CHAPTER I

INTRODUCTION

The well-known trigonometric functions cosine, sine and tangent satisfy the following functional equations

$$(1)\quad f(x+y)+f(x-y) = 2f(x)\,f(y),$$

$$(2)\quad f(x+y)+f(x-y) = 2f(x)\,f(y+\tfrac{\pi}{2}),$$

$$(3)\quad (1-f(x)\,f(y))f(x+y) = f(x)+f(y),$$

respectively. Observe that in these three equations we require the domain of f to be an algebraic system with one binary operation +, and require the range of f to be a subset of an algebraic system with two binary operations + and $\cdot$. If we consider f to be a function on a group G into a field K, both equations (1) and (2) are special cases of the functional equation

$$(S)\quad f(xy)+f(xy^{-1}) = 2f(x)\,f(y\theta),$$

where θ is a fixed element of G, and equation (3) is a special case of the functional equation

$$(T)\quad (1-f(x)\,f(y))f(xy) = f(x)+f(y).$$

The purpose of this thesis is to determine all the solutions of (S) and (T). In chapter IV, we characterize a class of solutions of (S) in terms of homomorphisms from G into the multiplicative group $\mathbb{C}^*$ of complex numbers. This class includes all the solutions of (S) in the case where G is commutative. Chapter V deals with characterization of continuous solutions of (S) on topological groups. Chapter VI deals with characterization of the solutions and continuous solutions of (T). In both chapters V and VI, we illustrate how our result can be applied to the case $G = \mathbb{R}^n$. In doing so, we need the knowledge of homomorphisms on $\mathbb{R}^n$ into certain subgroups of $\mathbb{C}^*$. Chapter III deals with the study of such homomorphisms.