce to the Newtonian theory. There are ten parameters for characterizing the Newtonian corrections of general metric theory of gravity [46]. These ten parameters are the so-called *parametized-post Newtonian* (PPN) parameters. Five of these parameters, $\xi_1, \xi_2, \xi_3, \xi_4$ and α_3, characterize energy momentum conservation of the theory. Therefore, all these five parameters automatically vanish in both æther model and Einstein's general relativity due to the consequence of the covariant action principle. The other two of PPN parameters, the Eddington-Robertson-Schiff parameters, β and γ, characterize the nonlinearity and the spatial curvature produced by gravity respectively. They are both unity for the Einstein's general relativity and time-like æther model but it has not been investigated for the space-like æther model. Next one is the Whitehead parameter, ξ, characterizes a peculiar sort of three-body interaction. This parameter also vanishes in both Einstein's general relativity and time-like æther model but it has not been investigated for the space-like æther model. Finally, α_1 and α_2 , which characterize the preferred frame effect, provide the different value comparable to the Einstein's general relativity. While α_1 and α_2 vanish in the Einstein' general relativity, in the time-like æther model, these parameters can be expressed in terms of β_i [48] as

$$\alpha_1 = \frac{-8(\beta_3^2 + \beta_1\beta_4)}{2\beta_1 - \beta_1^2 + \beta_3^2}, \tag{2.31}$$

$$\alpha_2 = \frac{\alpha_1}{2} - \frac{(\beta_1 + 2\beta_3 - \beta_4)(2\beta_1 + 3\beta_2 + \beta_3 + \beta_4)}{\beta_{123}(2 - \beta_{14})}. \tag{2.32}$$

The current observation suggests a very tiny value of these two parameters $\alpha_1 \leq 10^{-4}$ and $\alpha_2 \leq 4 \times 10^{-7}$ [49]. We have two ways to deal with this tiny value. First, we find the exact relation of the parameters up to the order of the observational constraints. In this way, we will encounter the fine tune problem. Second, since we have four independent parameters, we can eliminate two of them by setting $\alpha_1 = \alpha_2 = 0$. Thus there remain only two independent parameters. Conveniently, β_2 and β_4 will be eliminated by the following relations

$$\beta_2 = \frac{(\beta_3^2 - \beta_1\beta_3 - 2\beta_1^2)}{3\beta_1}, \tag{2.33}$$

$$\beta_4 = \frac{\beta_3^2}{\beta_1}. \tag{2.34}$$

2.4.2 Newton's constant

In this subsection, we will consider the modification of Newton's constant due to the existence of an æther field. In order to examine the æther models in Newtonian limit, we need to take the static-weak field limit. The corresponding metric will be obtained by using the metric with scalar perturbations in the Longitudinal gauge. This metric can be written as

$$ds^2 \;\; = \;\; -(1+2\Phi)dt^2 + (1-2\Psi)dx^i dx_i. \tag{2.35}$$

The time-like æther field corresponding to this metric with the constraint $A^\mu A_\mu = -v^2$ takes the form

$$A^\mu = ((1-\Phi)v, 0, 0, 0). \tag{2.36}$$

This solution satisfies the equation of motion (2.16). Note that the perturbation Φ, the gravitational potential, obeys the Poisson equation,

$$\nabla^2\Phi = 4\pi G_N \rho_m, \tag{2.37}$$

where G_N is the Newton's constant and ρ_m is the mass density of the object we are considering. Substituting the metric (2.35) and the corresponding æther solution (2.36) into the Einstein field equation (2.18) gives

$$2\nabla^2\Phi = 2\beta_{14}\nabla^2\Phi + 8\pi G_* \rho_m, \tag{2.38}$$

for $(0,0)$ component and

$$(\delta_{ij}\nabla^2 - \partial_i\partial_j)(\Phi - \Psi) = 0, \tag{2.39}$$

for (i,j) components. Where G_* is the Newton's constant of the æther model. From equation (2.39), we can assume that both Φ and Ψ vanish at the spatially infinity. This leads to the unique solution as $\Phi = \Psi$. Note that the energy momentum of the considering object acts as the source term in the Einstein field equation. By comparing to the Poisson equation above, we obtain the effective Newton's constant as

$$G_N = \frac{G_*}{1-\beta_{14}/2}. \tag{2.40}$$

The Newton's constant in a cosmological model, G_c is also modified by itself. Thus we can use similar strategy to figure out the modification of G_c. We begin with choosing the proper metric in the cosmological description. The universe is normally described by the spatially flat FRW metric

$$ds^2 \;\; = \;\; -dt^2 + a(t)^2 dx^i dx_i. \tag{2.41}$$

In the same manner with the calculation in the Newtonian limit, we will skip the explicit calculations and the result turns out to be that the effective Newton's constant in the cosmological model is

$$G_c = \frac{G_*}{1+(\beta_{13}+3\beta_2)/2}. \tag{2.42}$$

The Newton's constant should be the same at all scales. The corrections of the G_c are limited by the observation of the primordial ^4He abundance at the nucleosynthesis period [50]. This constraint can be expressed as $|G_c/G_N - 1| \leq 1/8$. This constraint is satisfied automatically by using the PPN constraints. In other words, by setting $\alpha_1 = \alpha_2 = 0$, it turns out that $G_c = G_N$.

This is one of the examples which show the consequences in the æther models. Most of other models satisfy the observational data when the PPN limits are taken. In other words, the PPN constraints are stronger than the others. Other tighter experimental suggestions also constrain the parameters in the æther models, for example, the gravitational Čerenkov radiation [26, 51] and modified power spectrum of CMB radiation [52]. More examples can be found in [45] and references therein.

We note that the experimental constraints above have been investigated only for the time-like æther model but not for the space-like æther model. This may be caused by the fact that the space-like æther model violates the rotational subgroup of Lorentz group. The most interesting constraint must be examined by the test of rotational invariance. However, the rotational invariance is strongly confirmed by many experiments, for example, statistical anisotropy of CMB radiation. The theoretical model of the space-like æther field is constructed in order to compare to the observational data from CMB radiation. Unfortunately, it encounters instabilities as we have mentioned in previous section. We also note that it is interesting to investigate the space-like æther model with higher-dimensional spacetime since this can avoid the rotational invariance in three-dimensional space.

Chapter III

COSMOLOGICAL MODELS

The standard evolution of the universe begins with inflationary period. In this period, the universe extremely expands. This phenomena can solve the horizon and flatness problems. Therefore, matter and radiation contents are diluted and the universe is cooled. The dynamical approach of the inflation can be provided by a scalar field slow rolling on the flat potential. After end of the inflation, all elementary matter and radiation are created by the description of the inflaton oscillation at the minimum of the potential. The universe in this period is extremely hot and all matter and radiation act as the hot soup and then we call this period as *reheating period*. The universe after this period is dominated by radiation and then matter respectively. During the matter-dominated period, there is a crucial event in which photons are decoupled with electrons. At this time, atoms are formed and photons can propagate freely. In others words, it is the farthest that we can observe the photons. Thus we will observe these photons as the background radiation that comes from all directions in the sky. This radiation is commonly called *cosmic microwave background* (CMB) radiation. After the matter-dominated period, the universe enters the accelerated expansion until nowadays. The mysterious thing that makes accelerating expansion of the universe is called *dark energy*. There are many models of dark energy and then we call the model that is responsible for the late-time acceleration of the universe as dark energy model. In this chapter, we will focus on two cosmological models, inflationary models and dark energy models.

3.1 Inflationary models

The Big Bang model of cosmology was successful to explain the non-static universe which was observed by Hubble in 1929. However, it encounters some cosmological problems such as horizon and flatness problems. The horizon problem comes from nearly perfect uniform temperature of CMB radiation while the Big Bang model cannot naturally provide such uniform temperature. The flatness problem comes from the observation that the universe nowadays is almost flat while we need to unnaturally fine tune the initial conditions in order to get this flat universe. These problems can be solved by using the interpretation that the universe accelerately expanded at the very early time. In order to obtain how much the universe needs to expand, we introduce the number of e-fold to characterize this expansion, $N \equiv \int H_I dt$ where H_I is the Hubble parameter at the inflationary period defined as $H = \dot{a}/a$ and a is the scale factor. From the observations, N needs to satisfy the constraint $N \gtrsim 65$. A vital part of inflationary model is the providing of tiny anisotropy in CMB radiation. Quantum fluctuations during inflationary period will seed the primordial perturbations in the cosmological scale and ultimately make the structure of the universe we observe nowadays. It is convenient to introduce the dynamical approach of inflation before discussing this topic in detail. The review contents in this section are found in common cosmological text books [53, 54, 55, 56, 57]. Some of them are collected from the review articles [58, 59, 60, 61, 62, 63, 64].

3.1.1 Dynamical models of inflation

A simple dynamical model of inflation is provided by a single scalar field slowly rolling on the flat potential. The action of this scalar field for the inflationary universe with FRW spacetime can be written as

$$S = \int d^4x\sqrt{-g}\Big(\frac{M_{pl}^2}{2}R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\Big). \tag{3.1}$$

For the homogeneous and isotropic universe, this scalar field depends only on time $\phi = \phi(t)$. The equation of motion for this scalar field is

$$\ddot{\phi} + 3H\dot{\phi} + \partial_\phi V = 0. \tag{3.2}$$

By varying action (3.1) with respect to the metric tensor, the non-zero components of the energy momentum tensor for the scalar field can be written as

$$\begin{aligned} T^0_{\ 0} &= -\Big(\frac{1}{2}\dot{\phi}^2 + V(\phi)\Big), \\ T^i_{\ j} &= \Big(\frac{1}{2}\dot{\phi}^2 - V(\phi)\Big)\delta^i_{\ j}. \end{aligned} \tag{3.3}$$

We interpret this scalar field as perfect fluid in the universe. This leads to the energy density and the pressure

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi), \tag{3.4}$$

$$p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi). \tag{3.5}$$

The Einstein field equation will provide the Friedmann and acceleration equation respectively

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{pl}^2}\rho_\phi = \frac{1}{3M_{pl}^2}\Big(\frac{1}{2}\dot{\phi}^2 + V(\phi)\Big), \tag{3.6}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{pl}^2}(\rho_\phi + 3p_\phi) = -\frac{1}{3M_{pl}^2}\Big(\dot{\phi}^2 - V(\phi)\Big). \tag{3.7}$$

From equation (3.7), it turns out that the condition for accelerating universe is

$$\dot{\phi}^2 < V(\phi). \tag{3.8}$$

This means that the kinetic term needs to be smaller than the potential term. In other words, inflaton field, corresponding to scalar field which is responsible for the inflation of the universe, needs to move slowly. The constraint from the observations can be written in terms of the potential as

$$N = \int_{t_i}^{t_f} H dt = \frac{1}{M_{pl}^2}\int_{\phi_i}^{\phi_f} \frac{V(\phi)}{V'(\phi)} d\phi \gtrsim 65. \tag{3.9}$$

This condition tells us that the potential needs to be flat at the inflationary period. This is the reason why we mentioned that inflationary models are provided by a scalar field slowly rolling on the flat potentials. However, this simple model encounters some problems, for example, the potential is not natural to yield the condition above, the mechanism for ending the inflation is not achieved for the power law model, many of the simple models cannot provide the running spectral indices. Many theoretical models are introduced to provide the explanation of observation such as k-inflation [67, 68], multi-field inflation [65, 66], DBI inflation [69, 70, 71], vector inflation [72], $f(R)$ inflation [73, 74]. Although, the number of e-fold alone is enough to indicate how much the universe expands during inflation, it is more convenient to consider the other sufficient but not necessary conditions, so-called *slow-roll conditions.* These conditions are expressed as

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{M_{pl}^2}{2}\Big(\frac{V'(\phi)}{V(\phi)}\Big)^2 \ll 1, \tag{3.10}$$

$$\eta = M_{pl}^2\Big(\frac{V''(\phi)}{V(\phi)}\Big) \ll 1, \tag{3.11}$$

where ϵ and η are the slow-roll parameters. ϵ characterizes the flatness of the potential and η characterizes how long of the flatness is.

3.1.2 Primordial power spectrum

In this subsection, we will show how can the structures in the universe such as galaxies, clusters and stars form by using the inflation. In order to show that, we need to consider the perturbation of inflaton field in the quantum level. The fluctuation modes of inflaton field will be strengthened by the accelerated expansion of the universe during inflation period. This makes the transition from quantum fluctuations to cosmological perturbations. In particular, inflation can provide the primordial power spectra which is scale-invariant. Note that the scale-invariant power spectra was assumed in order to get the proper initial conditions before inflationary models were introduced.

Due to the coupling of gravity to the other contents in the universe, the perturbation of inflaton field will couple to the metric. Therefore, we need to include the metric perturbations in our consideration. The general form of the metric perturbations can be written as

$$g_{\mu\nu} = a^2(\tau)\begin{pmatrix} -1 - 2\Phi & \partial_i B + S_i \\ \partial_i B + S_i & (1 - 2\Psi)\delta_{ij} + \partial_i\partial_j E + \partial_{(i}F_{j)} + h_{ij} \end{pmatrix}, \quad (3.12)$$

where τ is conformal time related to cosmic time as $dt = ad\tau$. This form of the metric perturbations is very convenient due to the decomposition of the perturbation variables. They are decomposed into three parts; scalar, vector, and tensor perturbation variables. The advantage of this form is that the perturbation modes are decoupled, allowing us to calculate the power spectra separately. In 4-dimensional spacetime, there are ten degrees of freedom due to the symmetry of the metric tensor. In this perturbation form, there are four degrees of freedom for the scalar type, Φ, B, Ψ and E. For the vector type, there are two of three-vectors, S_i and F_i. However, there are the transverse constraints for each vector, $\nabla_i S^i = 0$, and $\nabla_i F^i = 0$. These constraints come from the fact that we have to exclude the scalar degrees of freedom in the form of gradient of the scalar from these vector perturbations, for example, $\nabla_i F$. Thus, the remaining degrees of freedom of vector perturbations are four degrees of freedom, each vector has two degrees of freedom. For the tensor perturbations, there are six degrees of freedom of symmetric three-tensor perturbation, h_{ij}. However there are four constraints from the transverse and traceless constraints,$\nabla^i h_{ij} = 0$ and $h^i_i = 0$. Thus the remaining degrees of freedom are two. Note that these tensor perturbations are gauge invariant and correspond to the gravitational wave.

Considering scalar perturbations of the system, we have to include the perturbation of inflaton into this part,

$$\phi = \bar{\phi}(\tau) + \delta\phi(x, \tau). \quad (3.13)$$

Thus it is added up to five degrees of freedom. However, we have the gauge constraints to eliminate some degrees of freedom. From the general coordinate transformations in 4-dimensional spacetime, there are four gauge freedoms corresponding to two scalar and two vector gauge freedoms. Therefore, the remaining scalar and vector degrees of freedom of the system are reduced to three and two respectively.

Some degrees of freedom are eliminated by using the constraints from Einstein equations. There are two constraints for scalar and two constraints for vector. Therefore, the scalar perturbations have only one degree of freedom and there are no remaining degrees of freedom for vector perturbations. The vanishing of the vector perturbations come from the fact that the universe with FRW metric is homogeneous and isotropic. Finally, we end up with two degrees of freedom for the tensor perturbations corresponding to two polarizations of gravitational wave and one scalar perturbation which depends on the choices of the gauge we choose, for example, the curvature perturbation in co-moving gauge.

