Journal of Research Methodology, Volume 20, Number 1 (January-April 2007)

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Equalizing Gain Scores: Relative Gain Score vs. Cumulative Relative Gain Score

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ABSTRACT

This paper presented three common types of learning and development curves. The mathematical models of these learning curves were also presented. Next, the issue of measuring gain along the learning curves was discussed. In the past, a Relative Gain Score (RGS)—defined as "$(Y_2 - Y_1)/(F - Y_1) \times 100$" where Y_2 is post-evaluation score, Y_1 is pre-evaluation score, and F is full score of the evaluation, was formulated to compensate for difficult gain near the end of the learning curves. It was shown in this paper that RGS equalizes the gains at equal time intervals of the first common type of learning and development curves only. The present research paper presented an approach—the Cumulative Relative Gain Score (CRGS)—that equalizes the gains at equal time intervals for all three common types of learning and development curves. The mathematical development of the Cumulative Relative Gain Score (CRGS) was described in this paper. The RGS and CRGS were compared and contrasted. Finally, the application of the two indexes to empirical data collected from two past research studies was presented.

Introduction

Human learning or development as a function of time is not linear (see Figure 1). Instead, it is curvilinear. The curve could be in the form of decrease in acceleration as shown in Figure 2, or first increase in acceleration and then decrease in acceleration (S-shape) as shown in Figures 3 and 4. Note that in Figure 3, the lower portion of the S-shape is smaller than the upper portion while in Figure 4, the two portions are quite symmetrical.

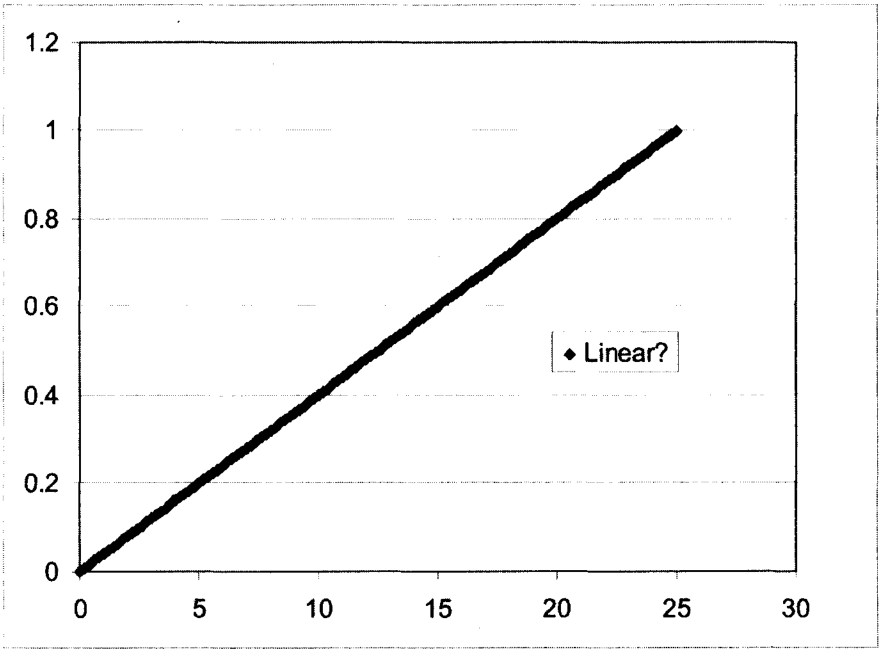

Figure 1 Is learning a linear function of time?

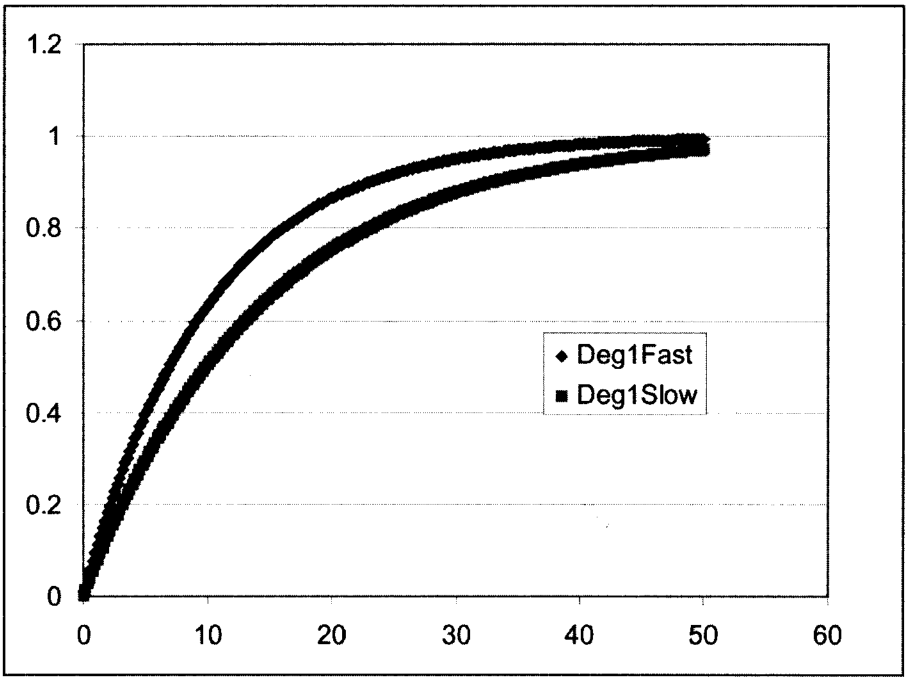

Figure 2 Learning curves modeled by $Y=1-e^{-aX}$. In the upper curve a = 0.1 and in the lower curve a = 0.07.

