# CHAPTER VI

# HEAT PIPE HEAT EXCHANGER DESIGN

## 6.1 Theory of Heat Exchanger Design from Performance of Single Heat Pipe

The problem types in general heat exchanger design may be divided into two catagories, that is the rating and the sizing problem. The rating problem predicts the performance of a specified or existing design. The heat transfer rate and effectiveness of the heat exchanger are predicted from the given specifications of the core geometry, the flow rates and the entering fluid temperatures. While the sizing problem is the typical design problem, which aims to fulfill the desired heat transfer rate and exchanger effectiveness dictated by given design conditions, such as the flow rates and the entering and leaving temperatures.

There are two main approaches for predicting the heat transfer performance of the heat exchanger composed of two-phase closed thermosyphons or heat pipes.

The first method is based on the conductance model. It estimates the heat transfer coefficients involved in each step of the transfer process. The rate equation and the principle of energy balance are required in the analysis. The method is applicable to exchangers made of heat pipes as well as two-phase closed thermosyphons (46, 47,).

The second method is based on the development made by Kays and London (48). It uses the effectiveness number of transfer unit as applied to the liquid-coupled indirect-type heat exchanger. Here, the thermosyphons or heat pipes in the heat exchanger core are considered as a pumped fluid loop that couples the hot side and cold side.(49, 50)

### 6.1.1 Analysis Based on the Conductance Model

In this method, the total heat transfer rate of the entire heat exchanger is defined by

$$Q_t = \sum_{p=1}^{n} Q_p = UA_t \, \Delta T_m \tag{6.1}$$

where

$Q_t$ = the total heat transfer rate of the heat exchanger

$Q_p$ = the heat transfer rate of a single heat pipe

$U$ = the overall heat transfer coefficient of the heat exchanger

$A_t$ = the total heat transfer area of the heat exchanger

$\Delta T_m$ = the log-mean temperature difference

For counter-current flow

$$\Delta T_m = \frac{(T_{hi}-T_{co})-(T_{ho}-T_{ci})}{\ln \dfrac{(T_{hi}-T_{co})}{(T_{ho}-T_{ci})}}$$

$T_{hi}$, $T_{ho}$ = the inlet and outlet temperatures of the hot fluid stream, respectively ($^{o}C$)

$T_{ci}$, $T_{co}$ = the inlet and outlet temperatures of the cold fluid stream, respectively ($^{o}C$)

The heat transfer rate of a single heat pipe is calculated using the equations presented in Chapter 3. The film heat transfer coefficients of the outer surface of the pipe are estimated using the following standard correlations for the fluid flow normal to a tube bank or rod bundle.

$$Nu = B.C_z \, Re^m \, Pr^n \tag{6.2}$$

Here B and m are functions of the longitudinal and transverse pitch ratios, $C_z$ is a correction factor for the geometry and the number of rows in the tube bank, and n is about 1/3 (46).

The overall heat transfer coefficient of the heat exchanger based on the total outside area of heat pipes are expressed as follows:

$$\frac{1}{U_o A_{ot}} = \frac{1}{h_{oh} A_{oht}} + \frac{\ln (r_o/r_i)}{2 \pi k L_{ht}} + \frac{1}{h_{ih} A_{iht}} + \frac{1}{h_{ic} A_{ict}} + \frac{\ln (r_o/r_i)}{2 \pi k L_{ct}} + \frac{1}{h_{oc} A_{oct}} \tag{6.3}$$



$$\frac{1}{U_o} = \frac{A_{ot}}{h_{oh} A_{oht}} + \frac{A_{ot} \ln(r_o/r_i)}{2 \pi k L_{ht}} + \frac{A_{ot}}{h_{ih} A_{iht}} + \frac{A_{ot}}{h_{ic} A_{ict}} + \frac{A_{ot} \ln (r_o/r_i)}{2 \pi k L_{ct}} + \frac{A_{ot}}{h_{oc} A_{oct}} \tag{6.4}$$

$U_o$ = the overall heat transfer coefficients of the heat exchanger based on the total outside area of heat pipe (Watt/$m^2$)

$A_{ot}$ = total outside area of heat pipe ($m^2$)

$A_{oht}$, $A_{oct}$ = total outside area of the evaporator and condenser sections of heat pipes, respectively.

$A_{iht}$, $A_{ict}$ = total inside area of the evaporator and condenser sections, respectviely.($m^2$)

$L_{ht}$, $L_{ct}$ = total length of evaporator and condenser sections, respectively (m)

k = thermal conductivity of the container material (Watt/$m^oC$)

6.1.2 Analysis Based on the Effective-Number of Transfer Unit

In this method the heat pipe heat exchanger is considered to be a liquid-coupled indirect type heat exchanger, with the heat pipes providing the coupling.