In order to get the power spectra, we have to find the equations of motion of perturbation fields and solve for the solutions of them. Then power spectra are obtained by using the amplitude of the variance of the fluctuation fields. We will skip explicit calculations and show only the step and important results of calculations for brevity. There are two ways to obtain the equations of motion at the first order perturbations. For the first way, one substitutes the perturbation in (3.12) and (3.13) into action (3.1) and then keeps the second order terms in the perturbed action. After that, the equations of motion at linear level in Fourier space can be obtained by using the Lagrange equation and Fourier transformation. For the second way, one finds the equations of motion first and then perturbs the equations of motion and keeps only the first order perturbations. In this chapter, we choose the first choice for our convenience.

- **Tensor modes**

For the tensor perturbations, each of degrees of freedom of h_{ij} can be characterized by one scalar, h. Thus one can find the equation of one scalar and then add the factor 2 into the power spectrum in the final result. The action and the equation of motion in Fourier space can be written as

$$S^{(2)} = \frac{M_{pl}^2}{2}\int d\tau d^3x \frac{a^2}{2}\Big(h'^2 - \partial_i h \partial^i h\Big), \tag{3.14}$$

$$\mu_k'' + (k^2 - \frac{a''}{a})\mu_k = 0, \tag{3.15}$$

where $\mu_k = M_{pl} a h_k$ and h_k is the amplitude of Fourier transformation. At the small scale ($k^2 \gg a''/a \simeq 2/\tau^2$), μ_k behaves as the simple harmonic oscillator. Thus, it implies the oscillation behavior of the gravitational wave at the small scale. The solution of equation (3.15) can be expressed as

$$\mu_k = \frac{e^{ik\tau}}{\sqrt{2k}}\Big(1 - \frac{i}{k\tau}\Big). \tag{3.16}$$

The power spectrum of this fluctuation can be defined via the correlation of each mode as

$$\begin{aligned} < \hat{h}_k^\dagger \hat{h}_{k'} > &= \frac{(2\pi)^3}{M_{pl}^2}\frac{|\mu_k|^2}{a^2}\,\delta^3(\vec{k} - \vec{k}'), \\ &\equiv (2\pi)^3 k^{-3} P_h\,\delta^3(\vec{k} - \vec{k}'), \end{aligned} \tag{3.17}$$

where, $\hat{h}_k$ is the quantum operator corresponding to the perturbation modes h_k. Therefore, the power spectrum of the tensor perturbation will be written as

$$P_h = \frac{k^3}{M_{pl}^2}\frac{|\mu_k|^2}{a^2} = \frac{1}{a^2\tau^2 M_{pl}^2} = \frac{H^2}{M_{pl}^2}, \tag{3.18}$$

where H is approximately constant at the inflation period and evaluated at the horizon crossing. Note that we use the large scale limit solution, $\mu_k \sim -\frac{e^{ik\tau}}{\sqrt{2k}}\frac{i}{k\tau}$ to calculate this power spectrum. This tells us that the power spectrum decays in the inflation period until the tensor modes cross the horizon, they are constant at the super-horizon scale. This is the scale-invariant power spectrum as we have mentioned.

- **Scalar modes**

The second order perturbations of the action for scalar modes can be written as

$$S^{(2)} = \frac{1}{2}\int d\tau d^3x \Big(v'^2 - \partial_i v \partial^i v + \frac{z''}{z}v^2\Big), \tag{3.19}$$

where $v = a(\delta\phi + \frac{\dot{\phi}}{H}\Phi)$ and $z = a\frac{\dot{\phi}}{H}$. The equation of motion in Fourier modes can be obtained,

$$v_k'' + (k^2 - \frac{z''}{z})v_k = 0. \tag{3.20}$$

We can defined the gauge invariant variable called *co-moving curvature perturbation* as $\mathcal{R} = v/z$. $\mathcal{R}$ is conserved in the super-horizon scale as long as adiabatic conditions are hold. Thus this quantity is directly related to the adiabatic perturbation and some literatures use this perturbation as the adiabatic perturbation. The non-adiabatic perturbation corresponding to the entropy perturbation is called iso-curvature perturbation. For the simple models, such as single field inflation, the iso-curvature perturbation vanishes automatically. This is also compatible with the observational data which gives the nearly perfected adiabatic initial perturbation. By comparing to the tensor case, the power spectrum of the scalar perturbation can be written as

$$P_{\mathcal{R}} = k^3 \frac{|v_k|^2}{z^2} = \frac{2}{9}\frac{aH}{\dot{\phi}} M_{pl}^2 P_h = \frac{H^2}{9\epsilon M_{pl}^2}. \tag{3.21}$$

To obtain this power spectrum, we use the definition of slow-roll parameters in (3.10) and the relation of $\mathcal{R}$ and $\delta\phi$

$$\mathcal{R}|_{post\ inflation} = \frac{2}{3} aH \frac{\delta\phi}{\dot{\phi}}|_{horizon\ crossing}. \tag{3.22}$$

The slow-roll condition, $\epsilon \ll 1$, implies that the power spectrum of the scalar modes has bigger value than the power spectrum of tensor modes. This is also compatible with the observational data. Conveniently, the ratio of the power spectrum of the tensor and scalar, the so-called *consistency relation*, can be defined as

$$r = \frac{P_h}{P_{\mathcal{R}}} = 9\epsilon. \tag{3.23}$$

3.2 Dark energy models

According to the observations [75, 76], it is found that the universe is expanding accelerately. The ordinary matter and radiation we know cannot explain this phenomena since they are gravitational attractive and then lead to the collapsing universe. It is necessary to introduce a new content of the universe in order to describe the accelerating universe nowadays which is called *dark energy*. From the recent observations [77], the dark energy contributes 72% to the total energy of the universe. They also suggest that about 23% contributes to *dark matter*. Dark matter is exotic matter that feels only the gravitational force. We will not discuss this topic in this thesis for brevity. Therefore, it is only about 5 % contributing to the matter and radiation that we have already known. There are many models of dark energy but the popular and simple basic one is *cosmological constant*. This model of dark energy properly fits to the observational data. However, it encounters some problems. Therefore, the dynamical models of dark energy are introduced in order to solve these problems. Not only the fluid models but also the modified gravity models are introduced. In this section, we will review the important idea of dark energy models including cosmological constant, fluid dark energy models and modified gravity models. The contents of this section are collected from the review articles and the lecture notes on cosmology schools [78, 79, 80, 81, 82].

Before considering dark energy models, we will introduce the general idea to obtain the accelerating universe. The homogeneous and isotropic universe in large scale suggests us that the large scale matter in the universe intends to be perfect fluid. Therefore, we begin the general consideration of the dark energy with the Einstein Hilbert action including the perfect fluid.

$$S = \int d^4x\sqrt{-g}\Big(\frac{M_{pl}^2}{2}R + \mathcal{L}_m\Big), \tag{3.24}$$

where $\mathcal{L}_m$ is the Lagrangian density of the perfect fluid which plays the role of the dark energy. Varying this action with respective to the metric, one gets the Einstein field equation as

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{1}{M_{pl}^2}T_{\mu\nu}, \tag{3.25}$$

where $G_{\mu\nu}$ is the Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar and $T_{\mu\nu}$ is the energy momentum tensor of the perfect fluid. The general form of this energy momentum tensor can be expressed as

$$T^{\mu}_{\ \nu} = \begin{pmatrix} -\rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}, \tag{3.26}$$

where ρ is the energy density and p is the pressure of the perfect fluid. The Friedmann and acceleration equation are

$$\Big(\frac{\dot{a}}{a}\Big)^2 = \frac{1}{3M_{pl}^2}\rho, \tag{3.27}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{6M_{pl}^2}(\rho + 3p) = -\frac{1}{6M_{pl}^2}\rho(1+3w), \tag{3.28}$$

where $w = \frac{p}{\rho}$ is the equation of state parameter. Thus the condition for accelerating universe is

$$w < -\frac{1}{3}. \tag{3.29}$$

The ordinary matter in cosmological scale can be interpreted as dust which has the equation of state parameter, $w = 0$. Thus it cannot yield the accelerating universe. Radiation which has $w = 1/3$ cannot yield the accelerating universe. This is the reason why ordinary matter and radiation cannot be dark energy. Next subsection, we will show that cosmological constant can be the dark energy and discuss the problems of the cosmological constant.

3.2.1 Cosmological constant

Cosmological constant was first introduced by Einstein in order to obtain the static universe as it has repulsive gravitational force . Then it was soon abandoned after discovery the expanding universe. Nevertheless, it was reintroduced again as a candidate for dark energy. To see how cosmological constant drives the accelerating expansion, we consider the Einstein Hilbert action with cosmological constant term

$$S = \int d^4x\sqrt{-g}\frac{M_{pl}^2}{2}\Big(R - 2\Lambda\Big), \tag{3.30}$$

where Λ is the cosmological constant. The energy momentum tensor of cosmological constant is

$$T^{\mu}_{\nu} = -\rho_\Lambda g^{\mu}{}_{\nu}, \tag{3.31}$$

where $\rho_\Lambda = M_{pl}^2\Lambda$. By comparing to the energy momentum tensor of the perfect fluid in (3.26), the equation of state parameter of the cosmological constant is

$$w_\Lambda = -1 < -\frac{1}{3}. \tag{3.32}$$

It is clear that cosmological constant satisfies the condition for accelerating universe. Cosmological constant also plays the role of the vacuum energy since it is a constant background energy that exists even when there are no matter and radiation in the universe. The observations suggest that the total energy density of the universe is about the critical energy density which is approximately $\rho_{cri} \sim 10^{-47}GeV^4$ [77]. Dark energy contributes to this value about 72 %. Thus we can estimate the vacuum energy density as $\rho_\Lambda \sim 10^{-47}GeV^4$. Theoretically, this vacuum energy density can be calculated from the quantum field theory as

$$\rho_{vac} = \frac{1}{2}\sum_{all\ particles} g_i \int_0^{k_{max}} \frac{d^3k}{(2\pi)^3}\sqrt{k^2+m^2} \sim \sum_i \frac{g_i k_{max}^4}{16\pi^2}, \tag{3.33}$$

where $g_i = (-1)^{2j}(2j+1)$ is the degeneracy factor for a particle of spin j. k_{max} is the maximum energy scale that quantum field theory is expected to be viable. If k_{max} is order of Planck scale, the vacuum energy density is order $\rho_{vac} \sim 10^{74}GeV^4$.

This extreme difference in order magnitude between the observational data and the theoretical prediction is called *cosmological constant problem*. Note that the problem still exists if the cut off scale is set as the QCD scale, $\rho_{vac} \sim 10^{-3}GeV^4$.

The other problem of cosmological constant is called the cosmic coincidence problem, which asks why dark energy began to dominate at this epoch. The evolution of cosmological constant is very different from other components such as matter. How is the energy density of cosmological constant comparable to the energy of matter at this period? In other words, we need to fine-tune the initial value of the energy density of every components of the universe in order to get the present acceleration period. If cosmic acceleration of the universe began earlier, the structures such as galaxies would never have had existed.

These two problems seem impossible to solve if cosmological constant is the dark energy unless we use the anthropic arguments. The idea of string theory landscape is one of the examples [133]. Since string theory compactification predicts a large number of de Sitter vacua, it is possible to choose the vacuum with the appropriate value of cosmological constant. However, using cosmological models with extra dimensions may solve the cosmological constant problem. Casimir dark energy model, which we will discuss later in detail, is one of the examples. For the cosmic coincidence problem, it can be solved by using the so-called tracker behavior, which may exist in some dark energy models such as quintessence models [83, 84]. The cosmic coincidence problem is beyond the scope of this thesis.

3.2.2 Fluid dark energy

Inspired by inflationary models, most of fluid dark energy models are scalar field models. The simplest scalar field models for dark energy are called *quintessence models* [83, 84]. The crucial difference between quintessence and inflationary models is that quintessence models is not constrained by the e-folding condition, (3.9). Actually, they require the flat potential but does not need the long flatness. It is possible to get the accelerating universe by putting a stationary scalar field at the minimum of the potential. It looks easier than inflationary models because one does not need to find an unusual potential. However, it is not easy since we have to track the evolution of the universe. In other words, dark energy needs to subdominate the matter and radiation during the matter- and radiation-dominated period.

To see how the scalar field provides the accelerating universe, we will start with the inflaton action in (3.1) which gives us the energy density and pressure in (3.4) and (3.5). Thus the equation of state parameter can be written as

$$w = \frac{p}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} = -1 + \frac{\dot{\phi}^2}{\dot{\phi}^2 + 2V(\phi)}. \qquad (3.34)$$

From this equation, the accelerating universe is provided by $\dot{\phi}^2 < V(\phi)$. The advantage of these models is their equation of state parameter and the dark energy density can vary with time. This enables the possibility to solve the cosmological constant and coincidence problems. To solve the coincidence problem, the value of the dark energy density at the present time must be an attractor of dynamical equations of dark energy. It is found that the potential in this model need to be runaway-type. The simple and popular runaway potentials are $V = M^{\alpha+4}\phi^{-\alpha}$ and $V = M^4 e^{-\lambda\phi/M_{pl}}$.

For the inverse power law potential, $V = M^{\alpha+4}\phi^{-\alpha}$, there are no the attractor solutions corresponding to the same order of the dark energy and dark matter, $\rho_{DE}/\rho_{DM} \sim O(1)$. Thus it cannot solve the coincidence problem. However, it can mitigate the fine tuning of the initial values relative to the cosmological constant. The cosmological evolutions are properly provided in this potential. For the exponential potential, $V = M^4 e^{-\lambda\phi/M_{pl}}$, it can provide the attractor solution but encounters unacceptable evolution behavior, there is no matter-dominated period. However, this behavior can be cured by introducing another scalar field or assuming the coupling between the quintessence field and the dark matter.

The crucial problem of quintessence models comes from the fact that effective mass of quintessence field need to be light in order to drive the accelerating universe,

$$m_{eff}^2 = \frac{d^2}{d\phi^2}V(\phi) \sim H_0^2 \sim 10^{-86} GeV^2. \qquad (3.35)$$

In the solar system scale, the scalar field with very light mass subjects to tight constraints from test of the equivalence principle and fifth force [85]. Generally, the fifth force interaction is characterized by the Yukawa potential, $V \propto \frac{e^{r/\lambda}}{r}$ where λ is a parameter characterized the range of interaction. In order to illustrate the effect of the fifth force, one can roughly estimate that λ is inversely proportional to the mass of the scalar field, $\lambda \propto \frac{1}{m_{eff}}$. Since the mass of the quintessence is very light, the interaction range is very long. Thus the fifth force must be observed by experiments. Unfortunately, there are no evidences of this force in the recent experiments. This is the main problem of the scalar field dark energy models. This problem will be solved by using the *chameleon mechanism* [85, 86]. This mechanism provides the effective mass of the scalar fields which depends on the energy density of environments. To see explicitly how this mechanism is, we begin with the chameleon action,

$$S = \int d^4x\sqrt{-g}\Big(\frac{M_{pl}^2}{2}R + \mathcal{L}_\phi\Big) - \int d^4x\mathcal{L}_m(\psi^{(i)}, g_{\mu\nu}^{(i)}). \tag{3.36}$$

$\psi^{(i)}$ is the matter field and $g_{\mu\nu}^{(i)}$ is the metric related to the metric in Einstein's frame, $g_{\mu\nu}$, as

$$g_{\mu\nu}^{(i)} = e^{2\beta_i\phi/M_{pl}}g_{\mu\nu}, \tag{3.37}$$

where β_i is dimensionless constant. By considering the equation of motion for scalar field, ϕ, one obtains the effective potential and mass

$$V_{eff}(\phi) = V(\phi) + \sum_i \rho_i e^{\beta_i\phi/M_{pl}}, \tag{3.38}$$

$$m_{eff}^2 = \frac{d^2}{d\phi^2}V(\phi) + \sum_i \frac{\beta_i^2}{M_{pl}^2}\rho_i e^{\beta_i\phi/M_{pl}} = m_\phi^2 + m_i^2. \tag{3.39}$$

For the experiments on the earth, the mass square in the second term can be estimated as $m_i^2 = m_{earth}^2 \sim \rho_{earth}/M_{pl}^2 \sim 10^{-55}GeV^4$ which dominates the effective mass square, m_ϕ^2. Thus the fifth force are hidden for the terrestrial experiments. Note that even in the astronomical experiments, m_i^2 still dominates m_ϕ^2, $m_i^2 = m_{galaxy}^2 \sim 10^{-80}GeV^4$.

