

CHAPTER 4

DESIGN EXAMPLES

4.1 Controlling Resultants

The section at the fixed support apropos of either an S-beam or a Z-beam is always critical since the beam at this location carries maximum bending moment, sizable torsional moment, and maximum shearing force. These resultants which have direct bearing on proportioning of the cross-sectional dimensions of the beam are cast thus.

The S-beam:

$$M_A = M_O \cos \phi + T_O \sin \phi - wR^2(1 - \cos \phi) \quad (4.1)$$

$$T_A = M_O \sin \phi - T_O \cos \phi - wR^2(\phi - \sin \phi) \quad (4.2)$$

$$V_A = wR\phi \quad (4.3)$$

The Z-beam:

$$M_A = T_O - \frac{1}{2} wL_1(L_1 + L_2) \quad (4.4)$$

$$T_A = \frac{1}{8} wL_2^2 - M_O \quad (4.5)$$

$$V_A = \frac{1}{2} w(2L_1 + L_2) \quad (4.6)$$

4.2 Design Criteria

The ultimate load criteria instituted by the American Concrete Institute, as contained in 1977 ACI Code¹, are employed. A list of rules governing proportioning of reinforced concrete members under the conjunctive influence of bending, torsion, and shear is given in Appendix C.

4.3 S-beam Design Example

Let it be required to install a curved corridor between buildings, in the form of a slab supported by a pair of S-beams, such as depicted in Figure 4.1. Each beam, by virtue of its spanwise characteristics commensurate with those of adjoining beams forming part of the buildings, is regarded as having fixed supports, the stipulation propelling applicability of the presented graphical aids. Marginal brickwork parapets are incorporated into the construction.

1. ACI Committee 318, Building Code Requirements for Reinforced Concrete (Detroit: American Concrete Institute, 1977).

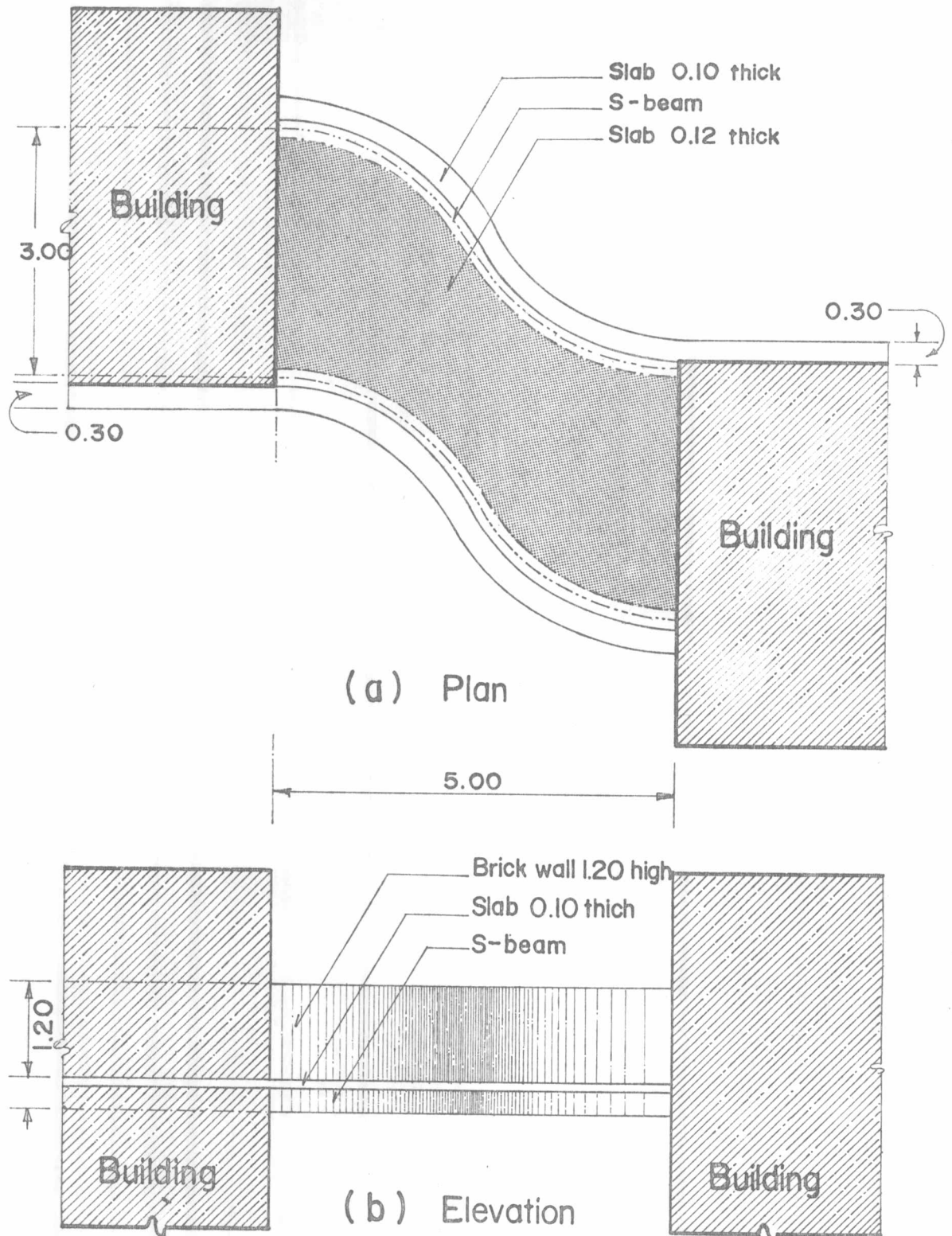


FIGURE 4.1 Corridor Supported by S-beams in Design Example

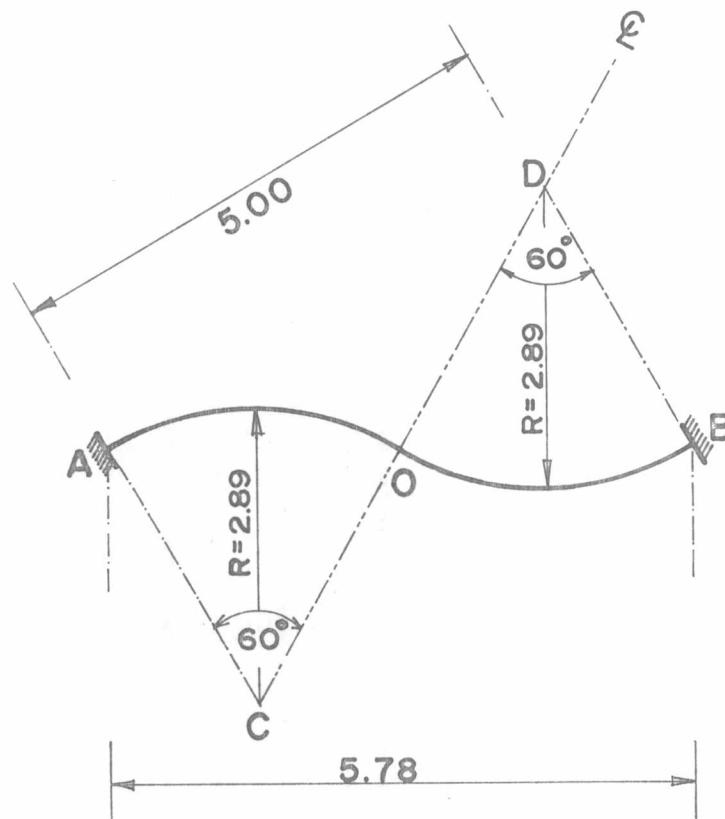


FIGURE 4.2 Skeletal Geometry of S-beam in Example

Figure 4.2 portrays the skeletal geometry of the beam identifying with $R = 2.89$ metres and $\phi = 60$ degrees. The aisle will be commissioned to carry an imposed load of 500 kilogrammes per square metre in addition to the self-weight of the facility. The following strength data apply: 28 days' strength of standard concrete cylinder,

210 kilogrammes per square centimetre; yield strength of steel, 3,420 kilogrammes per square centimetre.

