CHAPTER III

TYPE I AND TYPE II SUPERCONDUCTORS

Type I Superconductors

In general high temperatures and high magnetic fields destroy superconductivity. If we fix the temperature to a value less than the zero field T_c (0) then we have the following relationship between the internal and external fields as shown in fig 3.1.

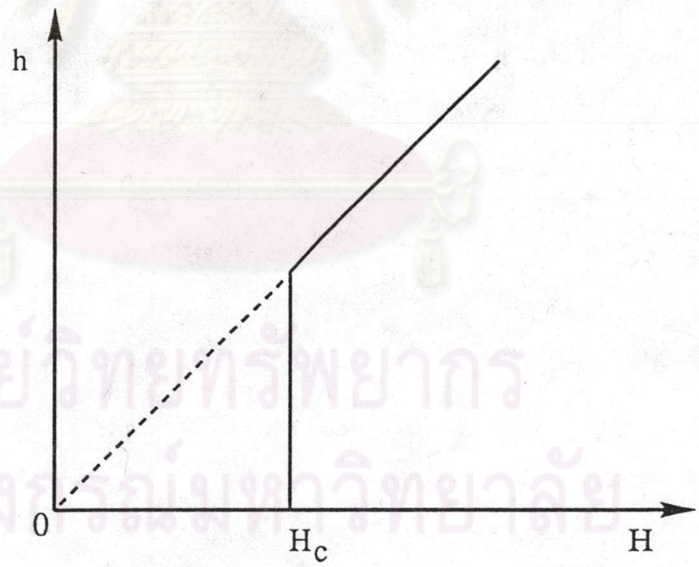

Fig 3.1 The relationship between the external field(H) and the internal field(h) in a type I superconductor.

For $H > H_c$ we have the the normal state which is assumed to be non-magnetic (internal magnetic field $\boldsymbol{h} = \boldsymbol{H}$, where H is the external magnetic field). For $H < H_c$ we have the Meissner effect ($h = 0$). If the sample is superconducting at temperature T in zero field, there is a unique critical field $H_c(T)$ above which the sample becomes normal. This transition is reversible, for superconductivity reappears as soon as H is reduced below $H_c(T)$. Experiments (Fetter and Walecka, 1971) on pure superconductors show that the curve $H_c(T)$ is roughly parabolic as shown in fig 3.2 and according to the empirical formula

$$H_c(T) = H_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right] \qquad (3.1)$$

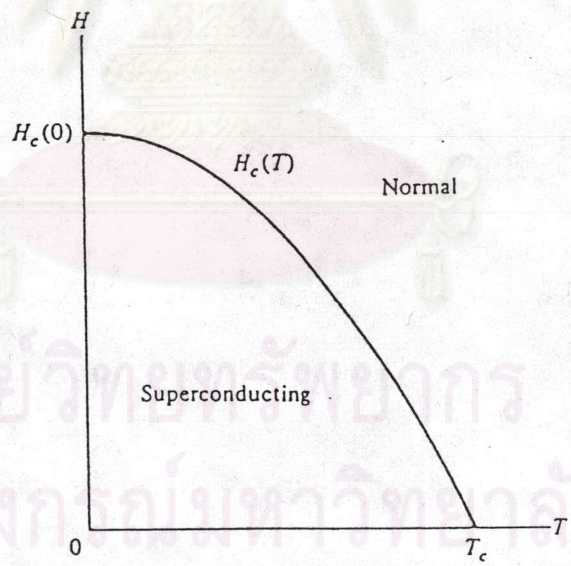

Fig. 3.2 Phase diagram in the H-T plane, showing superconducting and normal regions, and the critical curve $H_c(T)$ or $T_c(H)$ between them (Tinkham, 1975).

