# CHAPTER II

## THEORY OF RECIPROCATING ENGINE VIBRATION

In transforming the reciprocating motion of a piston to a uniform rotation of a crank, the pistons and connecting rods in a reciprocating engine move at widely varying speeds. The gas pressures in the cylinder also vary widely as the piston moves. The periodic characteristic of the gas-pressure force and the inertia forces of the moving parts of the mechanism cause engine vibration.

The prediction of vibratory forcing functions generated by internal combustion engines is limited because there are other sources of vibration in engines which cannot easily be accounted for theoretically. Among these are residual unbalances of numerous rotating parts, gear teeth impact forces, exhaust gas impulses and variations in the weights of similar parts such as pistons and connecting rods. Some of the assumptions upon which a theoretical treatment **based on** often are not satisfied exactly in the practical cases. One of the most important of these assumptions is that each cylinder produces an identical pressure-time history. Any irregularity in the engine cycle violates the assumption. These irregularities may arise from variations in ignition, fuel distribution to the cylinders, irregular valve operation etc.

The primaries objectives of the control of vibration in automobile engines are to achieve acceptable environmental conditions for passengers such as smooth riding, low noise level and to obtain conditions to meet operational requirements such as reducing stresses on the engine components, limit vibration level to sensitive parts.

The following works deal with vibratory forcing functions of single cylinder followed by 4-cylinder in-line engine and finally the innovation of counterbalancing system applied in 4-cylinder in-line automotive engine.

Single Cylinder

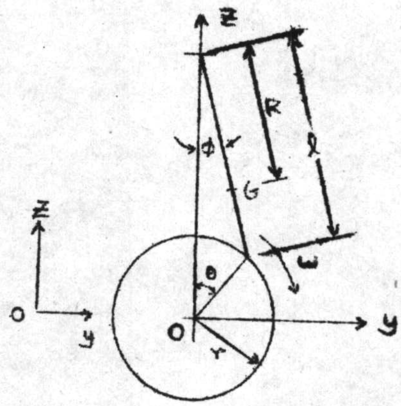


Fig. 2-1 Single cylinder

The crank and connecting rod mechanism is illustrated in Fig. 2-1. The followings are the forcing functions which act on the frame of the single cylinder engine:

1. Reciprocating inertia force acts along line of stroke.

$$F = -m\ddot{z}$$

2. Rotating inertia force rotates with crank. It can be eliminated by counterweight.

$$F_R = -m_3 r\omega^2$$

3. Residual couple acts on connecting-rod and reacts on engine frame.

$$M_R = m_2 R\ddot{\phi}\left(\frac{k_e^2}{R} - 1\right)$$

4. Gas-pressure torque acts on crankshaft and reacts on engine frame.

$$M_P = P\left(\frac{\dot{z}}{\omega}\right)$$

5. Reciprocating inertia torque acts on crankshaft and reacts on engine frame.

$$M_E = - m\ddot{z}(\frac{\mathring{z}}{\omega})$$

6. Correcting couple acts on crankshaft and reacts on engine frame.

$$M_M = M_R\{ \ p \sec\phi \cos\theta \ \}$$

where m = weight of the piston plus portion of connecting rod weight borne by piston pin

$\ddot{z}$ = acceleration of piston

$m_3$ = weight of crank effective at its radius plus portion of connecting rod weight borne by the crank pin

$\omega$ = rotational speed of the crank

$m_2$ = weight of the connecting rod

R = distance from centre of piston pin bearing on rod to its centre of gravity

$\ddot{\phi}$ = angular acceleration of connecting rod

$k_e$ = radius of gyration of connecting rod with respect to axis through piston pin bearing

l = lenght of connecting rod, distance between crankpin and piston pin

r = crankshaft radius

P = gas pressure force on piston head

$\mathring{z}$ = velocity of piston

p = r/l ratio of crank radius to lenght of connecting rod

$\theta$ = crank angle, measured from line of stroke with piston at top dead centre

To express the above forces and couples in a series of harmonic terms, it is necessary to express $\ddot{z}$, $\ddot{\phi}$, $\mathring{z}$ and $\sec\phi$ in terms of $\theta$.