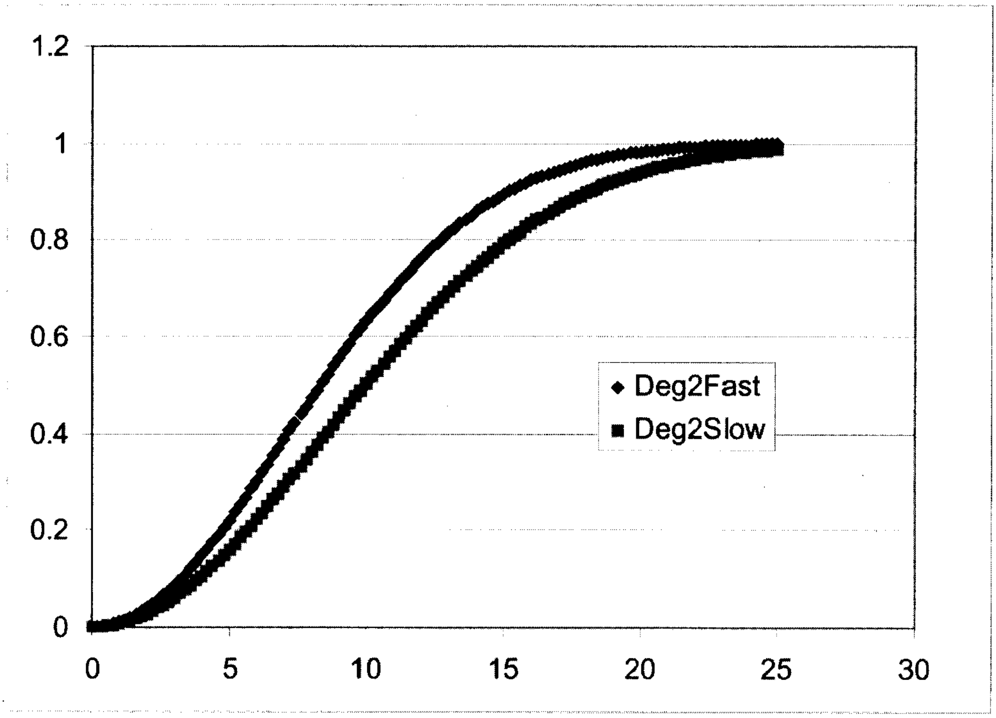

Figure 3 Learning curves modeled by $Y = 1-e^{-(aX)^2}$. In the upper curve a = 0.1 and in the lower curve a = 0.07.

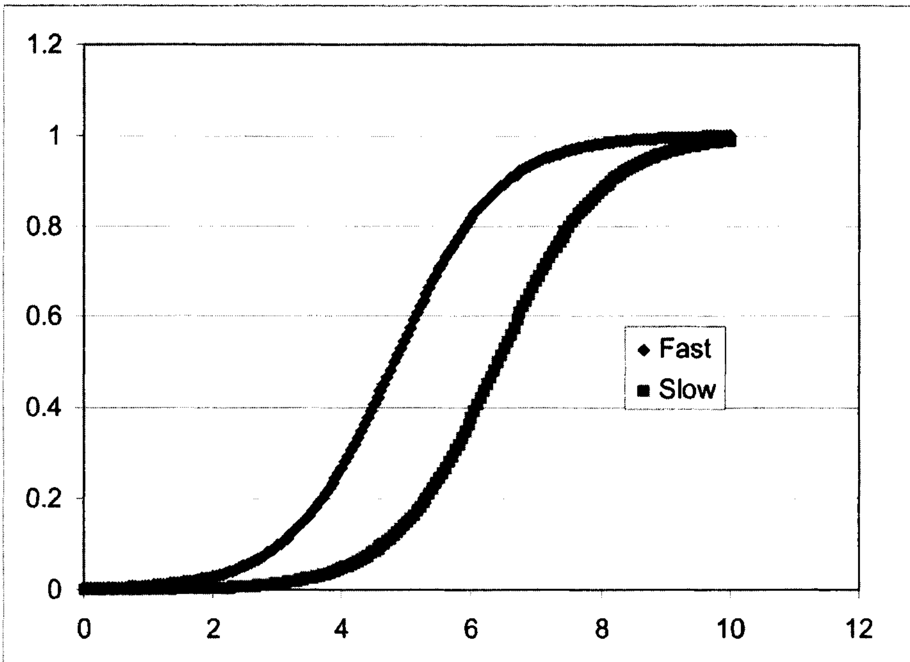

Figure 4 Learning curves modeled by $Y = \frac{1}{1+e^{-aX+b}}$. In the upper curve a = 1.25 and b = 6. In the lower curve a = 1.25 and b = 8.