There are many fluid dark energy models that we have not mentioned, for example, chaplygin gas [91], phantom field [92], tachyon field [89, 90], vector field [93], k-essence [87, 88]. We do not consider them here for brevity. However, the readers can follow on these topics in the review paper or the lecture notes of the dark energy models or the given references.

Finally, due to the light mass of the dark energy fields, they acquire the quantum fluctuations. This suggests us to include the cosmological perturbations into our consideration. This issue is very important because they can affect the structure formation. Moreover, some models are ruled out by their instability. We also skip this issue in this thesis since it is not our main probe.

3.2.3 Gravitational dark energy

While the fluid dark energy models correspond to a modification of the energy momentum tensor on the right hand side of Einstein field equation, the gravitational dark energy models correspond to a modification of Einstein tensor on the left hand side of the Einstein field equation. The simple and popular models are $f(R)$ gravity models. The idea of these models is to introduce an arbitrary function of the Ricci scalar, $f(R)$, instead of Ricci scalar R in the Einstein Hilbert action. These models can be interpreted as the more general model of cosmological constant while $f(R) = R - 2\Lambda$. For the inflationary models, The proper function takes the form $f(R) = R + \alpha R^2$, where α is a positive constant. In this subsection, we will discuss the late-time accelerating universe from $f(R)$ gravity models by following [94, 95, 96].

There are two approaches for $f(R)$ gravity models. First, it is the standard formalism which uses the metric, $g_{\mu\nu}$, depending on the affine connection, $\Gamma^{\alpha}_{\beta\gamma}$. In this approach, we derive the equation of motion by varying the action with respect to the metric. The other is Palatini formalism. In this formalism, the metric and the affine connection are treated as an independent dynamical variable and we have to vary the action with respect to both the metric and connection in order to obtain the equations of motion. These two approaches provide identical equations of motion in the case of Einstein Hilbert action. However, they provide the different results when we include the non-linear term of R into the action. We will consider only the standard approach in this thesis. For Palatini formalism, one can follow reference [96] and references therein.

The idea of $f(R)$ gravity models for dark energy is different from ones for the inflation because we need the mechanism to end the inflation but does not need for dark energy models. R^2 term in $f(R)$ gravity models for inflation guarantees that the acceleration will end since inflation enforces the flat universe in order to solve the flatness problem. Hence, R^2 term will vanish at the end of inflation and the standard general relativity is recovered. For the dark energy models, the additional terms must be dominated when the universe is enforced to be flat. Thus the simple $f(R)$ gravity models for dark energy can be written as $f(R) = R - \alpha R^{-n}$, where α and n are positive real constant. Note that the unified models of dark energy and inflation will be investigated by including both two terms, $f(R) = R + \alpha_1 R^2 - \alpha_2 R^{-n}$ [97].

For the $f(R)$ dark energy models, the action with matter can be expressed as

$$S = \frac{M_{pl}^2}{2} \int d^4x \sqrt{-g} f(R) - \int d^4x \mathcal{L}_m(\psi, g_{\mu\nu}). \tag{3.40}$$

By varying this action with respect to the metric, the equation of motion can be written as

$$\Sigma_{\mu\nu} \equiv F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)F(R) = \frac{1}{M_{pl}^2}T_{\mu\nu}^{(m)}, \qquad (3.41)$$

where $F(R) = \partial f/\partial R$, $T_{\mu\nu}^{(m)}$ is the energy momentum tensor of matter and

$$\Box F = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu F). \qquad (3.42)$$

It is useful to consider the trace of this equation of motion,

$$3\Box F(R) + F(R)R - 2f(R) = \frac{1}{M_{pl}^2}T^{(m)}. \qquad (3.43)$$

For Einstein's general relativity, $f(R) = R$, we obtain $F(R) = 1$. This leads to $M_{pl}^2 R = -T^{(m)}$ as the usual gravity theory while $\Box F(R) = 0$. For the $f(R)$ gravity, $\Box F(R)$ does not vanish. This leads to the propagating scalar degree of freedom, $\varphi \equiv F(R)$, named *scalaron*. To obtain the equation of state parameter of these dark energy models, we rewrite equation (3.41) as

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{1}{M_{pl}^2}(T_{\mu\nu}^{(m)} + T_{\mu\nu}^{(D)}), \qquad (3.44)$$

where

$$T_{\mu\nu}^{(D)} = M_{pl}^2\Big(\frac{f - R - 2\Box F}{2}g_{\mu\nu} + \nabla_\mu\nabla_\nu F + (1-F)R_{\mu\nu}\Big). \qquad (3.45)$$

In the FRW metric, one obtains two equations of motion as

$$3FH^2 = \frac{1}{2}(FR - f) - 3H\dot{F} + \frac{1}{M_{pl}^2}\sum_i \rho_i, \qquad (3.46)$$

$$-2F\dot{H} = \ddot{F} - H\dot{F} + \frac{1}{M_{pl}^2}\sum_i(\rho_i + p_i). \qquad (3.47)$$

Generally, we add all possible fluid contents in the universe in these equations through the summation terms. This leads to the energy density and the pressure of gravitational dark energy as

$$\rho_d = M_{pl}^2\Big(\frac{1}{2}(FR - f) - 3H\dot{F} + (A - F)3H^2\Big), \qquad (3.48)$$

$$p_d = M_{pl}^2\Big(-\frac{1}{2}(FR - f) + 2H\dot{F} + \ddot{F} - (A - F)(3H^2 - 2\dot{H}\Big), \qquad (3.49)$$

where A is a constant. Thus the equation of state parameter for dark energy can be expressed as

$$w_d = -\frac{M_{pl}^2(2A\dot{H} + 3AH^2) + \sum_i p_i}{3AH^2M_{pl}^2 - \sum_i \rho_i} \simeq \frac{w_{eff}}{1 - (F/A)\Omega_m}, \qquad (3.50)$$

where $\Omega_m = \rho_m/(3M_{pl}^2H^2)$ is the density parameter of the matter and $w_{eff} \equiv -1 - 2\dot{H}/(3H^2)$ is effective equation of state parameter. The last approximation comes from the negligence of the all contents of the universe except the matter. The universe which is filled with the cosmological constant, the so-called de-Sitter universe, is reproduced by taking $H \sim constant$ and $\Omega_m \sim 0$ and then it turns out that $w_d = -1$. To recover the standard evolution of the universe, A must be unity. This equation of state parameter provides the possibility to reach the phantom phase naturally. In other words, it provides the state that $w_d < -1$ while there are no the ghost fields in the theory. Note that the recent observations suggest that w_d will slightly less than -1, $w_d = -1.1 \pm 0.14$ [98].

The simple model, $f(R) = R - \alpha R^{-n}$, also provides dark energy description. However, it encounters the instability of the perturbations [99, 100, 101] and it is not easy to satisfy the local gravity constraint [102]. It is found that the viable $f(R)$ dark energy models need to satisfy the constraints [100, 103]:

$$\partial_R f > 0, \tag{3.51}$$

$$\partial_R^2 f > 0. \tag{3.52}$$

The first and the second conditions satisfy the requirements for avoiding the existence of ghost and tachyon respectively. There are many observational constraints on $f(R)$ dark energy models, for example, local gravity constraint. We do not discuss this topic in this thesis.

In addition to Ricci scalar, the modification of Einstein-Hilbert action can be written in terms of Ricci tensor, $R_{\mu\nu}$, and Riemann tensor, $R_{\mu\nu\rho\sigma}$. The specific choice which is topologically invariant in 4-dimensional spacetime is Gauss-Bonnet term, $\mathcal{G} \equiv R^2 - 4R^{\mu\nu}R_{\mu\nu} + R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}$. The crucial property of this term is that it does not contribute to the equation of motion when we vary the action with respect to the metric. Therefore, it is convenient to couple this term to the scalar field in order to modify the Friedmann equation. It is possible to construct $f(\mathcal{G})$ dark energy models. However, they suffer the instability during the matter and radiation dominated period. The progress of these models are active now. We leave the references [94, 104, 105, 106] and references therein for interested readers.

Chapter IV

EFFECTS OF TIME-LIKE ÆTHER FIELD ON COSMOLOGICAL MODELS

Æther theory is based on effective field theory. The effects of the æther field are relevant to the physics at the Planck scale and suppressed by factor $(M/M_{pl})^2$, where M is the mass scale of the æther theory. The most powerful mechanism that can encode quantum gravity effects is the inflation since quantum fluctuations are strengthen to cosmological perturbations. Therefore, it is useful to investigate the effects of the æther field in inflationary models. It was first investigated by Lim [28]. In this work, the power spectra of both tensor and scalar modes are calculated and the results are slightly different from the standard inflationary models. It is also found that the isocurvature perturbation is not generated and the vector perturbation modes are not relevant to the cosmological scale. Note that β_4 term is not included in this investigation.

The inflationary models with the time-like æther field including β_4 term are investigated in [114]. The perturbation equations are calculated in alternative way by using covariant and gauge invariant (CGI) formalism. The results in this investigation provide the primordial power spectra which are different from the results in [28] even though β_4 vanishes. The effects of the time-like æther field in the late-time evolution of the universe are also investigated [114, 119].

By using the ΛCDM model, it is found that the matter power spectrum and the CMB power spectrum are slightly modified. The constraints of æther parameters with observational data are also examined in [116]. In this work, by taking into account the constraints æther parameters from PPN, Cerenkov radiation and stability, the numerical constraints from CMB and Large Scale Structure data are performed by using modified CMBEASY code with Monte-Carlo Markov Chain method. We also note that the generalized æther model, in which the kinetic terms are generalized as $K \to f(K)$, is used for these investigations. Recently, the results of the perturbations in the inflationary models with the time-like æther field are carefully calculated in [113]. These results are different from the previous investigations. The isocurvature perturbation and the vector modes of the perturbation are generated. The investigation of [113] also connects to the recent development of gravity theory named *Horava gravity theory* [122, 123]. The Horava gravity theory provides the time-like Lorentz violation in the same fashion with the time-like æther model [127]. In this chapter, we review and discuss the results of this article.

The inflationary models with the time-like æther field are also investigated in the context of the coupling between the æther field and the scalar inflaton field [115, 117, 118]. The coupling terms can be performed by introducing the æther parameters depending on the inflaton field, $\beta \to \beta(\phi)$. In [115], the æther parameters are set as $\beta = \beta_{13} + 3\beta_2 = \beta(\phi) = \zeta\phi^2$ and the simple potential is $V = m^2\phi^2$. It is found that the inflation and reheating can be properly obtained in both with and without inflaton potential. The power spectra of tensor perturbations are calculated and the result is the same with the non-coupling case. The various potentials and coupling forms are also investigated in [117] by using the first-order formalism. The coupling form can be obtained by interpreting the degree of freedom of the æther field as the local expansion θ defined as $\theta \equiv \nabla_a A^a$ where θ relates to the Hubble parameter by $\theta = 3H$. Then the coupling term takes the form $\theta\phi = -A^a\nabla_a\phi +$ total derivative. The consequent effect of this term in the equation of motion is that there is an external force. The various constraints including stability, Cerenkov, positive energy and PPN conditions are derived and summarized in [118].

Recently, the generalized æther model is investigated in order to play the role of dark energy and dark matter [121]. For dark matter, it is proposed to be the sources of the structure formation. However, it does not fit with observational data. For dark energy, it can provide the accelerating universe and properly fit with observational data. However, we will not consider them here.

4.1 Inflationary model with the æther field

In this section, we review the inflationary model which is driven by scalar inflaton field and contains the æther field by following [113]. We begin this section with adding æther field action into inflaton field action. This action is expressed as

$$S = \int d^4x\sqrt{-g}\Big(\frac{M_{pl}^2}{2}(R + \mathcal{L}_A) - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\Big). \qquad (4.1)$$

We set the norm of the time-like æther as unity, $A_\mu A^\mu = -1$. Thus the Lagrangian of the time-like æther field can be written as

$$\begin{aligned}\mathcal{L}_A &= -\beta_1(\nabla_\mu A_\nu)(\nabla^\mu A^\nu) - \beta_2(\nabla_\mu A^\mu)^2 - \beta_3(\nabla_\mu A_\nu)(\nabla^\nu A^\mu) \\ &\quad -\beta_4 A^\mu A^\nu(\nabla_\mu A_\rho)(\nabla_\nu A^\rho) + \lambda(A_\mu A^\mu + 1). \end{aligned} \tag{4.2}$$

It is convenient for us to evaluate the calculations by using conformal time since most of the cosmological perturbations are calculated by using the conformal time. FRW metric with the conformal time can be written as

$$ds^2 = a^2(\tau)(-d\tau^2 + \delta_{ij}dx^i dx^j). \tag{4.3}$$

The æther field is highly constrained by homogeneity and isotropy. One of the allowed choices is $A^\mu = (1/a(\tau), 0, 0, 0)$. By using this form of the æther field and the energy momentum tensor defined in Chapter II, equation (2.19), the energy density and the pressure of the æther field can be written as

$$\rho_A = -\frac{3}{2}\alpha_A M_{pl}^2 \frac{\mathcal{H}^2}{a^2}, \;\; p_A = \frac{\alpha_A}{2} M_{pl}^2 \Big(\frac{\mathcal{H}^2}{a^2} + 2\frac{\mathcal{H}'}{a^2}\Big), \tag{4.4}$$

where $\mathcal{H} = a'/a$, $\alpha_A = \beta_1 + 3\beta_2 + \beta_3$ and prime denotes the derivative with respect to the conformal time. The Lagrange multiplier can be written as

$$\lambda = \frac{3}{a^2}(\beta_2\mathcal{H}' - \beta_{123}\mathcal{H}^2). \tag{4.5}$$

By substituting the energy density of æther and inflaton field into Einstein field equation, Friedmann and acceleration equation are

$$3\mathcal{H}^2 = M_{pl}^{-2}a^2(\rho_A + \rho_\phi) = \frac{M_{pl}^{-2}}{1+\alpha_A/2}a^2\rho_\phi, \tag{4.6}$$

$$6\mathcal{H}' = -M_{pl}^{-2}a^2\Big((1+3w_A)\rho_A + (1+3w_\phi)\rho_\phi\Big) = -\frac{M_{pl}^{-2}}{1+\alpha_A/2}a^2(1+3w_\phi)\rho_\phi. \tag{4.7}$$

By comparing these two equations with equations (3.6) and (3.7), we can see that the modification is to rescale of the Planck mass $M_{pl}^2 \to (1+\alpha_A/2)M_{pl}^2$. We note that the equation of motion for inflation field in (3.2) is not modified by the æther field. By using this rescaling, the slow-roll parameter will be obtained

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{M_{pl}^2(1+\alpha_A/2)}{2}\Big(\frac{V'(\phi)}{V(\phi)}\Big)^2. \tag{4.8}$$

It is important to note that the factor $1+\alpha_A/2$ need to be positive in order to avoid the negative gravity. This condition leads to

$$\alpha_A > -2. \tag{4.9}$$

The qualitative analysis of this slow-roll parameter is very useful in order to characterize the dynamics of the inflation modified by the æther field. If $\alpha_A + 2$ is small enough, the potential of inflaton field does not need to be flat. For convenient calculations with the cosmological perturbations, we choose the potential of the inflaton as the exponential potential. This potential leads to the power law inflation, $a \propto \tau^q$. Thus the slow-roll parameter can be expressed in terms of the equation of state parameter, $\epsilon = 3(1+w_\phi)/2$. To see explicitly how the æther field affects inflation dynamics, we consider the number of e-fold,

$$N = \int_{t_i}^{t_f} H dt = \frac{1}{M_{pl}^2(1+\alpha_A/2)} \int_{\phi_i}^{\phi_f} \frac{V(\phi)}{V'(\phi)} d\phi \gtrsim 65. \tag{4.10}$$

From this condition, we will see that the potential does not need to be flat in order to solve horizon and flatness problem.

