INTRODUCTION

It is well-known in the theory of matrices that for every positive integer n and for every field F, the multiplicative semigroup of all $n \times n$ matrices over F is regular. A theorem which extends this result is given in [1] as follows : For any positive integer n and for any ring R, the multiplicative semigroup of all $n \times n$ matrices over R is regular if and only if R is a regular ring. Also, regular matrix semigroups over a commutative idempotent semiring with 0,1 were studied in [2]. (By a <u>matrix semigroup</u> over an additively commutative semiring S, we mean a semigroup whose elements are matrices over S and operation is the matrix multiplication.) It was proved in [2] that for any positive integer n and for any commutative idempotent semiring S with 0,1, the multiplicative semigroup of all $n \times n$ matrices over S is regular if and only if either $n = 1$ or $n = 2$ and S is a Boolean algebra.

In this research, we continue the study of regular matrix semigroups extensively.

The first chapter gives some preliminaries which are used for this work. In Chapter II, we give a generalization of the result in [2] stated above. Necessary and sufficient conditions of a positive integer n and a Boolean semiring S such that the multiplicative semigroup of all $n \times n$ matrices over S is regular are given in Chapter II. (Boolean semirings are assumed to be commutative and have 0,1.) Semifields are a generalization of fields. In order to generalize the well-known result mentioned above, we give in Chapter III necessary and sufficient conditions of a positive integer n and a semifield S in order that the multiplicative semigroup of all n×n matrices over S is regular.