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APPENDICES

APPENDIX A

DERIVATION OF HEAT TRANFER EQUATION

The objective of this appendix is to present a derivation for the governing equation for gas-to-particle heat transfer in 2DSB. These equations are fundamental to the development of numerical models for multiphase flows that based the work of Crowe, Sommerfeld, and Tsuji (1998). The derivation results are presented below.

Continuous phase (gas phase)

Equation of thermal energy

$$
\begin{aligned}
&\frac{\partial\left[\alpha_c \langle\rho_c\rangle \bar{i}_c\right]}{\partial t}+\frac{\partial\left[\alpha_c \langle\rho_c\rangle \bar{u}_i \bar{h}_c\right]}{\partial x_i}=\langle p\rangle \frac{\partial}{\partial x_i}(\alpha_c \bar{u}_i)-\left(\frac{1}{V}\right)\sum_k \dot{m}_k\left(h_{s,k}+\frac{\left|v_{i,k}-\bar{u}_i^2\right|}{2}+\frac{w_k'^2}{2}\right) \\
&+\sum_k \beta_{v,k}\left|\bar{u}_i-v_{i,k}\right|^2-\langle p\rangle \frac{\partial}{\partial x_i}\left(\alpha_d\langle v_i\rangle\right)+\left(\frac{1}{V}\right)\sum_k \dot{V}_{d,k} p_{s,k}+\langle\tau_{ij}\rangle\left[\frac{\partial}{\partial x_j}\left(\alpha_d\langle v_i\rangle\right)+\alpha_c \bar{u}_i\right] \\
&+\frac{\partial}{\partial x_i}\left(k_{eff}\frac{\partial\langle T_c\rangle}{\partial x_i}\right)+\frac{2\pi k_c}{V}\sum_k\left(\frac{Nu}{2}\right)_k d_k\left(T_{d,k}-\langle T_c\rangle\right)
\end{aligned}
$$

(A.1)

The terms in right side in this equation can be identified as follows: (1) reversible work, (2) energy influx from the dispersed phase, (3) energy dissipation (always positive) from the particle-fluid interaction, (4) flow work due to motion of disperse phase, (5) work rate associated with droplet dilation, (6) energy dissipation due to shear stress in the continuous phase, (7)heat transfer through the mixture, and (8) heat transfer from the dispersed phase.

The term (2) is neglected due to no mass transfer from a particle. The terms (3), (4), and (5) is neglected because the thermal energy is very small when it is compared to term (8).

$$\frac{\partial[\alpha_c\langle\rho_c\rangle\bar{i}_c]}{\partial t}+\frac{\partial[\alpha_c\langle\rho_c\rangle\bar{u}_i\bar{h}_c]}{\partial x_i}=\langle p\rangle\frac{\partial}{\partial x_i}(\alpha_c\bar{u}_i)+\frac{\partial}{\partial x_i}\left(k_{eff}\frac{\partial\langle T_c\rangle}{\partial x_i}\right)+\frac{2\pi k_c}{V}\sum_k\left(\frac{Nu}{2}\right)_k d_k\left(T_{d,k}-\langle T_c\rangle\right) \tag{A.2}$$

Assumption of constant pressure

$$\frac{\partial}{\partial t}\left(\alpha_c\langle\rho_c\rangle C_{pc}\langle T_c\rangle\right)+\frac{\partial}{\partial x_i}\left(\alpha_c\langle\rho_c\rangle u_i C_{pc}\langle T_c\rangle\right)=\frac{2\pi k_c}{V}\sum_k\left(\frac{Nu}{2}\right)_k d_k\left(T_{d,k}-\langle T_c\rangle\right) \tag{A.3}$$

$$\langle\rho_c\rangle C_{pc}\frac{\partial}{\partial t}\left(\alpha_c\langle T_c\rangle\right)+\langle\rho_c\rangle C_{pc}\frac{\partial}{\partial t}\left(\alpha_c u_i\langle T_c\rangle\right)=\frac{2\pi k_c}{V}\sum_k\left(\frac{Nu}{2}\right)_k d_k\left(T_{d,k}-\langle T_c\rangle\right) \tag{A.4}$$

Divided by ρ_c

$$\frac{\partial}{\partial t}\left(\alpha_c\langle T_c\rangle\right)+\frac{\partial}{\partial x_i}\left(\alpha_c u_i\langle T_c\rangle\right)=\frac{2\pi k_c}{V\langle\rho_c\rangle C_{pc}}\sum_k\left(\frac{Nu}{2}\right)_k d_k\left(T_{d,k}-\langle T_c\rangle\right) \tag{A.5}$$