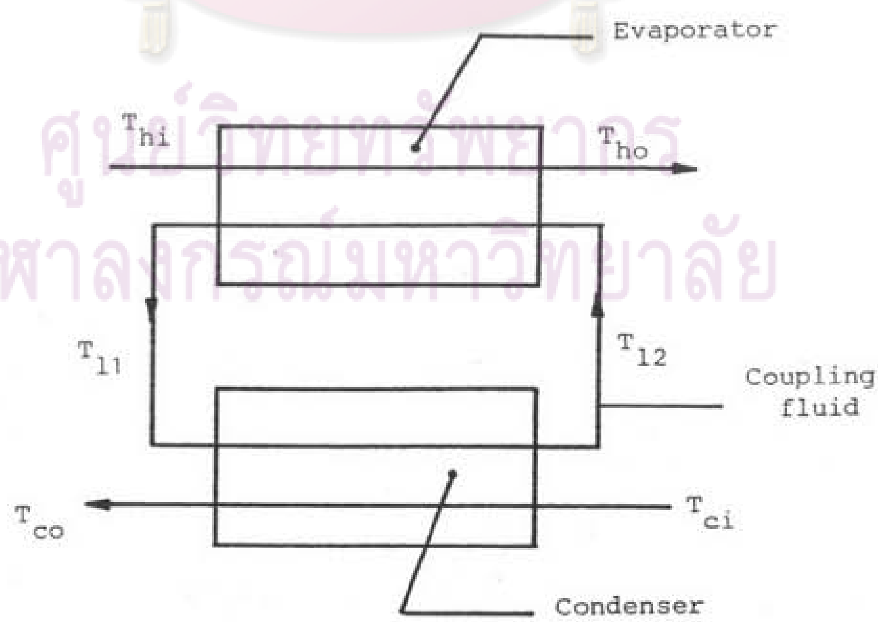



Figure 6.1 Liquid-coupled indirect type heat exchanger model

As shown in Figure 6.1, the hot fluid stream which has the inlet temperature $T_{hi}$, transfers heat to the heat pipe and its outlet temperature becomes $T_{ho}$. The heat causes the working fluid of the heat pipe, which has temperature $T_{12}$, to vaporize at the temperature $T_{11}$. The vapor flows to the condenser section, releases the heat at the temperature $T_{12}$ to the cold fluid stream. The heat released from the condensation of the working fluid causes the temperature of cold fluid stream to increase from $T_{ci}$ to $T_{co}$. In the case of heat-pipe heat exchangers, the vapor phase of the working fluid may be considered to be isothermal, so $T_{11}$ approximately equals $T_{12}$.

The assumptions used in this analysis are as follows:

1. Isothermal condition exists in the vapor phase inside the heat pipes, the temperature being the saturation temperature of the working fluid $T_s$.

2. Axial heat conduction through the heat pipe wall is negligible.

The effectiveness of the heat exchanger E is defined as

$$\text{Effectiveness (E)} = \frac{\text{(actual heat transfer rate)}}{\text{(maximum possible heat transfer rate from one stream to the other)}}$$



$$= \frac{Q}{Q_{max}} \tag{6.5}$$

Since $$Q_{max} = (mC_p)_{min}\,(T_{hi}-T_{ci}) \tag{6.6}$$

$$Q = E\,(mC_p)_{min}\,(T_{hi}-T_{ci}) \tag{6.7}$$

$$E = \frac{Q}{(mC_p)_{min}\ (T_{hi}-T_{ci})} \tag{6.8}$$

$$Q = m_h C_{ph}\ (T_{hi}-T_{ho}) = m_c C_{pc} (T_{co}-T_{ci}) \tag{6.9}$$

Substitution of equation (6.9) into (6.10) yields.

$$E_h = \frac{C_h\ (T_{hi}-T_{ho})}{C_{min}\ (T_{hi}-T_{ci})} \tag{6.10}$$

$$E_c = \frac{C_c\ (T_{co}-T_{ci})}{C_{min}\ (T_{hi}-T_{ci})} \tag{6.11}$$

where $C_h = m_h C_{ph}$ ; $C_c = m_c C_{pc}$ ; $C_{min} = (mC_p)_{min}$

The number of heat transfer units for the heat exchanger is defined as:

$$NTU = \frac{AU_{av}}{C_{min}} \quad \text{for the evaporator}$$

where A is the heat transfer area used in the definition of U.

For one row of heat pipe, the number of transfer unit (NTU) has the value :

$$NTU_h = \frac{U_h A_h}{C_h} \quad \text{for the evaporator}$$

$$NTU_c = \frac{U_c A_c}{C_c} \quad \text{for the condenser}$$

It is possible to have two working conditions, that is, $C_c > C_h$ or $C_h > C_c$.

a) Consider an operation with $C_c > C_h$

The effectiveness of heat transfer for the evaporator is

$$E_h = 1-e^{-NTUh} = (T_{hi}-T_{ho})/(T_{hi}-T_s) \qquad (6.14)$$

Similarly, the effectiveness of the condenser is

$$E_c = 1-e^{-NTUc}$$

$$= (T_{co}-T_{ci})/(T_s-T_{ci}) \qquad (6.15)$$

From equations (6.14) and (6.15), $T_s$ can be written as

$$T_s = T_{hi} - (T_{hi}-T_{ho})/E_h \qquad (6.16)$$

$$= T_{ci} + (T_{co}-T_{ci})/E_c$$



The overall effectiveness is defined as:



$$E = \frac{C_c}{C_h}(T_{co}-T_{ci})/(T_{hi}-T_{ci}) \qquad (6.17)$$

$$= (T_{hi}-T_{ho})/(T_{hi}-T_{ci})$$

From equations (6.16) and (6.17), the overall effectiveness of a single row of heat pipes can be deduced as:

$$E = \frac{1}{\frac{1}{E_h} + \frac{C_h/C_c}{E_c}} \qquad (6.18)$$

b) For the case of $C_h > C_c$

$$E = \frac{1}{E_c + \frac{C_c/C_h}{E_h}} \qquad (6.19)$$

Equations (6.18) and (6.19) can be generalized as:

$$E = \frac{1}{E_{min} + \frac{M}{E_{max}}} \qquad (6.20)$$

where $E_{min}$ and $E_{max}$ are the effectiveness of the fluid stream with minimum and maximum heat capacities, respectively. It is convenient to define



$$M = \frac{C_{min}}{C_{max}}$$

When the actual heat exchanger has 'N' rows of heat pipes as shown in Figure 6.2, the heat pipe heat exchanger can be considered as 'N' heat exchangers in series. The analysis for 'N' heat exchangers in series given by Kays and London is used here. Using the equilibrium line approach, the overall effectiveness for a heat-pipe heat exchanger with 'N' rows and counter-current flow can be calculated as follows:

$$E_N = \frac{\left[\frac{1-ME}{1-E}\right]^N - 1}{\left[\frac{1-ME}{1-E}\right]^N - M} \qquad (6.21)$$

