CHAPTER 3

REDUCTION OF SUNSPOT DATA

SUMMARY

Sunspot data taken with the 150 mm-Zeiss-Coude' refractor at the Bangkok Observatory are reduced. The Wolf's sunspot number and type of sunspot groups are found. The coordinates of sunspots are calculated and also approximately deduced by the use of solar grids prepared by the Fraunhofer Institute.

3.1. INTRODUCTION

The study of the detailed development of a sunspot group from observations cannot be made at a single observatory. It can be realized only by international cooperation, and the centre for such cooperation is Eidgenössische Sternwarte in Zürich. Every participating observatory is requested to send solar photographs and related data to the centre. These data include :-

(a) Date.

(b) Universal time.

(c) Number g of sunspot groups.

(d) Number f of individual spots.

(e) Types of sunspot groups, defined according to the Zürich classification.

(f) Image-quality classification.

(g) Heliographic latitude B and longitude L of the centre of the sunspot group.

All of this information can be obtained from observations with the

150 mm-Zeiss-Coudé' refractor at the solar observatory at Sukhumwit Road, Bang Kapi, Bangkok. Daily observations are being made to find Wolf's sunspot number on each day, to classify sunspot groups, to find the coordinates of sunspot groups, and to obtain images of good quality of every sunspot group appearing on the disk.

3.2. WOLF'S SUNSPOT NUMBER

The Wolf's sunspot number R is defined by

$$R = k(10g + f)$$

where g is the number of sunspot groups,

f is the number of individual spots,

k is the reduction factor which depends on the observer, on the method of counting the spots and of subdividing them into groups, on the size of the telescope and the magnification employed, and on the image quality.

Before the determination of reliable R numbers, each observer must find his reduction factor by comparing sunspot numbers on each day from his own observations with the standard sunspot numbers made at the Zürich Observatory. Counting can be based on the projected image or on direct observation through the eyepiece. The projected image is at the rear end of this Coude' refractor with an image size of about 150 mm. Direct visual observation is made at the front end of the telescope with the double eyepiece revolver for solar observation, containing a plane uncoated mirror, to reduce the intensity of radiation. The enlarging eyepiece found suitable for this purpose has a focal length of 40 mm, labelled by P 40. A neutral filter may be used to