The assumed cross-sectional dimensions of the beam are $t = 36$ and $b = 24$ centimetres, the ratio $\frac{t}{b}$ being 1.500. The following self-weights in kilogrammes per lineal metre hold: beam, 208; slab, 504; surface finish, 150; parapet, 262; and total, 1,124. The design imposed load registers $500 \times 1.50 = 750$ kilogrammes per metre. According to the ACI's rule the design ultimate load equals $1.4 \times 1,124 + 1.7 \times 750 = 2,850$ kilogrammes per metre.

With $\frac{t}{b} = 1.500$ and $\phi = 60$ degrees instrumentalisation of Charts 3.1 and 3.2 reveals:

$$\begin{aligned} M_o &= 0.165 wR^2 \\ &= 0.165 \times 2,850 \times (2.89)^2 = 3,927.58 \text{ m-kg} \end{aligned}$$

$$\begin{aligned} T_o &= 0.0667 wR^2 \\ &= 0.0667 \times 2,850 \times (2.89)^2 = 1,587.69 \text{ m-kg} \end{aligned}$$

Via relations (4.1), (4.2), and (4.3) the controlling resultants at the fixed support amount to

$$M_A = - 8,562.97 \text{ m-kg}$$

$$T_A = - 1,704.92 \text{ m-kg}$$

$$V_A = 8,625.27 \text{ kg}$$

Figures 4.3, 4.4, and 4.5 give respectively the distribution of the bending moment, torsional moment, and shearing force between the centre-span and the support. Of course the resultants occupy ultimate magnitudes to which the ACI's ultimate strength design rules can readily be applied.

