References

1. Prusinkiewicz, P., and Lindenmayer, A. The Algorithmic Beauty of Plants New York: Springer-Verlag, 1990.
2. Chuai-aree, S. An Algorithm for Simulation and Visualization of Plant Shoots Growth. Degree of Master of Science Computational Science Department of Mathematics Faculty of Science Chulalongkorn University, 2000.
3. Aono, M., and Kunii, T.L. Botanical Tree Image Generation. IEEE Computer Graphics and Applications Vol.4 No.5 (May 1984): 10-34.
4. Kawaguchi, Y. A Morphological Study of the Form of Nature. Computer Graphics Vol.16 No.3 (July 1982): 223-232.
5. Marshall, R., Wilson, R., and Carlson, W. Procedure Models for Generating Three-Dimensional Terrain. Computer Graphics Vol.14 No.3 (July 1980).
6. Smith, A.R. Plants, Fractals, and Formal Languages. ACM Computer Graphics Vol.18 No.3 (July 1984): 1-10.
7. Tomlinson, P.B. Tree Architecture. American Scientist Vol.71 (March 1983).
8. Oppenheimer, P.E. Real Time Design and Animation of Fractal Plants and Trees. ACM Computer Graphics Vol.20 No.4 (August 1986): 55-64.
9. Prusinkiewicz, P., Lindenmayer, A., and Hanan, J. Developmental Models of Herbaceous Plants for Computer Imagery Purposes. ACM Computer Graphics Vol.22 No.4 (August 1988): 141-150.
10. Hammel, M.S., Prusinkiewicz, P., and Wywill, B. Modelling Compound Leaves Using Implicit Contours. Proceedings of CG International '92 (June 1992): 119-212.

11. Prusinkiewicz, P., Modeling and Visualization of Biological Structures. Proceeding of Graphics Interface '93 (May 1993): 128-137.

12. Kaino, K. Geometry of Folded Pattern of Veins and Origami Model of Digitate Leaves. FORMA No.9 (1994): 253-257.

13. Kobayashi, H., Kresling, B., and Vincent, J.F.V. The Geometry of Unfolding Tree Leaves. Proceedings of Royal Society Vol.265 (1998): 147-154.

14. Lintermann, B., and Deussen, O. A Modelling and User Interface for Creating Plants. The Eurographics Association Vol.17 No.1 (March 1998): 73-82.

15. Lursinsap, C., Sophatsathit, P., Siripant, S., and Shinagawa, Y. Simulation of Leaf Growth Based On A Rewriting System: A Unified Leaf Model. Advanced Virtual and Intelligent Computing (AVIC) Center Technical Report No.01.01.2000 (January 2000).

16. Kaino, K., Yajima, K., and Chiba, N. Origami Modeling Method of Leaves of Plants and CG Image Generation of Flower Arrangement. IEEE Computer Graphics (2000): 207-212.

17. Chuai-aree, S., Siripant, S., and Lursinsap, C. Animating Plant Growth in L-System by Parametric Functional Symbols. Proceeding of International Conference on Intelligent Technology 2000 (December 2000) : 135-143.

18. Rodkaew, Y., Siripant, S., Lursinsap, C., and Chongstitvatana, P. Generate Leaf Shapes Using L-system and Genetic Algorithms. Proceedings of the First International Conference on Intelligent Technologies (InTech '2000) (2000).

19. Prusinkiewicz, P., Mündermann, L., Karwowski, R., and Lane, B. The Use of Positional Information in The Modeling of Plants. ACM Computer Graphics SIGGRAPH 2001 (August 2001): 289-300.

20. Rodkaew, Y., Siripant, S., Lursinsap, C., and Chongstitvatana, P. An Algorithm for Generating Vein Images for Realistic Modeling of a Leaf. Proceedings of Computational Mathematics and Modeling (CMM2002) (2002).

21. Maierhofer, S., and Tobler, R.F. Creating of Realistic Plants Using Semi-Automatic Parameter Extraction From Photographs. Research Center for Virtual Reality and Visualization (VRVis) Publications and Technical Reports (2002).

22. Bloomenthal, J. Modeling the Mighty Maple. ACM Computer Graphics Vol.19 No.3 (July 1985): 305-311.

23. Viennot, X.G., Eyrolles, G., Janey, N., and Arquès, D. Combinatorial Analysis of Ramified Patterns and Computer Imagery of Trees. ACM Computer Graphics Vol.23 No.3 (July 1989): 31-40.

24. Chiba, N., and Ohkawa, S. Visual Simulation of Botanical Trees Based on Virtual Heliotropism and Dormancy Break. The Journal of Visualization and Computer Animation Vol.5 (1994): 3-15.

25. West, G.B., Brown, J.H., and Enquist, B.J. A General Model for the Origin of Allometric Scaling Laws in Biology. The Science Magazine (Science) Vol.276 (4 April 1997).