When M = 1

$$E_N = \frac{NE}{1+(N-1)E} \qquad (6.22)$$

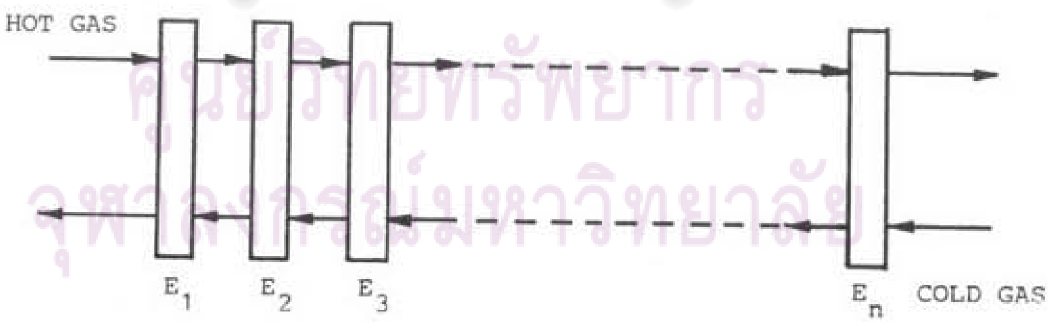



Figure 6.2 Schematic diagram of heat pipe heat exchanger with 'N' rows of pipes.

## 6.2 Design Procedure and Example of Calculations

### 6.2.1 Conditions Used in Heat Exchanger Design

In this section, the details of calculations made to size a heat exchanger will be shown under the following specified conditions.

1. The heat exchanger is designed to recover the cold energy from the ventilated air of a building air-conditioning system.

2. The air conditioner has a cooling load about 10 refrigeration tons. The sensible heat factor is 0.5, that is the cooling load consists of 5 tons of sensible heat load and 5 tons of latent heat load.

3. The heat exchanger aims to recover 10% of the sensible heat, or 0.5 tons.

4. The temperature of fresh (outside) air is 30$^{o}$C, whereas the temperature of ventilated air is 25$^{o}$C.

5. Number of air change is 2.5 times per hour.

### 6.2.2 Design Procedure

1. Specify the inlet and outlet temperatures of the heat exchanger.

2. Calculate $\Delta T_m$ of the heat exchanger.

3. Assume the value of the overall heat transfer coefficient.

4. Calculate the required heat transfer area and the required number of finned heat pipes.

5. Determine the heat pipe alignment in the heat exchanger and choose the type of fin to use.

6. Estimate the overall heat transfer coefficient using the empirical correlations obtained in the present work.

7. Compare the value of the overall heat transfer coefficient to that assumed in 3. Repeat steps 4 through 6 if the calculated value in 6 is less than, or much higher than, that assumed in 3.

8. The pressure drop in the designed heat exchanger must be less than the specified value, for example, 20 mm. $H_2O$. Otherwise, repeat the calculation from step 5.

### 6.2.3 Example of Heat-Pipe Heat Exchanger Design

1. Outlet temperature of ventilated air and fresh air

The properties of ventilated air and fresh air are specified as follows.

| Properties | Ventilated air | Fresh air |
|---|---|---|
| dry bulb temperature ($^{o}$C) | 25.0 | 30.0 |
| wet bulb temperature ($^{o}$C) | 19.6 | 26.0 |
| relative humidity (%) | 62 | 73 |
| absolute humidity (kg steam/kg dry air) | 0.012 | 0.02 |
| humid heat (kcal/kg dry air $^{o}$C) | 0.246 | 0.25 |
| humid volume ($m^3$/kg dry air) | 0.86 | 0.883 |



Estimate the room size from the air-conditioning capacity by using the following basis

cooling load : 400 Btu/hr per unit area of room

$$\text{floor area of room} = \frac{10 \times 12{,}000}{400} = 300 \text{ m}^2$$

Assume that the height of the room is 3.5 m.

So the volume will be $3.5 \times 300 = 1{,}050\ m^3$

Since number of air change is 2.5 times per hour, the volumetric flow rate of ventilated air $= 2.5 \times 1{,}050 = 2{,}625\ m^3/hr$

mass flow rate of ventilated air $= \dfrac{2{,}625}{0.86} = 3{,}052$ kg dry air /hr

By assuming that there is 20% air leakage from the room, the flow rate of fresh air also equals $(0.8)(3{,}052) = 2{,}440$ kg dry air /hr

the volumetric flow rate of fresh air $= 2{,}440 \times 0.883$

$= 2{,}150\ m^3/hr$

heat balance around the heat exchanger

$$Q = C_{pc}F_c\,\Delta T_c = C_{ph}F_h\,\Delta T_h$$

where $Q = 0.5$ tons $= 6{,}000$ Btu/hr

$= 1{,}512$ kcal/hr $= 1{,}758$ Watt

$C_{pc} = 0.246$ kcal/kg dry air °C

$F_c = 2{,}440$ kg dry air/hr

$C_{ph} = 0.25$ kcal/kg dry air °C

$1{,}512 = (0.246)(2{,}440)\ T_c = (0.25)(2{,}440)\ T_h$

$\Delta T_c = 2.52$ °C

$\Delta T_h = 2.48$ °C

outlet temperature of ventilated air $= 25 + 2.52 = 27.5$ °C

outlet temperature of fresh air $= 30 - 2.48 = 27.5$ °C

2. $\Delta T_m$ of heat exchanger

$$\Delta T_m = \frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln \dfrac{(T_{hi} - T_{co})}{(T_{ho} - T_{ci})}}$$

$$= 0.5\ [\Delta T_2 + \Delta T_1]$$

$$= \frac{(30 - 27.5) + (27.5 - 25)}{2} = 2.5$$

3. Estimate the overall heat transfer coefficient U of a single heat pipe

Calculate U of a single heat pipe without fin by using the following equation.

$$\frac{1}{UA} = \frac{1}{h_{oh}A_{oh}} + \frac{\ln (D_o/D_i)}{2\pi kL_h} + \frac{1}{h_{ih}A_{ih}} + \frac{1}{h_{ic}A_{ic}} + \frac{\ln (D_o/D_i)}{2\pi kL_c} + \frac{1}{h_{oc}A_{oc}}$$