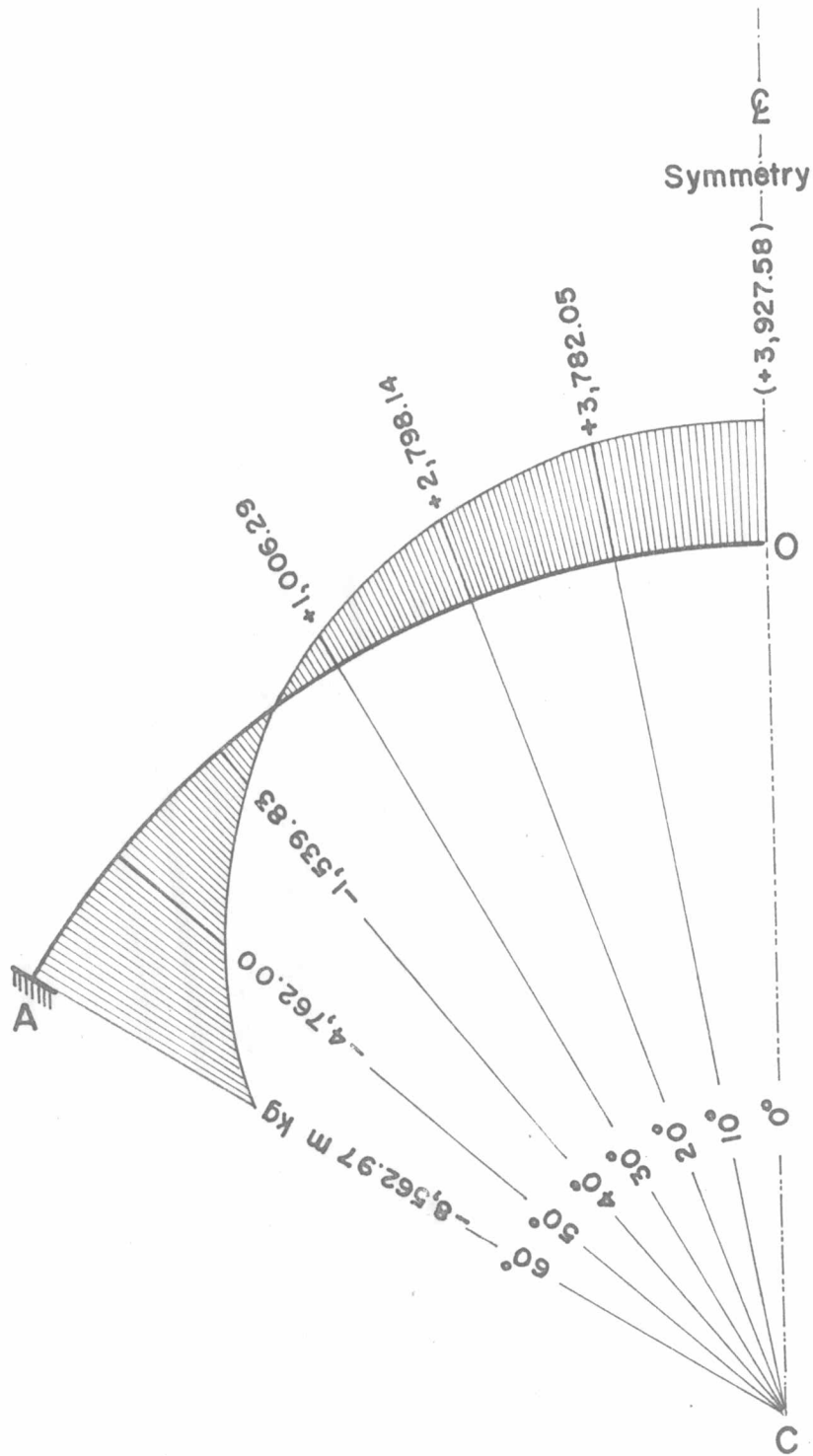


FIGURE 4.3 Distribution of Bending Monent

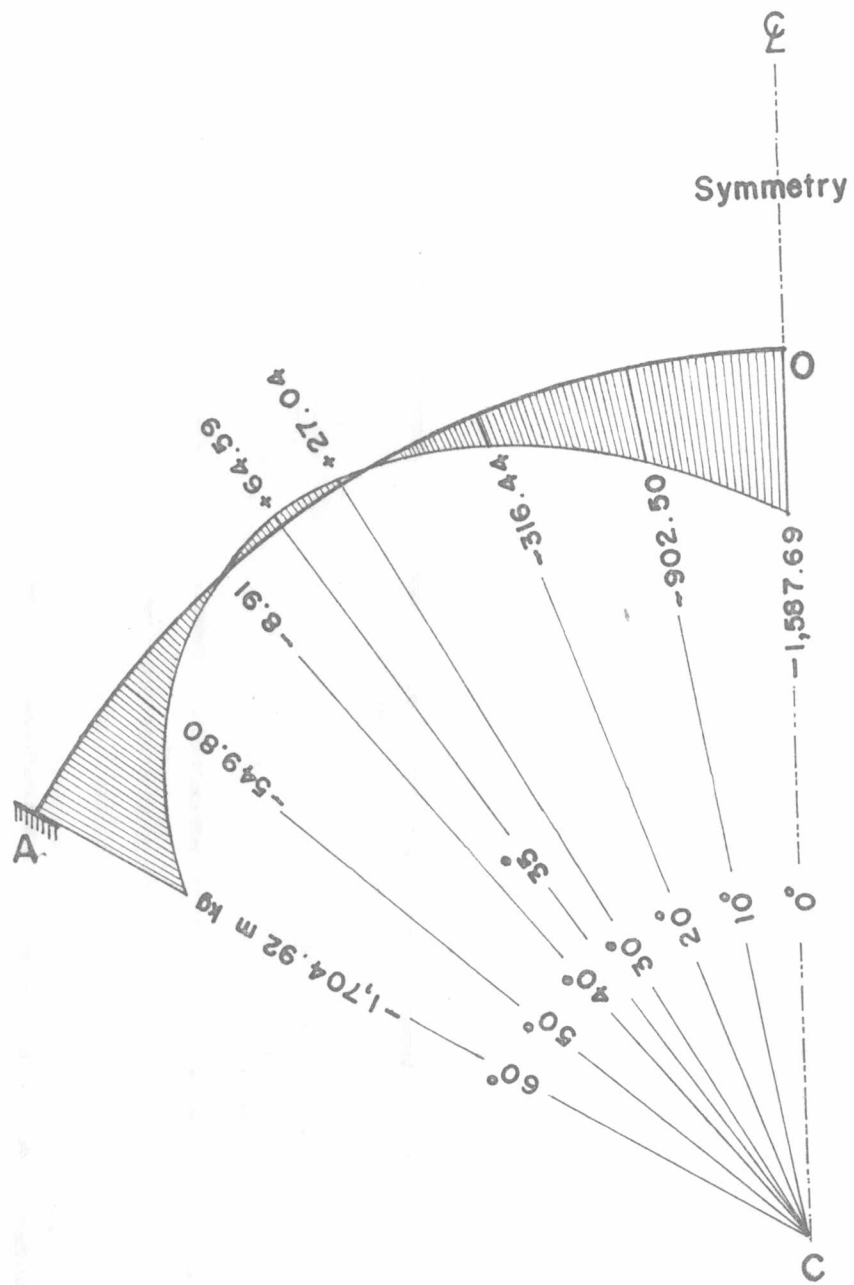


FIGURE 4.4 Distribution of Torsional Moment

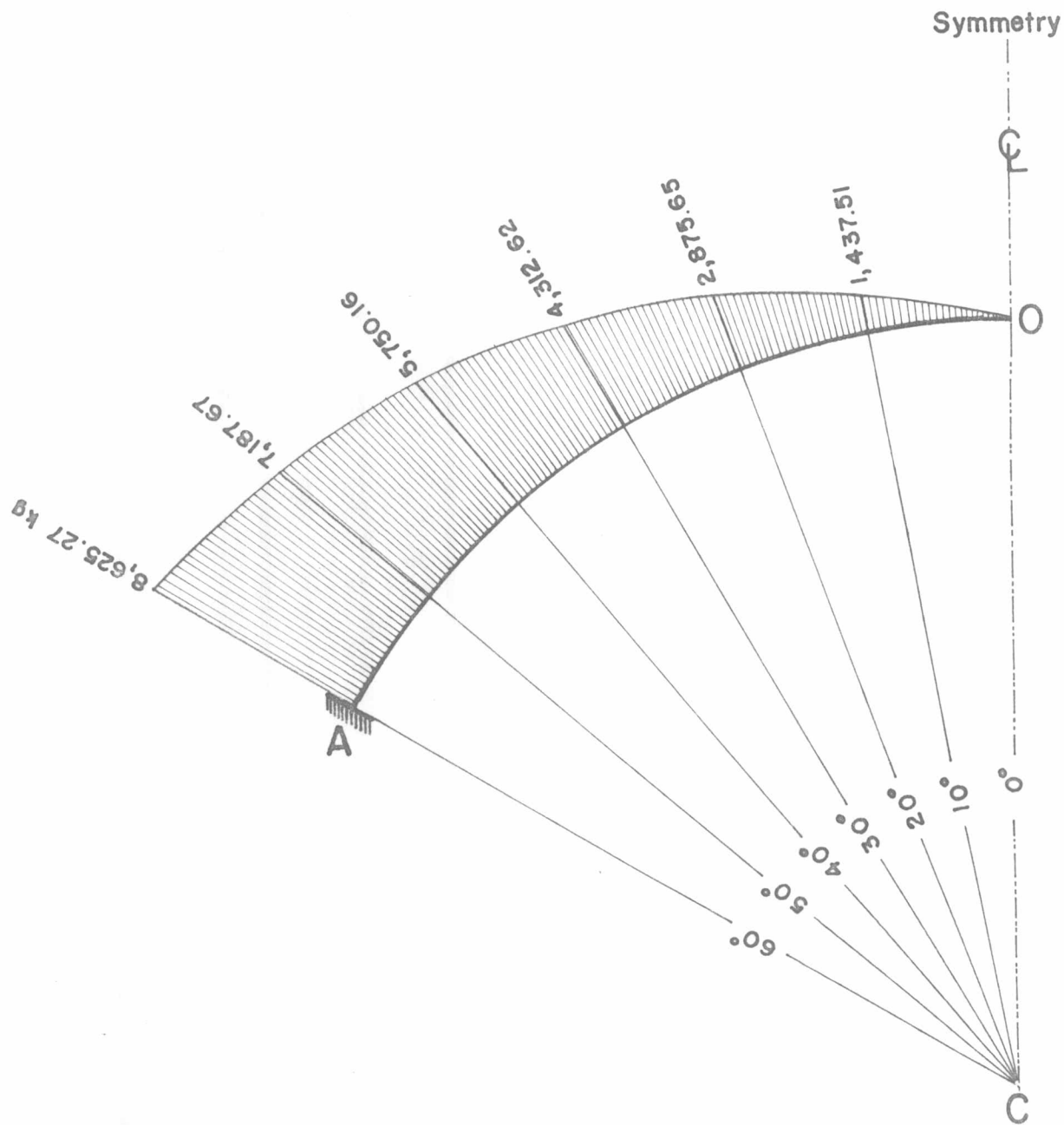


FIGURE 4.5 Distribution of Shearing Force

According to the ACI Code the following steel percentages corresponding to cube strength and yield strength of respectively 210 and 3,420 kg/cm² are calculated.

Balanced-beam percentage,

$$p_b = 0.85 \times 0.85 \times \frac{210}{3,420} \times \frac{6,117}{6,117+3,420} = 0.0285$$

Maximum allowable steel percentage,

$$p_{\max} = 0.75 p_b = 0.0214$$

Minimum allowable steel percentage,

$$p_{\min} = \frac{14}{3,420} = 0.0041$$

Thus with the chosen cross-section 24 by 36 cm, the effective depth being 31 cm, the longitudinal steel area limitation is governed by

$$A_{S_{\max}} = 0.0214 \times 24 \times 31 = 15.92 \text{ cm}^2$$

$$A_{S_{\min}} = 0.0041 \times 24 \times 31 = 3.05 \text{ cm}^2$$

For $A_{S_{\max}} = 15.92 \text{ cm}^2$ the thickness of the stress block is

$$a = \frac{15.92 \times 3,420}{0.85 \times 24 \times 210} = 12.71 \text{ cm}$$

and the upper-bound resisting moment of the section reads

$$M_{uR} = 0.90 \times 15.92 \times 3,420 \left(0.31 - \frac{1}{2} \times 0.1271 \right) = 12,076 \text{ m-kg}$$

At the support the bending moment to be resisted equals 8,562.97 m-kg (being smaller than the upper-bound demarkation 12,076 m-kg). A longitudinal steel area of 10.36 cm^2 is postulated to participate in carrying the commanding bending moment 8,562.97 m-kg. The stress block thickness congruent with the assumed steel area is

$$a = \frac{10.36 \times 3,420}{0.85 \times 24 \times 210} = 8.27 \text{ cm}$$

and the respective resisting moment is equal to

$$M_{uR} = 0.90 \times 10.36 \times 3,420 \left(0.31 - \frac{1}{2} \times 0.0827 \right) = 8,566.73 \text{ m-kg}$$

which is a trifle larger than the encroaching 8,562.97 m-kg. The designated steel area 10.36 cm^2 is therefore regarded as propitious for negotiating bending.