26. Karch, R., Schreiner, W., Neumann, F., and Neumann, M. Three-Dimensional Growth and Optimization of Arterial Tree Models. American Medical Informatics Association Annual Symposium (AMIA 1998) Poster.

27. Schreiner, W., Karch, R., Neumann, F., and Neumann, M. Optimization Targets for Computer Models of Blood Vessels. American Medical Informatics Association Annual Symposium (AMIA 1998) Poster.

28. Jirasek, C., and Prusinkiewicz, P. A Biomechanical Model of Branch Shape in Plants 1999 Western Computer Graphics Symposium (SKIGRAPH'99) (March 1999).

29. Reffye, (de) P., Edelin, C., Françon, J., Jaegger, M., and Puech, C. Plant Models Faithful to Botanical Structure and Development. ACM Computer Graphics Vol.22 No.4 (August 1988): 151-158.

30. Reffye (de) P. Modélisation de l'Architecture des Arbres Tropicaux par des Processus Stochastiques. Doctoral Degree in Computer Science University of Paris, Orsay, France, 1979.

31. Prusinkiewicz, P., Hammel, M.S., and Mjolsness, E. Animation of Plant Development. Proceedings of Computer Graphics SIGGRAPH '93 (August 1993): 351-360.

32. Hammel, M.S., and Prusinkiewicz, P. Simulating the Development of Fraxinus Pennsylvanica Shoots Using L-Systems. Proceedings of the Sixth Western Computer Graphics Symposium (March 1995): 49-58.

33. Hammel, M.S., and Prusinkiewicz, P. Visualization of Developmental Processes by Extrusion in Space Time. Proceedings of Graphics Interface'96: 246-258.

34. Steinberg, D., Sikora, S., Lattaud, C., Fournier, C., and Andrieu, B. Plant Growth Simulation in Virtual Worlds: Towards Online Artificial Ecosystems Proceedings of the 1st Workshop on Artificial Life Integration in Virtual Environments Lattaud C. ed., (September 1999): 19-25.

35. Wilson, H.D. "Vegetative Morphology II – The Leaf", website: http://www.csdl.tamu.edu/FLORA/Wilson/tfp/veg/tfplec3.htm

36. Virtual Classroom Biology, "The Microscopic World of Leaves", website: http://www-vcbio.sci.kun.nl/eng/virtuallessons/leaf/

37. Gille, U. "Analysis of Growth", website:

http://www.uni-leipzig.de/~vetana/growthe.htm

38. Gompertz, B. On the Nature of the Function Expressive of the Law of Human Mortality, and a New Mode of Determining the Value of Live Contengencies Philosophical Transactions Royal Society. Vol. 182 (1825): 513-585.

39. Verhulst, P.F. Notice sur la loi que la population suit dans son accroissement Correspondence Mathématique et Physique. Vol.10 (1838): 113-121.

40. Bertalanffy, L.v. Wachtum. In: Helmcke, J.G., Len-Gerken, H.v., and Starck, G. (Ed.) Hanndbuch der Zoologie. Berlin: W. de Gruyter, Bd.8, 10. Lieferung (1957): 1-68.

41. Brody, S. Bioenergetics and Growth. New York: Reinhold, 1945.

42. Richard, F.J. Aflexible Growth Curve for Empirical Use. Journal of Experimental Botany Vol.10 (1959): 290-300.

43. Janoschek, A. Das Reaktionskinetische Grundgesetz und seine Beziehungen zum Wachs-tums und Ertragsgesetz. Stat. Vjschr. Vol.10 (1957): 25-37.

44. Haykin, S. Neural Networks: A Comprehensive Foundation Second Edition. McMaster University Hamilton, Ontario, Canada: Prentice Hall International, Inc., 1999.

45. CGSC: "E.coli - Genetic Stock Center", website:

http://cgsc.biology.yale.edu/leafhelp.html

46. Chiba, N. "Computer Graphics Laboratory: Visual Simulation of Natural Phenomena and Traditional Arts", website:

http://www-cg.cis.iwate-u.ac.jp/lab/index-e.html

47. CPAI "Center for Plant Architecture Informatics", website:

http://www.cpai.uq.edu.au/

48. Gerald, C.F., and Wheatley, P.O. Applied Numerical Analysis Fifth Edition. California Polytechnic State University: ADDISON-WESLEY Publishing Company, 1994.

49. Greenworks "Home Page of The XFROG Modelling Software", website: http://www.greenworks.de/

50. Hammel, M.S. "BMV Publications", website: http://www.cpsc.ucalgary.ca/projects/bmv/papers/index.html

51. Hammel, M.S. "L-systems Software", website: http://www.cpsc.ucalgary.ca/projects/bmv/software.html

52. Hopkins, W.G. Introduction to Plant Physiology. Second Edition. The University of Western Ontario: John Wiley & Son, Inc., 1999.