Since the number of heat pipes in heat exchanger are not yet known, it may be assumed that the outer film coefficient is the same as that of the flow across a single cylinder and can be calculated by using Whitaker's equation (51).



$$Nu = (0.4\ Re^{0.5} + 0.06\ Re^{2/3})Pr^{0.4}$$

For ventilated air, let the air velocity = 3 m/s

$$G = \frac{(1+H)u}{V_H}$$

$$= \frac{(1+0.012)}{0.86}\ (3x3,3600)$$

$$= 12,700 \quad kg/m^2.hr$$

$$= 1.853 \times 10^{-5} \quad kg/m.s$$

$$C_p = 1.005 \quad kJ/kg\ ^oC$$

$$k = 2.614 \times 10^{-2} \quad W/m.^oC$$

$$Re = \frac{G\ D}{\mu} = \frac{(12,700)(9.52 \times 10^{-3})}{(1.853 \times 10^{-5})(3,600)}$$

$$= 1,812.4$$

$$Pr = \frac{C_p \mu}{k} = \frac{(1,005)(1.853 \times 10^{-5})}{2.614 \times 10^{-2}}$$

$$= 0.712$$

$$Nu = [(0.4)(1,812.4)^{0.5} + (0.06)(1,812.4)^{2/3}](0.712)^{0.4}$$

$$= 22.651$$

$$h'_{oc} = \frac{(22.651)(2.614 \times 10^{-2})}{9.52 \times 10^{-3}} = 62.19 \quad W/m^{2o}C$$

For fresh air $G = \frac{(1+H)u}{V_H}$

$$= \frac{(1+0.02)(3x3,600)}{0.883}$$

$$= 12,480 \quad kg/m^2\ hr$$

$$Re = \frac{(12,480)(9.52 \times 10^{-3})}{(1.853 \times 10^{-5})(3,600)}$$

$$= 1,781$$

$$Pr = \frac{(1,005)(1.853 \times 10^{-3})}{2.614 \times 10^{-2}}$$

$$= 0.712$$

$$Nu = [(0.4)(1,781)^{0.5} + (0.06)(1,781)^{2/3}](0.712)^{0.4}$$

$$= 22.432$$

$$h'_{oh} = \frac{(22.432)(2.614 \times 10^{-2})}{9.52 \times 10^{-3}} = 61.59 \quad W/m^{2\,o}C$$

Calculate the effective heat transfer area of outer surface

The total heat pipe length = 90 cm.

Since the evaporator and condenser length are equal

so the effective length $= \frac{90 - 3}{2} = 42$ cm.

outer surface area of the pipe without fin, $A_{tube}$

$$A_{tube} = \pi DL = \pi(9.52 \times 10^{-3})(0.42)$$

$$= 1.26 \times 10^{-2} \quad m^2$$

let the fin area is 10 times that of $A_{tube}$

$$A_{fin} = (10)(1.26 \times 10^{-2})$$

$$= 12.6 \times 10^{-2} \quad m^2$$

If the fin thickness is .33 mm. and the number of fins on each side is 132, the effective area of each side becomes

$$A'_o = (1.26 \times 10^{-2})\left[1 - \frac{(132)(0.33 \times 10^{-3})}{0.42}\right] + (0.126)(0.9)$$

$$= 0.1247 \quad m^2$$

Calculate the inner heat transfer coefficient of the evaporator section

let Q = 9 Watt

$$T_{bh} = \frac{30 + 27.5}{2} = 28.75 \quad {}^oC$$

$$T_{whi} = T_{bh} - \frac{Q}{h'_{oh}A'_{oh}} - \frac{Q \ln(D_o/D_i)}{2\pi kL_h}$$

$$= 27.577 \quad {}^oC$$

$$T_{bc} = \frac{25 + 27.5}{2} = 26.25 \ ^{o}C$$

$$T_{wci} = T_{bc} + \frac{Q}{h'_{oc}A'_{oc}} + \frac{Q\ln(D_o/D_i)}{2\pi kL_c} = 27.411 \ ^{o}C$$

$$T_s = \frac{T_{whi} + T_{wci}}{2} = 27.5 \ ^{o}C$$

$$h_{ih} = \left[\frac{C_p \ (Q/A)^{1-r}}{Pr^{1.7} \ C_{sf}}\right]\left[\frac{1}{\mu\lambda}\sqrt{\frac{g_c \sigma}{g(\rho - \rho_v)}}\right]^{-r}$$

where $C_{sf} = 7.021 \times 10^{-3}$

$r = 0.962$

$$Q = \frac{9}{\pi(8.7\times10^{-3})(0.42)} = 784.01 \ W/m^2$$

$C_p = 905.5 \ J/kg\ ^{o}C$

$\lambda = 153,024 \ J/kg$

$\mu = 6.408\times10^{-4} \ kg/m\ s$

$\rho = 1,558 \ kg/m^3$

$\rho_v = 3.994 \ kg/m^3$

$\sigma = 1.868\times10^{-2} \ N/m$

$k = 0.0964 \ W/m\ ^{o}C$

$$Pr = \frac{C_p\mu}{k} = \frac{(905.5)(6.408\times10^{-4})}{0.0964} = 6.019$$

$$h_{ih} = \frac{(905.5)(784.01)^{1-0.962}}{(153,024)(6.019)^{1.7}(7.021\times10^{-3})} \times \left[\frac{1}{(6.408\times10^{-4})(153,024)}\sqrt{\frac{1.868\times10^{-2}}{(9.8)(1558-3.994)}}\right]^{-0.962}$$

$= 2,949 \ W/m^2\ ^{o}C$

Calculate inner heat transfer coefficient of the condenser section.

