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APPENDIX A

In this appendix, we illustrate the construction of a graph of order rn that has the property $P(r, n)$ and has the maximum number of lines as given in the proof of Proposition 3.6 for the case $n = 4$, $r = 3$.

Let

$$T = \{v_{01}, v_{02}, v_{03}, v_{11}, v_{12}, v_{13}, v_{21}, v_{22}, v_{23}\}.$$

By definition of $A_1(i, j)$, $A_2(i, j)$, $A_3(i, j)$, we see that

$A_1(0, 1) = \varnothing$, $A_2(0,1) = \{v_{11}, v_{21}\}$, $A_3(0,1) = \{v_{12}\}$,
$A_1(0, 2) = \{v_{21}\}$, $A_2(0,2) = \{v_{12}, v_{22}\}$, $A_3(0,2) = \{v_{13}\}$,
$A_1(0, 3) = \{v_{22}\}$, $A_2(0,3) = \{v_{13}, v_{23}\}$, $A_3(0,3) = \{v_{14}\}$,
$A_1(0, 4) = \{v_{23}\}$, $A_2(0, 4) = \{v_{14}, v_{24}\}$, $A_3(0, 4) = \varnothing$,

$A_1(1, 1) = \varnothing$, $A_2(1, 1) = \{v_{01}, v_{21}\}$, $A_3(1, 1) = \{v_{22}\}$,
$A_1(1, 2) = \{v_{01}\}$, $A_2(1, 2) = \{v_{02}, v_{22}\}$, $A_3(1, 2) = \{v_{23}\}$,
$A_1(1, 3) = \{v_{02}\}$, $A_2(1, 3) = \{v_{03}, v_{23}\}$, $A_3(1, 3) = \{v_{24}\}$,
$A_1(1, 4) = \{v_{03}\}$, $A_2(1, 4) = \{v_{04}, v_{24}\}$, $A_3(1, 4) = \varnothing$,

$A_1(2, 1) = \varnothing$, $A_2(2, 1) = \{v_{01}, v_{11}\}$, $A_3(2, 1) = \{v_{02}\}$,
$A_1(2, 2) = \{v_{11}\}$, $A_2(2, 2) = \{v_{02}, v_{12}\}$, $A_3(2, 2) = \{v_{03}\}$,
$A_1(2, 3) = \{v_{12}\}$, $A_2(2, 3) = \{v_{03}, v_{13}\}$, $A_3(2, 3) = \{v_{04}\}$,
$A_1(2, 4) = \{v_{13}\}$, $A_2(2, 4) = \{v_{04}, v_{14}\}$, $A_3(2, 4) = \varnothing$,

Thus, by definition of η, we have

$\eta(v_{01}) = \{v_{11}, v_{21}, v_{12}, v_{13}, v_{14}\}$,
$\eta(v_{02}) = \{v_{21}, v_{12}, v_{22}, v_{13}, v_{14}\}$,
$\eta(v_{03}) = \{v_{21}, v_{22}, v_{13}, v_{23}, v_{14}\}$,
$\eta(v_{04}) = \{v_{21}, v_{22}, v_{23}, v_{14}, v_{24}\}$,

$\eta(v_{11}) = \{v_{01}, v_{21}, v_{22}, v_{23}, v_{24}\}$,
$\eta(v_{12}) = \{v_{01}, v_{02}, v_{22}, v_{23}, v_{24}\}$,
$\eta(v_{13}) = \{v_{01}, v_{02}, v_{03}, v_{23}, v_{24}\}$,
$\eta(v_{14}) = \{v_{01}, v_{02}, v_{03}, v_{04}, v_{24}\}$,

$\eta(v_{21}) = \{v_{01}, v_{11}, v_{02}, v_{03}, v_{04}\}$,
$\eta(v_{22}) = \{v_{11}, v_{02}, v_{12}, v_{03}, v_{04}\}$,
$\eta(v_{23}) = \{v_{11}, v_{12}, v_{03}, v_{13}, v_{04}\}$.
$\eta(v_{24}) = \{v_{11}, v_{12}, v_{13}, v_{04}, v_{14}\}$,

Therefore

$$X(G[\eta]) = \{\ v_{01}v_{11},\ v_{01}v_{21},\ v_{01}v_{12},\ v_{01}v_{13},\ v_{01}v_{14},\ v_{02}v_{21},$$

$$v_{02}v_{12}, v_{02}v_{22},\ v_{02}v_{13},\ v_{02}v_{14},\ v_{03}v_{21},\ v_{03}v_{22},$$

$$v_{03}v_{13},\ v_{03}v_{23},\ v_{03}v_{14},\ v_{04}v_{21},\ v_{04}v_{22},\ v_{04}v_{23},$$

$$v_{04}v_{14}, v_{04}v_{24},\ v_{11}v_{21},\ v_{11}v_{22},\ v_{11}v_{23},\ v_{11}v_{24},$$

$$v_{12}v_{22},\ v_{12}v_{23},\ v_{12}v_{24},\ v_{13}v_{23},\ v_{13}v_{24},\ v_{14}v_{24}\ \}.$$