53. Kurth, W. "Plant Modeling Group", website: http://www.uni-forst.gwdg.de/~wkurth/

54. Leaf Architecture Working Group. Manual of Leaf Architecture. Smithsonian Institution Washington, D.C., 1999.

55. Lee, C.W. "Horticulture Science Lab", website: http://www.ndsu.nodak.edu/instruct/chlee/plsc211/

56. Murray, J.D. Mathematical Biology. United States of America: Springer-Verlag Berlin Heidelberg Newyork, 1989.

57. Richardson, R. "Biology Classes (Biology)", website: http://scidiv.bcc.ctc.edu/rkr/Biology203/biology203.html

58. Richardson, R. "Biology Classes (Botany)", website: http://scidiv.bcc.ctc.edu/rkr/Botany110/botany.html

59. Watt, A. 3D Computer Graphics Second Edition. Instituto de Matemática Pura e Aplicada, Rio de Janeiro, Brasil and University of Sheffield, England: ADDISON-WESLEY Publishing Company, 1994.

60. กาญจนา สาลีติ๊ด, พฤกษศาสตร์ทั่วไป (กรุงเทพมหานคร: สำนักพิมพ์โอเดียนสโตร์, 2541) หน้า 97-115

61. ลิลลี่ กาวีต๊ะ, การเปลี่ยนแปลงทางสัณฐานและพัฒนาการของพืช (กรุงเทพมหานคร: สำนักพิมพ์ มหาวิทยาลัยเกษตรศาสตร์, มีนาคม 2546).

Appendix

Bezier Spline Curves

Bezier curves is widely used in computer graphics and computer-aided design. The curve is not really interpolating splines that it does not normally pass through all of the points. However, Bezier curves has the important property of staying within the polygon determined by the given points. In addition, Bezier spline curves have a nice geometric property in that in changing one of the points we change only one portion of the curve called a local effect. Finally, for the cubic splines just studied the points were given data points. For the two curves we study in this thesis the points in question are more likely control points that we select to determine the shape of the curve that we are working on.

For simplicity, we consider mainly the cubic version of the Bezier curves. We will express $y = f(x)$ in parametric form. The parametric form represents a relation between x and y by two other equations, $x = F_1(u)$, $y = F_2(u)$. The independent variable u is called the parameter. For example, the equation for a circle can be written with θ as the parameter as

$$x = r \cdot \cos(\theta), \tag{1}$$

$$y = r \cdot \sin(\theta). \tag{2}$$

When y and x are expressed in terms of a parameter u, $(x(u), y(u))$, $0 \le u \le 1$, defines a set of points (x, y), associated with the values of u.

Suppose we are given a set of control points, $p_i = (x_i, y_i)$, $i = 0, 1, \ldots, n$. (These points are also referred to as Bezier Points.) Figure A-1 is an example.

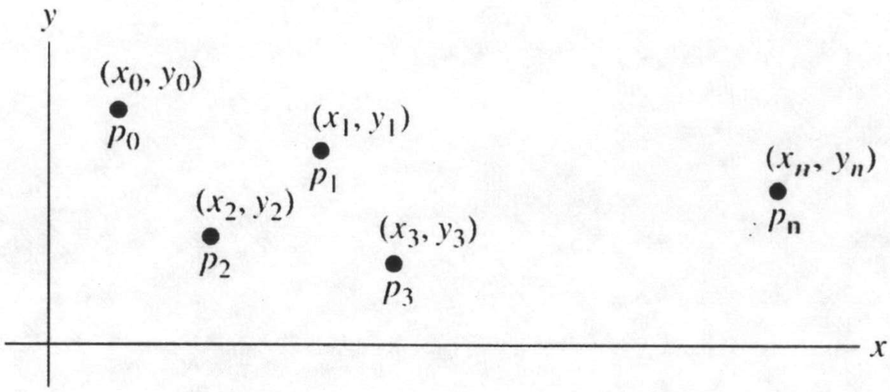

Figure A-1: Represent points on a 2D plane.

The points do not necessarily progress from left to right. We treat the coordinates of each point as a two-component vector,

$$p_i = \begin{pmatrix} x_i \\ y_i \end{pmatrix}. \tag{3}$$

The set of points, in parametric form, is

$$P(u) = \begin{pmatrix} X(u) \\ Y(u) \end{pmatrix}, \quad 0 \le u \le 1. \tag{4}$$

The m th-degree Bezier polynomial determined by n points (where $n = m + 1$) is given by

$$P(u) = \sum_{i=0}^{m} \binom{m}{i} (1-u)^{m-i} u^i p_i , \quad \text{where} \tag{5}$$

$$\binom{m}{i} = \frac{m!}{i!(m-i)!} \tag{6}$$

The preceding formula really represents two other scalar equations, one for X_i and the other for Y_i.