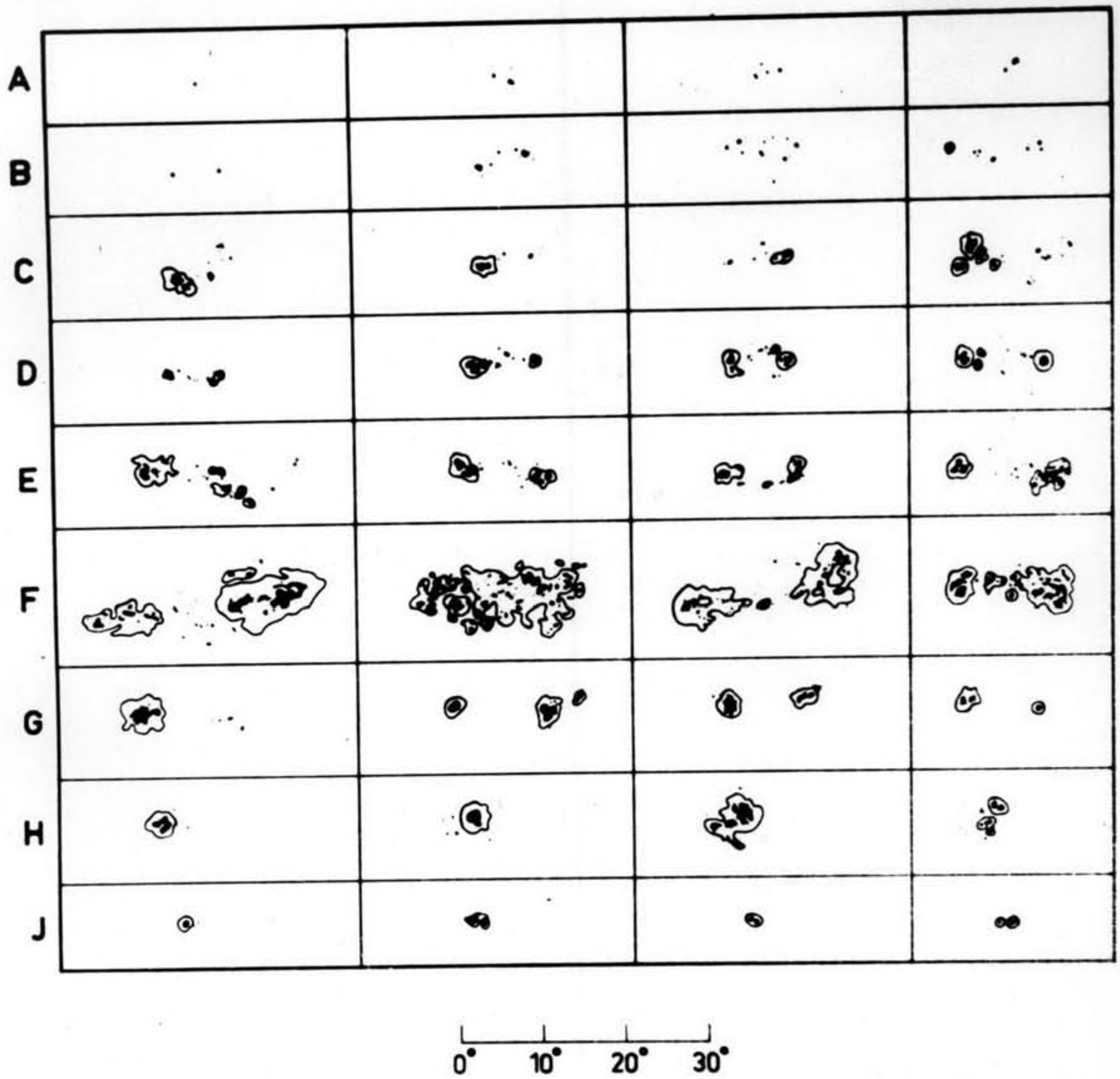


Figure 3.1.– Zürich classification of sunspot groups.

reduce the intense radiation if desired.

3.3. CLASSIFICATION OF SUNSPOT GROUPS

The classification of sunspot groups is made by comparison with the figures illustrating the individual types in classes A to J of the Zürich classification as shown on Figure 3.1.

3.4 CLASSIFICATION OF IMAGE QUALITY

Image quality is classified in five grades :

e	:	excellent;	p	:	poor;
g	:	good;	vp	:	very poor.
f	:	fair;			

3.5. DETERMINATION OF COORDINATES OF SUNSPOT GROUPS

3.5.1 Photographic observation

The position of the centre of a sunspot or a sunspot group can be found from the projected images or from whole disk photographs. Whole disk or first focus photographs are made at the front end of the telescope. A 35-mm Exacta camera is attached to the double eyepiece revolver for solar observation with the orange OG2 filter. In taking photographs of the solar disk using 35 mm Gevaert Duplo Pan Rapid roll film, the exposure time is 1/150 second. When the electronic guider for the telescope is stopped for a minute, the solar image drifts toward the west, and part of the east limb of the later image overlaps with the west limb of the former. The later image is then exposed on the same frame as the earlier image. The local time of taking the photograph is noted and changed to universal time by using the relationship:

Universal time = Local time - 7h (1)

The resulting image is shown on Figure 3.2. The line joining the intersections of the two circles on the plate is parallel to the north-south direction of the solar disk. Draw a line perpendicular to the north-south direction passing through the centre of the disks. This is the east-west direction and the west is on the side of the second image. With the optical path used in the Zeiss-Coudé refractor at the observatory, the north-south direction is reversed from that on the actual sun, as shown on Figure 3.2.

3.5.2 Determination of P, B_o, and L_o at the observed time

Before calculating the latitudes B and longitudes L of sunspot groups, the position angle P of the sun's axis of rotation, the heliographic latitude B_o and longitude L_o of the centre of the sun at the observed time must be known. These values are tabulated in the 1968 Astronomical Ephemeris (H.M. Stationery Office 1966) at 0h universal time for every day. Interpolation to the observed time is carried out using the most general interpolation formula, the Bessel interpolation formula:

$$f_p = f_o + p\delta_{\frac{1}{2}} + B_2(\delta_o^2 + \delta_1^2) + B_3\delta_{\frac{1}{2}}^3 + B_4(\delta_o^4 + \delta_1^4) + \ldots\ldots (2)$$

where f_p is the desired interpolated function;

f_o is the initial function;

p is the interpolating factor; when the tabular interval is one day or 24 hours, p can be found from the expression

$$p = \frac{\text{the observed universal time in hours}}{24} \qquad (2a);$$

$\delta_{\frac{1}{2}}$ is the first order difference;

δ_0^2, δ_1^2 are the second order differences;

$\delta_{\frac{1}{2}}^3$ is the third order difference;

δ_0^4, δ_1^4 are the fourth order differences;

B_2, B_3, B_4 are Bessel's coefficients.

Knowing f_o, p, and the differences, the first-order-difference correction $p\,\delta_{\frac{1}{2}}$ can be found from calculation, and the second-order-difference correction $B_2(\delta_0^2 + \delta_1^2)$, the third-order-difference correction $B_3\,\delta_{\frac{1}{2}}^3$, and the fourth-order-difference correction $B_4(\delta_0^4 + \delta_1^4)$ can be found from the interpolation tables: Table XV, Table XVI, and Table XVII in the 1968 Astronomical Ephemeris. f_p or P, B_o, L_o at the observed time are found in this way.