The torsional moment and the vertical shearing force to be accounted for are respectively 1,704.92 m-kg and 8,625.27 kg. With a strength reduction factor of 0.85 the nominal values of these resultants, in other words their modified values according to the ACI's law, are

$$T_n = 2,005.79 \text{ m-kg}$$

$$V_n = 10,147.38 \text{ kg}$$

With the factor C_t valued at

$$C_t = \frac{31}{24 \times 36} = 0.0359 \text{ cm}^{-1}$$

the torsional moment permitted to be carried by concrete alone is

$$T_c = 0.21 \times \sqrt{210} \times (24)^2 \times 36 \left[1 + \left(\frac{0.4 \times 8,625.27}{0.0359 \times 1,704.92 \times 100} \right)^2 \right]^{-\frac{1}{2}}$$

$$= 549.72 \text{ m-kg}$$

The torsional moment to be allocated to the web reinforcement thus equals

$$T_s = 2,005.79 - 549.72 = 1,456.07 \text{ m-kg}$$

which is admissible as the magnitude registers under the limit

$$4T_c = 4 \times 549.72 = 2,198.88 \text{ m-kg}$$

Using two-legged binders each having dimensions

$$b_s = 19 \text{ cm and } t_s = 31 \text{ cm the coefficient } \alpha_t \text{ is}$$

$$\alpha_t = 0.66 + \frac{0.33 \times 31}{19} = 1.198$$

and the area of single-legged binders required to carry the torsional moment T_s is computed to be

$$A_t = \frac{1,456.07 \times 100}{1.198 \times 19 \times 31 \times 3,420} S = 0.060 S \text{ cm}^2$$

in which S denotes the binder spacing in cm.

The maximum shearing force permitted to be borne by concrete alone is

$$V_c = 0.53 \times \sqrt{210} \times 24 \times 31 \left[1 + \left(\frac{2.5 \times 0.0359 \times 1,704.92 \times 100}{8,625.27} \right)^2 \right]^{-\frac{1}{2}}$$

$$= 2,805.94 \text{ kg.}$$

The shearing force to be allocated to the web reinforcement therefore amounts to

$$V_s = 10,147.38 - 2,805.94 = 7,341.44 \text{ kg}$$

which is passable as it registers below the limit

$$2.12 \sqrt{f_c} b d = 2.12 \times \sqrt{210} \times 24 \times 31 = 22,856 \text{ kg}$$

The area of two-legged binders required to carry the shearing force V_s is calculated as

$$A_v = \frac{7,341.44}{3,420 \times 31} S = 0.069 S \text{ cm}^2$$

The total area of two-legged binders necessary for resisting the torsional moment T_s in conjunction with the shearing force V_s is

$$2A_t + A_v = (2 \times 0.060 + 0.069) S = 0.189 S \text{ cm}^2$$

Bars with 9-mm diameter are called upon to form double two-legged binders at and near the support. The maximum allowable spacing of these doubly looped binders then reads

$$S_{\max} = \frac{4 \times 0.6362}{0.189} = 13.5 \text{ cm}$$

A spacing of 12 cm on centres is adopted.

The additional longitudinal steel required to participate in withstanding the torsional moment T_s is quantified as

$$A_{\ell} = \frac{1}{S} \times 2 \times 0.060(19+31) = 6.00 \text{ cm}^2$$

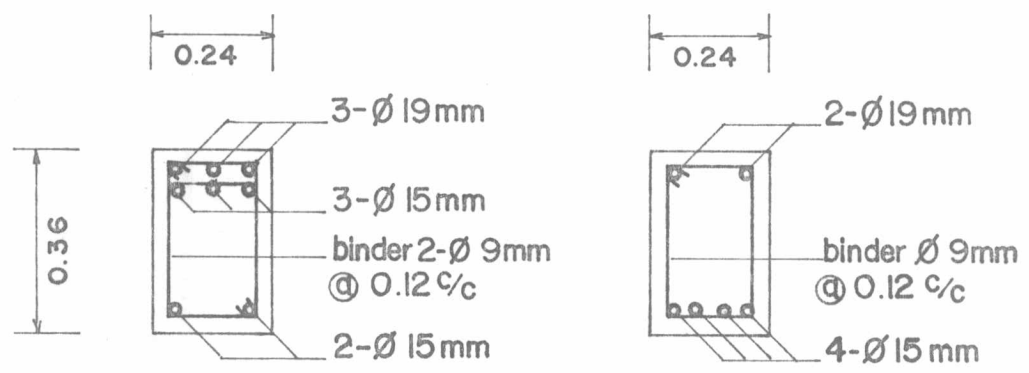
It follows that the total longitudinal reinforcement necessitated to justify the combined influence of the bending and the torsion consists of: $10.36 + 3.00 = 13.36 \text{ cm}^2$ in proximity to the top fibre of the beam; and 3.00 cm^2 in proximity to the bottom fibre. Accordingly three 19-mm-diameter and three 15-mm-diameter bars arranged in two layers are commissioned to constitute the top steel and two 15-mm-diameter bars, the bottom steel.

Table 4.1 enwraps information on variation in minimum reinforcement requirement between the centre-span and the support, guiding an economical overall reinforcing arrangement. The drawings of Figure 4.6 represent details of the executed design.

Table 4.1 - Variation in Reinforcement Requirement for S-beam in Example

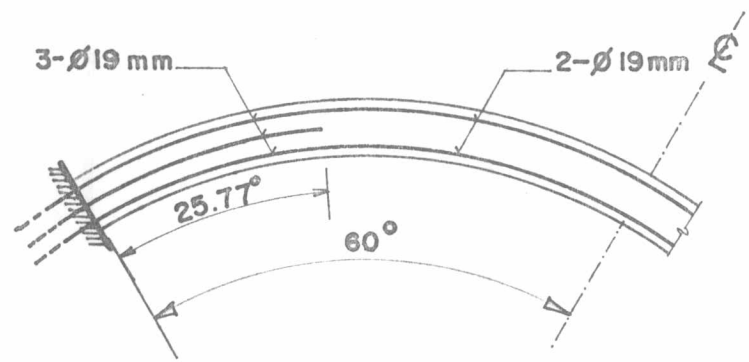
θ deg	M m-kg	T m-kg	V kg	Minimum Required Area of Reinforcement cm ²		
				Longitudinal		Binding
				Top	Bottom	
0	+ 3,927.58	- 1,587.69	0	2.530	6.750	0.102S
10	+ 3,782.05	- 902.50	1,437.51	3.320	7.370	0.046S
20	+ 2,798.14	- 316.44	2,875.65	2.670	5.610	0.026S
30	+ 1,006.29	+ 24.04	4,312.62	0.313	3.050	0.026S
35	- 176.87	+ 64.59	5,031.39	3.050	0.576	0.026S
40	- 1,539.83	- 8.91	5,750.16	3.050	0.075	0.026S
50	- 4,762.00	- 549.80	7,187.67	6.800	1.590	0.063S
60	- 8,562.97	- 1,704.92	8,625.27	13.36	3.000	0.189S