4.2 Cosmological perturbations with æther fields

The goal of this section is to find the primordial power spectra and compare the results to the standard scalar inflation. We use the same strategy as we done in Chapter III. In the first step, we will add the perturbations from the æther field into the perturbation of the metric (3.12) and the inflaton field (3.13). The æther field with its perturbations can be written as

$$A^0 = \frac{1}{a} + \delta A^0, \quad A^i = \frac{1}{a}(\partial_i C + V_i - S_i). \tag{4.11}$$

This form of the perturbations gives us two degrees of freedom from scalar perturbations $\delta A^0, C$ and two degrees of freedom from the transverse vector perturbation V_i. Notice that the transverse vector V_i is gauge-invariant. However, one degree of freedom for scalar perturbations will be eliminated by the fixed norm constraint. By perturbing the constraint equation, $g_{\mu\nu}A^\mu A^\nu = 1$, one obtains $\delta A^0 = -\Phi/a$. Thus we have one scalar and two vector degrees of freedom. Conveniently, we separate the calculations and discussions for each mode of perturbations.

4.2.1 Tensor perturbations

Substituting the æther field, the metric and the inflaton field with their perturbations into the action (4.1) and expanding up to the second order in h_{ij}, one obtains the action for the tensor perturbations as

$$S_t^{(2)} = \frac{M_{pl}^2}{2} \int d\tau d^3x \frac{a^2}{2}\Big((1-\beta_{13})h'^2 - \partial_i h \partial^i h\Big) \tag{4.12}$$

From this action, we see that the æther field effect is only in the first term. On the sub-horizon scale, the gravitational wave will propagate at the sound speed square,

$$s_t^2 = \frac{1}{1-\beta_{13}}. \tag{4.13}$$

To avoid the classical instability, one obtains the condition

$$1-\beta_{13} > 0. \tag{4.14}$$

Note that this condition also yields the solutions with no instability from ghost field since the kinetic term is not wrong sign. This result agrees with the result in the references [27, 113, 114, 28]. Note that the differences in the expression from various references come from the different notation they use. By using the same strategy in Chapter III, the primordial spectra from the tensor modes is

$$P_h = \sqrt{1-\beta_{13}}\frac{H^2}{M_{pl}^2}. \tag{4.15}$$

4.2.2 Scalar perturbations

It is important to consider the recent development of the gravity theory which is renormalizable named *Horava gravity theory* [122, 123] since it provides the Lorentz violation like æther theory [127]. However, Horava gravity encounters the instabilities and strong coupling at low energy [124]. The extensions of the Horava gravity are investigated in order to avoid such problems [125, 126]. This extension version is related with the æther theory and named as BPSH theories [127]. From point of view of the BPSH theories, the scalar degree of freedom for the æther field can be interpreted as the means of auxiliary scalar field, $\mathcal{T}$, through the identity [125, 126]

$$A_\mu \equiv \frac{-\partial_\mu \mathcal{T}}{(-\partial_\nu \mathcal{T}\partial^\nu \mathcal{T})^{1/2}}. \tag{4.16}$$

$\mathcal{T}$ can be interpreted as a time variable since the gradient of $\mathcal{T}$ is time-like everywhere. Analogous to BPSH theories, the surface of constant $\mathcal{T}$ will define a foliation of the space-like surface [113]. Consequently, the perturbation of this scalar field is related to the æther scalar perturbation C

$$\frac{\delta \mathcal{T}}{\mathcal{T}'} = -(B+C). \tag{4.17}$$

From this new degree of freedom, the curvature perturbation on the surface of constant inflaton field ζ can be decomposed into two new gauge invariant variables

$\zeta = \zeta_a + \delta N$ where

$$\begin{aligned} \zeta_a &\equiv \Psi - \mathcal{H}(B+C), \qquad (4.18)\\ \delta N &\equiv \frac{\mathcal{H}}{\phi'}\delta\phi + \mathcal{H}(B+C). \qquad (4.19) \end{aligned}$$

Geometrically, ζ_a is the curvature perturbation on the surface of constant æther field and δN can be interpreted as the isocurvature perturbation since it corresponds to the velocity of the æther relative to the inflaton field. δN is also interpreted as the differential e-folding number between the surface of constant æther field and inflaton field [113].

- **Sub-horizon limit**

In the short wavelength limit, ζ_a and δN are decoupled at the first order. Therefore, Lagrangian of them can be written as

$$\mathcal{L} = \frac{a^2}{2Z_N}(\delta N'^2 - k^2\delta N^2) + \frac{a^2}{2Z_a}(\zeta_a'^2 - s_a^2 k^2 \zeta_a^2) + ..., \qquad (4.20)$$

where the ellipsis denotes sub-leading terms and

$$Z_N = \frac{1}{2\epsilon(1+\alpha_A/2)M_{pl}^2}, \quad Z_a = \frac{2\pi s_t^2 \beta_{123}}{(1+\alpha_A/2)M_{pl}^2}, \quad s_a^2 = \frac{(2-\beta_{14})\beta_{123}s_t^2}{(2+\alpha_A)\beta_{14}}. \qquad (4.21)$$

In order to avoid the instabilities, one obtains the conditions

$$0 \le \beta_{14} < 2, \text{ and } 0 < \beta_{123}. \qquad (4.22)$$

From this Lagrangian, one can find the equations of motion and their solutions. We summarize that there are two modes of the solutions, inflaton perturbation mode and æther perturbation mode which are respectively expressed as

$$\zeta_a \to 0, \qquad \delta N \to \frac{Z_N^{1/2}}{a}\frac{e^{ik\tau}}{\sqrt{2k}}, \qquad (4.23)$$

$$\zeta_a \to \frac{Z_a^{1/2}}{a}\frac{e^{is_a k\tau}}{\sqrt{2s_a k}}, \qquad \delta N \to 0. \qquad (4.24)$$

Consequently, the power spectra associated to δN and ζ can be written respectively as

$$P_{\delta N} = Z_N\left(\frac{k}{a}\right)^2, \qquad (4.25)$$

$$P_\zeta = P_{\zeta_a} + P_{\delta N} = (Z_a + Z_N)\left(\frac{k}{a}\right)^2. \qquad (4.26)$$

- **Super-horizon limit**

For the long wavelength limit, ζ_a and δN are coupled. However, Lagrangian can be decomposed into other two independent variables as ζ and δN. The leading terms in the Lagrangian of ζ and δN can be expressed respectively as

$$\mathcal{L}_\zeta = \Big(\frac{2}{\epsilon} + \beta_{123}s_t^2\Big)^{-1}\sqrt{4+2\alpha_A}M_{pl}^2 a^2 \zeta'^2, \tag{4.27}$$

$$\mathcal{L}_{\delta N} = \beta_{14}(1+3w_\phi)^2 \frac{M_{pl}^2 a^2 (k\tau)^2}{8}\Big(\delta N'^2 + \frac{\kappa}{\tau^2}\delta N^2\Big), \tag{4.28}$$

where

$$\kappa = -6\Big(1+\frac{\alpha_A}{\beta_{14}}\Big)\frac{1+w_\phi}{(1+3w_\phi)^2}. \tag{4.29}$$

The corresponding independent solutions will be classified into two classes and four modes. The first class corresponding to adiabatic perturbations contains two modes which $\delta N = 0$ and the second class corresponding to isocurvature perturbations contains two modes which $\delta N \neq 0, \zeta = 0$. For adiabatic perturbations, there are the constant and decaying mode. The constant mode yields the anisotropic stress as

$$\Psi - \Phi = -\beta_{13}s_t^2\zeta_1 = -\beta_{13}\frac{(5+3w_\phi)}{1+3w_\phi}\Phi. \tag{4.30}$$

During inflationary period, $w_\phi \sim -1$, then leads to $(\Psi-\Phi)/\Phi \sim \beta_{13}$. To obtain the small contribution from the anisotropic stress, the æther parameters must be restricted, $|\beta_{13}| \lesssim 1$. For the decaying mode, the results are $\zeta_2 = 0, \Psi = \Phi \propto \mathcal{H}a^{-2}$. In order to provide the initial conditions for the perturbations at the matter and radiation dominated period, the interesting modes are the constant modes since the decaying modes will vanish before perturbations reenter the horizon. The power spectra for the constant mode can be approximated as

$$P_\zeta \approx \frac{H^2}{9\epsilon(1+\alpha_A/2)M_{pl}^2}(1+O(\beta_{123}\epsilon c_t^2)). \tag{4.31}$$

The leading part of this power spectra is different from the usual one by the factor $(1+\alpha_A/2)^{-1}$. For the isocurvature modes, the solutions are

$$\delta N \propto (-\tau)^{t_\pm}, \tag{4.32}$$

where

$$t_\pm = -\frac{1}{2}\Big(\frac{5+3w_\phi}{1+3w_\phi}\Big) \pm \sqrt{\Big(\frac{5+3w_\phi}{2(1+3w_\phi)}\Big)^2 + \kappa}. \tag{4.33}$$

The behavior of these two solutions are characterized by a parameter κ. The observational data suggests that the primordial power spectra are almost adiabatic.

Thus the isocurvature modes should be sub-dominated relative to the adiabatic modes. This argument suggests us to eliminate the growing mode of the isocurvature modes. Then this suggestion leads to the constraints on a κ and æther parameters as

$$\kappa \leq 0, \quad \alpha_A \leq -\beta_{14}. \tag{4.34}$$

The most interesting case is $\kappa = 0 \rightarrow \alpha_A = -\beta_{14}$ where it gives the constant primordial power spectrum. Moreover, this constraint also satisfies the PPN coditions where $\alpha_2 = 0$. The amplitude of isocurvature perturbation modes is comparable to the adiabatic modes and also contributes to the same order for the anisotropic stress. However, it can be the dominant results when $\kappa > 0$.

4.2.3 Vector perturbations

There are three transverse vector perturbation, V_i, F_i and S_i. We have two constraints from the equation of motion of S_i and the components $(i\ j)$ of Einstein field equation. Thus this leaves only one dynamical transverse vector field in which its quadratic Lagrangian can be expressed as

$$\mathcal{L}_v = \frac{M_{pl}^2}{2}\Big(\beta_{14}\xi_i'\xi'^i + \alpha_A(\mathcal{H}^2 - \mathcal{H}')\xi_i\xi^i - \beta_{14}s_v^2\partial_j\xi_i\partial^j\xi^i\Big), \tag{4.35}$$

where $\xi_i = aV_i$ and

$$s_v^2 = \frac{\beta_1}{\beta_{14}}\Big(1 + \frac{\beta_{13}^2 s_t^2}{2\beta_1}\Big). \tag{4.36}$$

In order to avoid the instabilities, the vector perturbations need to satisfy the conditions

$$\beta_{14} \geq 0 \ \text{ and } \ 2\beta_1(1+\beta_{13}) \geq -\beta_{13}^2. \tag{4.37}$$

The corresponding equation of motion for the above Lagrangian can be written in the term of original variable, $V_i = \xi_i/a$, as

$$V_i'' + 2\mathcal{H}V_i' + c_v^2k^2V_i + \Big((1+\alpha_A/\beta_{14})\mathcal{H}^2 + (1-\alpha_A/\beta_{14})\mathcal{H}'\Big)V_i. \tag{4.38}$$

It is shown explicitly in [113] that this equation is the same expression with the equation of the longitudinal vector, C. Thus the transverse vector is proportional to the æther velocity perturbation since $v_i = c_t^{-2}\partial_i C$ at the supper-horizon scale. The evolution of the transverse vector will behave like the æther velocity perturbation and its amplitude relates to δN as $V \propto v \propto \frac{k}{a\mathcal{H}}\delta N = \frac{k}{a'}\delta N$. Considering the case $\alpha_A = -\beta_{14}$ where δN is constant at the supper-horizon scale, we observe that V will decay since a' increases during inflationary period. For the de-Sitter space, the vector perturbation will decay exponentially. This argument leads to the vanishing of the vector perturbations as suggested in [28]. However, after inflation, the universe decelerates and the vector perturbations grow consequently. Due to the strong relation between δN and v, the estimated amplitude of the vector perturbations at the time of horizon reentry can roughly written as

$$V \sim \left(\frac{\epsilon}{\beta_{14}}\right)^{1/2} v \sim \left(\frac{\epsilon}{\beta_{14}}\right)^{1/2} \delta N, \tag{4.39}$$

where amplitude of the velocity field v is of order δN at horizon crossing during inflation. If β_{14} is small enough comparing to ϵ, it will affect the angular power spectrum of CMB anisotropy. From the calculation in [113], the estimated value of the CMB power spectrum can be written as

$$C_l^V \sim \left(\frac{\beta_{13}^2}{\beta_{14}}\right)^{1/2} C_l^h \sim \epsilon \left(\frac{\beta_{13}^2}{\beta_{14}}\right)^{1/2} C_l^\zeta. \tag{4.40}$$

From the observational data [98], the contribution of the vector perturbations to the temperature anisotropy must be sub-dominant comparing to the scalar perturbations. Thus this requirement places the constraint on the æther parameters,

$$\beta_{13}^2 < \epsilon^2 |\beta_{14}|. \tag{4.41}$$

Notice that this vector source of the perturbations will affect the polarization on the temperature anisotropy. This issue is very interesting for further investigations. In the last of this chapter, we will end with the summary of the constraints on the æther parameters by using cosmological phenomena and the theoretical constraints. The results are shown in Table IV.1.

Condition	Constraint
Solution of Einstein's equations	$\alpha_A > -2$
Stability of Tensors	$\beta_{13} < 1$
Stability of Scalars	$0 < \beta_{14} \leq 2, \ \ 0 < \beta_{123}$
Stability of Vectors	$2\beta_1(1 + \beta_{13}) \geq -\beta_{13}^2$
Anisotropic stress	$\beta_{13} \lesssim 1$
Non-growing scalar isocurvature modes	$\alpha_A \leq -\beta_{14}$
Subdominant contribution of vectors to CMB	$\beta_{13}^2 < \epsilon^2 \lvert\beta_{14}\rvert$

Table IV.1: Summary of the theoretical and phenomenological conditions on the parameters of æther theories.

Chapter V

EFFECTS OF SPACE-LIKE ÆTHER FIELD ON COSMOLOGICAL MODELS

The presence of space-like æther field in cosmological models with standard (3+1)-dimensional spacetime would destroy isotropy of the three-dimensional space, i.e, breaks $SO(3)$ rotational symmetry subgroup of Lorentz symmetry. Rotational invariance is a well-established feature of low-energy physics. Violations of this symmetry must be extremely small nowadays. However, they could have been much larger in earlier epochs, for example, during the inflationary era. In order to investigate the consequences of a small breaking of rotational invariance from the early universe, the authors in [38] assume the existence of a space-like æther field that picks out a preferred direction during inflation period. They also study the effect of such breaking on CMB anisotropies. From the observational point of view, there is a phenomenon so called the *Axis of Evil*, an apparent alignment of the CMB multipoles on very large scales [128, 129], which may be related to breaking of rotational invariance in the early epoch. However, its statistical significance is hard to quantify and there is no clear explanation of this large-scale anomaly in terms of Lorentz violating theory.

