CHAPTER I

INTRODUCTION

The present work is concerned with subharmonic functions, mainly related to half-spaces in the euclidean (m+1)-dimensional space where $m \geq 1$. We prove some results of the behaviour of the hyperplane means of non-negative subharmonic functions. Then we study the behaviour of functions that preserve harmonicity in the euclidean space. Finally, we use concepts from Functional Analysis to define and study vector-valued subharmonic functions.

In Chapter II, we deal with hyperplane means of non-negative subharmonic functions in the half-space $D := R^m \times (0, +\infty)$. We consider a result of Rippon ([19]) which states that if u is a positive subharmonic function in D, u has a harmonic majorant in D, and $M(u,1) < +\infty$, then either the hyperplane mean

$$M(u,y) := \int_{R^m} u(X,y)dX$$

is decreasing in $[1,+\infty)$ or

$$\int_1^{+\infty} \min[1, (y/M(u,y))^{1/m}]dy < +\infty.$$

We show that if the hypotheses of the above theorem are replaced by the conditions that u is a non-negative subharmonic function in D,

u^p has a harmonic majorant in D for some $p > 1$, the Green's potential G^{μ} is positive, and $M(G^{\mu},y) < +\infty$, then $M(u,y)$ is decreasing and convex in $[1,+\infty)$ or $M(u,y)$ is identical to $+\infty$. We also show that if u is a non-negative subharmonic function in D and $M(u,y)$ is locally bounded in $(0,+\infty)$ then

$$\lim_{y\to+\infty} \frac{M(u,y)}{y^{m+1}} = +\infty.$$

In Chapter III, we deal with a characterization of the functions that preserve harmonicity in the euclidean space. Recently, Nualtaranee ([18]) proved that a function that preserves harmonicity in the plane must be conformal. We generalize this result to a more general case, i.e. in the euclidean space. We prove the following: Let $\Omega \subset R^m$ be a domain, $\varphi:\Omega \to R$ be continuous, and $f:\Omega \to R^m$ be a C^2 function. If $\varphi(h \circ f)$ is harmonic for all harmonic function h on $f(\Omega)$, then f must be conformal at all points $p \in \Omega$ such that $\nabla f_1(p) \neq 0$ where f_1 is the first component of f.

Finally in Chapter IV, we deal with vector-valued subharmonic functions. The motivation of this chapter comes from the fact that there is a concept of vector-valued holomorphic function f from a domain $\Omega \subset \mathbb{C}$ into a Banach space; and this concept is useful for studying spectral theory for linear operators. Then there should be a concept of vector-valued subharmonic functions from a domain $\Omega \subset R^m$ into an extended Banach lattice $\bar{E}$. We begin with an attempt to understand how a sequence of points in E converges to infinity. We define vector-valued semi-continuous functions. Then we define vector-valued subharmonic functions and use the Bochner integral to study its basic properties. Furthermore, we apply these concepts to study the behaviour of the hyperplane means of vector-valued subharmonic functions.

A few words should be said about the terminology employed. For the definition of a symbol, the sign := has been used where it promised to increase readability. We use iff (meaning if and only if) occasionally without being defined. We denote by N, R the sets of integers > 0 and of all real numbers, respectively. We write $|x|$ for the euclidean norm of element $x \in R^m$.