$$h_{ic} = C_1 \left[ \frac{g \rho (\rho - \rho_v) \lambda k^3}{\mu (T_s - T_{wci}) L_c} \right]^{C_2}$$

where $C_1 = 1.705\times10^{-6}$

$C_2 = 0.744$

$$h_{ic} = 1.705\times10^{-6} \left[ \frac{(9.8)(1{,}558)(1{,}558-3.994)(153{,}024)(0.0964)^3}{(6.408\times10^{-4})(27.5-27.411)(0.42)} \right]^{0.744}$$

$$h_{ic} = 55{,}793 \ \ W/m^2\ {}^oC$$

Thus the overall heat transfer coefficient of a single heat pipe

$$\frac{1}{(UA)_p} = \frac{1}{(61.59)(0.1247)} + \frac{\ln(9.52/8.7)}{2\pi(386)(0.42)} + \frac{1}{(2{,}949)(\pi)(8.7\times10^{-3})(0.42)}$$

$$+ \frac{1}{(55{,}793)(\pi)(8.7\times10^{-3})(0.42)} + \frac{\ln(9.52/8.7)}{2\pi(386)(0.42)} + \frac{1}{(62.19)(0.1247)}$$

$$= 0.29$$

$$(UA)_p = 3.45 \ \ W/{}^oC$$

$$Q = UA\,\Delta T = (3.45)(2.5) = 8.62 \simeq 9 \ \ \text{Watt}$$

4. Calculate the required number of heat pipes

$$\text{number of heat pipes} = \frac{\text{total heat transfer rate}}{\text{heat transfer of a single heat pipe}}$$

$$= \frac{1{,}758}{9}$$

$$= 195$$

5. Heat pipe alignment and fin characteristics.

cross-sectional area of air flow = 45x42 $cm^2$

pipe arrangement : staggered

pipe pitch , $S_T$ = 2.54 cm.

$S_L$ = 2.2 cm.

fin thickness = 0.33 mm.

fin pitch = 0.3 mm.

number of fins on each side = 132

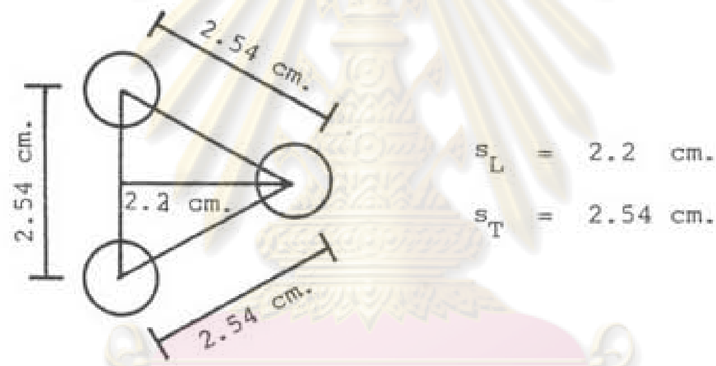





Figure 6.3 Pipe alignment



cross-sectional area of air flow = 45x42 $cm^2$

number of heat pipe in first row = 45/2.54 $\cong$ 17

number of heat pipe in second row = 16

the heat exchanger consists of 6 rows of 17 heat pipes

and 6 rows of 16 heat pipes

Total number of heat pipes = 198

$D_e$ = 4(net free volume)/heat transfer surface

net free volume = $(0.45)(0.42)(0.27) - (198)(\pi/4)(9.52\times10^{-3})^2(0.42-132\times0.33\times10^{-3}) - (132)(0.45)(0.33\times10^{-3})(0.27)$

$= 0.05103 - 5.305\times10^{-3} - 5.293\times10^{-3}$

$= 0.0404 \quad m^3$

heat transfer surface = $(198)(\pi)(9.52\times10^{-3})(0.42-132\times0.33\times10^{-3}) + 2(132)[(0.0254)(0.022) - (\pi/4)(9.52\times10^{-3})^2]$

$= 2.229 + 25.488 = 27.718 \quad m^2$

$$D_e = \frac{4(0.0404)}{27.718}$$

$= 5.830\times10^{-3}$ m.

$A_{min} = (0.45)(0.42)-(17)(9.52\times10^{-3})(0.42-132\times0.33\times10^{-3}) - (0.33\times10^{-3})(0.45)(132)$

$= 0.189 - 0.0609 - 0.0196$

$= 0.1085 \quad m^2$

$$G_{max} = \frac{m}{A_{min}}$$

For fresh air

$$G_{max} = \frac{m_h}{A_{min}}$$



$$m_h = \frac{(1+0.02)(2,440)}{3,600}$$

$= 0.691 \quad kg/s$

$$G_{max} = \frac{0.691}{0.1085} = 6.369 \quad kg/m^2.s$$

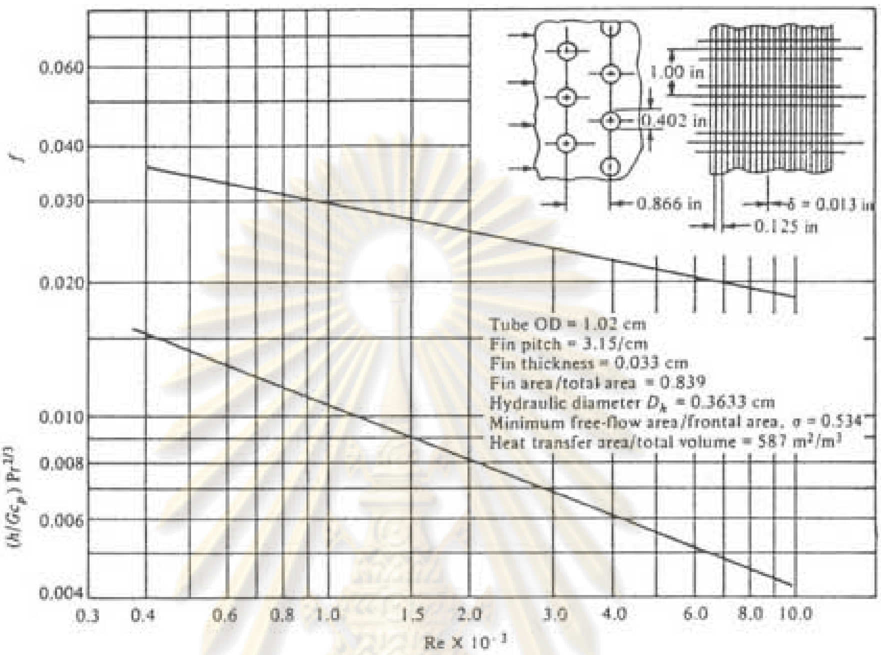



Figure 6.4 Heat transfer and friction factor for flow across plate-finned circular tube matrix (48)



