CHAPTER 0

INTRODUCTION

The theory of distributions appears to have first been formulated in 1936 by S.L.Soboleff and then developed in a systematic and thorough way by L.Schwartz,whose books appeared in 1950 and 1951. The Schwartz's theory of distributions had two important effects in mathematical analysis. First of all, it provided a rigorous justification for a number of formal manipulations that had become quite common in the technical literature. The second and more important effect was that it opened up a new area of mathematical research, which is turn provided as impetus in the development of number of mathematical disciplines, such as ordinary and partial differential,operational calculus, transformation theory, functional analysis, and Potential theory.

The purpose of this thesis is to apply distributions into Potential theory. It is hoped that the result of this study will help to popularize Potential theory.

Throughout this thesis Ω will stand for an open subset of $\mathbb{R}^n$ and μ will be a positive Radon measure. Some knowledge of topology and real analysis are also assumed.

Briefly, the structure of this thesis is as follows :

Chapter I recalls some notions and facts which are going to be used repeatedly without proof.

Chapter II introduces the space $\mathcal{D}(\Omega)$, which is the space of test functions on which distributions in Ω are defined, and the Schwartz functions.

Chapter III is devoted to the definition, the basic properties, support and differentiation of distributions, and Radon measures.

Chapter IV characterizes the convolution of functions and of distributions.

Chapter V studies some properties of the semicontinuous functions and the representation of Radon measures in term of Borel measures. Finally some well-known integration theorems are also discussed.

Chapter VI ends the thesis with an application of the previous results to Potential theory. It acts in harmonic functions, the Poisson integrals, superharmonic functions, potentials and finally the Riesz decomposition theorem. It is hoped that this study will be enough to provide tools for further studies by others who feel interested in this area.