From eq.(2.28) for the supercurrent , we have

$$J = \frac{c}{4\pi}\nabla\times h = -\frac{e^{*2}}{m^{*}c}\left(\frac{\hbar c}{e^{*}}\nabla\phi + A\right)|\Psi|^{2}$$

For a uniform external field H, and $h = H$, we have

$$\left(\frac{\hbar c}{e^{*}}\nabla\phi + A\right)|\Psi|^{2} = 0 \tag{3.2}$$

We cannot have

$$A = -\frac{\hbar c}{e^{*}}\nabla\phi$$

since $\nabla\times\nabla\phi = 0$ and $\nabla\times A = h = H \neq 0$. It follows that $\Psi = 0$ (normal phase).

On the other hand, if we set $h = 0$ in the equation for J, then we can have a superconducting phase. In that case, $A = -\frac{\hbar c}{e^{*}}\nabla\phi$ and $|\Psi|^{2} \neq 0$. The free energy density is

$$g_s(H) = g_n(H) + a|\Psi|^{2} + \frac{b}{2}|\Psi|^{4} + \frac{1}{2m^{*}}\left|\left(-i\hbar\nabla + \frac{e^{*}A}{c}\right)\Psi\right|^{2} \tag{3.3}$$

where $g_n(H) = f_n + \frac{(h-H)^{2}}{8\pi}$ is the free energy density for the normal phase ($\Psi = 0$). The two phases are in thermodynamic equilibrium at the critical field H_c. This condition may be expressed by the equation

$$g_s(H_c) = g_n(H_c) \tag{3.4}$$

The normal state of most superconducting elements is nonmagnetic (**h** $=$ **H**), so we find

$$g_n(H) = g_n(0) \tag{3.5}$$

In contrast, **h** vanishes in the superconducting phase (**h** $= 0$), which yields

$$g_s(H) = g_s(0) + \frac{H^2}{8\pi} \tag{3.6}$$

The Gibbs free energy is constant in the superconducting state. In particular

$$g_s(0) + \frac{H_c^2}{8\pi} = g_s(H_c) = g_n(H_c) = g_n(0) \tag{3.7}$$

However, for an infinite sample with a uniform Ginzburg-Landau order parameter, we saw in chapter II that $\Psi = \sqrt{(|a|/b)}$. Therefore,

$$g_s(0) = g_n(0) - \frac{a^2}{2b} \tag{3.8}$$

Thus we find that

$$H_c{}^2 = \frac{4\pi a^2}{b} \; ; \; H_c = |a|\sqrt{\frac{4\pi}{b}} \tag{3.9}$$

Also,

$$g_s(H) = g_s(0) + \frac{H^2}{8\pi}$$

$$= g_n(0) + \frac{H^2}{8\pi} - \frac{H_c^2}{8\pi}$$

$$= g_n(H) + \frac{H^2 - H_c^2}{8\pi} \qquad (3.10)$$

Therefore the superconducting phase is stable for $H < H_C$.

Type II Superconductors

Before type II superconductors are described, it is important to emphasize that the coherence length ξ and the penetration depth λ are both phenomenological quantities defined in terms of the constants a and b. Recall that

$$\lambda = \sqrt{\frac{m^* c^2 b}{4\pi e^{*2} |a|}}$$

and

$$\xi = \sqrt{\frac{\hbar^2}{2m^* |a|}}$$

It is conventional to introduce the *Ginzburg-Landau parameter* , κ defined by

$$\kappa = \frac{\lambda}{\xi} = \frac{m^* c}{\hbar e^*} \sqrt{\frac{b}{2\pi}} = \frac{\sqrt{2} e^*}{\hbar c} H_c(T) \lambda^2(T) \qquad (3.11)$$

This is independent of temperature near T_C (Fetter and Walecka, 1971).

In 1957, Abrikosov published a remarkably significant paper (Abrikosov, 1957), in which he investigated what would happen in the Ginzburg-Landau theory if κ were very large ($\lambda >> \xi$). This should lead to a *negative surface energy* (Tinkham, 1975, Rose-Innes and Rhoderick, 1978, de Gennes, 1989), so that the process of subdivision into domains should proceed until it is limited by the microscopic length ξ. Because this behavior is so different from the classic behavior described earlier, he called these "*type II superconductors*". The Ginzburg-Landau parameter characterizes the superconductor according to

$$\begin{array}{|lcl|} \hline \textit{Type I} & \kappa & < \dfrac{1}{\sqrt{2}} \\ \textit{Type II} & \kappa & > \dfrac{1}{\sqrt{2}} \\ \hline \end{array}$$

In fig. 3.3 we show the penetration depth λ and the coherence length ξ of type I and type II superconductors.