3.5.3 Calculation of B and L of a spot

Smart (1962) gives the following method for determining B and L:

From the whole disk image, find the position angle θ of the spot whose coordinates are to be determined. The position angle is measured from the north point of the disk towards the east to the spot. The radius r_o and the distance of the spot from the centre of the disk r are measured in any convenient unit. The semi-diameter S of the sun on the day of observation is found in the Astronomical Ephemeris. Then ρ_1, the angle between the direction of the sun's centre and the direction of the spot, both viewed from the earth is

$$\rho_1 = \frac{r}{r_o} S \qquad (3)$$

Let ρ be the angle at the centre of the sun between the direction of the spot and the direction of the earth, ρ can be found from

$$\sin(\rho + \rho_1) = \frac{\rho_1}{S} = \frac{r}{r_o} \qquad (4)$$

Then the heliographic latitude and longitude of the spot are determined from

$$\sin B = \sin B_0 \cos \rho + \cos B_0 \sin \rho \cos (P - \theta) \qquad (5)$$

$$\sin (L - L_0) = \sin \rho \sin(P - \theta) \sec B \qquad (6)$$

The following example is the reduction of observations on 6 February 1968. The whole disk photographs were taken at local time 09h 56 min as shown on Figure 3.3 and 10h 02 min as shown on Figure 3.2.

The count is made from the projected image, diameter about 150 mm, at the rear end of the telescope.

$$f = 47$$

$$g = 12$$

therefore $R = K(10 \times 12 + 47)$

$$= 167K$$

The reduction factor of the observer is 0.79

$$R = 167 \times 0.79$$

$$= 132$$

The image quality is fair.

Sample calculations are given in the Appendix 1.

3.5.4 Approximate practical procedure for obtaining B and L

The latitude and longitude of a spot or a sunspot group can be estimated by using the solar grids prepared by the Fraunhofer Institute. Fifteen grids have been constructed for the heliographic latitude B_0 of the centre of the sun from $+7^{\circ}$ to -7°. The solar image on the day of observation is printed at the same size as the grid. The north-south axis of the solar disk is then drawn. The selected grid for the latitude of the centre of the sun on the day of observation is placed