$$Re_h = \frac{G_{max} D_e}{\mu}$$

$\mu$ = $1.853 \times 10^{-5}$ kg/m.s

$D_e$ = $5.830 \times 10^{-3}$ m.

$$Re_h = \frac{(6.369)(5.830 \times 10^{-3})}{1.853 \times 10^{-5}}$$

$$= 2,004$$

From Figure 6.4

$$StPr^{2/3} = 0.008$$

$$St = \frac{h}{GC_p} \quad ; \quad Pr = \frac{C_p \mu}{k}$$

$$h = (0.008)(6.369)(1,005)\left[\frac{2.614 \times 10^{-2}}{(1005)(1.853 \times 10^{-5})}\right]^{2/3}$$

$$= 64.195 \quad W/m^2.^{o}C$$

$$R_f \text{ (fouling resistance)} = 0.00035 \quad m^2\ ^{o}C/W$$

$$\frac{1}{h_{oh}} = R_f + \frac{1}{h}$$

$$= 0.00035 + 0.01558$$

$$h_{oh} = 62.775 \quad W/m^2.^{o}C$$

$$h_{oh} \cong h'_{oh}$$

For ventilated air

$$G_{max} = \frac{m_c}{A_{min}}$$

$$m_c = \frac{(1+0.012)(2,440)}{3,600} = 0.686 \quad kg/s$$

$$G_{max} = \frac{0.686}{0.1085} = 6.323 \quad kg/m^2\ s$$

$$Re_c = \frac{(6.323)(5.83 \times 10^{-3})}{1.853 \times 10^{-5}} = 1,989$$

$$StPr^{2/3} = 0.008$$

$$h = (0.008)(6.323)(1,005)\left[\frac{2.614 \times 10^{-2}}{(1,005)(1.853 \times 10^{-5})}\right]^{2/3}$$

$$h = 63.73 \quad W/m^2.^{o}C$$

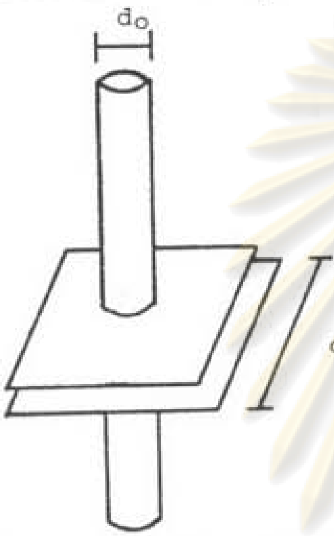
$R_f \;=\; 0.0035 \quad m^2.^oC/W$

$$\frac{1}{h_{oc}} = R_f + \frac{1}{h} = 0.00035 + 0.0157$$

$h_{oc} \;=\; 62.31 \quad W/m^2.^oC$

$h_{oc} \cong h'_{oc}$

Calculate fin eficiency




$d_o \;=\; 9.52\text{x}10^{-3} \quad m.$

$r_o \;=\; 4.76\text{x}10^{-3} \quad m.$

$d^* \;=\; 2.54\text{x}10^{-2} \quad m.$

$S \;=\; d^*/2$

$\;=\; 0.0127 \quad m.$

$$r^* = \left(\frac{2\sqrt{3}}{\pi}\right)^{0.5} S$$

$\;=\; 1.33\text{x}10^{-2} \quad m.$

Thermal conductivity of fin (aluminium) = $204 \quad W/m.^oC$

fin thickness (t) = $0.33\text{x}10^{-3}$ m.

$$mL = (r^* - r_o)\sqrt{\frac{2h}{kt}}$$

$$= (1.33\text{x}10^{-2} - 4.76\text{x}10^{-3})\sqrt{\frac{(2)(62.31)}{(204)(3.3\text{x}10^{-4})}}$$

$$= 0.367$$

$$\eta_{fin} = \frac{\tanh(mL)}{mL}$$

$$= \frac{\tanh(0.367)}{0.367}$$

$$= 0.96$$

6. Calculate the overall heat transfer coefficient of the heat exchanger

$$\frac{1}{(UA)_t} = \sum_{i=1}^{7} R_i$$

$$R_1 = \frac{1}{h_{oh}A_{oht}}$$

$$A_{ot} = (198)(\pi \times 9.52\times10^{-3})(0.42-132\times0.33\times10^{-3}) + (2\times132\times198)(0.96)[(0.0254)(0.022)-\frac{\pi}{4}(9.52\times10^{-3})^2]$$

$$= 2.229+24.469$$

$$= 26.698 \quad m^2$$

$$R_1 = \frac{1}{(62.775)(26.698)} = 5.967\times10^{-4} \quad {}^{o}C/W$$

$$R_2 = R_6 = \frac{\ln(D_o/D_i)}{2\pi kNL} = \frac{\ln(9.52/8.7)}{2\pi(386)(0.42)(198)}$$

