CHAPTER I

INTRODUCTION

The question of integrability of a dynamical system was raised soon after Newton formulated the equation of motion of three bodies in a gravitational field. By integrability of a Hamiltonian system with N degrees of freedom we mean the existence of N integrals of the motion which are independent and in involution . When the equations of motion are separable, the solutions can be obtained by the method of quadratures [1].

Up to now, there exists no general method for determining the integrability of a given dynamical system. A number of results have been obtained in this direction, for example, Jacobi's solution of the motion on an ellipsoid, and Kovalevskaya's integration of the rotation of a rigid body in some special cases. More recently, several examples of integrable Hamiltonian systems have been discovered by the theory of inverse scattering transforms.

In this thesis we review and examine the usefulness of two methods as a tool for identifying integrable Hamiltonian systems.

The first method, due to Bertrand [2], involves a direct search for integrals of motion, making the assumption that these constants are polynomials in the velocities (momenta) [3,4,5,6]. The coefficients of the polynomial are functions of the coordinates and are obtained by solving partial differential equations. This direct method is powerful in the simpler cases, when the integrals of motion are first or second order polynomials in the momenta.

The second method, associated to the name of Painlevé, has been used in order to investigate integrability of nonlinear first and second order ordinary differential equations. The interest in this method has been kindled by the conjecture of Ablowitz, Ramani, and Segur (ARS) [7,8,9] who related integrability to the singularity structure through the Painlevé property of the solutions, i.e. the solutions possess movable singularities in the complex time plane, which are poles. The ARS conjecture has led to the identification of two-dimensional integrable Hamiltonian systems.

In the next chapter, we review a description of the two approaches in order to identify the integrable cases: the Painlevé property and the direct search of the integral of motion. In chapter 3 we are devoted to the study of integrability of the Hénon-Heiles hamiltonian. An integrable case of Toda lattice [10,11] will be studied in chapter 4. Conclusion and outlook will be given in the last chapter.