Disperse phase

Equation of thermal energy

$$V_{d,k}\rho_d c_{p,d}\frac{dT_{d,k}}{dt}=-2\pi k_c\left(\frac{Nu}{2}\right)_k d_k\left(T_{d,k}-\langle T_c\rangle\right) \tag{A.6}$$

In case of monodisperse particles, d_k and $V_{d,k}$ are constant.

$$\frac{dT_{d,k}}{dt}=-\frac{\pi k_c d_k}{\left(\frac{\pi}{6}d_k^3\right)\rho_d c_{p,d}}Nu_k\left(T_{d,k}-\langle T_c\rangle\right) \tag{A.7}$$

$$\frac{dT_{d,k}}{dt}=-\frac{6k_c}{\rho_d c_{p,d}d_k^2}Nu_k\left(T_{d,k}-\langle T_c\rangle\right) \tag{A.8}$$

APPENDIX B

DISCRETIZATION OF HEAT TRANSFER EQUATION

Continuous phase (gas phase)

Thermal equation

$$\frac{\partial}{\partial t}\left(\alpha_c\langle T_c\rangle\right)+\frac{\partial}{\partial x_i}\left(\alpha_c u_i\langle T_c\rangle\right)=\frac{Q_s}{\langle\rho_c\rangle C_{pc}}+\frac{1}{\langle\rho_c\rangle C_{pc}}\frac{\partial}{\partial x_i}\left(k_c\frac{\partial T_c}{\partial x_i}\right) \qquad \text{(B.1)}$$

Coupling of $\frac{Q_s}{\langle\rho_c\rangle C_{pc}}$ is $\sum_k \frac{Q_{sk}}{\langle\rho_c\rangle C_{pc}}$

$$Nu=\frac{h_d d_k}{k_c} \qquad \text{(B.2)}$$

$$Q_s=\sum_k Q_{sk}=\frac{2\pi k_c}{V}\sum_k\left(\frac{Nu}{2}\right)d_k\left(T_{d,k}-\langle T_c\rangle\right) \qquad \text{(B.3)}$$

$$Q_s=\frac{\pi d_k^2}{V}h_d\left(\sum_k T_{d,k}-m_p\langle T_c\rangle\right)$$

$$Q_s=\frac{\pi d_k^2}{V_p}\left(1-\alpha_c\right)h_k\left(\sum_k T_{d,k}-m_p\langle T_c\rangle\right)$$

$$Q_s=\frac{6\left(1-\alpha_c\right)}{d_k}h_d\left(\langle T_{d,k}\rangle-\langle T_c\rangle\right) \qquad \text{(B.4)}$$

where

$$Nu=2.0+0.6\,\mathrm{Pr}^{1/3}\,\mathrm{Re}^{1/2}$$

$$\mathrm{Re}=\frac{\left|\bar{v}_p-u\right|\rho_c\varepsilon D_k}{\mu}$$

$$\mathrm{Pr}=\frac{C_{pc}\mu}{k_c}$$

Disperse phase

$$\frac{dT_{d,k}}{dt}=-\frac{V}{V_{d,k}\rho_d C_{pd}}Q_{sk}$$

(B.5)

$$\frac{dT_{d,k}}{dt}=-\frac{1}{\alpha_d\rho_d C_{pd}}Q_{sk} \qquad \text{(B.6)}$$

where

$$Q_s = \sum_k Q_{sk} = \frac{2\pi k_c}{V}\sum_k \left(\frac{Nu}{2}\right) d_k \left(T_{d,k} - \langle T_c \rangle\right) \tag{B.7}$$

$$Q_{sk} = \frac{2\pi k_c}{V}\left(\frac{Nu}{2}\right) d_k \left(T_{dk} - \langle T_c \rangle\right) \tag{B.8}$$