$$F = mr\omega^2 \Sigma A_n \cos n\theta \tag{1}$$

$$M_R = -m_2 r\omega^2 (R - \frac{k_e}{l}) \sum_{n=1,3}^{\infty} E_n \sin n\theta \quad (n \text{ odd}) \tag{2}$$

$$M_E = mr^2\omega^2 \Sigma B_n \sin n\theta \tag{3}$$

$$M_M = m_2 r\omega^2 (R - \frac{k_e}{l}) \sum_{n=2,4}^{\infty} E_n \sin n\theta \quad (n \text{ even}) \tag{4}$$

$A_n$, $B_n$, $E_n$ the harmonic coefficients are given in Table 2-1 where $p=r/l$

| Harmonic order | $A_n$ | $E_n$ | $B_n$ |
|---|---|---|---|
| 1 | 1 | $(1+\frac{1}{8}p^2+\frac{3}{64}p^4 ---)$ | $-(\frac{1}{4}p+\frac{1}{16}p^3+\frac{15}{512}p^5 ---)$ |
| 2 | $(p+\frac{1}{4}p^3+\frac{15}{128}p^5 ---)$ | $(\frac{1}{2}p-\frac{1}{32}p^5 ---)$ | $(\frac{1}{2}+\frac{1}{32}p^4 ---)$ |
| 3 | 0 | $-(\frac{3}{8}p^2+\frac{27}{128}p^4 ---)$ | $(\frac{3}{4}p+\frac{9}{32}p^3+\frac{81}{512}p^5 ---)$ |
| 4 | $-(\frac{1}{4}p^3+\frac{3}{16}p^5 ---)$ | $-(\frac{1}{4}p^3+\frac{1}{8}p^5 ---)$ | $(\frac{1}{4}p^2+\frac{1}{8}p^4 ---)$ |
| 5 | 0 | $(\frac{15}{128}p^4 ---)$ | $-(\frac{5}{32}p^3+\frac{75}{512}p^5 ---)$ |
| 6 | $(\frac{9}{128}p^5 ---)$ | $(\frac{3}{32}p^5 ---)$ | $-(\frac{3}{32}p^4 ---)$ |

Table 2-1 Harmonic coefficients where $p=r/l$

4-Cylinder In-line Engine

The balanced crankshaft designs will normally have cranknagle spacing of 0 - 180 - 180 - 0 , by substituing these values into above formula neglecting $M_R$ and $M_M$ which are usually small resulting in the equations that follow:

$$F_x = F_y = 0 \tag{5}$$

$$F_z = 4mr\omega^2 (C + A_2 \cos 2\theta + 0 + A_4 \cos 4\theta + ------) \tag{6}$$

$$M_x = 4mr^2\omega^2( 0 + B_2\sin 2\theta + 0 + B_4\sin 4\theta + \text{-----}) \tag{7}$$

$$M_z = M_y = 0 \tag{8}$$

$$M_{comb} = - 4 \frac{\pi D^2}{4} r \{ ( 0 + a_2\sin 2\theta + 0 + a_4\sin 4\theta + \text{----}) + ( 0 + b_2\cos 2\theta + 0 + b_4\cos 4\theta + \text{----}) \} \tag{9}$$

where $F_x$, $F_y$, $F_z$ : Vibromotive force in the direction of x, y and z axes, respectively, due to reciprocating masses

$M_x$, $M_y$, $M_z$ : Vibromotive moment around x, y and z axes, respectively, due to reciprocating masses

$M_{comb}$ : Vibromotive moment, due to explosion, around x axis

D : Diameter of cylinder

r : Radius of crank

m : Reciprocating weight per cylinder

$\theta$ : Rotating angle of crankshaft from the top dead centre

$\omega$ : Rotational speed of the crank

l : Lenght of connecting rod

Harmonic coefficients $A_2$, $A_4$ -----, $B_2$, $B_4$ ----- are listed in Table 2-1. $a_2$, $a_4$ -----, $b_2$, $b_4$ ------- are shown in Fig. 2-2.

<u>Analysis of Counter Balancing System</u>

Theoretically if second mode unbalance existing in 4-cylinder in-line engine can be eliminated, the smoothness of 8-cylinder in line engine will result.

Thus, second mode vibration to be got rid of are as follow:

$$F_z = 4A_2mr\omega^2\cos 2\theta \tag{10}$$

$$M_x = 4B_2mr^2\omega^2\sin 2\theta \tag{11}$$

$$M_{comb} = - \pi D^2 r \ ( a_2\sin 2\theta + b_2\cos 2\theta ) \tag{12}$$

In order to get rid of the above vibromotive forces, a counter-balancing system with N shafts of eccentric mass $m_i$ and eccentrity $r_i$, i = 1, 2, ----- N are arranged parallel with the x axis and the centres of gravity of the eccentric masses on the plane x = 0 as shown in Fig. 2-3 and let the speed of revolution of the shafts are twice that of crankshaft.