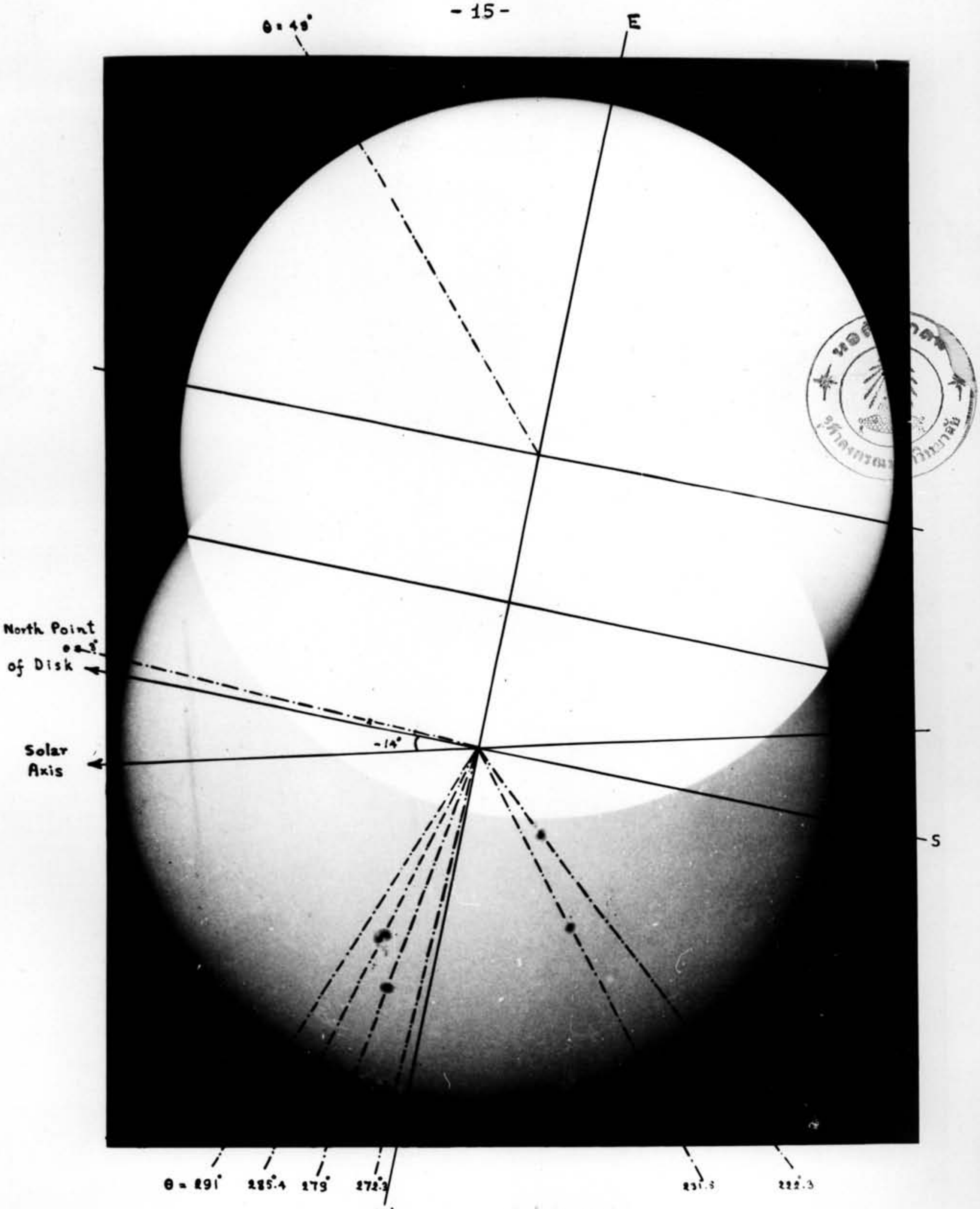
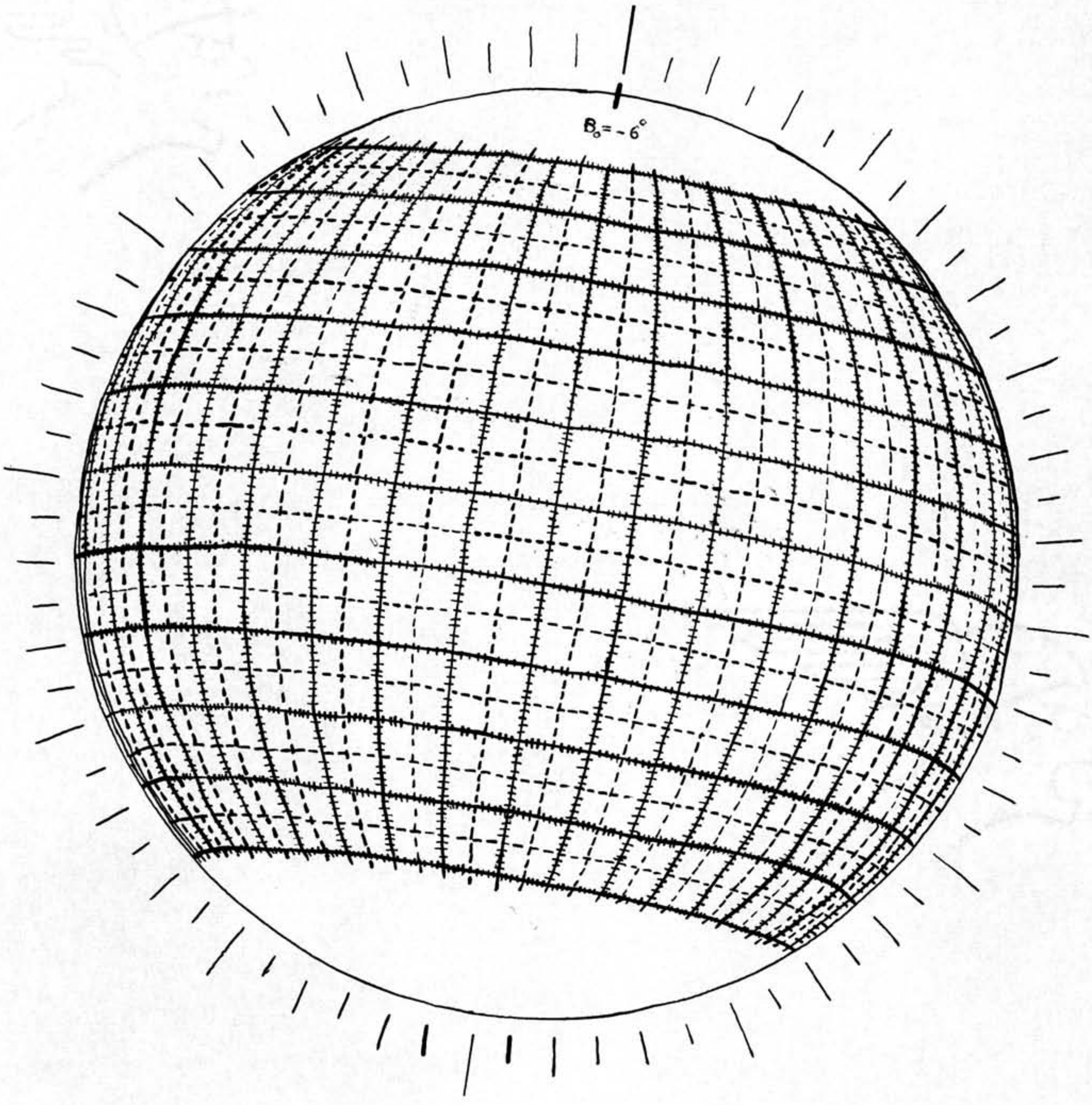


Figure 3.2.- Solar disk taken at Sukhumwit Observatory on 6. February 1968. (03h 02 min U.T.)