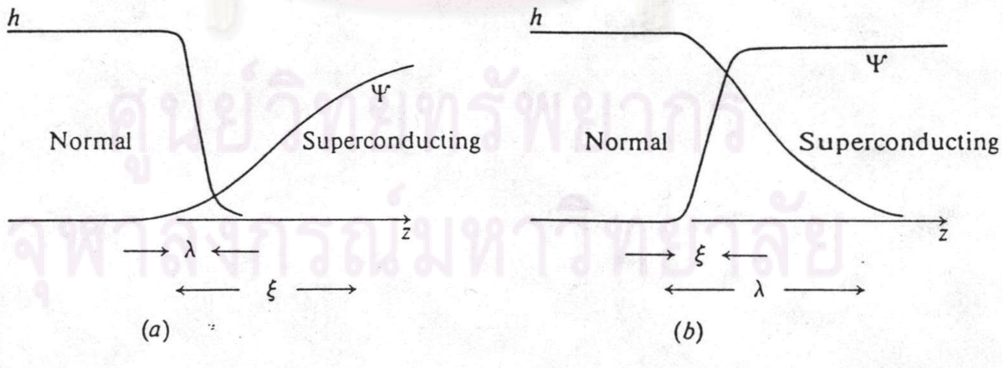

Fig.3.3 Schematic plot of the penetration depth λ and the coherence length ξ of superconductor (a). type I : $\xi > \lambda\sqrt{2}$, (b). type II : $\xi < \lambda\sqrt{2}$ (Fetter and Walecka, 1971)

For type II superconductors the degree of flux penetration is shown in fig. 3.4. For $H > H_{c2}$ the sample is normal ($\boldsymbol{h} = \boldsymbol{H}$), while for $H < H_{c1}$ we have a complete Meissner effect ($\boldsymbol{h} = 0$). In the intermediate state ($H_{c1} < H < H_{c2}$) the sample is in a mixed condition with non-uniform flux penetration, commonly called the *mixed state or vortex state*. H_{c1} and H_{c2} are called respectively the lower and upper critical fields. A brief of calculation H_{c1} and H_{c2} by the Ginzburg-Landau theory will be given latter (Tinkham, 1975., de Gennes, 1989, more elegant mathematical details were described carefully by J. Poulter "*The Lecture Notes on Ginzburg-Landau Theory*", 1991).

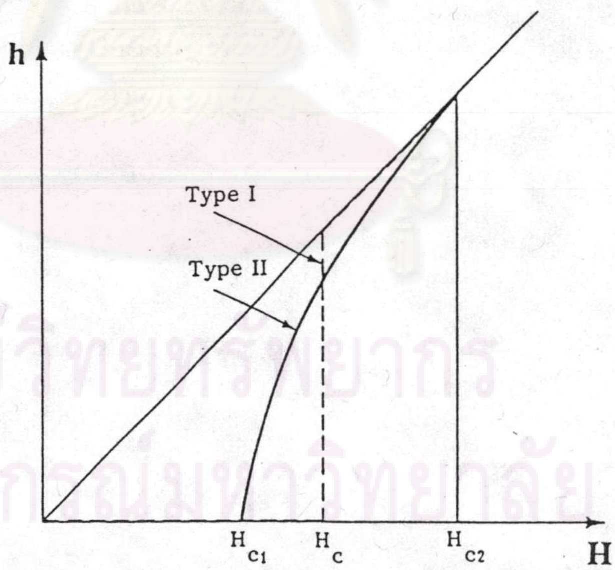

Fig. 3.4 The internal magnetic field (h) versus the external magnetic field (H) for type I (dashed line) and type II (solid line) superconductors (de Gennes, 1989).