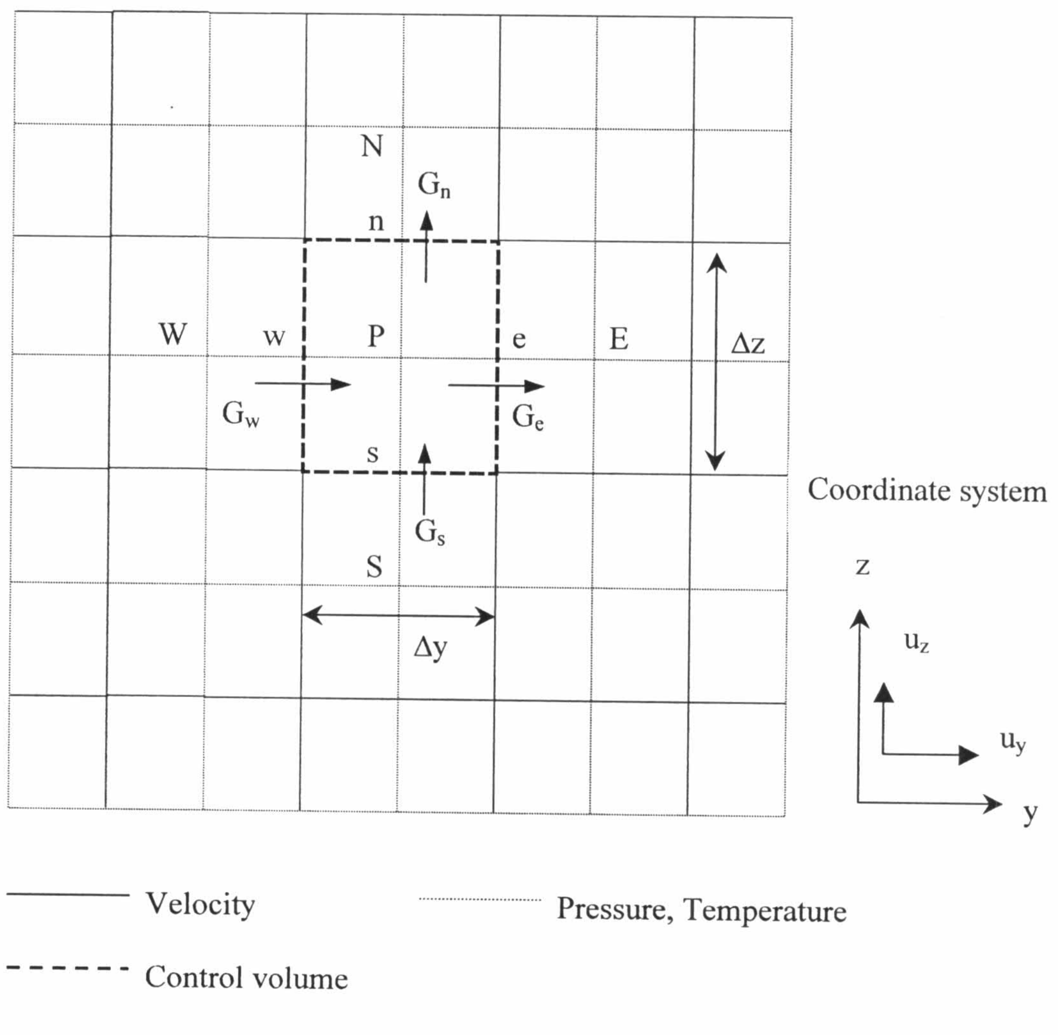

Figure B1 Control volume for discretization of thermal equation in continuous phase

Discretization of thermal equation in continuous phase

Thermal equation

$$\frac{\partial}{\partial t}(\alpha_c\langle T_c\rangle)+\frac{\partial}{\partial y}(\alpha_c u_y\langle T_c\rangle)+\frac{\partial}{\partial z}(\alpha_c u_z\langle T_c\rangle)=\frac{Q_s}{\langle\rho_c\rangle C_{pc}}+\frac{\partial}{\partial y}\left(k_c\frac{\partial T_c}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_c\frac{\partial T_c}{\partial z}\right)$$

(B.9)

Integrate equation of thermal in the control volume

Given $\langle T_c\rangle = T_c$ and $\langle \rho_c\rangle = \rho_c$

$$\int_s^n\int_w^e\int_t^{t+\Delta t}\frac{\partial}{\partial t}(\alpha_c T_c)dtdydz+\int_t^{t+\Delta t}\int_s^n\int_w^e\frac{\partial}{\partial y}(\alpha_c u_y T_c)dydzdt+\int_t^{t+\Delta t}\int_w^e\int_s^n\frac{\partial}{\partial z}(\alpha_c u_z T_c)$$
$$=\int_t^{t+\Delta t}\int_s^n\int_w^e\left(\frac{Q_s}{\rho_c C_{pc}}\right)dydzdt+\frac{1}{\rho_c C_{pc}}\int_t^{t+\Delta t}\int_s^n\int_w^e\frac{\partial}{\partial y}\left(k_c\frac{\partial T_c}{\partial y}\right)dydzdt$$
$$+\frac{1}{\rho_c C_{pc}}\int_t^{t+\Delta t}\int_w^e\int_s^n\frac{\partial}{\partial z}\left(k_c\frac{\partial T_c}{\partial z}\right)dzdydt$$