The vibromotive forces given by these N shafts are as follows:

$$F_{Bx} = 0 \tag{13}$$

$$F_{By} = 4\omega^2 \sum_{i=1}^{N} m_i r_i \sin(2k_i\theta + \alpha_i) \tag{14}$$

$$F_{Bz} = 4\omega^2 \sum_{i=1}^{N} m_i r_i \cos(2k_i\theta + \alpha_i) \tag{15}$$

$$M_{Bx} = 4\omega^2 \sum_{i=1}^{N} m_i r_i \{z_i \sin(2k_i\theta + \alpha_i) - y_i \cos(2k_i\theta + \alpha_i)\} \tag{16}$$

$$M_{By} = M_{Bz} = 0 \tag{17}$$

$F_{Bx}, F_{By}, F_{Bz}$ : Vibromotive force given by counter-balance shafts along x, y and z axes, respectively

$M_{Bx}, M_{By}, M_{Bz}$ : Vibromotive moment given by counter-balance shafts around x, y and z axes, respectively

$k_i$ : Constant values being +1 when shaft i turns in the same direction as the crankshaft and -1 when shaft i turns in the opposite direction

$\alpha_i$ : Phase angle from the top dead centre of the eccentric mass $m_i$ when the first cylinder stays at top dead centre

$y_i, z_i$ : Coordinates of the centre of revolutions of the shaft i

It is necessary that Eq. (14) must be zero so $N \geq 2$. Taking N = 2 for the simplicity of the system, then from Eq. (14)

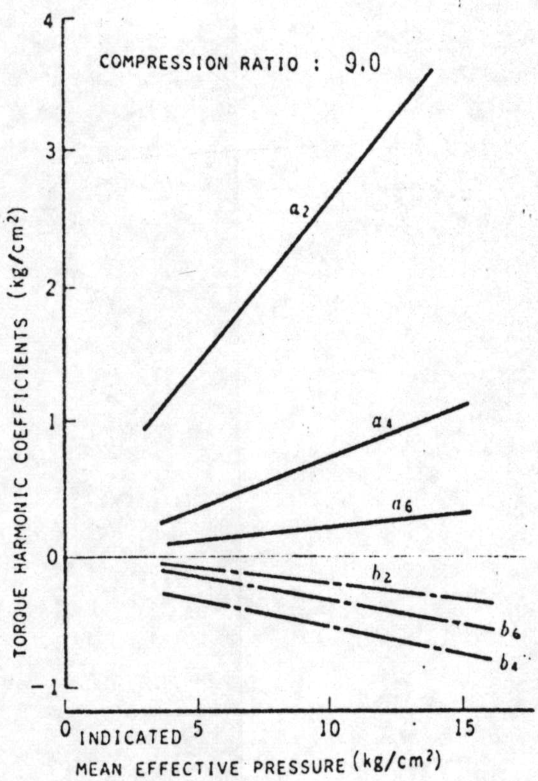


Fig. 2-2 Harmonic coefficients of torque fluctuation by combustion in 4-cylinder engine

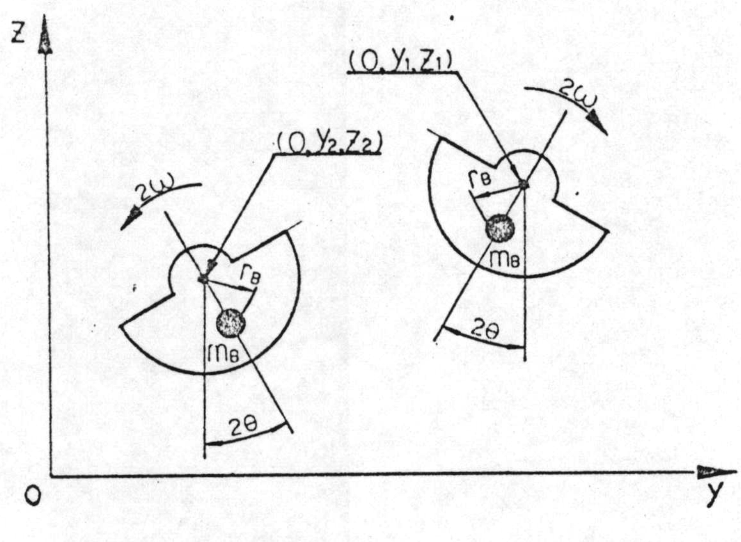


Fig. 2-3 Counter balancing shafts

Let $$m_1 r_1 = m_2 r_2 = m_B r_B \tag{18}$$

So that $$\sin(2\theta + \alpha_1) + \sin(-2\theta + \alpha_2) = 0 \tag{19}$$

Eq. (19) satisfies when $$\alpha_1 = \alpha_2 = \alpha = 0 \text{ or } \pi \tag{20}$$