$B_0 = -6°$

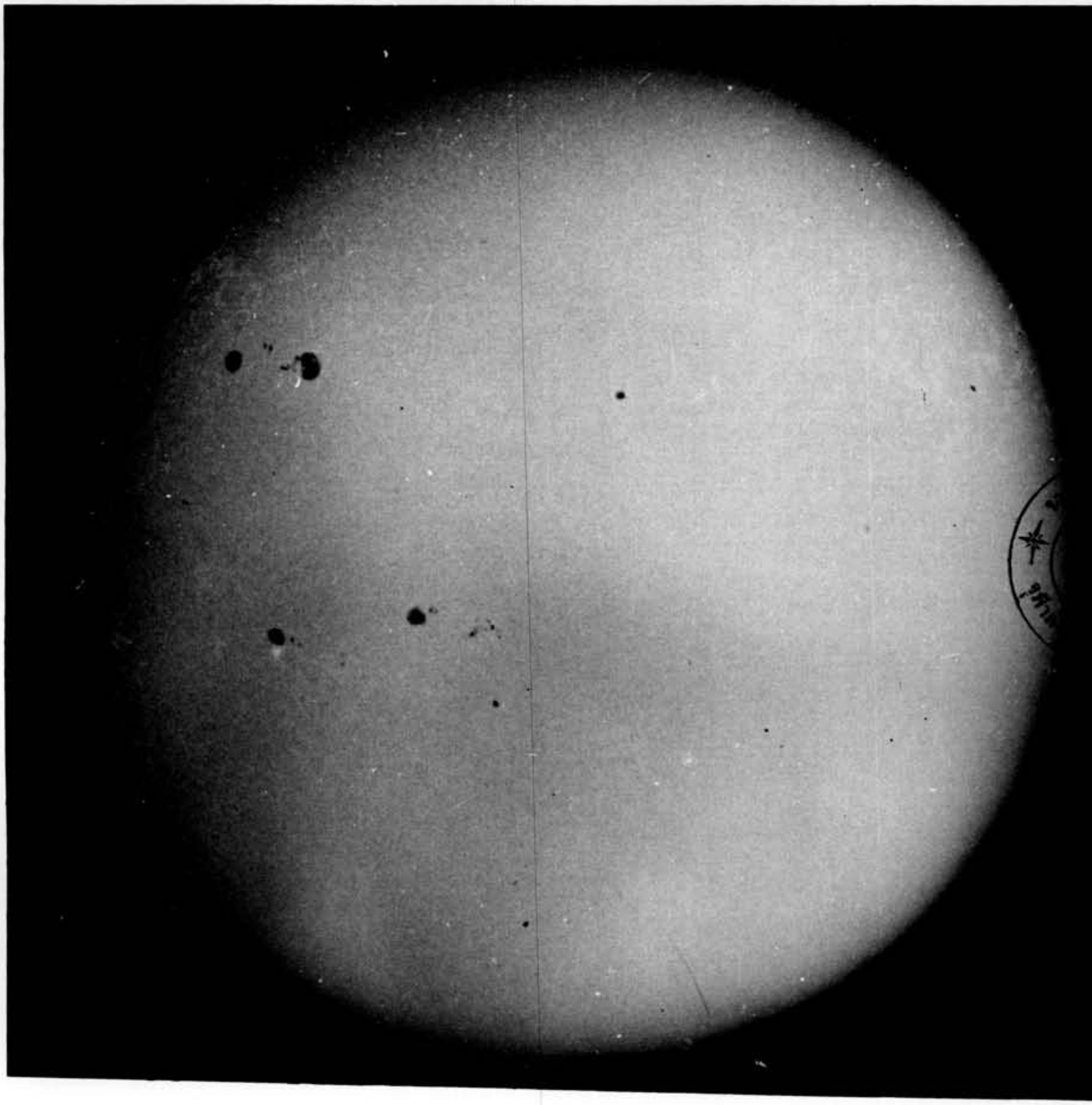


Figure 3.3. - Solar disk taken on 6 February 1968. (02h. 56min. U.T.)

over the image by superimposing the axis of the grid on the north-south axis of the disk. The grid is then rotated through an angle equivalent to the position angle P of the sun's axis of rotation on the day of observation, in a clockwise direction if P is positive and in an anti-clockwise direction if P is negative. The latitude B of a spot and the difference between the longitude of the spot and the centre of the sun $L - L_o$ are then read from the grid.