(B.10)

Term 1

$$\int_s^n\int_w^e\int_t^{t+\Delta t}\frac{\partial}{\partial t}(\alpha_c T_c)dtdydz=\left\{(\alpha_c T_c)_p-(\alpha_c T_c)_p^0\right\}\Delta y\Delta z \tag{B.11}$$

where $(\ \ )^0$ is value at the previous time

Term2

$$\int_t^{t+\Delta t}\int_s^n\int_w^e\frac{\partial}{\partial y}(\alpha_c u_y T_c)dydzdt=(\alpha_c u_y T_c)_e\Delta z\Delta t-(\alpha_c u_y T_c)_w\Delta z\Delta t \tag{B.12}$$
$$=G_e\Delta t-G_w\Delta t$$

where $(\alpha_c u_y T_c)_{e-w}=(\alpha_c u_y T_c)_e-(\alpha_c u_y T_c)_w$

$G_e=(\alpha_c u_y T_c)_e\Delta z$

$G_w=(\alpha_c u_y T_c)_w\Delta z$

Term3

$$\int_{t}^{t+\Delta t}\int_{w}^{e}\int_{s}^{n}\frac{\partial}{\partial z}(\alpha_c u_z T_c)dzdydt = (\alpha_c u_z T_c)_n \Delta y \Delta t - (\alpha_c u_z T_c)_s \Delta y \Delta t \qquad \text{(B.13)}$$

$$= G_n \Delta t - G_s \Delta t$$

where $(\alpha_c u_z T_c)_{n-s} = (\alpha_c u_z T_c)_n - (\alpha_c u_z T_c)_s$

$$G_n = (\alpha_c u_z T_c)_n \Delta y$$

$$G_s = (\alpha_c u_z T_c)_s \Delta y$$

Term 4

$$\int_{t}^{t+\Delta t}\int_{s}^{n}\int_{w}^{e}\left(\frac{Q_s}{\rho_c C_{pc}}\right)dydzdt = \left(\frac{Q_s}{\rho_c C_{pc}}\right)\Delta y \Delta z \Delta t \qquad \text{(B.14)}$$

Term 5

$$\frac{1}{\rho_c C_{pc}}\int_{t}^{t+\Delta t}\int_{s}^{n}\int_{w}^{e}\frac{\partial}{\partial y}\left(k_c \frac{\partial T_c}{\partial y}\right)dydzdt = \frac{1}{\rho_c C_{pc}}\left[\left(k_c \frac{\partial T_c}{\partial y}\right)_e - \left(k_c \frac{\partial T_c}{\partial y}\right)_w\right]\Delta z \Delta t \qquad \text{(B.15)}$$

$$\frac{1}{\rho_c C_{pc}}\int_{t}^{t+\Delta t}\int_{s}^{n}\int_{w}^{e}\frac{\partial}{\partial y}\left(k_c \frac{\partial T_c}{\partial y}\right)dydzdt = \frac{1}{\rho_c C_{pc}}\left[\frac{k_c (T_{ce} - T_{cp})}{(\Delta y)_e} - \frac{k_c (T_{cp} - T_{cw})}{(\Delta y)_w}\right]\Delta z \Delta t \qquad \text{(B.16)}$$

Because of $(\Delta y)_e = (\Delta y)_w$

$$\frac{1}{\rho_c C_{pc}}\int_{t}^{t+\Delta t}\int_{s}^{n}\int_{w}^{e}\frac{\partial}{\partial y}\left(k_c \frac{\partial T_c}{\partial y}\right)dydzdt = \frac{1}{\rho_c C_{pc}}\left[\frac{k_c (T_{ce} - 2T_{cp} + T_{cw})}{\Delta y}\right]\Delta z \Delta t \qquad \text{(B.17)}$$

Term 6

$$\frac{1}{\rho_c C_{pc}}\int_{t}^{t+\Delta t}\int_{w}^{e}\int_{s}^{n}\frac{\partial}{\partial y}\left(k_c \frac{\partial T_c}{\partial z}\right)dzdydt = \frac{1}{\rho_c C_{pc}}\left[\frac{k_c (T_{cn} - 2T_{cp} + T_{cs})}{\Delta z}\right]\Delta y \Delta t \qquad \text{(B.18)}$$