On the other hand, a presence of the space-like æther field in the universe with compact extra-space dimensions is less problematic. One can assume that the *vev* of the æther field aligns only in the compact directions. This will leave the isometry group $SO(1,3)$ of the non-compact dimensions unbroken and rotational invariance of low-energy physics is well preserved. Moreover, the æther field has unexpected behavior that can help to stabilize the extra-dimensional space. This behavior was first pointed out in [108] for 5-dimensional spacetime. There is a suggestion that the interplay between the æther field and the dynamical moduli field, describing size of extra dimensions may shed some light on the deep and previously unknown connection between the dimensionality of spacetime and the violation of Lorentz symmetry. Perhaps nature allows us to observe only the large three-dimensional space that preserves Lorentz symmetry but conceals the Lorentz violating directions in the compact space.

In this chapter, we will follow the discussion in [108] by consider five-dimensional cosmological model with one compact extra dimension. Casimir energy from various field fluctuations in compact extra dimensions could play a crucial role of dark energy (a cosmological constant) and the potential for stabilizing the size of the extra dimensions. In the first part of this chapter, we review the Casimir dark energy model [107]. Then we go on to consider the effects of the space-like æther field on this model. Note that, although the mechanism that we consider in this chapter involves the potential from the Casimir energy, the behavior of the æther field is totally independent from the potential. Hence, this æther stabilization mechanism can apply to other types of potential.

5.1 Casimir dark energy (CDE) models

Casimir effect was originally predicted by Casimir in 1948 [130]. He considered electromegnetic vacuum between two conducting plates. In quantum field theory, vacuum states contain the virtual particles which are in a continuous state of fluctuations. Casimir realized that the boundary condition is imposed to the system by the conducting plates. This causes momentum of the virtual particles to be discrete. Only those virtual particles that form standing wave between conducting plates are allowed. The energy density of this vacuum fluctuation decreases as the plates are moved closer. This implies that there is a small attractive force between the two plates. The Casimir force was first measured in 1958 by Sparnaay but with large experimental errors [131]. The more accurate measurement was performed by Lamoreaux in 1997 [132].

In theory with the compact extra dimensions, all fields satisfy the periodic boundary condition in compact directions. We will see that the Casimir energy from various field fluctuations in the compact directions can play the role of dark energy [107]. Moreover, it provides a mechanism to stabilize the extra dimension [109].

5.1.1 Casimir energy with an extra dimension

We begin with considering the Casimir energy of bosonic degrees of freedom. It is convenient to consider a massive scalar field since the other bosonic degrees of freedom give rise the same expression [110, 111]. It is instructive to consider the model with one extra dimension compactified on a circle, S^1. In this 5-dimensional spacetime, the metric can be written as

$$ds^2 = -dt^2 + a^2(t)dx^i dx^j \delta_{ij} + b^2(t)dy^2, \tag{5.1}$$

where $a(t)$ is the scale factor, $b(t)$ characterizes the radius of the extra dimension and the coordinates on S^1 are $0 \leq y \leq 2\pi$. Note that we allow the time-dependence of the radius for generality and compatibility with FRW metric. For a massive scalar field, the equation of motion is the Klein-Gordon equation,

$$(\partial_a \partial^a - m^2)\phi = 0. \tag{5.2}$$

The Latin indices, $a, b, c, ...$ are five spacetime indices running as $\{0, 1, 2, 3, 4\}$. The scalar field is set to satisfy the periodic boundary condition in the compact direction, $\phi(y = 0) = \phi(y = 2\pi)$. Its associated dispersion relation in the sub-horizon limit can be written as

$$-k^\mu k_\mu = m^2 + \frac{n^2}{b^2}, \tag{5.3}$$

where, $n \in \mathbb{Z}$ is the momentum number in the compact direction. Then the total vacuum energy contributing to Casimir energy can be written as

$$E_{cas} \;=\; \frac{1}{2}\left(\frac{L}{2\pi}\right)^3 \int d^3k \sum_n \sqrt{k^2 + m^2 + \frac{n^2}{b^2}}, \tag{5.4}$$

where $V = L^3$ be the spatial volume of non-compact space. By using the identity $\int f(k) d^n k = 2\pi^{n/2}/\Gamma(n/2) \int k^{n-1} f(k) dk$, we obtain

$$E_{cas} \;=\; \frac{1}{2}\left(\frac{L}{2\pi}\right)^3 \frac{2\pi^{3/2}}{\Gamma(3/2)} \int k^2 dk \sum_n \sqrt{k^2 + m^2 + \frac{n^2}{b^2}}, \tag{5.5}$$

$$\;=\; \frac{1}{2}\left(\frac{2}{L}\right)^{2s+1} \frac{\Gamma(s)}{\Gamma(-1/2)} b^{2s} \pi^{(2s+1)/2} \sum_n \left((bm)^2 + n^2\right)^{-s}, \tag{5.6}$$

where we define $s = -(3+1)/2$. Let us consider the massless case, $m = 0$. By using the zeta function regularization procedure, the Casimir energy density per one bosonic degree of freedom for massless scalar field can be written as

$$\rho_{cas}^{massless} = \frac{\widehat{E}_{cas}}{V 2\pi b} = \frac{\Gamma(-2s+1)}{\Gamma(-1/2)} 2^{2s} b^{2s-1} \pi^{3s-1} \zeta(-2s+1), \tag{5.7}$$

where ζ denotes the zeta function and we take $2\pi b$ to be the volume of compact dimension. For the massive case, we apply the Chowla-Selberg zeta function [111] in our regularization procedure and obtain the Casimir energy density per one degree of freedom for the massive scalar field:

$$\rho_{cas}^{massive} = -2(2\pi b)^{2s-1}(mb)^{(1-2s)/2}\sum_{n=1}^{\infty} n^{(2s-1)/2}K_{(1-2s)/2}(2\pi bmn), \qquad (5.8)$$

where $K_\nu(x)$ is the modified Bessel function. The energy density of these two types of the scalar field depends only on the dynamical variable, $b(t)$. The prefactor and the summation of the modified Bessel function indicate that the energy density of the bosonic part is negative. In order to get the positive finite minimum value of the total Casimir energy, one has to add the positive contributions into the system. It is found that the fermionic degrees of freedom will contribute to the total Casimir energy density with the same expression except for an extra minus sign. Thus the total Casimir energy can be written as

$$\rho_{Cas} = N_b\rho_{boson}^{massless} + N_f\rho_{fermion}^{massless} + \tilde{N}_b\rho_{boson}^{massive} + \tilde{N}_f\rho_{fermion}^{massive}, \qquad (5.9)$$

where N_b (N_f) and $\tilde{N}_b$ ($\tilde{N}_f$) are the numbers of bosonic (fermionic) degrees of freedom for massless and massive fields respectively. The qualitative nature of the total Casimir energy density depends on the relative magnitude of N_b, N_f, $\tilde{N}_b$ and $\tilde{N}_f$.

Phenomenologically, we take the numbers of all degrees of freedom as, $N_b = 5, \tilde{N}_b = 8, N_f = 8, \tilde{N}_f = 8$ [108]. The massless bosonic degrees of freedom come from the massless graviton in $(1+d)$-dimensional spacetime, $N = (d-2)(d+1)/2$. The massive bosonic degrees of freedom can interpret as the 8 massive scalar fields. The contribution from the other bosons such as electromagnetic vector field can be ignored by the phenomenological notion that they do not obey the boundary conditions we impose. For the fermionic contributions, the number of degrees of freedom is chosen in order to get the positive minimum of the Casimir energy density.

The positive minimum value of the Casimir energy density is obtained from not only by choosing the number of degrees of freedom but also from by choosing the mass ratio $\bar{\lambda} = m_b/m_f$, where m_b is the scalar mass and m_f is the Dirac fermion mass. It is found that the mass ratio must satisfy the condition $\bar{\lambda} \geq 0.516$. In Figure 5.1, we show the plot of the Casimir energy density as a function of b. By setting $\bar{\lambda} = 0.516$, the positive minimum of the energy density is obtained. This minimum of the energy density allows the possibility to stabilize the extra dimension. The stabilized radius of the extra dimension corresponds to the radius at the minimum of the potential, $b = b_{min}$. b_{min} also relates to the mass of the

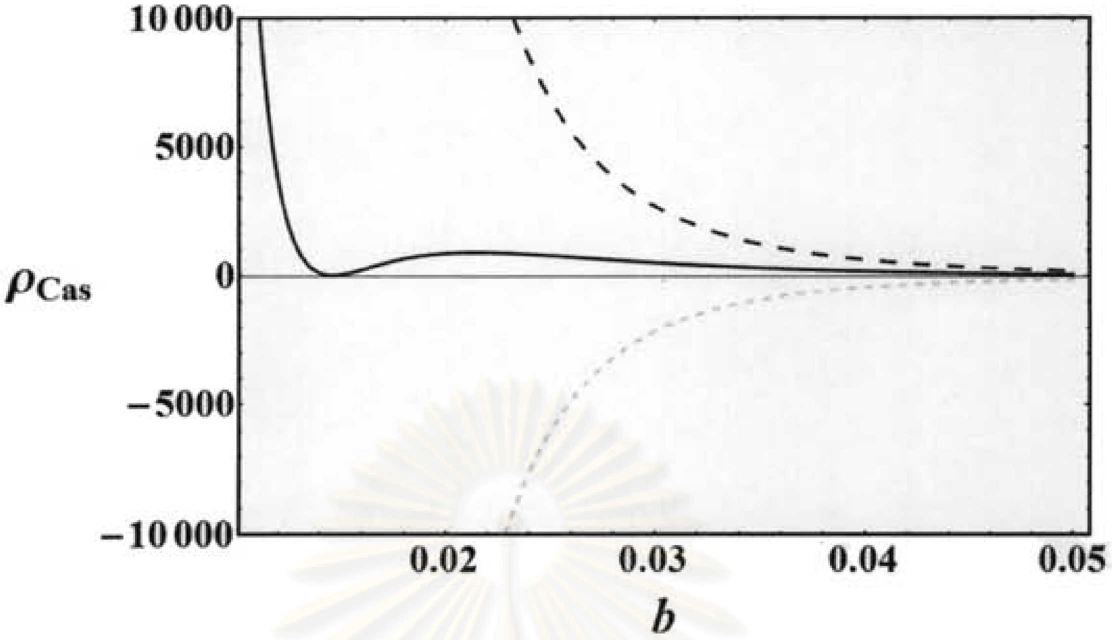

Figure 5.1: This figure shows ρ_{Cas} as a function of b which has the local minimum at $b = 0.0146$. The blue long-dashed line denotes the contributions from fermions. The green short-dashed line denotes the contributions from bosons. The black solid line denotes the total contribution.

fermion and we will calculate this mass in the next subsection. Note that there are no unique choices for choosing the numbers of degrees of freedom, for example, the choice in reference [107].

5.1.2 Dynamics of Casimir dark energy

In order to obtain the dynamics of the Casimir dark energy, we add the energy momentum tensor contributed from the Casimir effect into the Einstein field equation. The general form of the Casimir energy momentum tensor which is compatible with the metric in the equation (5.1) can be written as [107]

$$T^{\mu}_{\ \nu(Cas)} = \begin{pmatrix} -\rho_{Cas} & 0 & 0 & 0 & 0 \\ 0 & p_a & 0 & 0 & 0 \\ 0 & 0 & p_a & 0 & 0 \\ 0 & 0 & 0 & p_a & 0 \\ 0 & 0 & 0 & 0 & p_b \end{pmatrix}, \tag{5.10}$$

where p_a and p_b are the Casimir pressure in the non-compacted and compacted dimension respectively. These pressures can be defined as

$$p_a \equiv -\frac{\partial}{\partial V_a}\Big(\rho_{Cas} V_a\Big), \tag{5.11}$$

$$p_b \equiv -\frac{\partial}{\partial V_b}\Big(\rho_{Cas} V_b\Big), \tag{5.12}$$

where $V_a \propto a^{d-n}$ and $V_b \propto b^n$. Here, d is the numbers of all spatial dimensions and n is the numbers of the extra dimensions and $d = 4, n = 1$ for this model. These definitions automatically yield the cosmological constant behavior in 4-dimensional spacetime while $p_a = -\rho_{Cas}$ and $p_b = -\rho_{Cas} - b\partial_b \rho_{Cas}$. The conservation equation of the energy momentum tensor reads

$$\dot{\rho}_{Cas} + 3H_a(\rho_{Cas} + p_a) + H_b(\rho_{Cas} + p_b) = 0, \tag{5.13}$$

where $H_a = \dot{a}/a$ and $H_b = \dot{b}/b$. Substituting the energy momentum tensor into the Einstein field equation, one obtains

$$3H_a^2 + 3H_aH_b = M_*^{-3}\rho_{Cas}, \tag{5.14}$$

$$3\frac{\ddot{a}}{a} - 3H_aH_b = -M_*^{-3}(\rho_{Cas} + p_b), \tag{5.15}$$

$$3\frac{\ddot{b}}{b} + 9H_aH_b = M_*^{-3}(\rho_{Cas} + 2p_b - 3p_a). \tag{5.16}$$

The behavior of the scale factor and the radius of the extra dimension are shown in Figure 5.2. From this figure, we will see that the extra dimension can be stabilized and the universe accelerately expands. The Casimir energy can be interpreted as the dark energy and also provides the mechanism to stabilize extra dimension.

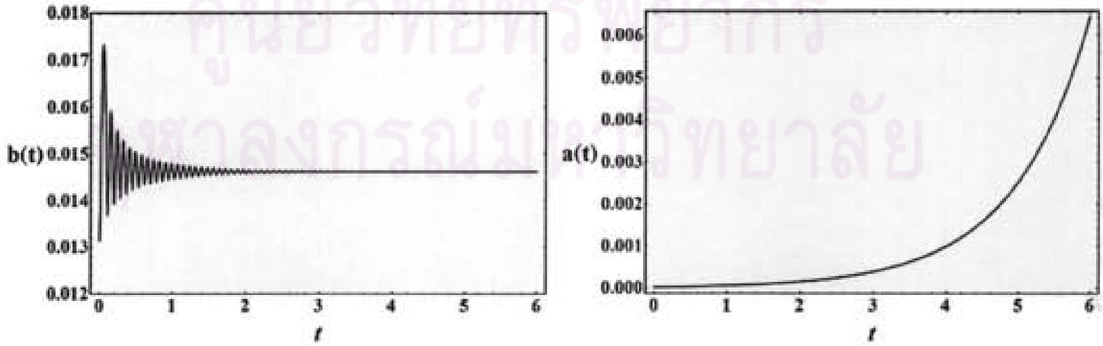

Figure 5.2: This figure shows the behavior of the radius of the extra dimension (left panel) and the scale factor (right panel).