FIGURE 4.6 S-beam Details

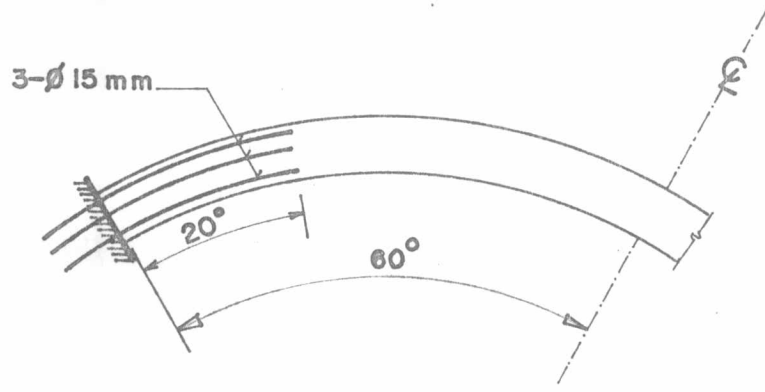


(a) Support section

(b) Centre-span section

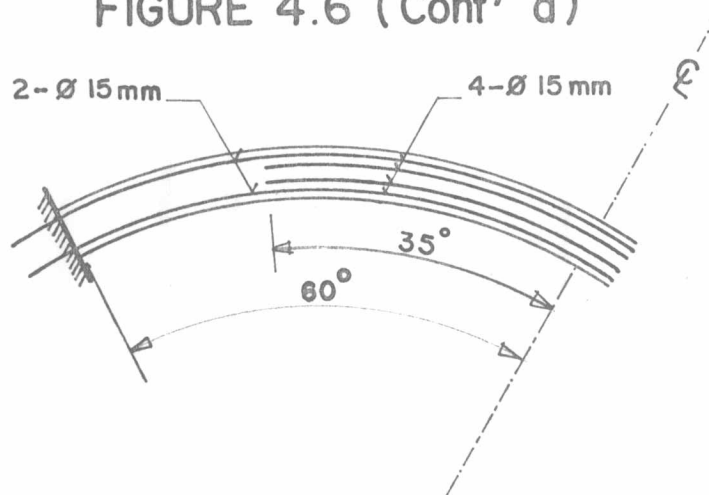


(c) Plan of upper layer of top longitudinal bars

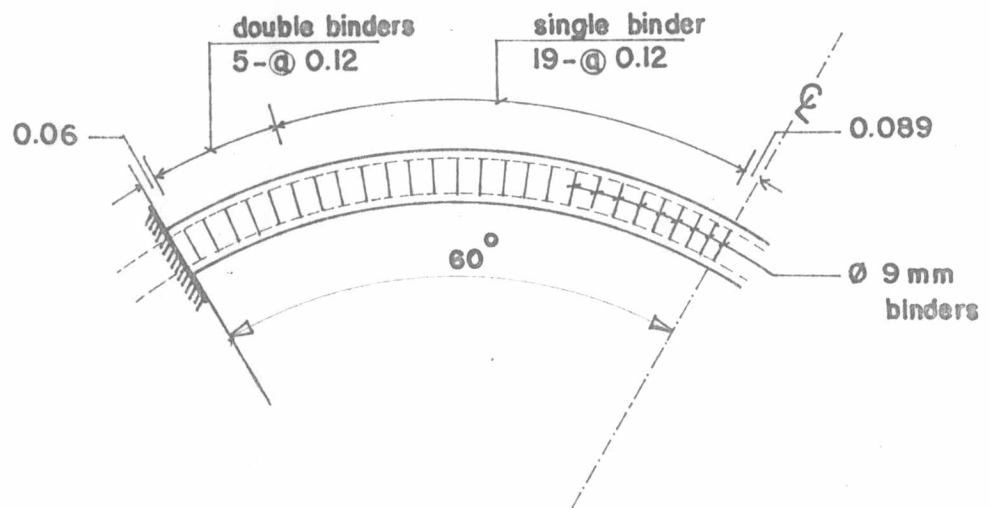


(d) Plan of lower layer of top longitudinal bars

FIGURE 4.6 (Cont' d)