Substitute Eq. (15), (16), (17) in Eq. (14) and divided by Δt

$$\frac{\left\{(\alpha_c T_c)_p - (\alpha_c T_c)_p^0\right\}}{\Delta t}\Delta y \Delta z + G_e + G_w + G_n + G_s = \frac{Q_s}{\rho_c C_{pc}}\Delta y \Delta z + \frac{1}{\rho_c C_{pc}}\left[\frac{k_c\left(T_{ce} - 2T_{cp} + T_{cw}\right)}{\Delta y}\right]\Delta z \Delta t + \frac{1}{\rho_c C_{pc}}\left[\frac{k_c\left(T_{cn} - 2T_{cp} + T_{cs}\right)}{\Delta z}\right]\Delta y \Delta t$$

(B.19)

Equation of continuity

$$\frac{\partial}{\partial t}\alpha_c + \frac{\partial}{\partial y}\left(\alpha_c u_y\right) + \frac{\partial}{\partial z}\left(\alpha_c u_z\right) = 0 \quad \text{(B.20)}$$

Integrate the equation of continuity in the same control volume (Figure 1)

$$\int_w^e \int_s^n \int_t^{t+\Delta t} \frac{\partial}{\partial t}(\alpha_c) dt dy dz + \int_t^{t+\Delta t} \int_s^n \int_w^e \frac{\partial}{\partial y}\left(\alpha_c u_y\right) dy dz dt + \int_t^{t+\Delta t} \int_w^e \int_s^n \frac{\partial}{\partial z}\left(\alpha_c u_z\right) dz dy dt = 0$$

(B.21)

$$\left\{\frac{\alpha_{cp} - \alpha_{cp}^0}{\Delta t}\right\}\Delta y \Delta z + \left(\alpha_c u_y\right)_e \Delta z - \left(\alpha_c u_y\right)_w \Delta z + \left(\alpha_c u_z\right)_n \Delta y - \left(\alpha_c u_z\right)_s \Delta y = 0 \quad \text{(B.22)}$$

$$\left\{\frac{\alpha_{cp} - \alpha_{cp}^0}{\Delta t}\right\}\Delta y \Delta z + F_e - F_w + F_n - F_s = 0 \quad \text{(B.23)}$$

where $F_e = \left(\alpha_c u_y\right)_e \Delta z$, $F_w = \left(\alpha_c u_y\right)_w \Delta z$, $F_n = \left(\alpha_c u_z\right)_n \Delta y$, $F_s = \left(\alpha_c u_z\right)_s \Delta y$

From integrated thermal equation(B.19) and equation of continuity equation(B.23)

$$\frac{\left\{(\alpha_c T_c)_p - (\alpha_c T_c)_p^0\right\}}{\Delta t}\Delta y \Delta z + G_e + G_w + G_n + G_s = \frac{Q_s}{\rho_c C_{pc}}\Delta y \Delta z + \frac{1}{\rho_c C_{pc}}\left[\frac{k_c\left(T_{ce} - 2T_{cp} + T_{cw}\right)}{\Delta y}\right]\Delta z \Delta t + \frac{1}{\rho_c C_{pc}}\left[\frac{k_c\left(T_{cn} - T_{cp} + T_{cs}\right)}{\Delta z}\right]\Delta y \Delta t \quad \text{(B.24)}$$

$$\left\{\frac{\alpha_{cp} - \alpha_{cp}^0}{\Delta t}\right\}\Delta y \Delta z + F_e - F_w + F_n - F_s = 0 \quad \text{(B.25)}$$

Arrange the equations

T_{cp} x (B.25)

$$\left\{\frac{\alpha_{cp}T_{cp}-\alpha_{cp}^{0}T_{cp}}{\Delta t}\right\}\Delta y\Delta z+T_{cp}F_{e}-T_{cp}F_{w}+T_{cp}F_{n}-T_{cp}F_{s}=0 \tag{B.26}$$

Eq.(B.24)-Eq.(B.26)

$$\frac{\Delta y\Delta z}{\Delta t}\left(-\alpha_{cp}^{0}T_{cp}^{0}+\alpha_{cp}^{0}T_{cp}\right)+\left(G_{e}-T_{cp}F_{e}\right)-\left(G_{w}-T_{cp}F_{w}\right)+\left(G_{n}-T_{cp}F_{n}\right)-\left(G_{s}-T_{cp}F_{s}\right)$$
$$=\frac{Q_{s}}{\rho_{c}C_{pc}}\Delta y\Delta z+\frac{1}{\rho_{c}C_{pc}}\left[\frac{k_{c}\left(T_{ce}-2T_{cp}+T_{cw}\right)}{\Delta y}\right]\Delta z\Delta t+\frac{1}{\rho_{c}C_{pc}}\left[\frac{k_{c}\left(T_{cn}-2T_{cp}+T_{cs}\right)}{\Delta z}\right]\Delta y\Delta t \tag{B.27}$$