Substitute Eqs. (18) and (20) into Eqs. (15) and (16) resulting in :

$$F_{Bz} = 8 m_B r_B \omega^2 \cos(2\theta + \alpha) \tag{21}$$

$$M_{Bx} = 4 m_B r_B \omega^2 \{(z_1 - z_2)\sin(2\theta + \alpha) - (y_1 + y_2)\cos(2\theta + \alpha)\} \tag{22}$$

Three cases can be considered.

(1) Elimination of vertical force only.

From Eqs. (1[illegible]) and (21)

$$F_z + F_{Bz} = 0$$

$$4A_2 mr\omega^2 \cos 2\theta + 8 m_B r_B \omega^2 \cos(2\theta + \alpha) = 0$$

It satisfies when

$$m_B r_B = \frac{1}{2} A_2 mr$$

And $$\alpha = \pi$$

In order to eliminate vibromotive moments from counterbalance shafts,

From Eq. (22) $$M_{Bx} = 0$$

If $$z_1 = z_2$$

$$y_1 = - y_2$$

This system is shown in Fig. 2-4

(2) Elimination of vibromotive force and moment originating from cranking motion.

In the similar manner as above to eliminate vertical force

$$m_B r_B = \frac{1}{2}A_2 mr$$

$$\alpha = \pi$$

To eliminate vibromotive moment from Eqs. (11) and (22)

$$M_x + M_{Bx} = 0 \tag{23}$$

And let $$y_1 = - y_2$$

$$\alpha = \pi$$

Then Eq. (23) becomes

$$4B_2 mr^2\omega^2 \sin 2\theta - 2A_2 mr\omega^2(z_1 - z_2) \sin 2\theta = 0$$

$$z_1 - z_2 = 2r\frac{B_2}{A_2}$$

Harmonic coefficients $A_2$ and $B_2$ are given in Table 2-1 as

$$B_2 = \frac{1}{2} + \frac{1}{32}p^4 + - - - -$$

$$A_2 = p + \frac{1}{4}p^3 \quad \frac{15}{128}p^5 + - - - -$$

In the majority of engines l/r has a value of 3.5 to 4 or more[5] and takes a value of 3.692 for the balancing engine in this case, then the values for $B_2$ and $A_2$ are

$$B_2 = \frac{1}{2} + \frac{1}{32(3.692)^4} = 0.500168$$

$$A_2 = \frac{1}{3.692} + \frac{1}{4(3.692)^3} + \frac{15}{128(3.692)^5} = 0.276$$

000247

$$\frac{B_2}{A_2} = 1.8119$$

Hence $$z_1 - z_2 = 2 \times 1.8119\, r$$

$$= 3.6238\, r$$

$$\simeq 1/r \times r$$

$$= 1$$

This system is similar to Fig. 2-5.

(3) Elimination of vibromotive force and moment originating from cranking motion as well as moment exerted by explosion

Similarly to above in eliminating vertical force

$$m_B r_B = 1/2\, A_2 m r$$

$$\alpha = \pi$$

In order to eliminate moment

$$M = M_x + M_{comb} + M_{Bx}$$

$$M = \{\, 4B_2 m r^2 \omega^2 - \pi D^2 a_2 r - 4m_B r_B \omega^2 (z_1 - z_2)\} \sin 2\theta$$

$$+ \{\, -\pi D^2 b_2 r + 4(y_1 + y_2) m_B r_B \omega^2\} \cos 2\theta$$

Select $$y_1 = - y_2$$

$$M = \{\, 4B_2 m r^2 \omega^2 - \pi D^2 a_2 r - 4m_B r_B \omega^2 (z_1 - z_2)\} \sin 2\theta$$

$$- \pi D^2 b_2 r \cos 2\theta$$

Nakamura[6] stated that in general automotive engine operating at road load, the sine and cosine components of $M_{comb}$ exerted by combustion are equal to about 30 and 3% of that originating from cranking motion.