Examples of curves like Figure 2 appear in Brown and Saks (1985, p. 124), in Hulse, Deese and Egeth (1975, pp. 24-26), in Haber and Fried (1975, pp. 172 & 185), and in LeFrancois (1995, p. 39). Examples of curves like Figure 3 appear in Restle and Greeno (1970, p. 22), and in Zajonc and Markus (1975, p. 77). Examples of curves like Figure 4 appear in Brown and Saks (1985, p. 121), and in Hilgard and Bower (1966, p. 153). Mayer (1999, p. 42) also shows a multi-stage S-shape learning curve.

Curves like Figure 2 could be modeled by a first-degree sigmoid equation:

$$Y = 1 - e^{-aX} \tag{1}$$

where
Y is a measure of learning or development
X is a measure of time
a indicates the speed the curve approaches maximum

Curves like Figure 3 could be modeled by a second-degree sigmoid equation:

$$Y = 1 - e^{-(aX)^2} \tag{2}$$

where Y is a measure of learning or development

X is a measure of time

a indicates the speed the curve approaches maximum

According to Zajonc and Markus (1975), Equation 2 could be used to model intellectual development. For spatial intellectual development, the value of coefficient a = 0.1 whereas for verbal intellectual development, a = 0.07 which indicates a slower development.

Curves like Figure 4 could be represented by IRT models that provide an S-shaped ogive (see Embretson & Reise, 2000, p. 298). One such model is a logistic function shown in Equation 3:

$$Y = \frac{1}{1 + e^{-aX+b}} \tag{3}$$

where Y is a measure of learning or development

X is a measure of time

a indicates the speed of the curve as it rises

b indicates the position of the S-shape along the X axis

The many implications of these three types of curves on the study of learning and development were thoroughly discussed in Archwamety, Tangdhanakanond, and Pitiyanuwat (2005). The most important of these implications is probably the implication on measuring learning and development gains at various intervals in time. This is discussed in the following section.

Learning Curves and Measuring Gains

If learning or development Y were a linear function of time X, the use of $Y_2 - Y_1$ (the traditional gain score) as the indicator of gain at every time interval would have been all right. However, learning or development Y is NOT a linear function of time X. The function is curvilinear (most likely S-shaped) as shown in the introduction of this paper. If we take gain $Y_2 - Y_1$ at every equal interval from the beginning to the end of the curve, we will notice that the gain becomes smaller and smaller towards the end of the curve. This means an accomplished learner will have difficulty making gain! Bereiter (1963) [as cited in Embretson & Reise, 2000] emphasized this point as one of the many problems of the traditional gain score $Y_2 - Y_1$. Should we not then give higher and higher weights to gains made towards the end of the learning curve? In many Olympic competitions a higher level of difficulty receives more weight in scoring.

Improving the Traditional Gain Score

Problems of traditional gain score ($Y_2 - Y_1$) were noted in many studies (see, e.g., Bereiter, 1963; Cronbach and Furby, 1970; Embretson & Riese, 2000; Rogosa, Brandt, & Zimowski, 1982). Several improvements (for example, residual change measures) have been suggested but remained unsatisfactory (see Embretson & Riese, 2000, p. 296; Rogosa, Brandt, & Zimowski, 1982). A more promising recent attempt to weight a traditional gain score throughout the range of the learning curve was proposed by Kanjanawasee (1989) [see also Ruengtrakul, 2002]. The weighted gain score is known as the "Relative Gain Score (RGS)" which is defined as:

$$\text{Relative Gain Score} = \frac{Y_2 - Y_1}{F - Y_1} \times 100 \qquad (4)$$

where

Y_2 = Score of post-evaluation

Y_1 = Score of pre-evaluation

F = full score of the evaluation

Note that $F - Y_1$ becomes smaller and smaller as the gain score is taken towards the end of the learning curve—thus giving more and more weight to $Y_2 - Y_1$. This is an improvement over the traditional gain score considering the nature of the learning curve. Relative Gain Score is applicable to either a single learner (corresponding to only one learning curve shown in Figure 2, 3 or 4), or a group of learners (corresponding to both learning curves shown in Figure 2, 3 or 4 as well as other curves imagined between the two curves). In the case of a group of learners, we refer to gain score as "M_2-M_1" (Mean2 - Mean1, or *"group gain score"*) rather than Y_2-Y_1. A "group learning curve" would be one imagined to be somewhere between the two curves shown in Figure 2, 3, or 4.