For $n = 6$, this would give the equation defined by six points p_0, p_1, p_2, p_3, p_4, and p_5:

$$P(u) = (1-u)^5 p_0 + 5(1-u)^4 u p_1 + 10(1-u)^3 u^2 p_2 + 10(1-u)^2 u^3 p_3 + 5(1-u)u^4 p_4 + u^5 p_5,$$

since, for $m = 5$ and $i = 0, 1, ..., 5$, we have $\binom{5}{0} = 1$, $\binom{5}{1} = 5$, $\binom{5}{2} = 10$, $\binom{5}{3} = 10$, $\binom{5}{4} = 5$, and $\binom{5}{5} = 1$. The preceding equation represents the pair of equations

$$X(u) = (1-u)^5 x_0 + 5(1-u)^4 u x_1 + 10(1-u)^3 u^2 x_2 + 10(1-u)^2 u^3 x_3 + 5(1-u)u^4 x_4 + u^5 x_5,$$
$$Y(u) = (1-u)^5 y_0 + 5(1-u)^4 u y_1 + 10(1-u)^3 u^2 y_2 + 10(1-u)^2 u^3 y_3 + 5(1-u)u^4 y_4 + u^5 y_5.$$

Observe that, if $u = 0$ then $X(0)$ is identical to x_0 and similarly for $Y(0)$. If $u = 1$, the point referred to is (x_5, y_5). As u takes on values between 0 and 1, a curve is traced that goes from the first point to the sixth point of the set. Ordinarily the curve will not pass through the central point of the six except if they are collinear then the curve is the straight line through them all. An example of the Bezier spline curve, which is constructed by six control points shows in Figure A-2.

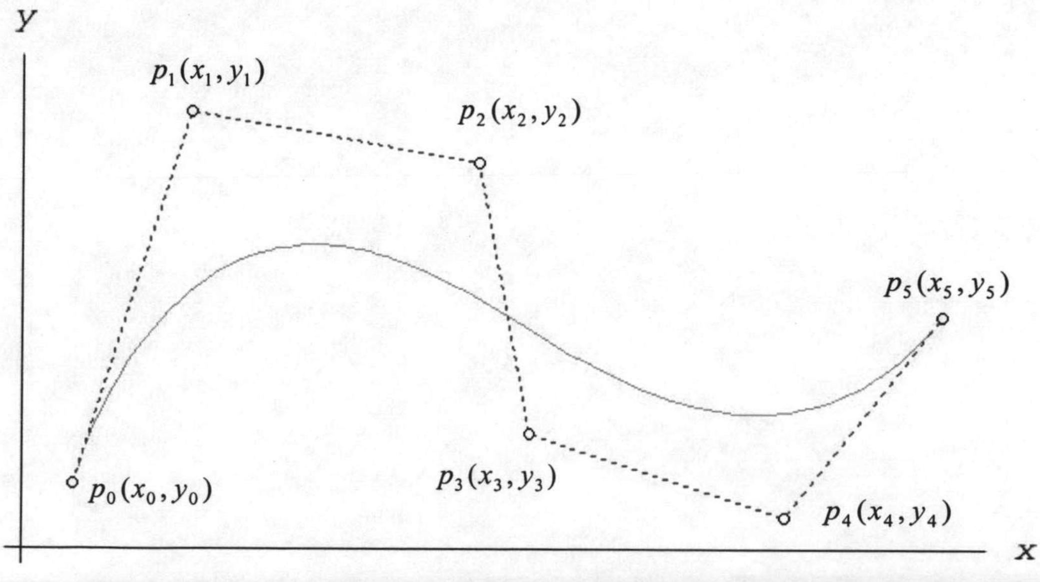

Figure A-2: An example of 5th degree of Bezier spline curve.

An algorithm for drawing a piece of a 5th degree of Bezier spline curve is shown below:

Algorithm of the 5th degree - Bezier Spline Curve

1. Given six control points, $p_i(x_i, y_i)$, $i = 0,...,5$
2. **FOR** $u = 0$ **TO** 1 STEP 0.01 **DO**
3. Compute
4. $X = (1-u)^5 x_0 + 5(1-u)^4 u x_1 + 10(1-u)^3 u^2 x_2 + 10(1-u)^2 u^3 x_3 + 5(1-u)u^4 x_4 + u^5 x_5$,
5. $Y = (1-u)^5 y_0 + 5(1-u)^4 u y_1 + 10(1-u)^3 u^2 y_2 + 10(1-u)^2 u^3 y_3 + 5(1-u)u^4 y_4 + u^5 y_5$.
6. Plot (X, Y).
7. **END FOR.**

Vitae

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