(e) Plan of bottom longitudinal bars



(f) Arrangement of binders

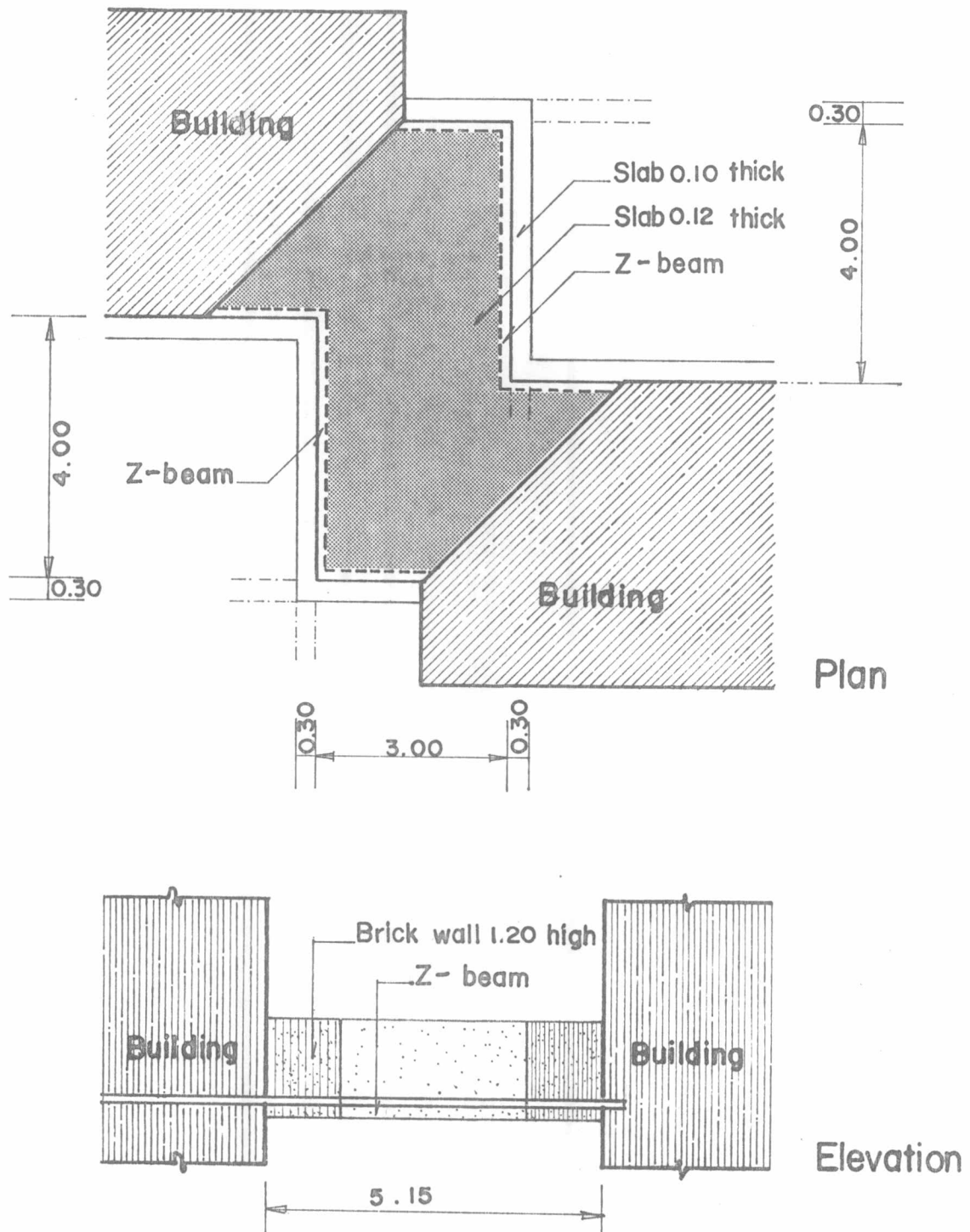


FIGURE 4.7 Corridor Supported by Z-beams in Design Example

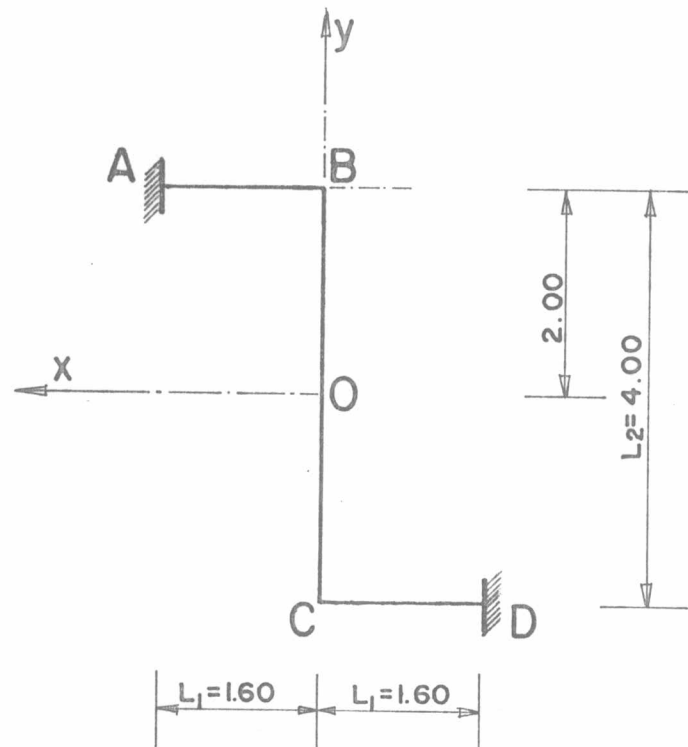


FIGURE 4.8 Skeletal Geometry of Z-beam in Example

4.4 Z-beam Design Example

A zigzagged corridor of Figure 4.7 enters into this Z-beam example, the beam's skeletal geometry being portrayed in Figure 4.8. Each of the commissioned pair of beams identifies with the following parameters:

$$L_1 = 1.60 \text{ m}, L_2 = 4.00 \text{ m}, b = 24 \text{ cm}, t = 36 \text{ cm},$$

$$k = L_1/L_2 = 0.40, \text{ and } t/b = 1.5$$

An imposed load of 750 kg/m^2 on the corridor's floor in conjunction with the facility's self-weight is set forth. Per lineal metre of the beam's axis the imposed load and the total self-weight respectively read 750 and 1,124 kg.

The corresponding ultimate load thus equals

$1.4 \times 1,124 + 1.7 \times 750 = 2,850$ kg/m. Making use of Charts 3.3 and 3.4 the centre-span redundants are quantified as

$$M_O = 0.0948 \times 2,850 \times (4.00)^2 = 4,322.88 \text{ m-kg}$$

$$T_O = 0.0337 \times 2,850 \times (4.00)^2 = 1,536.72 \text{ m-kg}$$

By dint of relations (4.4), (4.5), and (4.6) the values of the controlling resultants are

$$M_A = -11,232 \text{ m-kg}$$

$$T_A = 1,377.12 \text{ m-kg}$$

$$V_A = 10,260 \text{ kg}$$

Figures 4.9, 4.10, and 4.11 furnish information on the distribution of the analysed bending moment, torsional moment, and shearing force, respectively.

Again concrete cylinder strength of 210 kg/cm^2 and steel yield strength of $3,420 \text{ kg/cm}^2$ find exploitation. The proportioning of the reinforcement for the beam, which pursues a similar course to that associated with the foregoing S-beam example, is not elaborated. Table 4.2 reports variation in reinforcement requirement between the centre-span and the support. The sketches of Figure 4.12 bring forward details of the beam.