$$\frac{\Delta y\Delta z}{\Delta t}\alpha_{cp}^{0}\left(T_{cp}-T_{cp}^{0}\right)+\left(G_{e}-T_{cp}F_{e}\right)-\left(G_{w}-T_{cp}F_{w}\right)+\left(G_{n}-T_{cp}F_{n}\right)-\left(G_{s}-T_{cp}F_{s}\right)$$
$$=\frac{Q_{s}}{\rho_{c}C_{pc}}\Delta y\Delta z+\frac{1}{\rho_{c}C_{pc}}\left[\frac{k_{c}\left(T_{ce}-2T_{cp}+T_{cw}\right)}{\Delta y}\right]\Delta z\Delta t+\frac{1}{\rho_{c}C_{pc}}\left[\frac{k_{c}\left(T_{cn}-2T_{cp}+T_{cs}\right)}{\Delta z}\right]\Delta y\Delta t \tag{B.28}$$

Given: $\alpha_c = \varepsilon$

$$F_{e}=\left(\alpha_{c}u_{y}\right)_{e}\Delta z=\left(\varepsilon u_{y}\right)_{e}\Delta z=\frac{\varepsilon_{i,j}+\varepsilon_{i+1,j}}{2}u_{yi,j}\Delta z \tag{B.29}$$

$$F_{w}=\left(\alpha_{c}u_{y}\right)_{w}\Delta z=\left(\varepsilon u_{y}\right)_{w}\Delta z=\frac{\varepsilon_{i,j}+\varepsilon_{i-1,j}}{2}u_{yi-1,j}\Delta z \tag{B.30}$$

$$F_{n}=\left(\alpha_{c}u_{z}\right)_{n}\Delta y=\left(\varepsilon u_{z}\right)_{n}\Delta y=\frac{\varepsilon_{i,j}+\varepsilon_{i,j+1}}{2}u_{zi,j}\Delta y \tag{B.31}$$

$$F_{s}=\left(\alpha_{c}u_{z}\right)_{s}\Delta y=\left(\varepsilon u_{z}\right)\Delta y=\frac{\varepsilon_{i,j}+\varepsilon_{i,j-1}}{2}u_{zi,j-1}\Delta y \tag{B.32}$$

From equation (B.28)

$$\frac{\Delta y\Delta z}{\Delta t}\alpha_{cp}^{0}\left(T_{cp}-T_{cp}^{0}\right)+\left(G_{e}-T_{cp}F_{e}\right)-\left(G_{w}-T_{cp}F_{w}\right)+\left(G_{n}-T_{cp}F_{n}\right)-\left(G_{s}-T_{cp}F_{s}\right)$$
$$=\frac{Q_{s}}{\rho_{c}C_{pc}}\Delta y\Delta z+\frac{1}{\rho_{c}C_{pc}}\left[\frac{k_{c}\left(T_{ce}-2T_{cp}+T_{cw}\right)}{\Delta y}\right]\Delta z\Delta t+\frac{1}{\rho_{c}C_{pc}}\left[\frac{k_{c}\left(T_{cn}-2T_{cp}+T_{cs}\right)}{\Delta z}\right]\Delta y\Delta t \tag{B.33}$$

Term1

$$\frac{\Delta y \Delta z}{\Delta t}\varepsilon_p^0\left(T_{cp}-T_{cp}^0\right)=\frac{\Delta y \Delta z}{\Delta t}\varepsilon_{i,j}^0\left(T_{ci,j}-T_{ci,j}^0\right) \quad \text{(B.34)}$$

Term 2

$$G_e=\left(\varepsilon u_y T_c\right)_e \Delta z=\left(\varepsilon u_y\right)_e \Delta z T_{ce}=F_e T_{ce} \qquad upwind\begin{cases}F_e>0, & T_{ce}=T_{ci,j}\\ F_e<0, & T_{ce}=T_{ci+1,j}\end{cases}$$

$$G_e=T_{ci,j}\max\left[F_e,0\right]-T_{ci+1,j}\max\left[-F_e,0\right]$$

$$T_{cp}F_e=T_{ci,j}\max\left[F_e,0\right]-T_{ci,j}\max\left[-F_e,0\right]$$

So

$$\left(G_e-T_{cp}F_e\right)=\overbrace{\max\left[-F_e,0\right]}^{a_e}\left\{T_{ci,j}-T_{ci+1,j}\right\}=a_e\left\{T_{ci,j}-T_{ci+1,j}\right\} \quad \text{(B.35)}$$