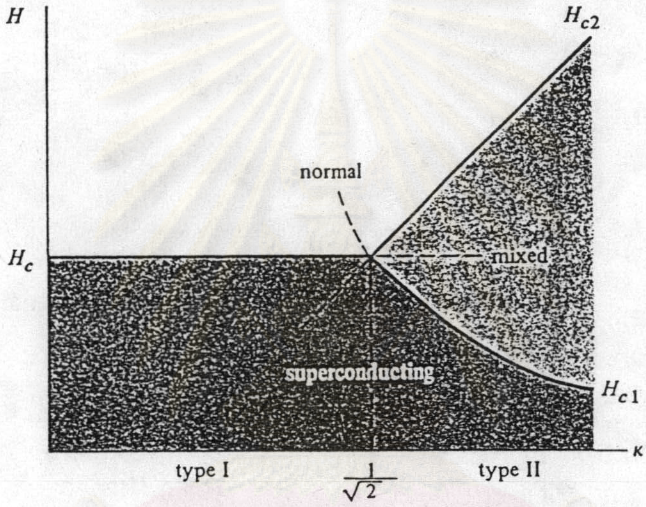

Fig. 3.5 The three phases of a superconductor for a given Ginzburg-Landau parameter κ and external magnetic field (H). (Taylor, 1970).

In fig. 3.5, we show the three phases of a superconductor which have the lowest free energy for a given Ginzburg-Landau parameter κ and applied magnetic field H.

Mixed State

In the previous section we have mentioned the formation of the mixed state for certain superconductors, due to the negative surface energy between two different regions. The configuration of the normal regions threading the superconducting material should be such that the ratio of surface to volume of normal material is a maximum (Rose-Innes and Rhoderick, 1978). It turns out that a favorable configuration is one in which the superconductor is threaded by cylinders of normal material lying parallel to the applied magnetic field. We shall refer to these cylinders as *normal cores* . These cores arrange themselves in a regular pattern, in fact a *triangular close-packed lattice* (Kleiner , *et al*., 1964).

We might expect the normal cores to have a very small radius, because the smaller the radius of the cylinders, the larger the ratio of its surface area to its volume. The bulk of the material is diamagnetic, the flux due to the applied field being opposed by a diamagnetic surface current which circulates around the perimeter of the specimen. This diamagnetic material is threaded by normal cores lying parallel to the applied magnetic field, and within each core is a magnetic field having the same direction as that of the applied magnetic field. The flux within each core is generated by a vortex of persistent current that circulates around the core with a sense of rotation opposite to that of the diamagnetic surface current. The pattern of currents and the resulting flux are illustrated in Fig. 3.6.

The vortex current encircling a normal core interacts with the magnetic field produced by the vortex current encircling any other and, as a result, any two cores repel each other. This is somewhat similar to the repulsion between parallel solenoids or bar magnets. Because of this mutual interaction the flux lines do not lie at random but arrange themselves into a regular periodic hexagonal array as shown in fig.3.7. This array is usually known as the *fluxon lattice* or *Abrikosov lattice* (Kleiner, *et al.*, 1964). The existence of the normal cores and their arrangement in a periodic lattice has been revealed by experiment. The decoration technique of Essman and Träuble (Träuble and Essmann, 1968) reveals the pattern of the normal cores by allowing very small (500 Å) ferromagnetic particles to settle on the surface of a type-II superconductor in the mixed state. The particles locates themselves where the magnetic flux is strongest, i.e. where the normal core intersects the surface, as shown in fig. 3.8.

Details of the mixed state

Imagine a type II material: In a low applied magnetic field $\boldsymbol{H}$, the sample exhibits a complete Meissner effect, and the magnetic induction $\boldsymbol{B}$ vanishes ($\boldsymbol{B} = \int \mathbf{h}(\mathbf{r})d\mathbf{r}$), as H is increased to a critical value H_{c1}. The penetration of magnetic flux becomes favorable, and the quantized flux lines (vortices) are formed. These filaments consist of a core region which is not sharply defined, but is spread out over an area with approximate radius equal to ξ (the coherence length) as shown in fig.3.6.