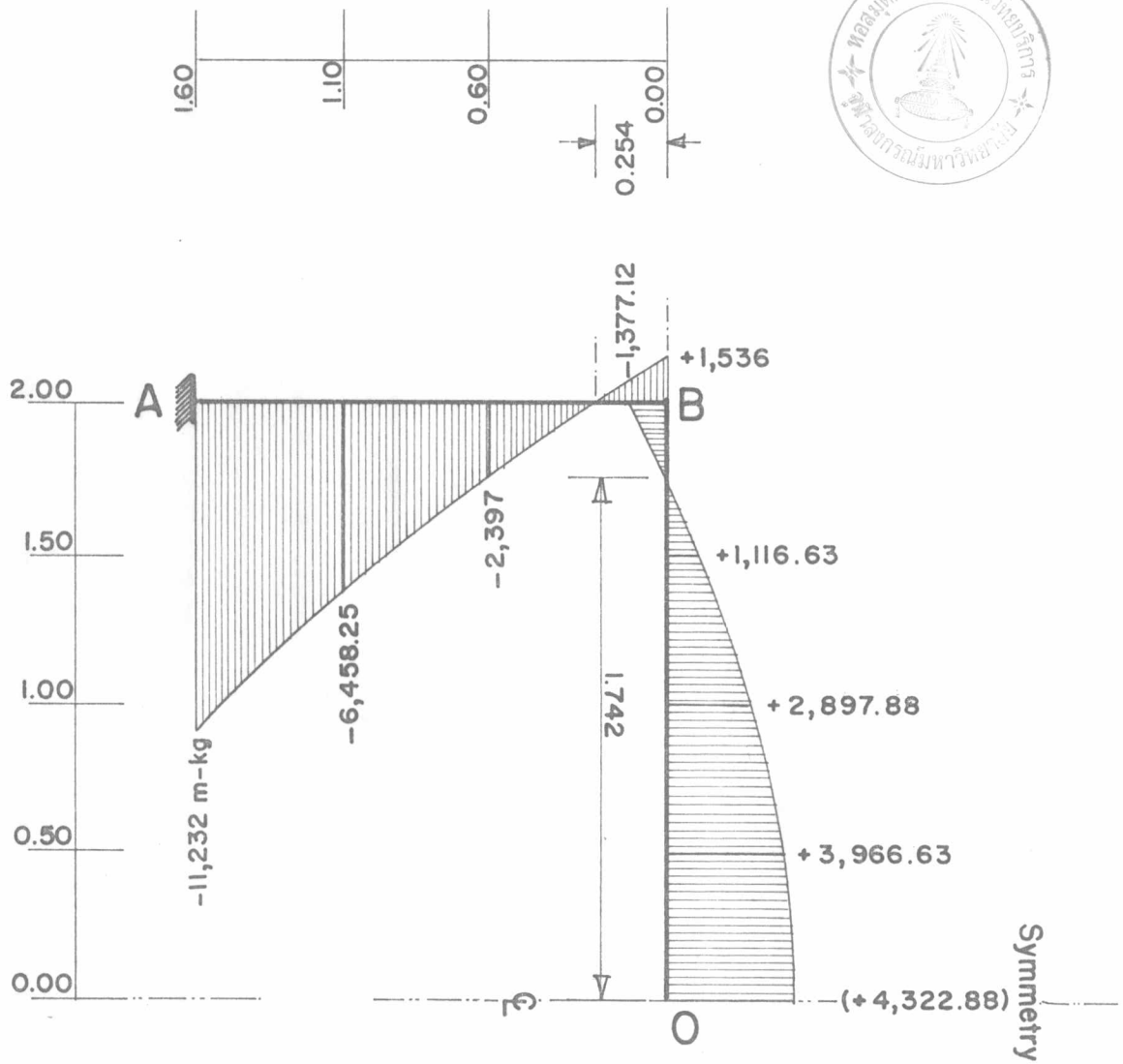


FIGURE 4.9 Distribution of Bending Moment

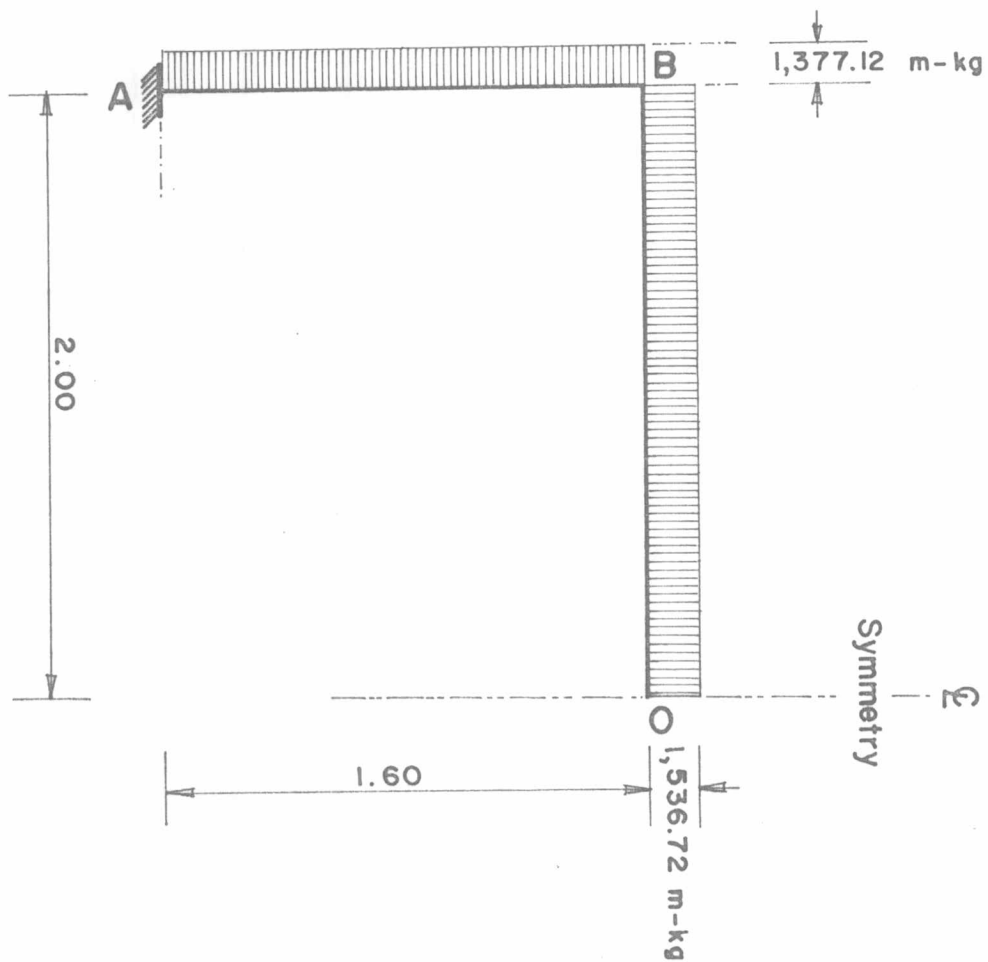


FIGURE 4.10 Distribution of Torsional Moment

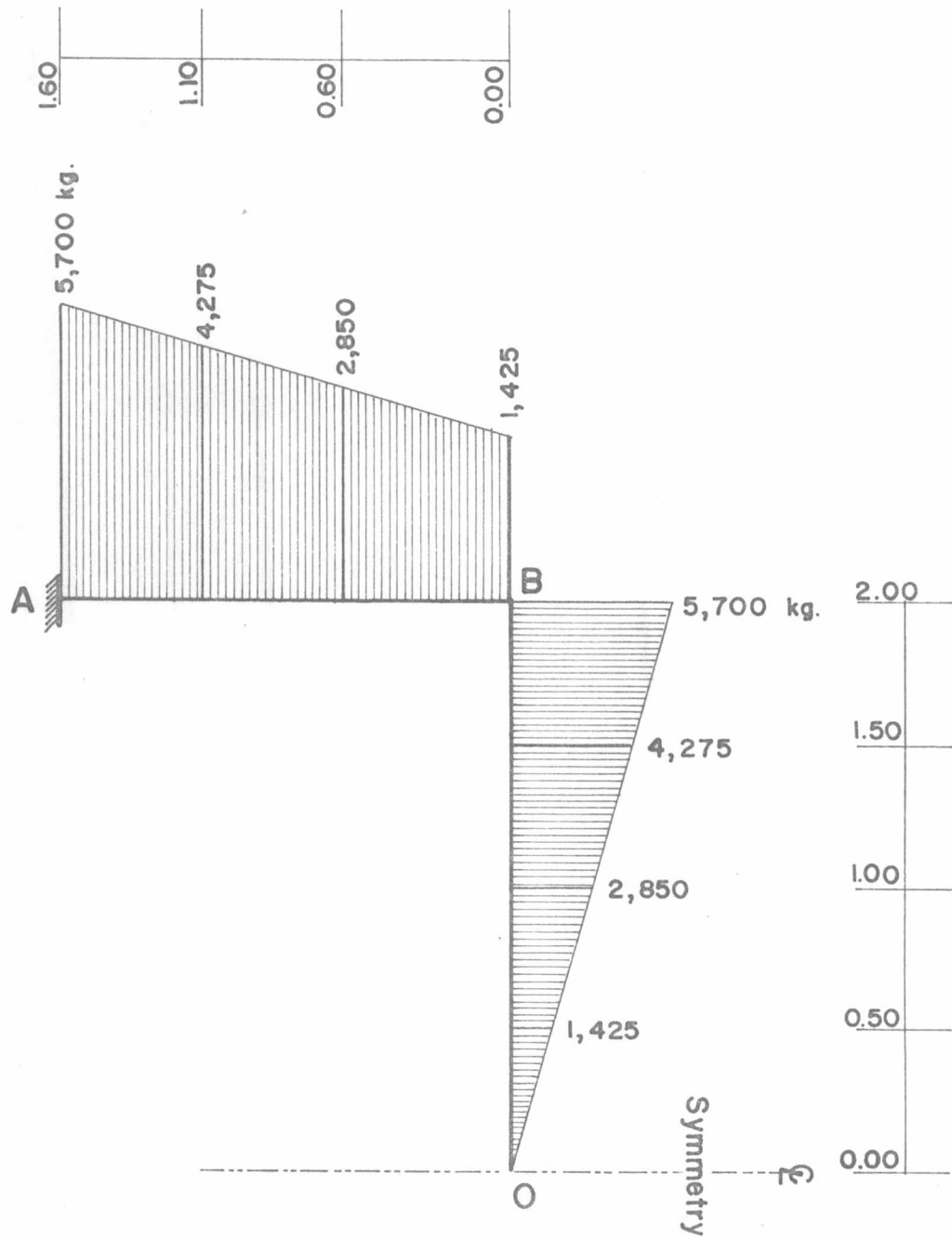
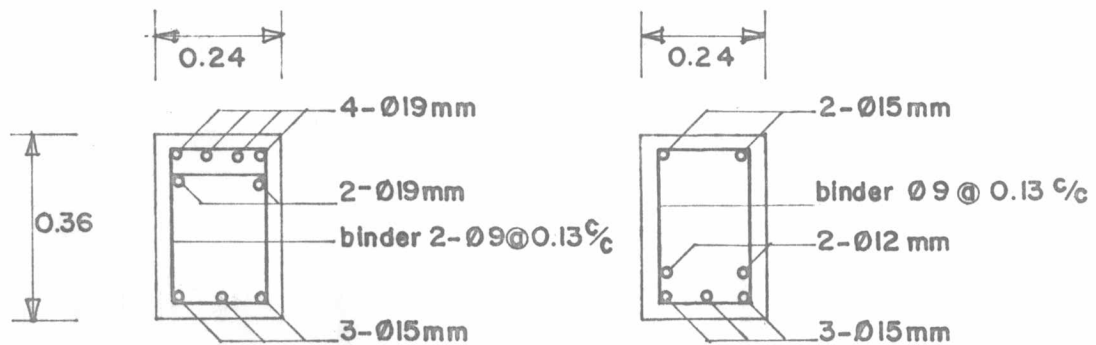