Kanjanawasee's Relative Gain Score (RGS) equalizes the gain throughout the learning/developmental curve represented by Equation (1) [$Y = 1 - e^{-aX}$]. See the proof of this in Appendix A. However, RGS does not equalize the gain throughout the learning/developmental curves represented by Equations (2) and (3). The present paper presents an approach that will equalize gain throughout the learning/developmental curves represented by all three equations. The resulting weighted gain score from this new approach will be called the "Cumulative Relative Gain Score (CRGS)."

A New Approach in Equalizing Traditional Gain Scores

Let x be a point on the x-axis in Figure 2 and Δx be a very small interval to the right of it. Then at x there corresponds a $\Delta y = y_2 - y_1$ or a very small gain score (in the traditional sense). Note that with fixed Δx throughout the x-axis, Δy is not fixed but varies according to the function $Y = 1 - e^{-aX}$. Now, at any point x on the x-axis, dividing the corresponding Δy (or y_2-y_1) by dy/dx the derivative of y with respect to x, gives the constant Δx = Δy/(dy/dx). This process equalizes the originally unequal infinitesimal gain score y_2-y_1.

The derivative of Y at any x = $d(1 - e^{-aX})/dx$

$$= ae^{-aX}$$

$$= a(1-Y) \qquad (5)$$

Thus, dividing each of the original unequal infinitesimal gain scores y_2-y_1 by $a(1-y_1)$ would equalize the gain score y_2-y_1 throughout the x-axis. Since a is constant for a particular learning curve $1-e^{-aX}$, dividing y_2-y_1 by $1-y_1$ instead of $a(1-y_1)$ would also equalize the gain score y_2-y_1 throughout the x-axis. It is most interesting to also note here that "dividing y_2-y_1 by $1-y_1$" in this infinitesimal situation is precisely Kanjanawasee's Relative Gain Score (RGS) when the maximum raw score is unitized (maximum raw score rescaled to 1).

For the learning curve represented by "$Y = 1 - e^{-(aX)^2}$ (Figure 3)," the derivative of Y is not "$a(1-y_1)$," but is:

$$d(1 - e^{-(aX)^2})/dx = 2a^2 X e^{-(aX)^2}$$

$$= 2a^2 X (1 - Y) \qquad \text{since } e^{-(aX)^2} = 1 - Y$$

$$= 2a^2 \frac{\sqrt{-\ln(1-Y)}}{a} . (1 - Y) \qquad \text{by expressing } X \text{ in terms of } Y$$

$$= 2a\sqrt{-\ln(1-Y)} . (1 - Y) \tag{6}$$

Thus, in the case of learning curve represented by $Y = 1 - e^{-(aX)^2}$, (Figure 3), dividing each of the original unequal infinitesimal gain scores y_2-y_1 by $2a\sqrt{-\ln(1-y_1)} . (1-y_1)$, not $a(1-y_1)$, would equalize the gain score y_2-y_1 throughout the x-axis. Since $2a$ is a constant for a particular learning curve $Y = 1 - e^{-(aX)^2}$; dividing y_2-y_1 by $\sqrt{-\ln(1-y_1)} . (1-y_1)$ would also equalize the gain score y_2-y_1 throughout the x-axis. It is clear that in this case the Relative Gain Score $(y_2-y_1)/(1-y_1)$ does not equalize the traditional gain score but "$(y_2-y_1)/(\sqrt{-\ln(1-y_1)}\ (1-y_1))$" does. Note the negative natural log. Since the full score of y is one, "$1 - y_1$" is less than one and its natural log is negative. The negative of this natural log is therefore positive—making the eventual square root possible.

For the learning curve represented by "$Y = \frac{1}{1+e^{-aX+b}}$ (Figure 4)," the derivative of Y is:

$$d\left(\frac{1}{1+e^{-aX+b}}\right)/dx = \frac{ae^{-aX+b}}{(1+e^{-aX+b})^2}$$

$$= a.\frac{e^{-aX+b}}{(1+e^{-aX+b})}\cdot\frac{1}{(1+e^{-aX+b})}$$

$$= a.(1-Y).Y$$

$$= a.Y.(1-Y) \qquad (7)$$

Thus, in the case of learning curve represented by $Y = \frac{1}{1+e^{-aX+b}}$ (Figure 4), dividing each of the original unequal infinitesimal gain scores y_2-y_1 by "$a.y_1.(1 - y_1)$" would equalize the gain score y_2-y_1 throughout the x-axis. As in the previous cases, the constant "a" is not needed for equalization. It is interesting to note that the derivative of $\frac{1}{1+e^{-aX+b}}$ at "$a = 1$" is the same as "the product of the vertical line above $(1 - y_1)$ and the vertical line below (y_1) the point on the learning curve corresponding to the point x_1 where the derivative is taken."