Consequently

$$M = \{ 4B_2mr^2\omega^2 - 0.3(4B_2mr^2\omega^2) - 4m_Br_B\omega^2(z_1 - z_2) \}$$

$$\sin 2\theta - 0.03(4B_2mr^2\omega^2) \sin 2\theta$$

$$= \{ 0.67 \times 4B_2mr^2\omega^2 - 4m_Br_B\omega^2( z_1 - z_2) \} \sin 2\theta$$

For $M = 0$

$$z_1 - z_2 = ( 0.67 \times 4B_2mr^2\omega^2 ) \div ( 4m_Br_B\omega^2 )$$

$$= ( 0.67 \times 4B_2mr^2\omega^2 ) \div ( 4 \times \tfrac{1}{2}A_2mr\omega^2 )$$

$$= 0.67 \times 2rB_2 \div A_2$$

$$\approx 0.7\ 1$$

This system arrangement is shown in Fig. 2-5 where vibromotive force is completely eliminated and vibromotive moment arising from inertia and gas pressure is lower as far as possible under frequently followed road load condition and at the same time to decrease the moment on an average over all other load encountered. This system is adopted in the counter balancing shaft engine being investigated.

The operational principle of the second-mode counter balancing system could be summarized as in Fig. 2-6 where one counter balance shaft is arranged above and