FIGURE 4.11 Distribution of Shearing Force

Table 4.2 - Variation in Reinforcement Requirement for Z-beam in Example

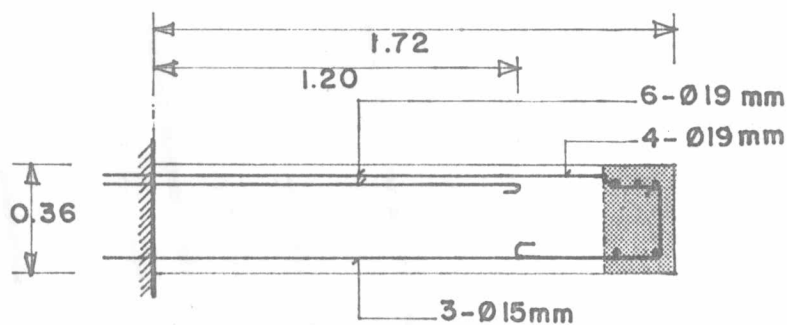
Longitudinal Part						
Distance m	M m-kg	T m-kg	V kg	Minimum Required Area of Reinforcement cm ²		
				Longitudinal		Binding
				Top	Bottom	
x = 1.60	- 11,232.00	1,377.12	10,260	16.820	4.298	0.173S
1.10	- 6,458.25	1,377.12	8,835	9.923	2.327	0.158S
0.60	- 2,397.00	1,377.12	7,410	4.856	2.276	0.145S
0	+ 1,536.00	1,377.12	5,700	2.219	3.849	0.129S
Transverse Part						
y = 2.00	- 1,377.12	1,536.72	5,700	4.035	2.580	0.145S
1.50	+ 1,116.63	1,536.72	4,275	2.544	3.720	0.132S
1.00	+ 2,897.88	1,536.72	2,850	2.513	5.661	0.120S
0.50	+ 3,966.63	1,536.72	1,425	2.497	6.904	0.109S
0	+ 4,322.88	1,536.72	0	2.575	7.415	0.098S

FIGURE 4.12 Z-beam Details

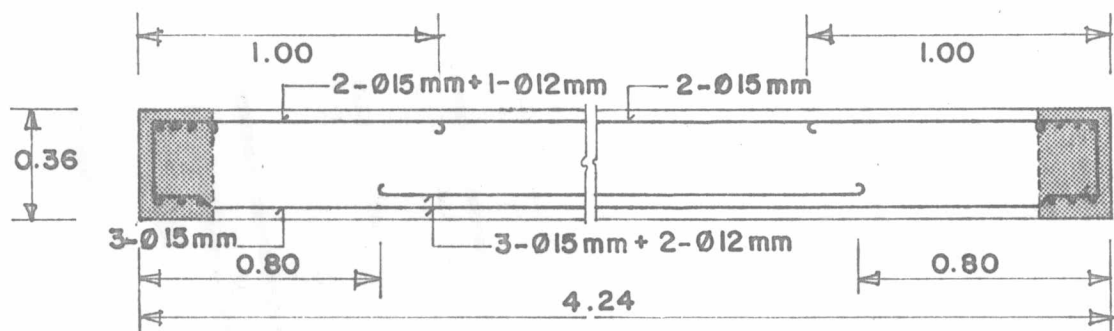


(a) Support section

(b) Centre-span section

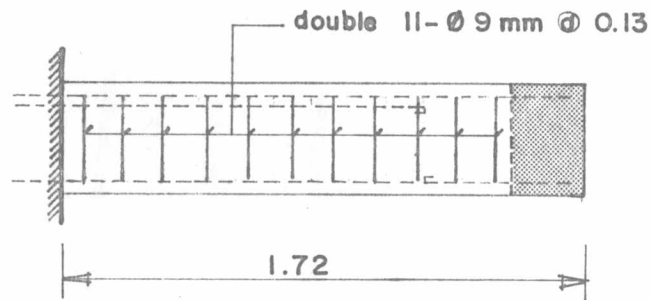


(c) Longitudinal section in longitudinal part

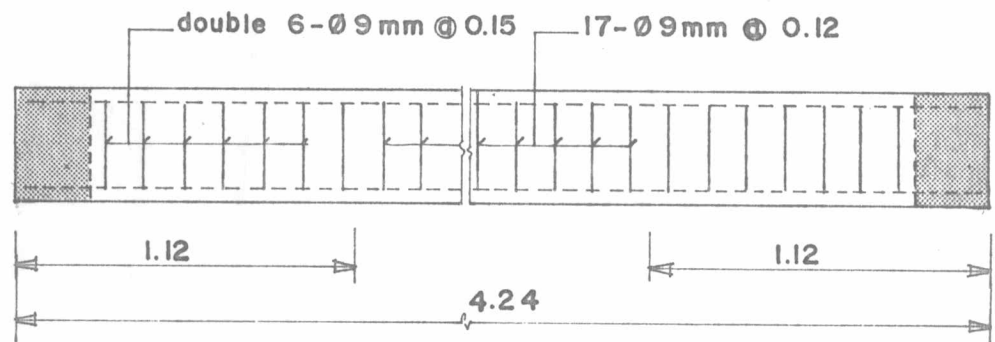


(d) Longitudinal section in transverse part

FIGURE 4.12 (Cont'd)



(e) Arrangement of binders in longitudinal part



(f) Arrangement of binders in transverse part