To summarize, when attempting to equalize infinitesimal gain score y_2-y_1, divide it by "$1 - y_1$" if the learning curve is modeled by "$Y = 1-e^{-aX}$ (Figure 2)," divide by

"$\sqrt{-\ln(1-y_1)}.(1 - y_1)$" if the learning curve is modeled by "$Y = 1-e^{-(aX)^2}$ (Figure 3)," and divide by "$y_1.(1 - y_1)$" if the learning curve is modeled by "$Y = \frac{1}{1+e^{-aX+b}}$ (Figure 4)."

Larger Than Infinitesimal Gain. The above discussion applies to infinitesimal gain score "y_2-y_1." To equalize gain score "Y_b-Y_a" larger than infinitesimal, taking the definite integral of the infinitesimal "$\Delta y = y_2-y_1$" is needed.

For the case of learning curve modeled by "$Y = 1-e^{-aX}$ (Figure 2)," the equalized "Y_b-Y_a" is:

$$\Delta y_1/(1-y_1) + \Delta y_2/(1-y_2) + \ldots + \Delta y_n/(1-y_n) = \int_a^b \frac{1}{1-Y}\,dY$$

$$= -\ln(1-Y)\Big]_a^b$$

$$= \ln(1-Y_a) - \ln(1-Y_b) \qquad (8)$$

where n goes from 1 to infinity, a is the beginning of Y and b the end of Y. The expression in Equation (8) is the first of three forms of "Cumulative Relative Gain Score (CRGS)." It is, in this case, a series of infinitesimal RGSs strung together. Compared with a RGS = "$\frac{Y_b-Y_a}{1-Y_a} = \Delta y_1/(1-y_1) + \Delta y_2/(1-y_1) + \ldots + \Delta y_n/(1-y_1)$" where Δy_1 is at Y_a and Δy_n is at Y_b(note the constant y_1 in the denominator), it is clear that a CRGS = "$\Delta y_1/(1-y_1) + \Delta y_2/(1-y_2) + \ldots + \Delta y_n/(1-y_n) = \ln(1-Y_a) - \ln(1-Y_b)$" will have a larger value since the y in the denominator keeps increasing. Thus, Kanjanawasee's formula RGS = $\frac{Y_b-Y_a}{1-Y_a}$ applied to a gain that is larger than infinitesimal will tend to underestimate the real relative gain.

For the case of learning curve modeled by "$Y = 1 - e^{-(aX)^2}$ (Figure 3)," the equalized "$Y_b - Y_a$" is:

$$\Delta y_1 / [\sqrt{-\ln(1-y_1)}.(1-y_1)] + \Delta y_2 / [\sqrt{-\ln(1-y_2)}.(1-y_2)] + \ldots$$

$$\ldots + \Delta y_n / [\sqrt{-\ln(1-y_n)}.(1-y_n)]$$

$$= \int_a^b \frac{1}{\sqrt{-\ln(1-Y)}} \frac{1}{1-Y} dY$$

$$= 2\sqrt{-\ln(1-Y)}\,\Big]_a^b$$

$$= -2\sqrt{-\ln(1-Y_a)} + 2\sqrt{-\ln(1-Y_b)} \qquad (9)$$

Equation (9) is the second of three forms of "Cumulative Relative Gain Score (CRGS)."

For the case of learning curve modeled by "$Y = \dfrac{1}{1+e^{-aX+b}}$ (Figure 4)," the equalized "$Y_b - Y_a$" is:

$$\Delta y_1/[y(1-y_1)] + \Delta y_2/[y(1-y_2)] + \ldots + \Delta y_n/[y(1-y_n)] = \int_a^b \frac{1}{Y(1-Y)} dY$$

$$= -\ln\left(\frac{1-Y}{Y}\right)\Bigg]_a^b$$

$$= \ln\left(\frac{1-Y_a}{Y_a}\right) - \ln\left(\frac{1-Y_b}{Y_b}\right) \qquad (10)$$

Equation (10) is the last of three forms of "Cumulative Relative Gain Score (CRGS)."

Summarizing the New Approach in Equalizing Traditional Gain Scores. The new approach in equalizing the traditional gain score is summarized as shown below:

Step 1. Differentiate the learning/development curve equation with respect to X.

Step 2. Convert Variable X in the expression obtained from Step 1 into Variable Y using the learning/development curve equation.

Step 3. Divide the infinitesimal gain dY by the expression in Y obtained from Step 2 and take a definite integral from $Y = a$ (first evaluation score) to $Y = b$ (second evaluation score).

The definite integral taken in Step 3 is the main reason behind naming the resulting index the "Cumulative" Relative Gain Score.

The Roles of the Constants *"a"* and *"2 a"* in Equations (5), (6), and (7)

Although, as mentioned previously, the constants "a" and "2 a" in Equations (5), (6), and (7) are not needed in equalizing gain scores within the same learning curve, each is needed in comparing gain scores across different learning curves of the same type. Take the two learning curves in Figure 2 as an example. An infinitesimal equalized gain score at any point "$(y_2-y_1)/(1 - y_1)$" without considering the constant "a" would be the same for both curves. However, if the constant "a" is considered, an infinitesimal equalized gain score "$(y_2-y_1)/[a.(1 - y_1)]$" for the lower curve ($a = 0.07$) will be larger than the upper curve ($a = 0.1$). Thus, in the present system, slow growing learning curves receive more weight that faster ones—as they should.