Further, the magnetic flux associated with each core spreads into the surrounding material over a distance approximately equal to the penetration depth λ. The properties of the material vary with position in a periodic manner. At the center of each vortex the order parameter (concentration of superelectrons) falls to zero, so that along the center of each vortex is a very thin core (strictly a line) of normal material.

The dips in the order parameter are about two coherence lengths wide. The flux density due to the applied magnetic field is not cancelled in the normal cores and falls to a small value over a distance about λ away from the core as shown in fig.3.7. The total flux generated at each core by the encircling current vortex is just one quantum of magnetic flux (Kittel, 1988).

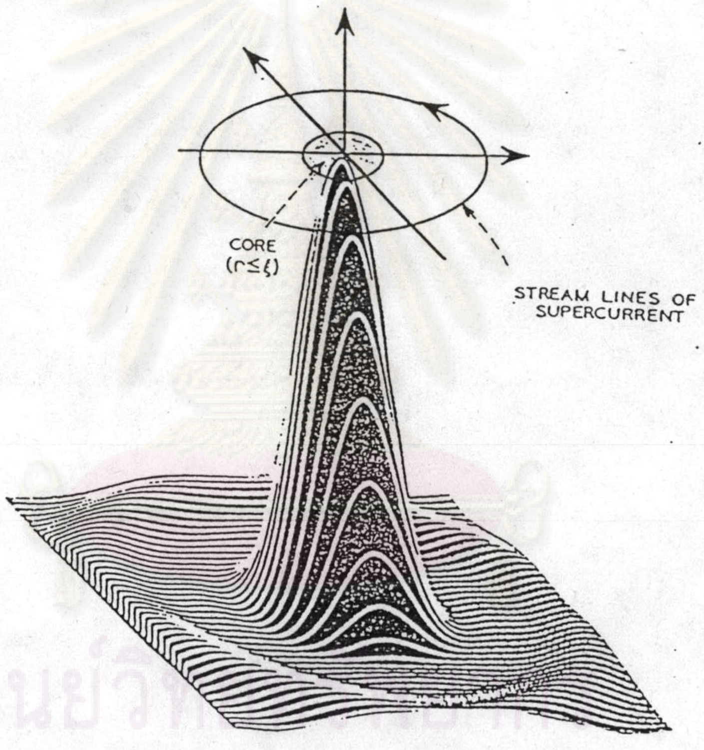

Fig. 3.6 An isolated flux line, carrying a single flux quantum, consisting of a cylindrical core whose radius is equal to the coherence length ξ of the material. A current circulates around the core out to the penetration depth λ.

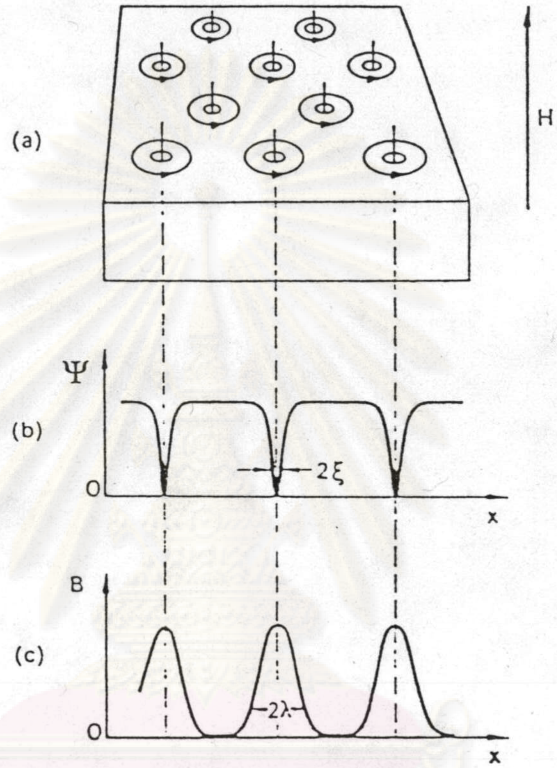

Fig 3.7 The mixed state in an applied magnetic field H just greater than H_{c1}. (a) Lattice of cores and associated vortices were arranged in a triangular pattern. (b) Variation of the order parameter Ψ with position. (c) Variation of internal magnetic field **h** with position (Rose-Innes and Rhoderick, 1978).