5.1.3 Dynamics in the radion picture

Since the observed universe is in 4-dimensional spacetime, we have to investigate this model by using the 4-dimensional effective field theory. The 4-dimensional action will be obtained by using the KK-dimensional reduction. Since we have the energy momentum tensor for the Casimir energy source, the corresponding Lagrangian density for the Casimir source must be formulated. Since the energy density and pressure of the Casimir source depend only on b, it is useful to write down the Lagrangian density as a function of b. In the spirit of KK-dimensional reduction, the degree of freedom of the radius b of the fifth direction will correspond to a scalar field, the so-called *radion field*, in 4-dimensional spacetime. Therefore, it is convenient to write down the Lagrangian density as 5-dimensional potential term. Thus the action of this model can be written as

$$S_{5D} = \int d^5x\sqrt{-g}\Big(\frac{M_*^3}{2}R - V(b)\Big). \tag{5.17}$$

$V(b) = \rho_{Cas}$ denotes the potential term in 5-dimensional spacetime. To obtain the 4-dimensional action, let us start with KK-dimensional reduction of the above action. In order to make the resulting effective action in the canonical form, we apply Weyl rescaling $g_{\mu\nu E} = \Omega g_{\mu\nu}$ $(\mu, \nu = 1, \ldots, 3)$ and define the new time variable $dt_E = \Omega^{1/2}dt$, $a_E(t_E) = \sqrt{\Omega}a(t)$; $\Omega = 2\pi b M_*^3/M_{pl}^2$. Note that M_* is the Planck mass in 5-dimensional spacetime defined via the relation $M_{pl}^2 = (2\pi b_{min})M_*^3$. Thus $\Omega = 1$ at $b = b_{min}$. The effective action takes the form

$$S_{4D} = \int d^4x\sqrt{-g_E}\Big(\frac{M_{pl}^2}{2}R_E - \frac{1}{2}g_E^{\mu\nu}\nabla_\mu\Psi\nabla_\nu\Psi - U(\Psi)\Big), \tag{5.18}$$

where $U(\Psi) = 2\pi b\Omega^{-2}V(b)$ is the 4-dimensional effective potential. Here we define the radion field $\Psi = \frac{M_{pl}}{\sqrt{2}}\sqrt{3}ln(b/b_{min})$. This is the usual action for a scalar field with specific potential. As we have mentioned in the previous chapter, the accelerating universe can be obtained when the scalar field is sitting at the minimum of its potential. The radion field corresponding to radius of the extra dimension may oscillate around the minimum of the potential during matter and radiation dominated period and settle at the minimum of the potential, then the universe reaches the accelerating phase at the present time. By varying the above 4-dimensional action with respect to the metric, we obtain

$$H_E^2 = \frac{1}{3M_{pl}^2}\Big(U(\Psi) + \frac{1}{2}\left(\frac{d\Psi}{dt_E}\right)^2\Big), \tag{5.19}$$

$$\frac{d^2\Psi}{dt_E^2} + 3H_E\frac{d\Psi}{dt_E} = -\frac{\partial U}{\partial\Psi}, \tag{5.20}$$

where H_E is the Hubble parameter in the Einstein frame defined as $H_E \equiv \frac{1}{a_E}\frac{da_E}{dt_E}$. Equation (5.19) is the constraint equation and equation (5.20) is the real dynamical equation of motion which acts as the simple harmonic equation with friction term. The gradient of the potential on the right-hand side represents the force acting on the radion. In other words, it is the slope of the potential characterizing the oscillation behavior of the redion.

From the equation (5.8), the energy density behaves as $\rho \propto m^5$ while $b \propto m^{-1}$. Thus the fermion mass is related to the $\rho^{(4)}_{observe}$ as

$$\rho^{(4)}_{obs} \approx (2.3 \times 10^{-3}\ eV)^4 = \rho^{(4)}_{min} = 2\pi b_{min}\rho^{(5)}_{min} = 2\pi\ 0.0146 \times 23.4\left(\frac{m_f}{40}\right)^4, \quad (5.21)$$

where 23.4 is the minimum value of the potential at $b = 0.0146$ and the number 40 comes from the setting of $m_f = 40$ in the numerical simulation. This relation gives the fermion mass $m_f \sim 7.6 \times 10^{-2}\ eV$ and then leads to the radius of the extra dimension $b_{min} = 0.0146 \times 40/m_f \sim 7.7\ eV^{-1} = 1.5 \times 10^{-6}\ m$. This value corresponds to the quantum gravity scale in 5-dimensional spacetime $M_* = (M_{pl}^2/2\pi b_{min})^{1/3} \sim 1.4 \times 10^9\ GeV$. All of these behaviors and the quantities in the model suggest us that this is the proper model for dark energy candidates. However, there are some problems in this model. We will discuss in the next subsection.

5.1.4 Problems of the CDE models

The crucial problem of CDE models is the destabilization of the extra dimension when matter is taken into account. In the previous subsection, we assume that the universe is filled only by the Casimir energy. The Casimir energy can interpret as dark energy and provides the mechanism to stabilize the extra dimension. However, the real universe does not contain only dark energy but also matter and radiation. Unfortunately, it is found that the extra dimension cannot be stabilized when matter is taken into account. To see this behavior, we start with adding the matter content into the model,

$$S = S_{5D} + \int d^5x\sqrt{-g}\mathcal{L}_{matter}. \quad (5.22)$$

By varying this action with respect to the metric, the equations of motion can be written as

$$3H_a^2 + 3H_aH_b = M_*^{-3}(\rho_{Cas} + \rho_m), \tag{5.23}$$

$$3\frac{\ddot{a}}{a} - 3H_aH_b = -M_*^{-3}\Big(\rho_{Cas} + \rho_m + p_b\Big), \tag{5.24}$$

$$3\frac{\ddot{b}}{b} + 9H_aH_b = M_*^{-3}\Big(\rho_{Cas} + \rho_m + 2p_b - 3p_a\Big). \tag{5.25}$$

ρ_m is the energy density of the non-relativistic matter in 5-dimensional spacetime. This matter includes baryon, electron and dark matter. Since this matter repre-

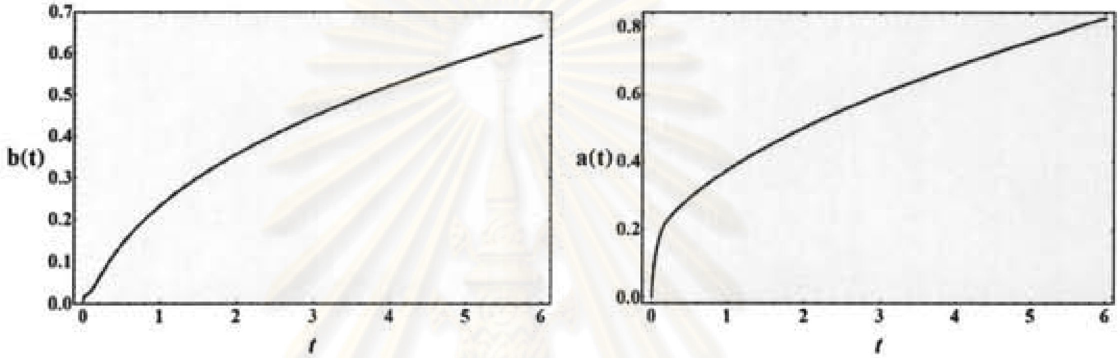

Figure 5.3: This figure shows the behavior of the radius of the extra dimension (left panel) and the scale factor (right panel) when the non-relativistic matter is taken into account.

sents dust in the 5-dimensional spacetime, it will evolve as $\rho_m \propto 1/(a^3 2\pi b)$. At the present time, the value of this energy density can be related to the energy density of the dark energy as $\rho_{m0} \approx \rho_{Cas0}2.8/7.2$. By using $a = (1+z)^{-1}$, ρ_m can be expressed as

$$\rho_m = \frac{2.8}{7.2}\rho_{min}\left(\frac{b_{min}}{b}\right)(1+z)^3, \tag{5.26}$$

where z is the red-shift. By using the numerical simulation, the evolution of the radius of the extra dimension and the scale factor can be shown in Figure 5.3. This figure shows explicitly that the extra dimension cannot be stabilized and the scale factor is not accelerated. In order to get more insight for this problem, we need to go to the radion picture in 4-dimensional spacetime. We use the conservation of the energy-momentum tensor in four and five dimensions to demonstrate that the radion field will be driven toward the minimum of the 4-dimensional effective potential as

$$U_{eff} = U_{Cas} + \frac{M_{pl}^2}{\Omega M_*^3}\frac{\rho_m}{4} = U_{Cas} + \frac{\rho_m^{(4)}}{4}\left(\frac{b_{min}}{b}\right)^2, \tag{5.27}$$

where $U_{Cas} \equiv 2\pi(b_{min}^2/b)\rho_{Cas}$. The matter density in 4-dimensional spacetime $\rho_m^{(4)} = \rho_m(2\pi b) = \frac{2.8}{7.2}\rho_{min}(2\pi b_{min})(1+z)^3$ is a function of $(1+z)^3$ and does not depend on the radius of the extra dimension b.

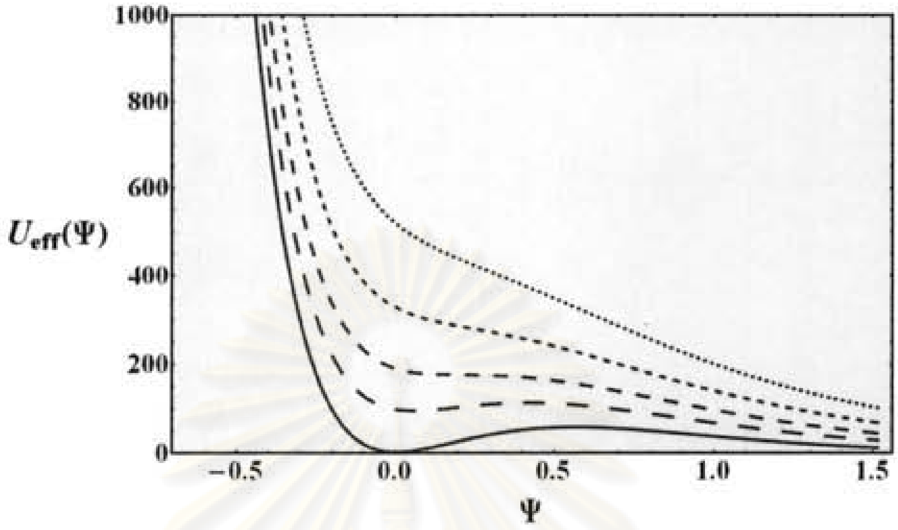

Figure 5.4: This is the plot of the effective potential $U_{eff}(\Psi)$ at red shift $z = 0.0$, 7.0, 9.0, 11.0, 13.0 in the unit of $(m_f/40)^4$ and $\lambda = 0.516$. The local minimum of $U_{eff}(\Psi)$ no longer exists when the red-shift increases.

The effective potential U_{eff} with the various red-shift is illustrated in Figure 5.4. At $z = 0$, the presence of non-relativistic matter will lift up the minimum of U_{eff} slightly. However, at early time, high red-shift, the $1/b^2$-term in equation (5.27) becomes dominant and destroys the existence of the minimum. This effect will drive b to expand even though there is a local minimum today since the radion field Ψ has already rolled pass the minimum and cannot get back to the stable point. Notice that this effect is the same if matter is confined to the brane. This situation can be shown by adding a 4-dimensional term into the 5-dimensional action (5.17)

$$S = S_{5D} + \int d^4x\sqrt{-g^{(4)}}\mathcal{L}_{matter}. \qquad (5.28)$$

There is no dimension reduction in the matter term but the conformal transformation rescale the matter energy density as $\rho_m^{(4)}/\Omega^2$. Thus, the total effective potential is the same as in equation (5.27).

5.2 CDE models with the æther field

In this section, we propose the way to solve the destabilization of the extra dimension in the CDE model by taking into account the simple form of æther fields

[108]. In the first part, the interactions of æther field with other fields are discussed by following references [36, 35]. The simple interactions will modify the dispersion relation of the ordinary particles and lead to the modification of the Casimir energy. However, the main contribution of the æther effects is the modification of the dynamical evolution. We demonstrate these effects in both numerical and qualitative analysis in the later part of this section.

5.2.1 The æther field and its interactions

The interactions between the æther field and the other fields are considered in this subsection. It is found that only non-minimally gravitational interaction can directly affect the dynamical evolution of the universe. The other interactions affect only on the Casimir energy. Phenomenologically, we can assume that the æther field does not interact with other fields except graviton field. For this assumption, the stabilization mechanism is still viable due to the minor effect of the modification. However, the possible simple interactions will be investigated in this section for generality.

We begin an investigation with choosing the simple form of the æther field. It is instructive to consider only the Maxwell-type kinetic term of the æther field due to the non-complicated calculation. In the sub-horizon scale, corresponding to the Minkowsky spacetime, this type of the æther field can avoid the instability issue [39]. However, it encounters the instability at the horizon crossing scale [40, 41]. For the particle interactions, it is convenient to consider only in sub-horizon scale.

By setting $\beta_1 = 1/2, \beta_3 = -1/2, \beta_2 = \beta_4 = 0$ and adding the interaction terms, the action of this system can be written as

$$S = \int d^5x\sqrt{-g}\Big(-\frac{1}{4}V_{ab}V^{ab} - \lambda(A_aA^a - v^2) + \sum_i \mathcal{L}_i\Big), \tag{5.29}$$

where $V_{ab} = \nabla_a A_b - \nabla_b A_a$. The terms in the summation of equation (5.29), $\mathcal{L}_i$, represent various interaction terms of the æther field with other fields that we will discuss later. If we neglect the interaction terms for the moment, the equation of motion for the æther field can be written as

$$\nabla_a V^{ab} + v^{-2}A^bA_c\nabla_d V^{cd} = 0. \tag{5.30}$$

Any solutions with $V_{ab} = 0$ will solve the equation of motion (5.30). In order to preserve Lorentz invariance in the 4-dimensional non-compact spacetime, we choose the background solution such that the æther is a space-like vector field which has non-vanishing components along the extra fifth dimension,

$$A^a = (0, 0, 0, 0, v). \tag{5.31}$$

We now consider the effect of the interaction terms $\mathcal{L}_i$ in (5.29) which can include the terms corresponding to the æther field coupled to scalars, vectors, fermions and gravity. Let us begin with the interaction effect of the æther with a real massive scalar field ϕ. By imposing $\mathbb{Z}_2$ symmetry, $A^a \to -A^a$, the Lagrangian for the scalar field with the lowest-order coupling is

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial_a\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\alpha_\phi^2}{2}\frac{A^aA^b}{v^2}\partial_a\phi\partial_b\phi, \tag{5.32}$$

where α_ϕ^2 is the dimensionless coupling constant characterized the strength of the scalar interaction. We insert the fraction v^2 into this term to normalize the æther field. In order to provide all effects in the system, including æther fields, scalar fields and their interaction, to be in the same order, the coupling constant should be order of unity. The corresponding equation of motion for the scalar field takes the form

$$\partial_a\partial^a\phi - m^2\phi = -\frac{\alpha_\phi^2}{v^2}\partial_a(A^aA^b\partial_b\phi). \tag{5.33}$$

Expanding the scalar field in Fourier modes $\phi \propto e^{ik_ax^a}$, one obtains the modified dispersion relation,

$$-k^\mu k_\mu = m^2 + (1+\alpha_\phi^2)k_5^2. \tag{5.34}$$

Note that the above interaction term is lowest order when the $\mathbb{Z}_2$ symmetry is imposed. However, if we ignore to impose this symmetry, the leading term will also vanish by using the integration by part [36].