Term3

$$G_w=\left(\varepsilon u_y T_c\right)_w \Delta z=\left(\varepsilon u_y\right)_w \Delta z T_{cw}=F_w T_{cw} \qquad upwind\begin{cases}F_w>0, & T_{cw}=T_{ci-1,j}\\ F_w<0, & T_{cw}=T_{ci,j}\end{cases}$$

$$G_w=T_{ci-1,j}\max\left[F_w,0\right]-T_{ci,j}\max\left[-F_w,0\right]$$

$$T_{cp}F_w=T_{ci,j}\max\left[F_w,0\right]-T_{ci,j}\max\left[-F_w,0\right]$$

So

$$\left(G_w-T_{cp}F_w\right)=\overbrace{\max\left[F_w,0\right]}^{a_w}\left\{T_{ci-1,j}-T_{ci,j}\right\}=a_w\left\{T_{ci-1,j}-T_{ci,j}\right\} \quad \text{(B.36)}$$

In the same way for Term 4 and Term 5

Term4

$$G_n=F_n T_{cn} \qquad upwind\begin{cases}F_n>0, & T_{cn}=T_{ci,j}\\ F_n<0, & T_{cn}=T_{ci,j+1}\end{cases}$$

$$\left(G_n-T_{cp}F_n\right)=\overbrace{\max\left[-F_n,0\right]}^{a_n}\left\{T_{ci,j}-T_{ci,j+1}\right\}=a_n\left\{T_{ci,j}-T_{ci,j+1}\right\} \quad \text{(B.37)}$$

Term5

$$G_s = F_s T_{cs} \qquad upwind \begin{cases} F_s > 0, & T_{cs} = T_{ci,j-1} \\ F_s < 0, & T_{cs} = T_{ci,j} \end{cases}$$

$$\left(G_s - T_{cp}F_s\right) = \overbrace{\max\left[F_s, 0\right]}^{a_s}\left\{T_{ci,j-1} - T_{ci,j}\right\} = a_s\left\{T_{ci,j-1} - T_{ci,j}\right\} \tag{B.38}$$

Substitute all terms in equation (B.33)

$$\frac{\Delta y \Delta z}{\Delta t}\varepsilon_{i,j}^{0}\left(T_{ci,j} - T_{ci,j}^{0}\right) + a_e\left\{T_{ci,j} - T_{ci+1,j}\right\} - a_w\left\{T_{ci-1,j} - T_{ci,j}\right\} + a_n\left\{T_{ci,j} - T_{ci,j+1}\right\}$$
$$- a_s\left\{T_{ci,j-1} - T_{ci,j}\right\} = \frac{Q_s}{\rho_c C_{pc}}\Delta y \Delta z + \frac{k_c}{\rho_c C_{pc}}\left[\frac{\left(T_{ci+1,j} - 2T_{ci,j} + T_{ci-1,j}\right)}{\Delta y}\right]\Delta z \Delta t +$$
$$\frac{k_c}{\rho_c C_{pc}}\left[\frac{\left(T_{ci,j+1} - 2T_{ci,j} + T_{ci,j-1}\right)}{\Delta z}\right]\Delta y \Delta t$$

(B.39)

Rearrange Equation (B.39)

$$\left(\frac{\Delta y \Delta z}{\Delta t}\varepsilon_{i,j}^{0} + a_e + a_w + a_n + a_s\right)T_{ci,j} = \frac{Q_s}{\rho_c C_{pc}}\Delta y \Delta z +$$
$$\frac{k_c}{\rho_c C_{pc}}\left[\left(\frac{T_{ci+1,j} - 2T_{ci,j} + T_{ci-1,j}}{\Delta y}\right)\Delta z \Delta t\right] + \frac{k_c}{\rho_c C_{pc}}\left[\left(\frac{T_{ci,j+1} - 2T_{ci,j} + T_{ci,j-1}}{\Delta z}\right)\Delta y \Delta t\right] +$$
$$\varepsilon_{i,j}^{0}T_{ci,j}^{0}\frac{\Delta y \Delta z}{\Delta t} + a_e T_{ci+1,j} + a_w T_{ci-1,j} + a_n T_{ci,j+1} + a_s T_{ci,j-1}$$

