CHAPTER II

THEORY

Models of modulus elasticity and ultimate strength for composite materials reinforced in unidirection arrangement of fibres under uniaxial tensile loads applied in the direction of the fibres can be established readily, if two assumptions are made. The first of these is that both constituent materials are linearly elastic, which is true for the glass fibres but only approximately true for the plastic matrices. The second assumption is that the strains in the two materials are equal, which implies perfect bonding between them and is sometimes dignified by the title "Theory of Combined Action".

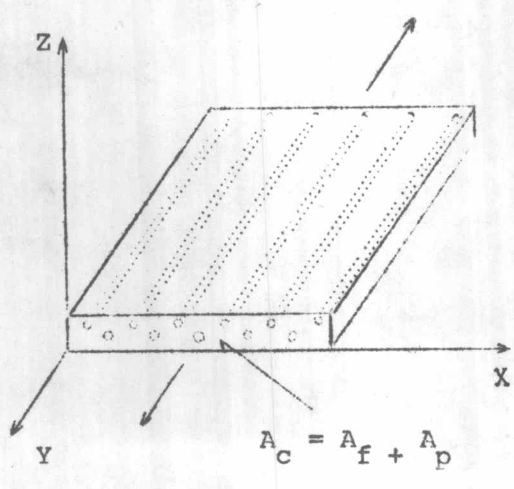

Fig. 2-1 DIAGRAMMATIC REPRESENTATION OF A UNIDIRECTIONAL LAMINATE

The first assumption is to imply [illegible] that both materials obey Hooke's law, i.e.

$$\epsilon = \frac{\sigma}{E} \qquad (1)$$

where ϵ = strain, σ = stress, E = tensile modulus. The second assupmtion implies that

$$\epsilon_c = \epsilon_f = \epsilon_p \qquad (2)$$

where the subscripts c,f,p apply to the composite, fibres and plastic respectively. Combining equations (1) and (2) with the equilibrium equation

$$A_c\sigma_c = A_f\sigma_f + A_p\sigma_p \qquad (3)$$

where A = cross-sectional area, leads to the following equation for the modulus of the composite,

$$E_c = E_f\frac{A_f}{A_c} + E_p\frac{A_p}{A_c} \qquad (4)$$

This means that the modulus of the composite is the volume weighted average of the moduli of the fibres and the plastic, which is often referred to as the "Law of mixtures: Equation (4) is, strictly, only correct if the Poisson's ratios of the constituent materials are the same but although this is not generally the case in the differences are small and the 'Law of mixtures' gives a good indication of the longitudinal stiffness of undirectional composites.

Following a very similar argument, it can be shown that if the plastics matrix is highly compliant, i.e. that its strain to fraceture is greater than that of the glass fibres, the ultimate strength of the composite is given by:

$$\sigma_{cu} = \sigma_{fu}\left(\frac{A_f}{A_c} + \frac{E_p A_p}{E_f A_c}\right) \qquad (5)$$

where suffix u signifies ultimate. If the plastics matrix has a lower strain to failure than the glass fibres, equation (5) reduces to :

$$\sigma_{cu} = \sigma_{fu}\frac{A_f}{A_c} \qquad (6)$$