Another point is worth noting. Since the parameter "a" is in general less than 1.0, an equalized relative gain score is generally larger when its value is known and used than when its value is unknown or not used, or assumed equal to 1.0. This indicates that when the parameter "a" is not considered, the equalized gain computed will tend to be on the conservative side.

Application of the Relative Gain Scores (RGS) and Cumulative Relative Gain Scores (CRGS) in Curriculum-Based Measurement (CBM) Data

Does an equalized relative gain score as outlined above have any practical application in existing research data? Curriculum-based measurement for writing (W-CBM) data from Malecki and Jewell (2003) provide an application example. The researchers collected six types of W-CBM scores from first graders to eighth graders in the Fall semester as well as Spring semester providing 16 data points for each W-CBM measure. Two of these six measures—Percentage of Words Spelled Correctly (%WSC) and Percentage of Correct Writing Sequences (%CWS), are directly applicable to the present paper since the maximum score is 100% or a full 1.0. The graph of development for each of them is displayed in Figure 5 and Figure 6, respectively. The fit between the graph and the learning curve modeled by "$Y = 1 - e^{-aX}$ (Figure 2)," is also shown. The coefficient a = 0.65 was used for the curve fitting in Figure 5, and a = 0.37 was used for the curve fitting in Figure 6—reflecting faster growth in %WSC.

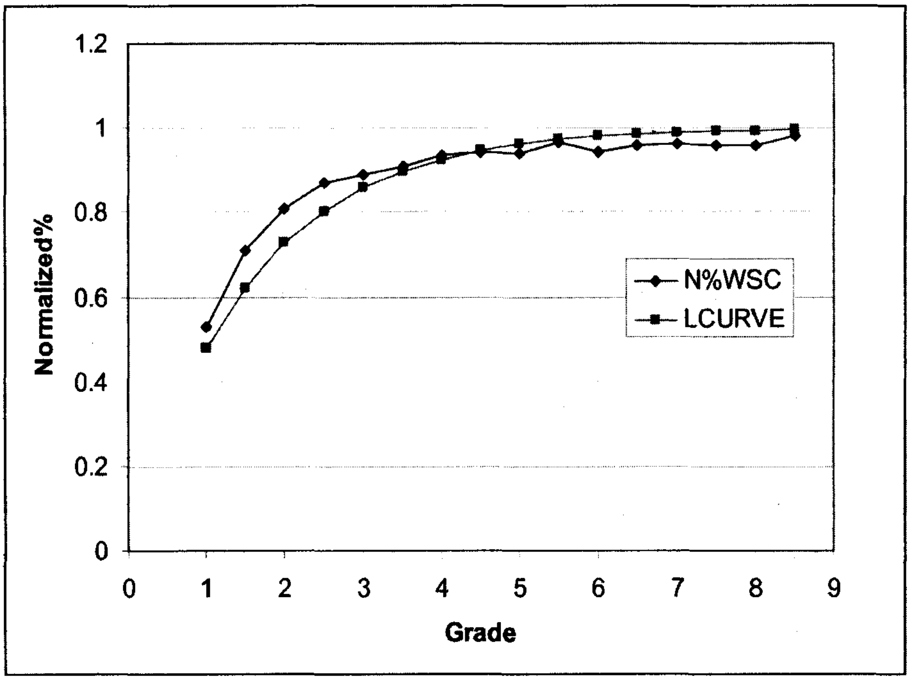

Figure 5 Learning curve (LCURVE) is modeled by $Y=1-e^{-aX}$ with a = 0.65. N%WSC is %WSC rescaled to minimum = 0 and maximum = 1 (normalized to unity).