(B.40)

$$\left(a_e + a_w + a_n + a_s + \varepsilon_{i,j}^{0}\frac{\Delta y \Delta z}{\Delta t} + \frac{6(1-\varepsilon)h_d}{d_k}\frac{\Delta y \Delta z}{\rho_c C_{pc}} + \frac{2k_c}{\rho_c C_{pc}}\frac{\Delta z \Delta t}{\Delta y} + \frac{2k_c}{\rho_c C_{pc}}\frac{\Delta y \Delta t}{\Delta z}\right)T_{ci,j} =$$
$$\frac{6(1-\varepsilon)h_d\langle T_d\rangle}{d_k}\frac{\Delta y \Delta z}{\rho_c C_{pc}} + \frac{k_c}{\rho_c C_{pc}}\frac{\Delta z \Delta t}{\Delta y}\left(T_{ci+1,j}\right) + \frac{k_c}{\rho_c C_{pc}}\frac{\Delta z \Delta t}{\Delta y}\left(T_{ci-1,j}\right) + \frac{k_c}{\rho_c C_{pc}}\frac{\Delta y \Delta t}{\Delta z}\left(T_{ci,j+1}\right)$$
$$+ \frac{k_c}{\rho_c C_{pc}}\frac{\Delta y \Delta t}{\Delta z}\left(T_{ci,j-1}\right) + \varepsilon_{i,j}^{0}\frac{\Delta y \Delta z}{\Delta t}T_{ci,j}^{0} + a_e T_{ci+1,j} + a_w T_{ci-1,j} + a_n T_{ci-1,j} + a_s T_{ci,j-1}$$

(B.41)

Given

$$a_p = a_e + a_w + a_n + a_s + \varepsilon_{i,j}^{0}\frac{\Delta y \Delta z}{\Delta t} + \frac{6(1-\varepsilon)h_d}{d_k}\frac{\Delta y \Delta z}{\rho_c C_{pc}} + \frac{2k_c}{\rho_c C_{pc}}\frac{\Delta z \Delta t}{\Delta y} + \frac{2k_c}{\rho_c C_{pc}}\frac{\Delta y \Delta t}{\Delta z}$$

$$b = 6(1-\varepsilon)h_d \langle T_d \rangle \frac{\Delta y \Delta z}{\rho_c C_{pc}} + \varepsilon_{i,j}^{0} T_{ci,j}^{0} \frac{\Delta y \Delta z}{\Delta t}$$

so the equation can be rewritten as

$$a_p T_{ci,j} = \left(a_e + \frac{k_c}{\rho_c C_{pc}} \frac{\Delta z \Delta t}{\Delta y} \right) T_{ci+1,j} + \left(a_w + \frac{k_c}{\rho_c C_{pc}} \frac{\Delta z \Delta t}{\Delta y} \right) T_{ci-1,j} + \left(a_n + \frac{k_c}{\rho_c C_{pc}} \frac{\Delta y \Delta t}{\Delta z} \right) T_{ci,j+1}$$
$$+ \left(a_s + \frac{k_c}{\rho_c C_{pc}} \frac{\Delta y \Delta t}{\Delta z} \right) T_{ci,j-1} + b$$

(B.42)

APPENDIX C

PROPERTIES OF AIR AND CORN

The physical properties of air and corn used for simulation are summarized in this appendix. Both physical and thermal properties are presented below.

Table B.1 Physical properties of grains used in the aerodynamics and heat transfer characteristics of grains

Parameter	Corn (Yellow dent)	Soybean (Maple arrow)	Wheat (Laval-19)
Particle diameter, m	0.00800	0.00663	0.00363
Sphericity	0.755	0.858	0.621
Angle of repose, degree	29.2	33.3	28.2
Particle density, kg/m^3	1,231	1,130	1,300
Specific heat, J/(kg K)	2,400	2,072	2,198
Coefficient of friction	0.30	0.20	0.33
Poisson's ratio	0.25	-	-

Table B.2 Physical properties of air at 101.325 kPa (1 Atm Abs), SI Units

T (^{o}C)	T (K)	ρ (kg/m^{3})	C_p (kJ/kg K)	$\mu \times 10^{5}$ (kg/m s or Pa s)	k (W/m K)
-17.8	255.4	1.379	1.0048	1.62	0.02250
0	273.2	1.293	1.0048	1.72	0.02423
10.0	283.2	1.246	1.0048	1.78	0.02492
37.8	311.0	1.137	1.0048	1.90	0.02700
65.6	338.8	1.043	1.0090	2.03	0.02925
93.3	366.5	0.964	1.0090	2.15	0.03115
121.1	394.3	0.895	1.0132	2.27	0.03323
148.9	422.1	0.838	1.0174	2.37	0.03531
176.7	449.9	0.785	1.0216	2.50	0.03721
204.4	477.6	0.740	1.0258	2.60	0.03894
232.2	505.4	0.700	1.0300	2.71	0.04084
260.0	533.2	0.662	1.0341	2.80	0.04258

VITA

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