$$= 4.466\times10^{-7} \quad {}^{o}C/W$$

$$R_3 = \frac{1}{h_{ih}A_{iht}}$$

$$= \frac{1}{(2,949)(\pi)(8.7\times10^{-3})(0.42)(198)}$$

$$= 1.492\times10^{-4} \quad {}^{o}C/W$$

$$R_5 = \frac{1}{h_{ic}A_{ict}}$$

$$= \frac{1}{(55,793)(\pi)(8.7\times10^{-3})(0.42)(198)}$$

$$= 7.886\times10^{-6} \quad {}^{o}C/W$$

$$R_7 = \frac{1}{h_{oc}A_{oct}}$$

$$= \frac{1}{(62.31)(26.698)} = 6.011\text{x}10^{-4} \quad {}^{o}C/W$$

$$\frac{1}{(UA)_t} = 5.967\text{x}10^{-4} + (2)(4.466\text{x}10^{-7}) + 1.492\text{x}10^{-4} +$$

$$7.886\text{x}10^{-6} + 6.011\text{x}10^{-4}$$

$$= 1.356\text{x}10^{-3} \quad {}^{o}C/W$$

$$(UA)_t = 737.46 \quad W/{}^{o}C$$

$$U_{ot} = \frac{737.46}{26.698} = 27.62 \quad W/m^2.{}^{o}C$$

7. Calculate pressure drop across the heat exchanger

$$D_e = \frac{4\ (\text{net free volume})}{\text{friction surface}}$$

$$\text{net free volume} = (0.45)(0.42)(0.27)-(198)(\pi/4)(9.52\text{x}10^{-3})^2(0.42-132\text{x}3.3\text{x}10^{-4})-(0.45)(0.27)(3.3\text{x}10^{-4})(132)$$

$$= 0.0404 \quad m^3$$

$$\text{friction surface} = (198)(\pi)(9.52\text{x}10^{-3})[0.42 - (132)(0.33\text{x}10^{-3})] + (2)(132)(198)[(0.0254)(0.022) - (\pi/4)(9.52\text{x}10^{-3})^2] + [(2)(0.45) + (2)(0.42)](0.27)$$

$$= 28.188 \quad m^2$$

$$D_e = \frac{(4)(0.0404)}{28.188} = 5.733\text{x}10^{-3} \quad m.$$

for fresh air, $G_{max} = 6.369 \quad kg/m^2.s$

$$Re = \frac{(6.369)(5.733\text{x}10^{-3})}{1.853\text{x}10^{-5}} = 1,970$$

From Figure 6.4

$$f = 0.025$$

$$\Delta P = \frac{fG^2A}{2\rho A_{min}}$$

$$\frac{A}{A_{min}} = \frac{4\text{xLength}}{D_e}$$

$$= \frac{(4)(0.27)}{5.733\text{x}10^{-3}}$$

$$= 188.38 \quad m^2$$

$$\Delta P = \frac{(0.025)(6.369)^2(188.38)}{(2)(1.1766)}$$

$$= 81.2 \quad N/m^2$$

$$= 8.3 \quad mm.H_2O$$

$$\Delta P < 20 \quad mm.H_2O$$

6.3 Results of Calculations

From the previous calculations, it can be concluded as follows:

a. The designed heat exchanger has the dimensions as shown in Figure 6.5

b. The heat exchanger consists of 12 rows of heat pipe with 6 rows of 17 heat pipes and 6 rows of 16 heat pipes in each row, and the total number of heat pipes are 198. The pipe arrangement is of staggered type, which has a transverse pitch of 2.54 cm. and longitudinal pitch of 2.2 cm.

c. The heat pipes are made of 90-cm.-long copper tube, 9.52 mm. in outside diameter, and use Freon-113 as the working fluid.

d. The inlet and outlet temperatures of ventilated air are 25 and 27.5 °C, whereas those of fresh air are 30 and 27.5 °C, respectively. The flow of the fluid are counter flow with respect to one another.

e. The heat exchanger can recover 0.5 refrigeration tons of cold energy, or 1,758 Watt and has the overall heat transfer coefficient of about 27.6 $W/m^{2o}C$ based on the total outside bare tube area.