3.6 RESULTS

The reduced sunspot data are tabulated below:

Date	Universal time	Image quality	B	L	Type	f
6.II.68	0302	f	$+11^\circ$	76°	J	1
			$+19^\circ$	28°	A	1
			-16°	96°	H	4
			-19°	114°	H	3
			$+15^\circ$	160°	E	10
			$+11^\circ$	123°	J	1
			$+11^\circ$	112°	C	12
			$+6^\circ$	99°	A	1

REFERENCES

H.M. STATIONERY OFFICE (1966).--"The Astronomical Ephemeris, 1968." United Kingdom ed. (London.)

SMART, W.M. (1962).--"Text-Book on Spherical Astronomy." 5th ed. p. 174. (Cambridge University Press: London.)

APPENDIX I

SAMPLE CALCULATIONS

Calculation to obtain P, B_o, and L_o at the observed time

The observed time is 3h 02 min U.T.

$$p = \frac{91}{24 \times 30} = 0.1264$$

Date	P at 0h U.T.	δ	δ^2	δ^3	δ^4
4.II	-13°.07				
		-40			
5.II	13.47		+1		
		39		-1	
6.II	13.86		0		+2
		39		+1	
7.II	14.25		+1		=2
		38		-1	
8.II	14.63		0		
		-38			
9.II	-15.01				

From equation (2)

$$\begin{aligned} P &= -13^{\circ}.86 + 0^{\circ}.1264(-0.39) + B_2(+0.01) + B_3(+0.01) + B_4(0) \\ &= -13^{\circ}.86 - 0^{\circ}.049296 - 0^{\circ}.0003 + 0^{\circ}.00006 \\ &= -13^{\circ}.909536 \\ &\simeq -13^{\circ}.91 \end{aligned}$$

Date	B_o at Oh U.T.	δ	δ^2	δ^3	δ^4
4.II	$-6^\circ.20$				
		-7			
5.II	6.27		+1		
		6		-2	
6.II	6.33		-1		+5
		7		+3	
7.II	6.40		+2		-6
		5		-3	
8.II	6.45		-1		
		-6			
9.II	-6.51				

From equation (2)

$$
\begin{aligned}
B_o &= -6^\circ.33 + 0^\circ.1264(-0.06) + B_2(+0.01) + B_3(+0.03) + B_4(-0.01) \\
&= -6^\circ.33 - 0^\circ.007584 - 0^\circ.0003 + 0^\circ.00018 - 0^\circ.00005 \\
&= -6^\circ.337754 \\
&\simeq -6^\circ.34
\end{aligned}
$$

Date	$L_{\odot}$ at 0h U.T.	δ	δ^2	δ^3	δ^4
4.II	107°.79				
		-1317			
5.II	94.62		+1		
		1316		-2	
6.II	81.46		-1		+3
		1317		+1	
7.II	68.29		+0		0
		1317		+1	
8.II	55.12		+1		
		-1316			
9.II	41.96				

From equation (2)

$$L_o = 81^\circ.46 + 0^\circ.1264(-13.17) + B_2(-0.01) + B_3(+0.01) + B_4(+0.03)$$

$$= 81^\circ.46 - 0^\circ.166468 + 0^\circ.0003 + 0^\circ.00006 + 0^\circ.00014$$

$$= 81^\circ.294$$

$$\simeq 81^\circ.29$$

Calculation of B and L

$P = -13^\circ.91$

$B_\odot = -6^\circ.34$

$L_\odot = 81^\circ.29$

$S = 16'14''.8$

$= 16'.25$

Group no.	$r/r_\odot$	θ in degree	P-θ in degree
1	0.31375	3.0	-16.91
2	0.85125	49.0	62.91
3	0.30000	222.3	236.21
4	0.56250	231.8	245.71
5	0.98750	272.3	286.21
6	0.70750	279.0	292.91
7	0.58250	285.4	299.31
8	0.37120	291.0	-304.91

$$\rho_1 = \frac{r}{r_o} S$$

$$\sin(\rho + \rho_1) = \frac{\rho_1}{S}$$

$$\sin B = \sin B_o \cos \rho + \cos B_o \sin \rho \cos(P - \theta)$$

$$\sin(L - L_o) = \sin \rho \sin(P - \theta) \sec B$$

(1)

$$\rho_1 = 0.31375 \times 16'.25$$

$$= 5'.098$$

$$\sin(\rho + \rho_1) = \frac{\rho_1}{S} = 0.31375$$

$$\rho + \rho_1 = 18^\circ.29$$

$$\rho = 18.19 - .08 = 18^\circ.21$$

$$\sin B = \sin(-6^\circ.34) \cos 18^\circ.21 + \cos(-6^\circ.34) \sin 18^\circ.21 \cos(-16^\circ.91)$$