This graph is shown in Figure A.1

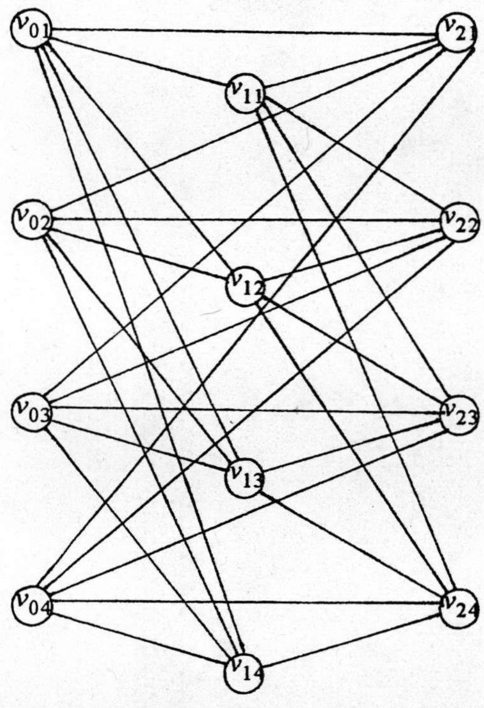

Figure A.1

APPENDIX B

In this appendix, we will show the construction of an example graph $G(p, r, n)$ as given in the proof of Proposition 3.8 for the case $p = 17$, $n = 4$ and $r = 3$.

According to Proposition 3.8, we have $m = 5$, $k = 2$, $s = 1$, $m_0 = 3$ and $m_1 = 2$. Therefore, the distinct elements to be used in our construction are

$$v_{01}, v_{02}, v_{03}, v_{04},$$

$$v_{11}, v_{12}, v_{13}, v_{14},$$

$$v_{21}, v_{22}, v_{23}, v_{24},$$

$$u_{01}, u_{02}, u_{03},$$

$$u_{11}, u_{12},$$

and the sets C_i, $i = 0, 1, 2$, T, U_i, $i = 0, 1$, and U are

$$C_0 = \{v_{01}, v_{02}, v_{03}, v_{04}\},$$

$$C_1 = \{v_{11}, v_{12}, v_{13}, v_{14}\},$$

$$C_2 = \{v_{21}, v_{22}, v_{23}, v_{24}\}$$

and $T = \{v_{01}, v_{02}, v_{03}, v_{04}, v_{11}, v_{12}, v_{13}, v_{14}, v_{21}, v_{22}, v_{23}, v_{24},\}.$

Let

$$U_0 = \{u_{01}, u_{02}, u_{03}\},$$

$$U_1 = \{u_{11}, u_{12}\}$$

and $U = \{u_{01}, u_{02}, v_{03}, u_{11}, u_{12}\}.$

By definition of $A_1(i, j)$, $A_2(i, j)$, $A_3(i, j)$, we see that

$A_1(0, 1) = \varnothing$, $A_2(0,1) = \{v_{11}, v_{21}\}$, $A_3(0,1) = \{v_{12}\}$,
$A_1(0, 2) = \{v_{21}\}$, $A_2(0,2) = \{v_{12}, v_{22}\}$, $A_3(0,2) = \{v_{13}\}$,
$A_1(0, 3) = \{v_{22}\}$, $A_2(0,3) = \{v_{13}, v_{23}\}$, $A_3(0,3) = \{v_{14}\}$,
$A_1(0, 4) = \{v_{23}\}$, $A_2(0, 4) = \{v_{14}, v_{24}\}$, $A_3(0, 4) = \varnothing$,

$A_1(1, 1) = \varnothing$, $A_2(1, 1) = \{ v_{01}, v_{21} \}$, $A_3(1, 1) = \{ v_{22} \}$,
$A_1(1, 2) = \{ v_{01} \}$, $A_2(1, 2) = \{ v_{02}, v_{22} \}$, $A_3(1, 2) = \{ v_{23} \}$,
$A_1(1, 3) = \{ v_{02} \}$, $A_2(1, 3) = \{ v_{03}, v_{23} \}$, $A_3(1, 3) = \{ v_{24} \}$,
$A_1(1, 4) = \{ v_{03} \}$, $A_2(1, 4) = \{ v_{04}, v_{24} \}$, $A_3(1, 4) = \varnothing$,