Next we consider the fermion terms. By imposing the $\mathbb{Z}_2$ symmetry, the Lagrangian for a fermionic field with the leading interaction term can be written as

$$\mathcal{L}_\psi = i\bar{\psi}\gamma^a\partial_a\psi - m\bar{\psi}\psi - i\alpha_\psi^2\frac{A^aA^b}{v^2}\bar{\psi}\gamma_a\partial_b\psi, \tag{5.35}$$

where α_ψ is the dimensionless coupling constant of the fermions. By using the same manner with the scalar field case, the corresponding modification of the dispersion relation for the fermionic case can be written as

$$-k^\mu k_\mu = m^2 + (1+\alpha_\psi^2)^2k_5^2, \tag{5.36}$$

The form of this equation is different from the analogous equation in the bosonic case: i.e. the second term on the right-hand side increases by the factor $(1+\alpha_\psi^2)^2$. However, if we do not impose the $\mathbb{Z}_2$ symmetry, it is found that the leading terms are expressed as

$$A_a\bar{\psi}\gamma^a\psi, \tag{5.37}$$

$$i\alpha_\psi\frac{A^a}{v}\bar{\psi}\partial_a\psi, \tag{5.38}$$

and the corresponding dispersion relations for each term are

$$-k^\mu k_\mu = m^2+(v+k_5)^2, \tag{5.39}$$

$$-k^\mu k_\mu = m^2-2m\alpha_\psi k_5+(1+\alpha_\psi^2)k_5^2. \tag{5.40}$$

For the vector field, we consider an Abelian gauge field B_a, with the strength tensor $F_{ab}=\nabla_a B_b-\nabla_b B_a$. The Lagrangian with leading interaction term is

$$\mathcal{L}_B=-\frac{1}{4}F_{ab}F^{ab}-\frac{\alpha_B^2}{2}\frac{A^aA^b}{v^2}g^{cd}F_{ac}F_{bd}. \tag{5.41}$$

By varying this Lagrangian with respect to the field B_a, one obtains two independent equations corresponding to the 5 and μ components. After we choose gauge choice as $B_5=0$, these two equations can be expressed as

$$k_5k_\mu\epsilon^\mu = 0, \tag{5.42}$$

$$\Big(k^\mu k_\mu+(1+\alpha_B^2)k_5^2\Big)\epsilon^\nu-k^\nu k_\mu\epsilon^\mu = 0, \tag{5.43}$$

where ϵ^μ is the polarization vector. These two equations can be decomposed into two modes. The first mode corresponding to $k_5=0$ provides us the usual result for the photons. The other corresponding to $k_\mu\epsilon^\mu=0$ yields the dispersion relation

$$-k^\mu k_\mu=(1+\alpha_B^2)k_5^2. \tag{5.44}$$

Finally, we consider the æther field which couples non-minimally to gravity. This can be described by the action

$$S_g=\int d^5x\sqrt{-g}\Big(\frac{M_*^3}{2}R+\tilde{\alpha}_gA^aA^bR_{ab}\Big), \tag{5.45}$$

where $\tilde{\alpha}_g$ is the dimensionless graviton coupling constant and M_* is the Planck mass in 5 dimensional spacetime. By varying this action with respect to the metric tensor, we obtain the equation of motion $G_{ab}=M_*^{-3}T_{ab(g)}$ with

$$\begin{aligned}T_{ab(g)}=\tilde{\alpha}_g\Big(R_{cd}A^cA^dg_{ab} &+ \nabla_c\nabla_a(A_bA^c)+\nabla_b\nabla_c(A_aA^c)\\ &- \nabla_c\nabla_d(A^cA^d)g_{ab}-\nabla_c\nabla^c(A_aA_b)\Big),\end{aligned} \tag{5.46}$$

Let us consider a small fluctuation of the metric

$$g_{ab} = \eta_{ab} + h_{ab}. \tag{5.47}$$

Following the explanation in the gauge field case, we have freedom to choose the gauge choice. Here the gauge $h_{\mu 5} = 0$ is chosen [36]. The metric perturbations can be decomposed into

$$h_{\mu\nu} = \bar{h}_{\mu\nu} + \bar{\Phi}\eta_{\mu\nu}, \quad h_{55} = \bar{\Psi}, \tag{5.48}$$

where $\eta^{\mu\nu}\bar{h}_{\mu\nu} = 0$, $\bar{h}_{\mu\nu}$ presents the propagating modes of the gravitational wave, $\bar{\Phi}$ denotes the Newtonian gravitational field and $\bar{\Psi}$ is a component associated with the radion field describing the mode of the extra dimension. By setting $\bar{\Phi} = 0 = \bar{\Psi}$, and considering transverse waves, $\partial^{\lambda}\bar{h}_{\lambda\mu} = 0$, the gravitational equation of motion becomes

$$-\frac{1}{2}\partial^c\partial_c\bar{h}_{\mu\nu} = \frac{\tilde{\alpha}_g v^2}{M_*^3}\partial_5^2\bar{h}_{\mu\nu}. \tag{5.49}$$

This equation gives the modified dispersion relation for graviton

$$-k^{\mu}k_{\mu} = \left(1 + \alpha_g^2\right)k_5^2, \tag{5.50}$$

where $\alpha_g^2 = 2\frac{\tilde{\alpha}_g v^2}{M_*^3}$.

5.2.2 Casimir energy with the æther field

Casimir energy can be modified by the effect of the interactions of the æther field with various fields through the dispersion relations. In the previous section, we have shown the modified dispersion relation of various fields. In cosmological background, the wave number in the extra direction can be written as

$$k_5^2 = \frac{n^2}{b^2}. \tag{5.51}$$

Thus the dispersion relation for scalar field in equation (5.3) will be modified as

$$-k^{\mu}k_{\mu} = m^2 + \left(1 + \alpha_\phi^2\right)\frac{n^2}{b^2}. \tag{5.52}$$

Then the Casimir energy in (5.4) and (5.5) is also modified as

$$\begin{aligned} E_{cas} &= \frac{1}{2}\left(\frac{L}{2\pi}\right)^3 \int d^3k \sum_n \sqrt{k^2 + m^2 + \left(1 + \alpha_\phi^2\right)\frac{n^2}{b^2}}, \\ &= \frac{1}{2}\left(\frac{L}{2\pi}\right)^3 \frac{2\pi^{3/2}}{\Gamma(3/2)} \int k^2 dk \sum_n \sqrt{k^2 + m^2 + (1 + \alpha_\phi^2)\frac{n^2}{b^2}}. \end{aligned} \tag{5.53}$$

It is just replacing $\frac{n^2}{b^2}$ with $\left(1+\alpha_\phi^2\right)\frac{n^2}{b^2}$. In order to regularize the Casimir energy in usual way, we rescale k and m as $k^2 = (1+\alpha_\phi^2)k'^2$ and $m^2 = (1+\alpha_\phi^2)m'^2$. Thus the Casimir energy will be rewritten as

$$E_{cas}(\alpha_\phi) = \frac{(1+\alpha_\phi^2)^2}{2}\left(\frac{L}{2\pi}\right)^3\frac{2\pi^{3/2}}{\Gamma(3/2)}\int k'^2dk'\sum_n\sqrt{k'^2+m'^2+\frac{n^2}{b^2}}. \quad (5.54)$$

Since the integrand of k' is the same as the usual one, we can evaluate in the same manner and it leaves the difference only in the pre-factor. Thus we can immediately write down the Casimir energy density per one bosonic degree of freedom with the æther coupling α_ϕ as

$$\rho_{boson}^{massless}(\alpha_\phi) = \frac{\Gamma(-2s+1)}{\Gamma(-1/2)}\frac{2^{2s}b^{2s-1}\pi^{3s-1}}{(1+\alpha_\phi^2)^s}\zeta(-2s+1), \quad (5.55)$$

$$\rho_{boson}^{massive}(\alpha_\phi) = -\frac{2(2\pi b)^{2s-1}}{(1+\alpha_\phi^2)^s}\left(\frac{mb}{\sqrt{1+\alpha_\phi^2}}\right)^{\frac{(1-2s)}{2}}\sum_{n=1}^{\infty}n^{(2s-1)/2}K_{(1-2s)/2}\left(\frac{2\pi bmn}{\sqrt{1+\alpha_\phi^2}}\right), \quad (5.56)$$

for the contributions from massless and massive scalar fields respectively. We can see that the æther effects are just rescaling the Casimir energy and scalar mass by factors $(1+\alpha_\phi^2)^2$ and $(1+\alpha_\phi^2)^{-1/2}$ respectively. In the other bosonic particles, we can evaluate in the same manner due to the same modification of dispersion relations. We just replace the coupling constant and their mass into the above expression and we do not show them here.

For the fermion, it can be classified into three cases depending on how the modified dispersion relations are. It is not easy to regularize the energy when $\mathbb{Z}_2$ symmetry is not imposed since we cannot rescale the mass m and the wave number k in the usual way as done in the scalar field case. We leave this investigation in further works. In this model, we will focus on the modification of the dynamical behavior in order to avoid the destabilization of the extra dimension. Thus we can leave the complicated regularization by imposing that the æther field does not interact with other fields except graviton which always minimally interacts with all fields. Thus the Casimir energy will be the same in this case. Furthermore, we can compactify the extra dimension by imposing the $\mathbb{Z}_2$ symmetry on the fifth direction. In this case, the regularization of Casimir energy can be evaluated in the same manner with the scalar field case by inserting minus sign and replacing $m_b \to m_f$, $(1+\alpha_\phi^2) \to (1+\alpha_\psi^2)^2$. Therefore, the Casimir energy densities per one degree of freedom of massless and massive fermion can be written respectively as

$$\rho^{massless}_{fermion}(\alpha_\psi) = -\frac{\Gamma(-2s+1)}{\Gamma(-1/2)}\frac{2^{2s}b^{2s-1}\pi^{3s-1}}{(1+\alpha_\psi^2)^{2s}}\zeta(-2s+1), \tag{5.57}$$

$$\rho^{massive}_{fermion}(\alpha_\psi) = \frac{2(2\pi b)^{2s-1}}{(1+\alpha_\psi^2)^{2s}}\Big(\frac{m_f b}{1+\alpha_\psi^2}\Big)^{(1-2s)/2}\sum_{n=1}^{\infty} n^{(2s-1)/2}K_{(1-2s)/2}\Big(\frac{2\pi b m_f n}{1+\alpha_\psi^2}\Big). \tag{5.58}$$

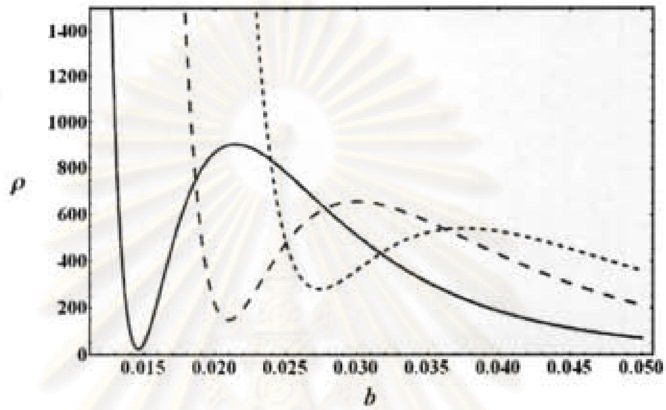

Figure 5.5: Interactions between bosons/fermions and æther field can affect the Casimir energy. In this figure, we fix the mass ratio $\bar{\lambda} = 0.516$. The Casimir energy density ρ is presented in the y-axis in units of $(m_f/40)^5$. The solid, long dashed and short dashed line denote the Casimir energy density when the coupling constants are $(\alpha_\phi, \alpha_\psi) = (0.0, 0.0), (1.0, 0.644)$, and $(1.5, 0.897)$ respectively. The value of b_{min} and ρ_{min} increase as we increase the value of the coupling constants. The shape of the potential well gets shallower as the coupling increases. We set $\alpha_g = \alpha_\phi$ for simplicity.

The total Casimir energy density can be rewritten as

$$\rho = N_b\rho^{massless}_{boson}(\tilde{\alpha}_g) + N_f\rho^{massless}_{fermion}(\alpha_\psi) + \tilde{N}_b\rho^{massive}_{boson}(\alpha_\phi) + \tilde{N}_f\rho^{massive}_{fermion}(\alpha_\psi), \tag{5.59}$$

where the numbers of degrees of freedom can be chosen in order to get the existence of the local minimum as we have done in the usual case. Note that the energy densities for each field depend on the coupling constant. Thus we have to specify this coupling by hand or neglect all of them for non-interacting case. In order to obtain how the interactions affect the Casimir energy density, we plot the energy density with various coupling constants as shown in Figure 5.5. As we have mentioned, the energy density of scalar fields modify as $\rho_{Cas(eff)} = (1+\alpha_\phi^2)^2\rho_{Cas}$. This modification yields the result that ρ_{min} increases when the coupling constant increases. In the same strategy, mass of scalar field will be modified as $m_{s(eff)} = (1+\alpha_\phi^2)^{-1/2} m_s$. Thus the mass will be decreased as increasing the coupling constant and yields increasing of b_{min} as shown in Figure 5.5.

5.2.3 Dynamics of CDE with æther fields

The dynamical behavior of the universe is analyzed by using the Einstein field equation. In order to take into account the effects of the æther field, we add the contributions from energy momentum tensor of the æther field and the effective gravity coupling in equation (5.46) into the Einstein field equation. Thus the total energy momentum tensor will be expressed as

$$T_{ab(total)} = T_{ab(Cas)} + T_{ab(A)} + T_{ab(g)} + T_{ab(m)}. \tag{5.60}$$

The energy momentum tensor of the æther field with Maxwell-kinetic term can be expressed as

$$T_{ab(A)} = V_{ac}V_b^c - \frac{1}{4}V_{cd}V^{cd}g_{ab} + v^{-2}A_aA_bA_c\nabla_dV^{cd}. \tag{5.61}$$

For the metric in equation (5.1), the solution of the æther field can be written as

$$A^a = \Big(0, 0, 0, 0, \frac{v}{b(t)}\Big). \tag{5.62}$$

Substituting this solution into equation (5.61), one obtains the non-zero components of the æther energy momentum tensor

$$T^0{}_{0(A)} = -\frac{v^2}{2}H_b^2,\ \ T^i{}_{j(A)} = \frac{v^2}{2}H_b^2\,\delta^i{}_j,\ \ T^5{}_{5(A)} = -\frac{\ddot{b}}{b} + \frac{1}{2}H_b^2 - 3H_aH_b. \tag{5.63}$$

From this expression, one can see that the energy density of the æther field is proportional to the time-derivative of b, $\rho_A \propto \dot{b}$. We expect that this contribution will slow down the radion field at the matter-dominated period. In other words, it slows down the radion field before the local minimum of the potential will exist. By adding all contents, the dynamical equations in (5.23)-(5.25) can be rewritten as

$$\begin{aligned}
3H_a^2 + 3H_aH_b &= M_*^{-3}(\rho_{Cas} + \rho_m + \frac{1}{2}v^2H_b^2), &(5.64)\\
3\frac{\ddot{a}}{a} - 3H_aH_b &= -M_*^{-3}\Big(\rho_{Cas} + \rho_m + p_b - (1-2\alpha_g)v^2D\Big), &(5.65)\\
3\frac{\ddot{b}}{b} + 9H_aH_b &= M_*^{-3}\Big(\rho_{Cas} + \rho_m + 2p_b - 3p_a - 2(1-2\alpha_g)v^2D\Big) &(5.66)
\end{aligned}$$

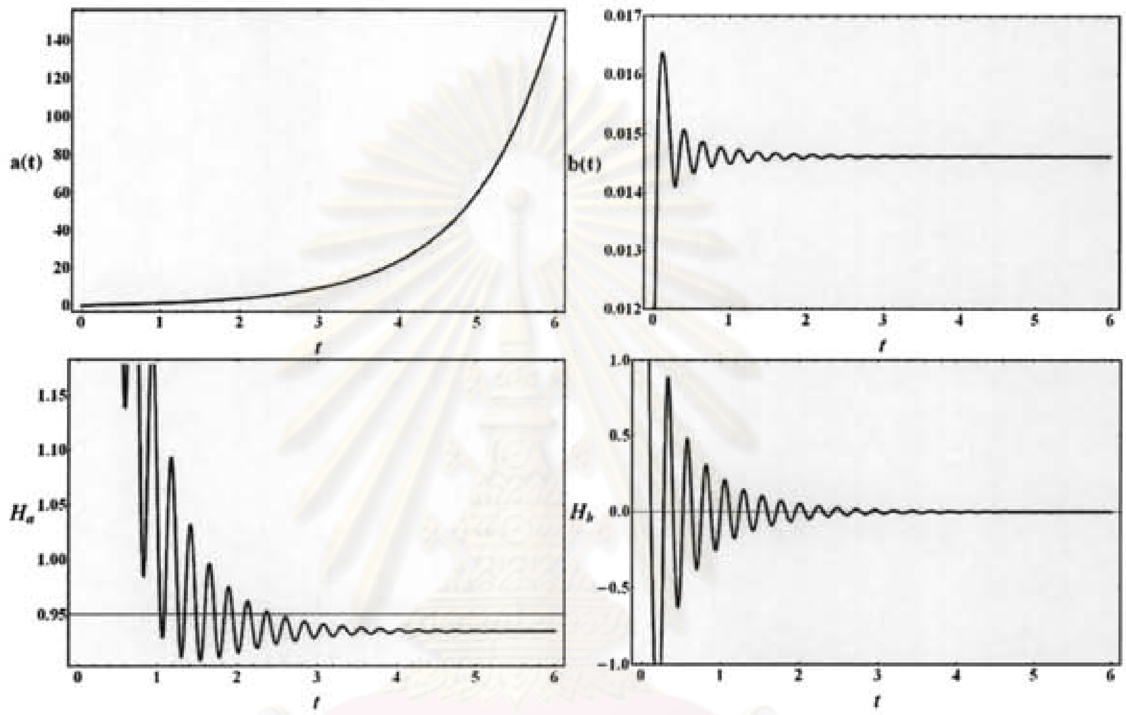

Figure 5.6: These graphs illustrate cosmological dynamics of the universe which includes non-relativistic matter content and the æther field in the extra dimension. Left: The scale factor a (upper) and the Hubble constant for the non-compact dimensions H_a (lower) as the function of time. H_a oscillates as the deceleration period of the universe and it settles down to the constant value when the universe enters a de Sitter phase. Right: The scale factor b (upper) and the Hubble constant for the compact extra dimension H_b (lower) as the function of time. H_b oscillates between positive and negative region before settles down to zero. The extra dimension is stabilized although non-relativistic matter is present.

where $D = (\frac{\ddot{b}}{b} + 3H_a H_b)$.