$$= -0.1104 \times 0.9499 + 0.9939 \times 0.3125 \times 0.9567$$

$$= -0.10487 + 0.29715$$

$$= 0.19228$$

$$B = 11^\circ.09$$

$$\sin(L - L_o) = \sin 18^\circ.21 \sin(-16^\circ.91) \sec 11^\circ.09$$

$$= \frac{-0.3125 \times 0.2909}{0.9813}$$

$$= -0.0926$$

$$L - L_o = -5^\circ.31$$

$$L = -5^\circ.31 + 81^\circ.29$$

$$= 75^\circ.98$$

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(2)

$$\rho_1 = 0.85125 \times 16'.25$$

$$= 13'.88$$

$$\sin(\rho + \rho_1) = \frac{\rho_1}{S}$$

$$= 0.85125$$

$$\rho + \rho_1 = 58^\circ.35$$

$$\rho = 58^\circ.35 - 0.2313 \qquad = 58^\circ.12$$

$$\sin B = \sin(-6^\circ.34) \cos 58^\circ.12 + \cos(-6^\circ.34) \sin 58^\circ.12 \cos(-62^\circ.91)$$

$$= -0.1104 \times 0.5281 + 0.9939 \times 0.8492 \times 0.4553$$

$$= -0.05830 + 0.38428$$

$$= 0.3260$$

$$B = 19^\circ.025$$

$$\sin(L - L_o) = \sin 58^\circ.12 \sin(-62^\circ.91) \sec 19^\circ.025$$

$$= \frac{-0.8492 \times 0.8903}{0.9454}$$

$$= -0.7997$$

$$L - L_o = -53^\circ.1$$

$$L = -53^\circ.1 + 81^\circ.29$$

$$= 28^\circ.19$$

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(3)

$$\rho_1 = 0.3000 \times 16'.25$$

$$= 4'.875$$

$$\sin(\rho + \rho_1) = 0.3000$$

$$\rho + \rho_1 = 17^\circ.46$$

$$\rho = 17^\circ.46 - 0.08$$

$$= 17^\circ.38$$

$$\sin B = \sin(-6^\circ.34) \cos 17^\circ.38 + \cos(-6^\circ.34) \sin 17^\circ.38 \cos(-236^\circ.21)$$

$$= -0.1104 \times 0.9543 + 0.((39 \times 0.2987(-\cos 56^\circ.21)$$

$$= -0.10542 - 0.9939 \times 0.2987 \times 0.5561$$

$$= -0.10542 - 0.1651$$

$$= -0.2705$$

$$B = -15^\circ.7$$

$$\sin(L - L_o) = \sin 17^\circ.38 \sin(-236^\circ.21) \sec(-15^\circ.7)$$

$$= \sin 17^\circ.38(+\sin 56^\circ.21) \sec(-15^\circ.7)$$

$$= +\frac{0.2987 \times 0.8311}{0.9627}$$

$$= +0.2578$$

$$L - L_o = +14^\circ.95$$

$$L = +14^\circ.95 + 81^\circ.29$$

$$= 96^\circ.24$$

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(4)

$$\rho_1 = 0.5625 \times 16'.25$$

$$= 9'.14$$

$$\sin(\rho + \rho_1) = 0.5625$$

$$\rho + \rho_1 = 34^\circ.23$$

$$\rho = 34^\circ.23 - 0^\circ.15$$

$$= 34^\circ.08$$

$$\sin B = \sin(-6^\circ.34) \cos 34^\circ.08 + \cos(-6^\circ.34) \sin 34^\circ.08 \cos(-245^\circ.71)$$

$$= -0.1104 \times 0.8283 + 0.9939 \times 0.5603 \times (-\cos 65^\circ.71)$$

$$= -0.1104 \times 0.8283 + 0.9939 \times 0.5603 \times (-0.4113)$$

$$= -0.09144 - 0.22904$$

$$= -0.3205$$

$$B = -18^\circ.7$$

$$\sin(L - L_o) = \sin 34^\circ.08 \sin(-245^\circ.71) \sec(-18^\circ.7)$$

$$= \sin 34^\circ.08(+\sin 65^\circ.71) \sec(-18^\circ.7)$$

$$= \frac{+0.5603 \times 0.9115}{0.9472}$$

$$= +0.5392$$

$$L - L_o = +32^\circ.63$$

$$L = +32^\circ.63 + 81^\circ.29$$

$$= 113^\circ.92$$

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(5) $$\rho_1 = 0.9875 \times 16'.25$$

$$= 16'.05$$

$$\sin(\rho + \rho_1) = 0.9875$$

$$\rho + \rho_1 = 80^\circ.93$$

$$\rho = 80^\circ.93 - 0.27$$

$$= 80^\circ.66$$

$$= 80^\circ.66$$

$$\sin B = \sin(-6^\circ.34) \cos 80^\circ.66 + \cos(-6^\circ.34) \sin 80^\circ.66 \cos(-286^\circ.21)$$