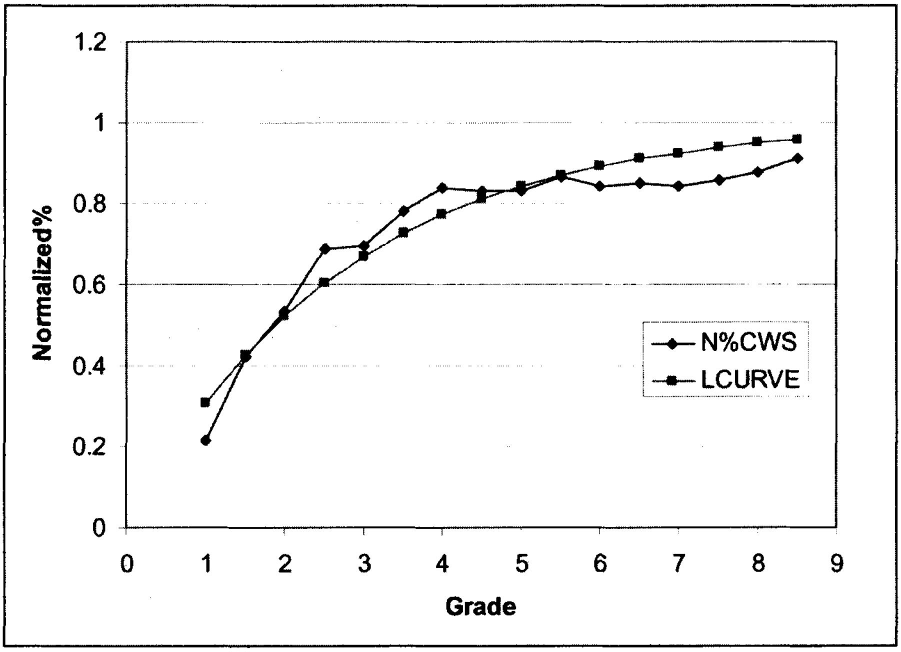

Figure 6 Learning curve (LCURVE) is modeled by $Y=1-e^{-aX}$ with a = 0.37. N%CWS is %CWS rescaled to minimum = 0 and maximum = 1 (normalized to unity).

The first three columns of Table 1 presents the numerical data used in constructing the graph shown in Figure 5. Note that both the actual data points (for the non-smooth curve) and the idealized data points (for the smooth curve) are shown. Kanjanawasee's Relative Gain Scores (RGS) and the present paper's Cumulative Relative Gain Scores (CRGS) are also given for adjacent data points at various steps for the smooth curve. Note the equalized gain computed for adjacent data points at various levels or steps.

Table 1

First through Eighth Grade Growth in Percentage of Words Spelled Correctly (%WSC) from the Malecki and Jewell (2003) study, Learning Curve(LCurve) Modeled by $Y = 1 - e^{-aX}$ with a = 0.65 and Y Rescaled to a Maximum of 100, Relative Gain Scores (RGS) and Cumulative Relative Gain Scores (CRGS) Computed at 1, 2 and 4 Steps Apart

Grade	%WSC	LCurve	RGS(LCurve)			CRGS(LCurve)		
			1-step	2-step	4-step	1-step	2-step	4-step
1.0	52.9	47.8						
1.5	70.9	62.3	27.7			32.5		
2.0	80.9	72.7	27.7	47.8		32.5	65	
2.5	86.9	80.3	27.7			32.5		
3.0	89.0	85.8	27.7	47.8	72.7	32.5	65	130
3.5	90.9	89.7	27.7			32.5		
4.0	93.6	92.6	27.7	47.8		32.5	65	
4.5	94.2	94.6	27.7			32.5		
5.0	94.1	96.1	27.7	47.8	72.7	32.5	65	130
5.5	96.8	97.2	27.7			32.5		
6.0	94.2	98.0	27.7	47.8		32.5	65	
6.5	95.9	98.5	27.7			32.5		
7.0	96.1	98.9	27.7	47.8	72.7	32.5	65	130
7.5	95.9	99.2	27.7			32.5		
8.0	95.7	99.4	27.7	47.8		32.5	65	
8.5	98.0	99.6	27.7			32.5		

Note. The RGS gain of 27.7 or CRGS gain of 32.5 is for half a grade gain. The RGS gain of 47.8 or CRGS gain of 65 is for one grade to the next grade gain. The RGS gain of 72.7 or 130 is for gain two grades apart.

Application of the Relative Gain Scores (RGS) and Cumulative Relative Gain Scores (CRGS) in Student Learning and Developmental Data at Darunsikkhalai School

Tangdhanakanond, Pitiyanuwat, and Archwamety (2005, 2006*a*, 2006*b*) reported research studies conducted at an experimental school in Bangkok--Darunsikkhalai School, in which students were taught under a full-scale constructionist approach. The students were evaluated three times over a nine-week learning period on academic outcomes (mathematics and the Thai language) and on personal desirable characteristics outcomes (five disciplines and four quotients). To compare the application of RGS and CRGS to these outcome data, it was assumed that all of the learning/development curves are of the first type [see Equation (1) and Figure 2]. Gain scores of each student from Time 1 evaluation to Time 2 evaluation as well as gain scores from Time 2 to Time 3 were obtained. These traditional gain scores were then converted into Relative Gain Scores (RGS) and Cumulative Relative Gain Scores (CRGS) for comparison. The RGS and CRGS averages were as shown in Table 2. Note that the CRGS is consistently larger than its RGS counterpart.