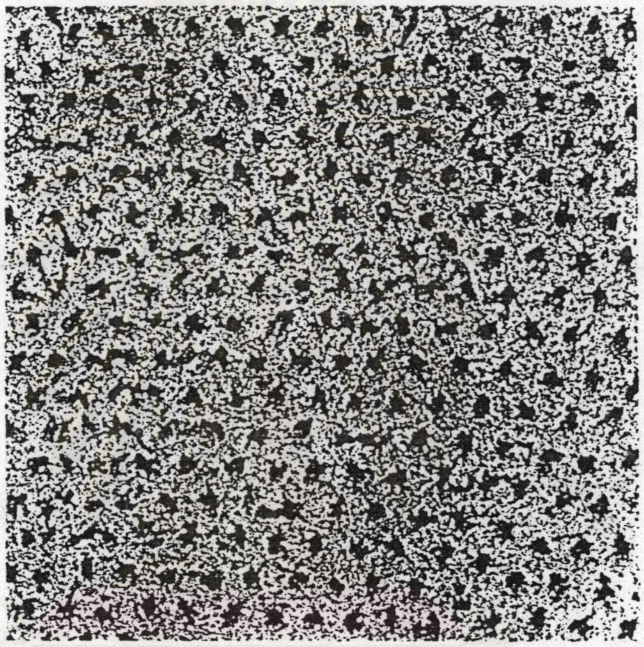

Fig 3.8 Triangular lattice of vortices containing dislocations as seen using an electron microscope and a replication technique. (Träuble and Essmann, 1968).

The Lower Critical Field H_{c1}

At the lower critical field H_{c1}, the transition to the mixed state occurs with the entry of the first few vortices (Duzer and Turner, 1981). The separation of these vortices is much greater than λ for fields near H_{c1}, making it possible to calculate the energy of an isolated vortex by neglecting the small contribution from interaction with other vortices. If the flux penetration were uniform then we would have $\Psi = 0$ everywhere. So we consider a non-uniform local flux penetration. Keeping as much symmetry as possible we choose a flux line $\mathbf{h} = h(r)\,\hat{\mathbf{z}}$. The function h(r) has its maximum value at r = 0 and tends to zero for large r. A suitable choice of vector potential is $\mathbf{A} = A(r)\,\hat{\theta}$. So $\mathbf{h} = \nabla\times\mathbf{A} = \hat{z}\frac{1}{r}\frac{d}{dr}(r\,A(r))$. For $\kappa >> 1$ we have $\lambda >> \xi$ and the order parameter Ψ is essentially equal to unity except for a core region of radius ξ. Then we will have (J. Poulter, 1991)

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dh(r)}{dr}\right) = \frac{1}{\lambda^2}h(r) \tag{3.12}$$

The solution of this equation is the modified Bessel function $h \sim K_0(r/\lambda)$. By definition, when $H = H_{c1}$ the Gibbs free energy must have the same value whether the first vortex is in or out of the sample. Thus, at H_{c1} we have (Tinkham, 1975)

$$G_s|_{\text{no flux}} = G_s|_{\text{first vortex}} \tag{3.13}$$

and also we have (J. Poulter, 1991)

$$\varepsilon_1 = \frac{1}{4}\int_0^\infty r\,dr\left[\, h^2(r) + \lambda^2\left(\frac{xh(r)}{dr}\right)^2 \right]$$

$$= \frac{H_{c1}}{2}\int_0^\infty r\,dr\,h(r) = \frac{H_{c1}}{2}\frac{\hbar c}{e^*} \tag{3.14}$$

where ε_1 is the free energy per unit of length (or line tension), and Φ_0 is a flux quantum ($\Phi_0 = 2\pi\hbar c/e^*$).