$$= -\sin 6^\circ.34 \cos 80^\circ.66 + \cos 6^\circ.34 \sin 80^\circ.66 \cos 73^\circ.79$$

$$= -0.1104 \times 0.1623 + 0.9939 \times 0.9868 \times 0.2792$$

$$= -.0179 + 0.2738$$

$$= +0.2559$$

$$B = +14^\circ.83$$

$$\sin(L - L_o) = \sin 80^\circ.66 \sin(-286^\circ.21) \sec 14^\circ.83$$

$$= \sin 80^\circ.66 \sin 73^\circ.79 \sec 14^\circ.83$$

$$= \frac{0.9868 \times 0.9603}{0.9667}$$

$$= 0.9803$$

$$L - L_o = 78^\circ.6$$

$$L = 78^\circ.6 + 81^\circ.29$$

$$= 159^\circ.89$$

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(6) $$\rho_1 = 0.7075 \times 16'.25$$

$$= 11'.497$$

$$\sin(\rho + \rho_1) = 0.7075$$

$$\rho + \rho_1 = 45^\circ.03$$

$$\rho = 45^\circ.03 - 0.19$$

$$= 44^\circ.84$$

$$\sin B = \sin(-6^\circ.34) \cos 44^\circ.84 + \cos(-6^\circ.34) \sin 44^\circ.84 \cos(-292^\circ.91)$$

$$= -\sin 6.34 \cos 44^\circ.84 + \cos 6^\circ.34 \sin 44^\circ.84 \cos 67^\circ.09$$

$$= -0.1104 \times 0.7091 + 0.9939 \times 0.7051 \times 0.3893$$

$$= -0.07828 + 0.27282$$

$$= 0.1945$$

$$B = 11^\circ.22$$

$$\sin(L - L_o) = \sin 44^\circ.84 \sin(-292^\circ.91) \sec 11^\circ.22$$

$$= \sin 44^\circ.84 \sin 67^\circ.09 \sec 11^\circ.22$$

$$= \frac{0.7051 \times 0.9211}{0.9809}$$

$$= 0.6621$$

$$L - L_o = 41^\circ.46$$

$$L = 41^\circ.46 + 81^\circ.29$$

$$= 122^\circ.75$$

(7) $\rho_1 = 0.5825 \times 16'.25$

$= 9'.466$

$\sin(\rho + \rho_1) = 0.5825$

$\rho + \rho_1 = 35^\circ.63$

$\rho = 35^\circ.63 - 0.16$

$= 35^\circ.47$

$\sin B = \sin(-6^\circ.34) \cos 35^\circ.47 + \cos(-6^\circ.34) \sin 35^\circ.47$
$\cos(-299.31)$

$= -\sin 6^\circ.34 \cos 35^\circ.47 + \cos 6^\circ.34 \sin 35^\circ.47$
$\cos 60^\circ.69$

$= -0.1104 \times 0.8144 + 0.9939 \times 0.5803 \times 0.4895$

$= -0.0899 + 0.2823$

$= 0.1924$

$B = 11^\circ.1$

$\sin(L - L_\odot) = \sin 35^\circ.47 \sin(-299^\circ.31) \sec 11^\circ.1$

$= \sin 35^\circ.47 \sin 60^\circ.69 \sec 11^\circ.1$

$= \dfrac{0.5803 \times 0.8720}{0.9813}$

$= 0.5157$

$L - L_\odot = 31^\circ.05$

$L = 31^\circ.05 + 81^\circ.29$

$= 112^\circ.34$

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(8) $\rho_1 = 0.3712 \times 16'.25$

$= 6'.439$

$\sin(\rho + \rho_1) = 0.3712$

$\rho + \rho_1 = 21^\circ.79$

$$\rho = 21^\circ.79 - 0.11$$

$$= 21^\circ.68$$

$$\sin B = \sin(-6^\circ.34) \cos 21^\circ.68 + \cos(-6^\circ.34) \sin 21^\circ.68 \cos(-304^\circ.91)$$

$$= -\sin 6^\circ.34 \cos 21^\circ.68 + \cos 6^\circ.34 \sin 21^\circ.68 \cos 55^\circ.09$$

$$= -0.1104 \times 0.9292 + 0.9939 \times 0.3694 \times 0.5720$$

$$= -0.1026 + 0.2100$$

$$= 0.1074$$

$$B = 6^\circ.16$$

$$\sin(L - L_o) = \sin 21^\circ.68 \sin(-304^\circ.91) \sec 6^\circ.16$$

$$= \sin 21^\circ.68 \sin 55^\circ.09 \sec 6^\circ.16$$

$$= \frac{0.3694 \times 0.8201}{0.9942}$$

$$= 0.3047$$

$$L - L_o = 17^\circ.74$$

$$L = 17^\circ.74 + 81^\circ.29$$

$$= 99^\circ.03$$