$A_1(2, 1) = \varnothing$, $A_2(2, 1) = \{ v_{01}, v_{11} \}$, $A_3(2, 1) = \{ v_{02} \}$,
$A_1(2, 2) = \{ v_{11} \}$, $A_2(2, 2) = \{ v_{02}, v_{12} \}$, $A_3(2, 2) = \{ v_{03} \}$,
$A_1(2, 3) = \{ v_{12} \}$, $A_2(2, 3) = \{ v_{03}, v_{13} \}$, $A_3(2, 3) = \{ v_{04} \}$,
$A_1(2, 4) = \{ v_{13} \}$, $A_2(2, 4) = \{ v_{04}, v_{14} \}$, $A_3(2, 4) = \varnothing$,

Thus, by definition of η, we have

$\eta(v_{01}) = \{ v_{11}, v_{21}, v_{12}, v_{13}, v_{14}\}$,
$\eta(v_{02}) = \{ v_{21}, v_{12}, v_{22}, v_{13}, v_{14}\}$,
$\eta(v_{03}) = \{ v_{21}, v_{22}, v_{13}, v_{23}, v_{14}\}$,
$\eta(v_{04}) = \{ v_{21}, v_{22}, v_{23}, v_{14}, v_{24}\}$,

$\eta(v_{11}) = \{ v_{01}, v_{21}, v_{22}, v_{23}, v_{24}\}$,
$\eta(v_{12}) = \{ v_{01}, v_{02}, v_{22}, v_{23}, v_{24}\}$,
$\eta(v_{13}) = \{ v_{01}, v_{02}, v_{03}, v_{23}, v_{24}\}$,
$\eta(v_{14}) = \{ v_{01}, v_{02}, v_{03}, v_{04}, v_{24}\}$,

$\eta(v_{21}) = \{ v_{01}, v_{11}, v_{02}, v_{03}, v_{04}\}$,
$\eta(v_{22}) = \{ v_{11}, v_{02}, v_{12}, v_{03}, v_{04}\}$,
$\eta(v_{23}) = \{ v_{11}, v_{12}, v_{03}, v_{13}, v_{04}\}$.
$\eta(v_{24}) = \{ v_{11}, v_{12}, v_{13}, v_{04}, v_{14}\}$, .

Hence, by definition of η' in proposition 3.8, we have

$\eta'(v_{01}) = \{ v_{11}, v_{21}, v_{12}, v_{13}, v_{14}, u_{11}, u_{12}\}$,
$\eta'(v_{02}) = \{ v_{21}, v_{12}, v_{22}, v_{13}, v_{14}, u_{11}, u_{12}\}$,
$\eta'(v_{03}) = \{ v_{21}, v_{22}, v_{13}, v_{23}, v_{14}, u_{11}, u_{12}\}$,
$\eta'(v_{04}) = \{ v_{21}, v_{22}, v_{23}, v_{14}, v_{24}, u_{11}, u_{12}\}$,

$\eta'(v_{11}) = \{ v_{01}, v_{21}, v_{22}, v_{23}, v_{24}, u_{01}, u_{02}, u_{03}\}$,
$\eta'(v_{12}) = \{ v_{01}, v_{02}, v_{22}, v_{23}, v_{24}, u_{01}, u_{02}, u_{03}\}$,
$\eta'(v_{13}) = \{ v_{01}, v_{02}, v_{03}, v_{23}, v_{24}, u_{01}, u_{02}, u_{03}\}$,
$\eta'(v_{14}) = \{ v_{01}, v_{02}, v_{03}, v_{04}, v_{24}, u_{01}, u_{02}, u_{03}\}$,

$\eta'(v_{21}) = \{ v_{01}, v_{11}, v_{02}, v_{03}, v_{04}\}$;
$\eta'(v_{22}) = \{ v_{11}, v_{02}, v_{12}, v_{03}, v_{04}\}$,
$\eta'(v_{23}) = \{ v_{11}, v_{12}, v_{03}, v_{13}, v_{04}\}$.
$\eta'(v_{24}) = \{ v_{11}, v_{12}, v_{13}, v_{04}, v_{14}\}$,

$$\eta'(u_{01}) = \{ u_{11}, u_{12}, v_{11}, v_{12}, v_{13}, v_{14}\},$$
$$\eta'(v_{02}) = \{ u_{11}, u_{12}, v_{11}, v_{12}, v_{13}, v_{14}\},$$
$$\eta'(v_{03}) = \{ u_{11}, u_{12}, v_{11}, v_{12}, v_{13}, v_{14}\},$$

$$\eta'(u_{11}) = \{ u_{01}, u_{02}, u_{03}, v_{01}, v_{02}, v_{03}, v_{04}\}$$
$$\eta'(u_{12}) = \{ u_{01}, u_{02}, u_{03}, v_{01}, v_{02}, v_{03}, v_{04}\}$$