We will follow the same step to characterize the evolution of the universe. The numerical simulation of the above coupling equations is illustrated in Figure 5.6. From this simulation, it is found that the extra dimension can be stabilized while the scale factor is accelerated. The Hubble parameters of both extra direction, H_b, and non-compact directions, H_a, are also shown in this figure. Note that we ignore all interactions in this simulation. In order to figure out the role of the æther field explicitly, we also simulate the evolution of the radius, b, with various values of the æther vev. This behavior is shown in Figure 5.7. Note that we exclude the matter content in this simulation to show the role of the æther field explicitly. From this figure, it is clear that the contribution from the æther field can slow down the radion field.

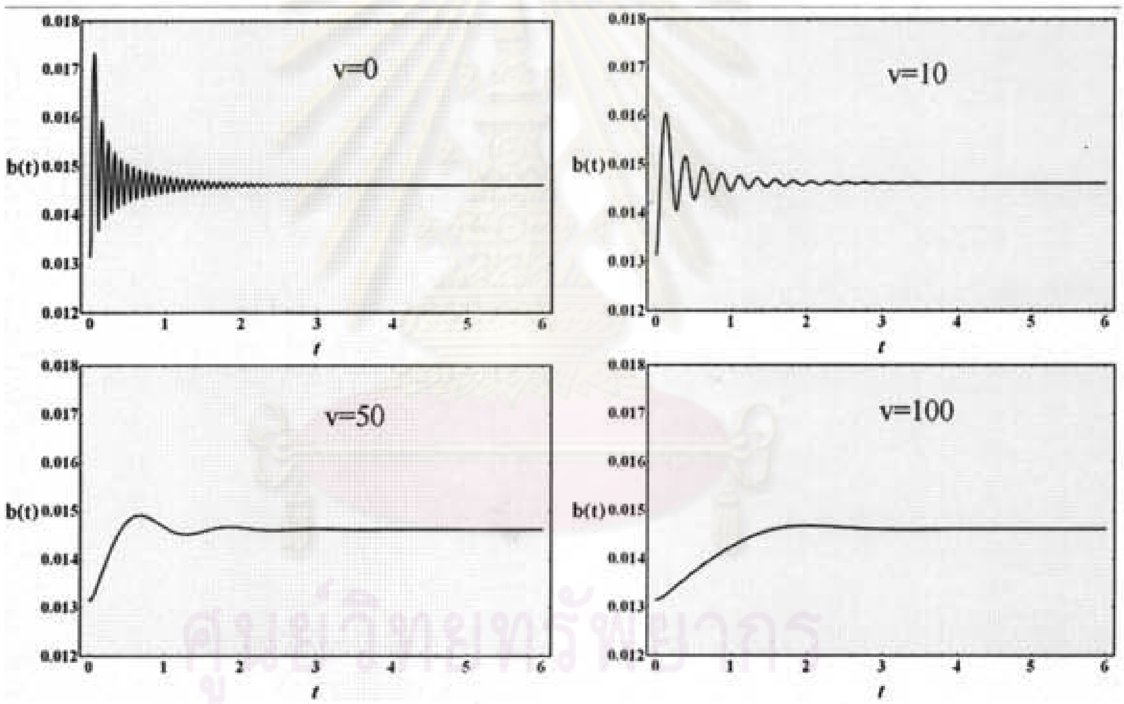

Figure 5.7: The dynamics of the radius $b(t)$ for the compact direction as a function of time with different values of parameter v. In the absence of æther field $v = 0$, $b(t)$ shows oscillation behavior around its critical value b_{min} before stabilizing at this value. Non-vanish value of v reduces the influence of Casimir force. As the value of v increases, the oscillation frequency and amplitude decrease. If the vev of the æther field is large enough, for example $v = 100$, oscillation behavior disappears. The extra dimension evolves smoothly to its stable fixed point. The time variable t is presented in the unit of Hubble time t_H. The time for stabilization to occur is around $\sim 6t_H$. The condition for stabilization of b is $\frac{\delta b}{b} \leqslant 10^{-5}$.

In the qualitative analysis, we consider the dynamics of the system by using the 4-dimensional effective field theory. For this proposal, we will investigate the system in the radion picture by considering the 5-dimensional action with the æther fields,

$$S_{5D} = \int d^5x\sqrt{-g}\Big(\frac{M_*^3}{2}R - \frac{1}{4}V_{ab}V^{ab} - V(b)\Big). \tag{5.67}$$

To see the role of the æther field explicitly, we omit the Lagrange multiplier, matter and gravity coupling terms in this consideration. Following in the same step, the 4-dimensional action is obtained by using the KK-dimensional reduction. Therefore, the 4-dimensional action can be written as

$$S_{4D} = \int d^4x\sqrt{-g_E}\Big\{\frac{M_{pl}^2}{2}R_E - \frac{1}{2}(1+\alpha^2)g_E^{\mu\nu}\nabla_\mu\Psi\nabla_\nu\Psi - U(\Psi)\Big\}, \tag{5.68}$$

where we define the dimensionless parameter $\alpha^2 = \frac{2v^2}{3M_*^3}$. This action gives rise to the following set of equations:

$$H_E^2 = \frac{1}{3M_{pl}^2}\Big(U(\Psi) + \frac{1}{2}(1+\alpha^2)\left(\frac{d\Psi}{dt_E}\right)^2\Big), \tag{5.69}$$

$$\frac{d^2\Psi}{dt_E^2} + 3H_E\frac{d\Psi}{dt_E} = -\frac{1}{(1+\alpha^2)}\frac{\partial U}{\partial\Psi}. \tag{5.70}$$

The æther factor $1/(1+\alpha^2)$ on the right-hand side of equation (5.70) reduces the influence of the potential gradient $-\partial U/\partial\Psi$. In other words, it reduces the force acting on the oscillator. As a consequence, it will slow down the oscillation frequency of Ψ around the minimum of the potential $U(\Psi)$. If this factor is big enough, Ψ will move down the potential at very slow speed since the friction term dominates. We can tune v such that there is enough time for the universe to create the minimum of $U_{eff}(\Psi)$ before the radion rolls pass it. By this mechanism, the stability of the extra dimension can restore.

Let us compare the stabilization time t_{stab} of the moduli field with the age of the universe. The age of the universe in our model is

$$t_{age} = \frac{1}{H_{a0}}\int_0^1 \frac{dx}{x\sqrt{\Omega_{Casimir} + \Omega_m x^3}} = \frac{1.5376}{H_{a0}} \approx 1.5376t_H, \tag{5.71}$$

where we set $\Omega_{Casimir} = 0.72$ and $\Omega_m = 0.28$. From Figure 5.6, the stabilization time $t_{stab} \approx 6t_H$. Then, $t_{stab} \approx 3.90t_{age}$ is greater than the age of the universe. The constancy of the 4-dimensional gravitational constant up to very early epoch of the universe will post strong constraint on the size of the extra dimension. The oscillation behavior of the moduli field may contradict with astronomical observations. In order to construct a more realistic cosmological model of this scenario, the extra dimension should reach its stable fixed point before the present time, i.e, $t_{stab} \lesssim t_{age}$ which require fine tuning of many parameters. The new possible solution is that we assume very high value of v so that the oscillation of b has a very long period. The moduli will evolve smoothly with no oscillating behavior. We can choose the value of v such that the size of the extra dimension changes so slowly and it cannot alter the results of the standard evolution of the universe [108].

In this analysis we assume homogeneous and isotropic distribution of non-relativistic matter. However, local matter distributions might perturb the radion and knock it over the minimum, causing the (local) catastrophic expansion of the fifth dimension. In [107], it was also noted that the minimum of the potential well is generally not deep enough to prevent the quantum tunneling of the radion. At this stage, it is not clear whether these two difficulties can be solved by the new mechanism. These aspects of instability in the presence of the æther are still open questions.

We note that this is only a toy model since we restrict our attention on 5-dimensional spacetime. It is useful to extend the consideration to the 6-dimensional spacetime because it is possible to solve the hierarchy problem. Moreover, in $\mathbb{M}^{1+3} \times \mathbb{T}^2$ spacetime, there exists the true minimum in the shape moduli direction and the usual local minimum in the 5-dimensional spacetime becomes the saddle point [112].

Chapter VI

DISCUSSIONS AND SUMMARIES

In this thesis, we briefly reviewed the theories of Lorentz violation by following [3]. Such theories are classified into kinematic and dynamical approaches. The crucial idea and the consequent effects of the interesting models in Lorentz violation theories, including modified dispersion relation, doubly special relativity, standard model extension and æther models, are discussed. The æther models are examined in detail since they are directly relevant to the cosmological models. In order to realize the æther models, the consistency of the theoretical model and the constraints from the observational data are considered. It is found that the time-like æther models are viable in some ranges of the æther parameters [45]. For the space-like æther models, it is not intensively investigated and the conclusive argument is only that the æther models with Maxwell-like kinetic term are not stable [40, 41]. For the other types of the Lagrangian, the conclusive arguments for the instability issue have not been investigated yet. This issue is very interesting to be investigated in further works.

The cosmological models are also reviewed in this thesis in order to be the basic knowledge for investigating the cosmological effects of the æther field. According to the standard evolution of the universe, there are two periods that the universe accelerately expands, the very early and the present eras. The successful theoretical models that can properly explain the accelerating universe at the early era are inflationary models. The simple dynamics of the inflationary models is provided by the slow-rolling of a scalar field on the flat potential. The crucial characteristic of the inflationary models is that the scale-invariant primordial power spectra are obtained naturally. These power spectra correspond to the constant amplitude of the perturbations. These perturbations are the initial cosmological perturbations to seed of the structure we observe nowadays. Since inflationary models take place at the very early time, they intend to encode the signal of the quantum gravity theories. It is useful to consider the æther effects in these models and we discuss this topic in Chapter IV.

The late-time acceleration of the universe is also reviewed in this thesis. Dark energy is adopted as an explanation for the late-time acceleration. We discuss the simple and useful models of dark energy in both fluid and gravitational approach. Although dark energy models share the same behavior with inflationary models, providing the accelerating universe, the crucial characteristics are different. In the inflationary models, it needs to end the accelerating expansion of the universe but it does not need for the late-time acceleration. Moreover, the dark energy must track the standard evolution of the universe in which there are the matter and radiation dominant periods before the present time. It is not easy to naturally obtain this behavior in dark energy models. The very simple and useful model of dark energy is cosmological constant model. It properly fits to the observational data but it encounters the cosmological constant problem and coincidence problem. The simple single scalar field dark energy models called quintessence models are proposed to eliminate these problems but they do not naturally provide the standard evolution of the universe. Alternative ways to obtain the late-time acceleration come from the modified gravity approach. However, the viable theoretical models do not provide the natural description of the local gravity. Phenomenologically, it is interesting to add the effects of the æther field into the dark energy models.

In Chapter IV, the effects of the time-like æther field on inflationary models are discussed. The inflationary models including the time-like vector field are intensively investigated due to the fact that the observations suggest the isotropic and homogeneous universe in spatial dimensions. The effects of the æther field on inflationary model are investigated in two approaches: interacting and non-interacting approaches. We briefly summarize the results of the interacting approach by following [115, 117, 118]. It is found that the æther field can affect the dynamics of the inflaton field. The inflaton potential is not necessarily flat in some forms of the interaction terms. Moreover, the inflation can be reached even without inflaton potentials. We consider the non-interacting approach in details by following [28, 114, 116, 113]. The results of all investigations are slightly different. The main content of that chapter follows the careful interpretations and analysis in [113]. It is found that the background evolution of the universe is modified in such a way that the slow-roll parameters can be controlled by the æther parameters. The primordial power spectra of the adiabatic scalar modes and tensor modes are slightly modified. The significant effects of the æther field are placed on the existence of the isocurvature perturbations and the vector modes of the perturbations. The phenomenological constraints on the æther parameters are obtained in order to suppress and exclude the effects that the observation data does not suggest. The constraints of æther parameters are summarized in Table IV.1.

The effects of the space-like æther field are not intensively investigated due to the violation of rotational invariance. However, it is investigated as a toy model in order to figure out the signals of the statistical anisotropy in CMB power spectrum [38]. The kinetic term of this model contains only the Maxwell term and the accelerating universe is driven by the cosmological constant. However, the æther field in this model encounters the theoretical inconsistency such as instabilities [31, 39, 40, 41].

In order to avoid the rotational symmetry violation in three spatial dimensions, one can introduce the extra dimensions and sets the norm of the æther field in such extra dimensions. The theories of the extra dimensions are motivated from one of the candidates of quantum gravity theories such as string theory. Thus it is convenient to consider the effects of the æther field in the theoretical models with extra dimensions. One of the dark energy models with the extra dimensions is the Casimir dark energy models [107]. The accelerating universe is driven by cosmological constant which is interpreted as Casimir energy. The Casimir energy naturally emerges from this system since the extra dimensions are compactified with some boundary conditions. This interpretation of these models can solve the cosmological constant problem automatically. Moreover, it also provides the mechanism to stabilize the extra dimensions. However, the extra dimensions will be destabilized when the non-relativistic matter is taken into account.

In Chapter V, we add the æther field into the Casimir dark energy models in order to provide the stabilization mechanism of the extra dimension [108]. It is found that the æther field can play the role of the friction and slow down the radion field before it passes the minimum of the potential. In other words, The extra dimension can be stabilized when we take into account the effects of the æther field. In this investigation, we consider only the Casimir dark energy model in 5-dimensional spacetime. It is more useful if the 6-dimensional spacetime are examined since it is possible to solve the hierarchy problem. The observational constraints of the æther parameters on this model is very interesting since it lacks of investigations. We leave these interesting topics to further works.

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