Thus

$$H_{c1} = \frac{4\pi\varepsilon_1}{\Phi_0} \tag{3.15}$$

The calculation of Ψ, h, and ε_1 for arbitrary κ unfortunately requires numerical solution of the Ginzburg-Landau equations. Thus, considerable attention has been given to the extreme type II limit,
in which $\kappa = \lambda/\xi >> 1$.

We will treat only materials with $\kappa >> 1$, so that the core is very small. We have found that (Tinkham, 1975)

$$\varepsilon_1 \approx \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \ell n\,\kappa \tag{3.16}$$

Now that we have evaluated the line tension ε_1, we can substitute back into (3.15) to get H_{c1}. The field at which flux first penetrates (for $\kappa >> 1$) is

$$\boxed{H_{c1} = \frac{4\pi\varepsilon_1}{\Phi_0} \approx \frac{\Phi_0}{4\pi\lambda^2}\,\ell n\,\kappa = \frac{1}{2}\frac{\hbar c}{e^*}\frac{1}{\lambda^2}\,\ell n\,\kappa} \tag{3.17}$$

The Upper critical Field H_{c2}

Near H_{c2}, the vortices are packed so tightly that the cores fill much of the volume and the vortices form a lattice so that the energy has the minimum value.

For H only just below H_{c2} the Ginzburg - Landau order parameter Ψ will be very small. From eq.(2.16) if we neglect all the terms of order Ψ^3 or smaller, then we have

$$\frac{1}{2m^*}\left(-i\hbar\nabla + \frac{e^*\boldsymbol{A}_0}{c}\right)^2\Psi + a\Psi = 0 \qquad (3.18)$$

This is the linearized Ginzburg-Landau equation. where the external field $\boldsymbol{H} = \nabla\times\boldsymbol{A}_0$.

For a particle of mass m* and charge - e* moving in a magnetic field $\boldsymbol{H} = \nabla\times\boldsymbol{A}_0$, the Hamiltonian is

$$\hat{H} = \frac{1}{2m^*}\left(-i\hbar\nabla + \frac{e^*\boldsymbol{A}_0}{c}\right)^2 \qquad (3.19)$$

If $\boldsymbol{H} = H\,\hat{z}$, then the eigenvalues of this Hamiltonian are known to be (Landau and Lifshitz, 1977),

$$E = \left(n + \frac{1}{2}\right)\hbar\omega + \frac{p_z^2}{2m^*} \qquad (3.20)$$

where

$$\omega = \frac{e^*H}{m^*c} \qquad (3.21)$$

The energy levels corresponding to motion perpendicular to the magnetic field are just the Landau levels (Landau and Lifshitz, 1977). For the case of the linearized Ginzburg-Landau equation

$$|a| = \left(n + \frac{1}{2}\right)\hbar\frac{e^*H}{m^*c} + \frac{p_z^2}{2m^*} \tag{3.22}$$

and

$$H = \frac{2m^*c}{(2n + 1)\hbar e^*}\left(|a| - \frac{p_z^2}{2m^*}\right) \tag{3.23}$$

In order to determine H_{c2} we must take the largest possible value of H. This means that we must choose $n = p_z = 0$, and

$$H_{c2} = \frac{2m^*c}{\hbar e^*}|a| \tag{3.24}$$

We have from before that

$$\sqrt{2}\kappa H_c = \frac{\sqrt{2}m^*c}{\hbar e^*}\sqrt{\frac{b}{2\pi}}\sqrt{\frac{4\pi}{b}}|a|$$

$$= \frac{2m^*c}{\hbar e^*}|a| \tag{3.25}$$

so that

$$\boxed{H_{c2} = \sqrt{2}\kappa H_c} \tag{3.26}$$

and $H_{c2} > H_c$ only if $\kappa > \frac{1}{\sqrt{2}}$