Therefore

$$\begin{aligned} X(G[\eta]) = \{ \ & v_{01}v_{11}, v_{01}v_{21}, v_{01}v_{12}, v_{01}v_{13}, v_{01}v_{14}, v_{02}v_{21}, \\ & v_{02}v_{12}, v_{02}v_{22}, v_{02}v_{13}, v_{02}v_{14}, v_{03}v_{21}, v_{03}v_{22}, \\ & v_{03}v_{13}, v_{03}v_{23}, v_{03}v_{14}, v_{04}v_{21}, v_{04}v_{22}, v_{04}v_{23}, \\ & v_{04}v_{14}, v_{04}v_{24}, v_{11}v_{21}, v_{11}v_{22}, v_{11}v_{23}, v_{11}v_{24}, \\ & v_{12}v_{22}, v_{12}v_{23}, v_{12}v_{24}, v_{13}v_{23}, v_{13}v_{24}, v_{14}v_{24}, \\ & u_{01}u_{11}, u_{01}u_{12}, u_{02}u_{11}, u_{02}u_{12}, u_{02}u_{11}, u_{02}u_{12}, \\ & u_{01}v_{11}, u_{01}v_{12}, u_{01}v_{13}, u_{01}v_{14}, \\ & u_{02}v_{11}, u_{02}v_{12}, u_{02}v_{13}, u_{02}v_{14}, \\ & u_{03}v_{11}, u_{03}v_{12}, u_{03}v_{13}, u_{03}v_{14}, \\ & u_{11}v_{01}, u_{11}v_{02}, u_{11}v_{03}, u_{11}v_{04}, \\ & u_{12}v_{01}, u_{12}v_{02}, u_{12}v_{03}, u_{12}v_{04} \}. \end{aligned}$$

This graph is shown in Figure B.1

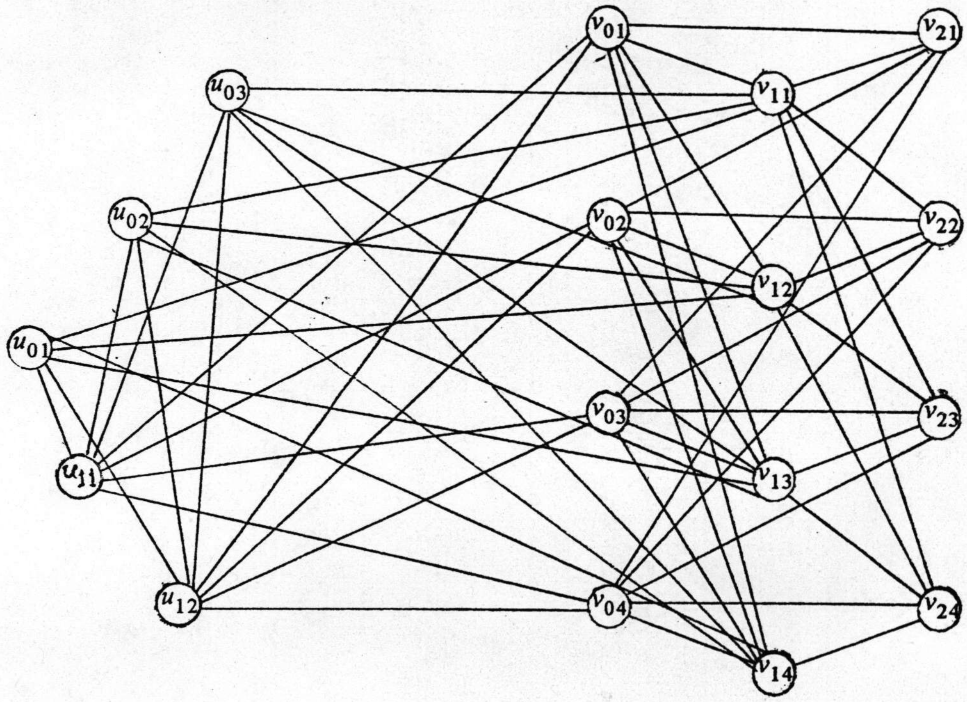
v_{01}
v_{21}
u_{03}
v_{11}
u_{02}
v_{02}
v_{22}
v_{12}
u_{01}
v_{03}
v_{23}
u_{11}
v_{13}
u_{12}
v_{04}
v_{24}
v_{14}

Figure B.1

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