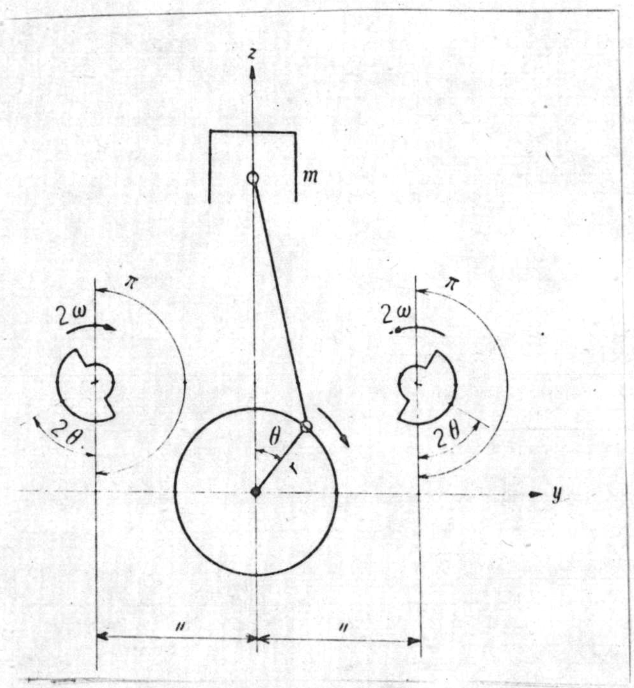


Fig.2-4 Counter balancing system for equilibration of vertical force only

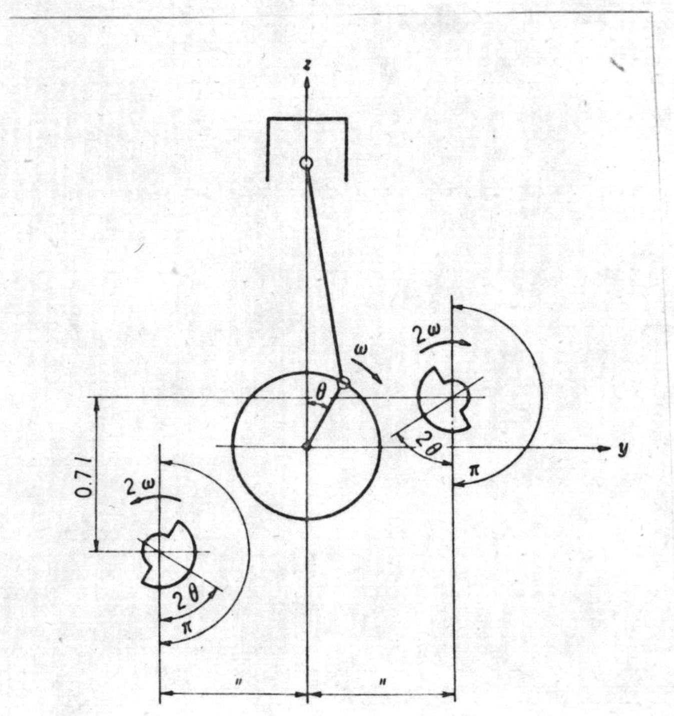


Fig.2-5 Optimum counter-balancing system

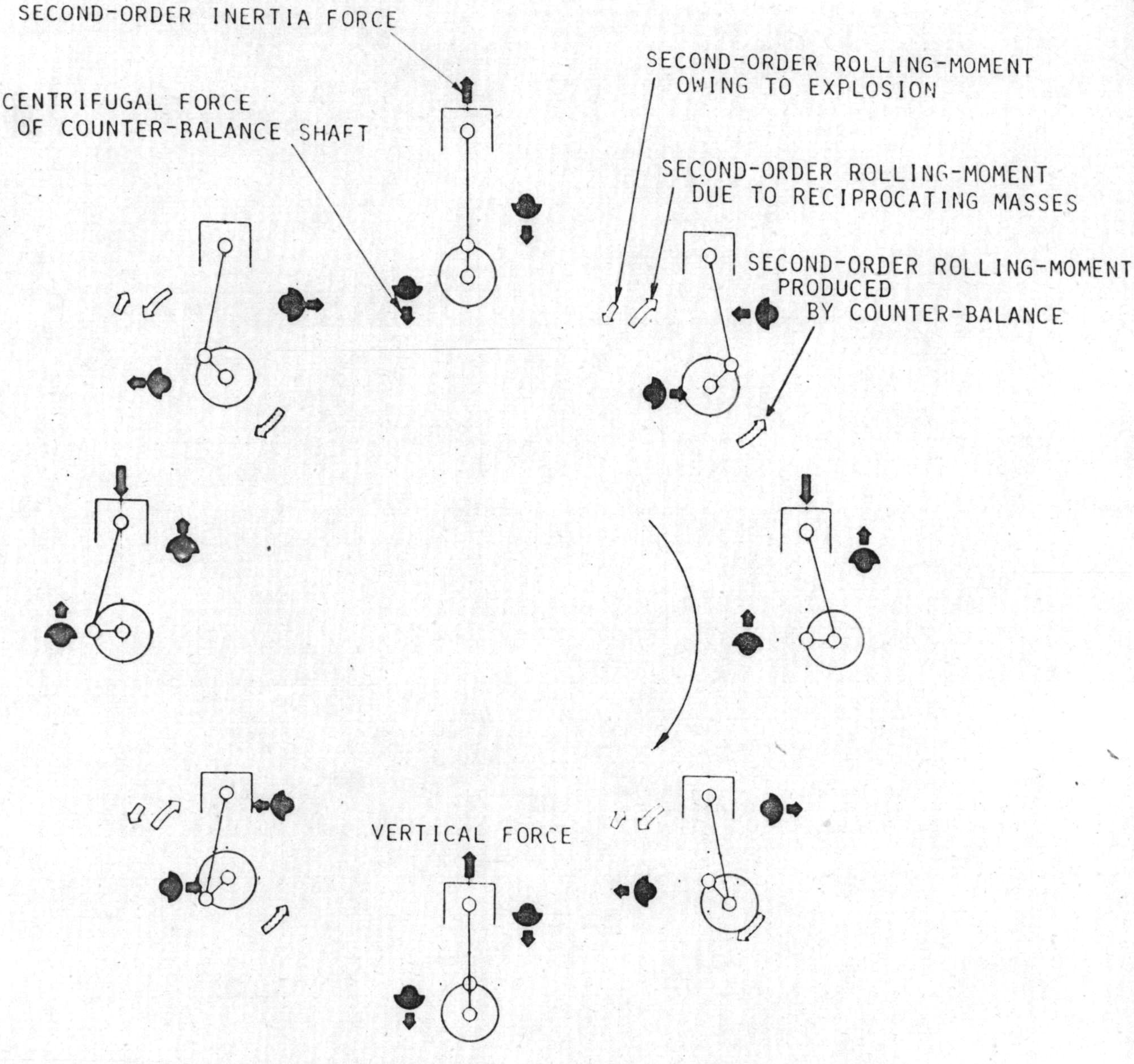


Fig.2-6 Operational principle of counter-balancing system

rotate in the same direction as that of the crankshaft while another being arranged below and made to rotate in the opposite direction by means of a reversing gear. The system is driven by chain connected to crankshaft at twice the speed of the crankshaft revolution.