Table 2

Time 1-2 and Time 2-3 Means of Relative Gain Scores (RGS) and Cumulative Relative Gain Scores (CRGS) of Students' Learning Evaluated at Three Points in Time (N = 12)

Students' learning	Time 1-2		Time 2-3	
	RGS	(CRGS)	RGS	(CRGS)
1. Academic achievements				
1.1 Mathematics				
1.1.1 calculating and problem solving skill (CAL)	35.39	(47.78)	36.57	(54.41)
1.1.2 data presentation and analytical skill (DAT)	27.05	(35.55)	39.46	(59.82)
1.2 Thai				
1.2.1 listening skill (LIS)	18.88	(23.77)	63.04	(167.07)
1.2.2 speaking skill (SPE)	13.47	(16.11)	54.13	(95.97)
1.2.3 reading skill (REA)	22.45	(27.63)	21.83	(29.80)
1.2.4 writing skill (WRI)	29.00	(39.60)	37.07	(64.41)
2. Desirable characteristics				
2.1 Five disciplines				
2.1.1 personal mastery (PM)	7.15	(10.81)	48.60	(101.76)
2.1.2 mental model (MM)	15.73	(21.53)	24.00	(40.72)
2.1.3 shared vision (SV)	19.46	(23.54)	24.69	(31.49)
2.1.4 team learning (TL)	13.75	(24.02)	36.20	(65.61)
2.1.5 systems thinking (ST)	23.73	(29.51)	27.53	(39.62)
2.2 Four quotients				
2.2.1 emotional quotient(EQ)	7.10	(11.87)	24.83	(32.79)
2.2.2 adversity quotient(AQ)	17.98	(20.64)	19.42	(26.10)
2.2.3 technology quotient(TQ)	10.98	(14.01)	52.42	(109.36)
2.2.4 moral quotient(MQ)	12.75	(17.91)	41.75	(74.38)

Relative Gain Scores vs. Cumulative Relative Gain Scores

As shown earlier, Kanjanawasee's Relative Gain Score (RGS) equalizes the gain throughout the learning curve represented by Equation (1) [$Y = 1-e^{-aX}$]--so does the present paper's Cumulative Relative Gain Score (CRGS). What, then, is the difference between the two indexes?

There are two basic differences between the GRS and the CRGS when applied to a learning curve represented by Equation (1) [$Y = 1-e^{-aX}$]. First, the RGS is not "additive" but the CRGS is. RGS over an interval plus RGS over an adjacent interval of equal size does NOT equal the RGS taken over both intervals at once. Note in Table 1 that the RGS for Grade 1 to Grade 1.5 is 27.7 and the RGS for Grade 1.5 to Grade 2 is also 27.7. However, the RGS for Grade 1 to Grade 2 is 47.8 rather than "27.7 + 27.7 = 55.4." The CRGS, on the other hand, is "additive." Note in Table 1 that the CRGS for Grade 1 to Grade 1.5 is 32.5 and the CRGS for Grade 1.5 to Grade 2 is also 32.5. Moreover, the CRGS for Grade 1 to Grade 2, unlike the RGS counterpart, is 32.5 + 32.5 = 65. The CRGS is additive while the RGS is not. The additivity property of a measurement scale was discussed in Coombs, Dawes, and Tversky (1970, p. 9). The second difference between CRGS and RGS is that CRGS is consistently larger (see Table 1).

In the case of learning curves represented by Equation (2) [$Y = 1-e^{-(aX)^2}$] or Equation (3) [$Y = \frac{1}{1+e^{-aX+b}}$], the CRGS equalizes the gain throughout the learning curve while the RGS does not. Therefore, in such a case, if equalization of gain is needed, the CRGS should probably be used instead of RGS.

Discussion

One of the drawbacks of the traditional gain score approach is the overlooking of the ceiling effect. In other words, for equal intervals toward the end of a learning curve, the gain becomes smaller and smaller. This means the accomplished learner will have difficulty making gain. The Relative Gain Score approach by Kanjanawasee (1989) has overcome this drawback. However the RGS equalizes the gain throughout the learning curve for only one of the three common types of learning/development curves. The present paper has taken a further step to equalize the gain throughout the learning curve for all three common types of learning/development curves through the use of the Cumulative Relative Gain Score (CRGS).

For the first type of learning curves [see Equation (1) and Figure 2], both the RGS and CRGS seem legitimate in equalizing gain. The application of both indexes to actual data from Malecki and Jewell (2003) was tried and shown in Table 1. The application of both indexes was also tried with actual data from Tangdhanakanond et al. (2005, 2006*a*, 2006*b*) and was as shown in Table 2. Further research should be conducted to evaluate how these indexes compare in determining statistical significant gains.

For the second or third type of learning curves [see Equations (2) and Figure 3 as well as Equation (3) and Figure 4], only CRGS seems legitimate in equalizing gain. Future research should be conducted on actual learning/development data to determine how sensitive CRGS is in detecting statistical significant gains compared with the traditional gain scores.

Another interesting point to consider is that although the theoretical learning/development curves assume continuous gain (see Figures 2, 3, and 4), actual data often show loss at certain intervals among many learners. Would the sensitivity of detecting statistical significant gain of either RGS or SRGS be affected if many learners show losses rather than gains over a learning interval?

The basic theoretical concept of CRGS has been formulated in the present paper. Only further research will be able to tell how well CRGS functions in the real world of data compared with traditional gain score. Only further research will be able to tell how well CRGS functions in the real world of data compared with RGS. It is hoped that a future paper would be able to provide some answers to those two questions.

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