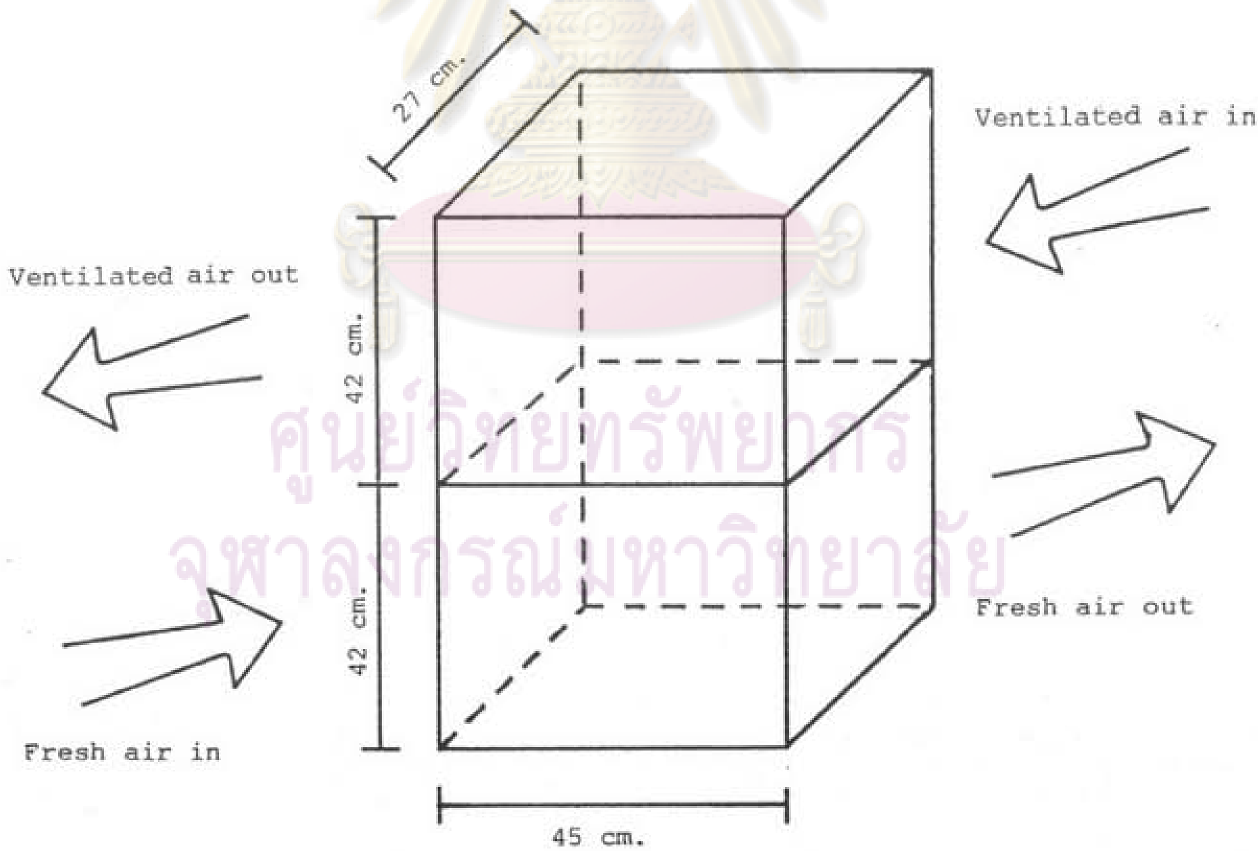



Figure 6.5 Dimensions of designed heat exchanger

From the previous calculations, it can be seen that the heat resistances of the outer surface are much larger than the internal resistances. This means that the controlling mechanism is the mechanism of heat transfer from the air to the bundle of heat pipes, instead of the internal mechanism, Hence the resulting overall heat transfer coefficient is very low.

In the step of calculating the overall heat transfer coefficient, an iterative method has been employed because the saturated temperature, $T_S$, is not known. To make the calculation easier, a computer program for calculating the heat transfer resistances and the number of needed heat pipes has been coded. The flow chart of the program is shown in Figure 6.6.

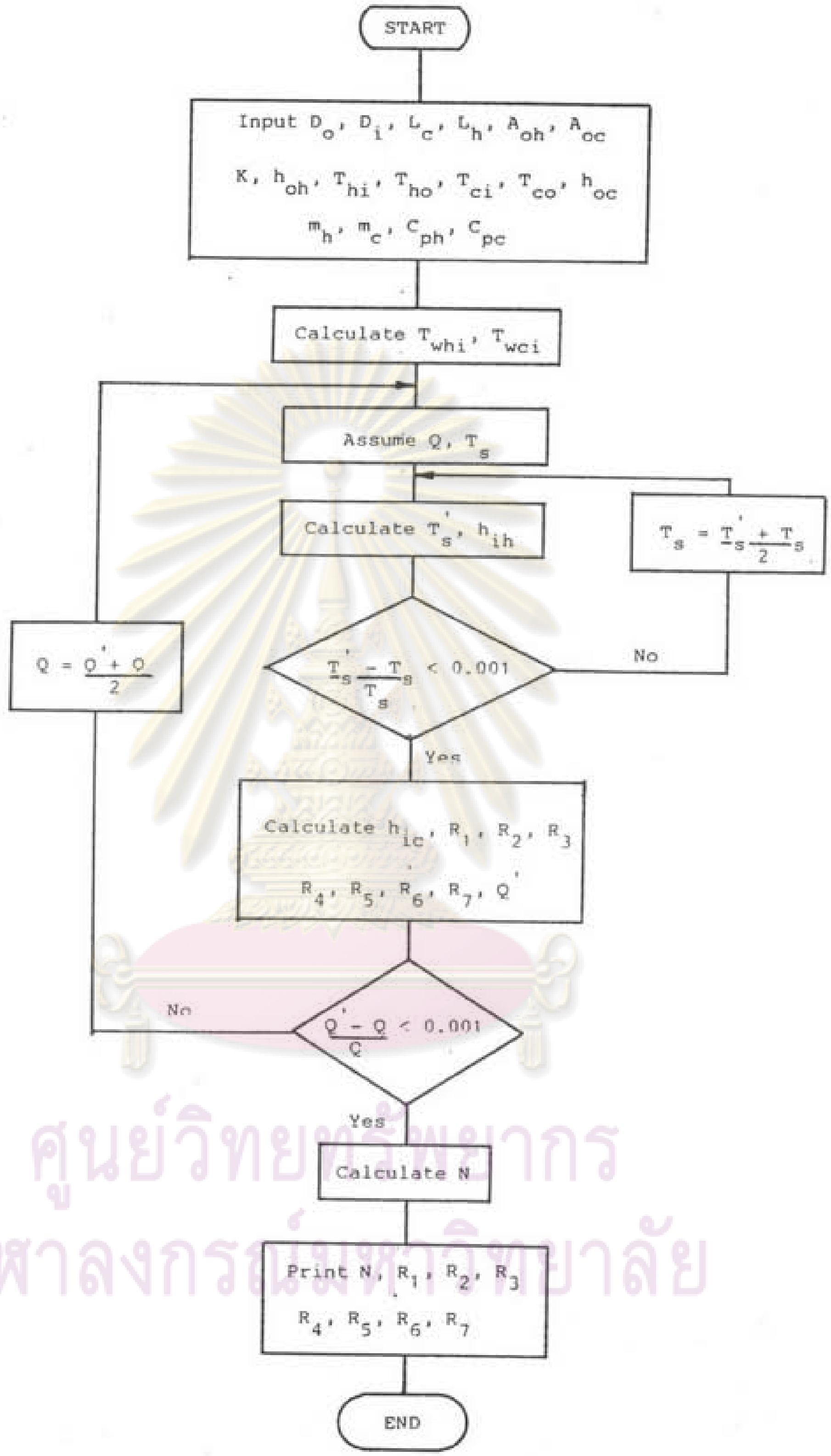



Figure 6.6 Flow chart of program for calculating heat resistances and the required number of heat pipes