## CHAPTER III

## ANALOG COMPUTER APPLICATIONS IN DESIGN OF NONUNIFORMLY SPACED ARRAY

As studies on the realization of the designated directional pattern by using unequally spaced array we have the one in which the infinite Fourier series component of a given directional pattern are approximated by the infinite set of series,but the need to invert the matrix equal to the number of elements contained in the array , this method is difficult to use in practice , as well as the one in which a directional pattern is obtained by relating a pattern function obtained from the continuous line source to a distribution function of element spacings , is only an approximate method and the radiation pattern obtained has some deviations from the desired values, especially in the far out sidelobes.

This thesis introduce the applications of an analog computer to the pattern synthesis of nonuniformly spaced arrays in order to obtain the more satisfied patterns. The value of this method is that a directional pattern can easily be observed or examined in the form of visual curve on the monitor ossilloscope of an analog computer.

Firstly, a likely configuration of element is chosen and the array factor is programmed into an analog computer.After the directional pattern has been displayed on the monitor oscilloscope the more satisfied patterns can be obtained by the perturbation method. The coefficient potentiometers providing on the analog computing panel which correspond to the element locations are adjusted to vary, or perturb, the location of each element one at a time and to select the position which produces the best pattern that can be - observed from the oscilloscope. After the location of all elements are perturbed , the process is repeated until perturbing the elem ent locations offer no significant improvement of the radiation pattern. The sucess of this method depends on the choice of the ini tial element locations and how freedom the computer is allowed in the examination of the possible alternative locations for each ele ment .

For convenience, the initial element location is based on the density tapering method(see appendix) of design nonuniformly spaced array.

Let consider again to Eq.(20). the array factor of nonunifor mly spaced array

$$f(u) = 1 + 2\sum_{n=1}^{N} \cos 2\pi z_n u$$

If $z_n$ the distance of the $n^{th}$ element expressed in wavelength from the center are given, the mathematical operation of algebratic summation can be performed by an analog computer and f(u) will be simultaneously plotted on the monitor oscilloscope. In so doing, the block diagrams corresponding to the array factor equation must be first constructed .

The variable u or $\sin\theta$ in this equation is chosen to be the time variable or computed time of an analog computerwhich is denoted by t . Thus f(u) will correspond to the voltage level for instantaneous value of t .

## Preparation of Block Diagram

Now we will illustrate by block diagrams how to solve the array factor equation by the combination of computing elements.

### Generation of $2\cos(2\pi z_n t)$

The generalized $\cos \omega t$ can be obtained as a solution of the following eq$^n$.

$$\frac{d^2x}{dt^2} + \omega^2 x = 0 \quad ; \quad \text{if } t=0 \text{ , } x=1$$

The above equation can be made into the following Block Diagram :

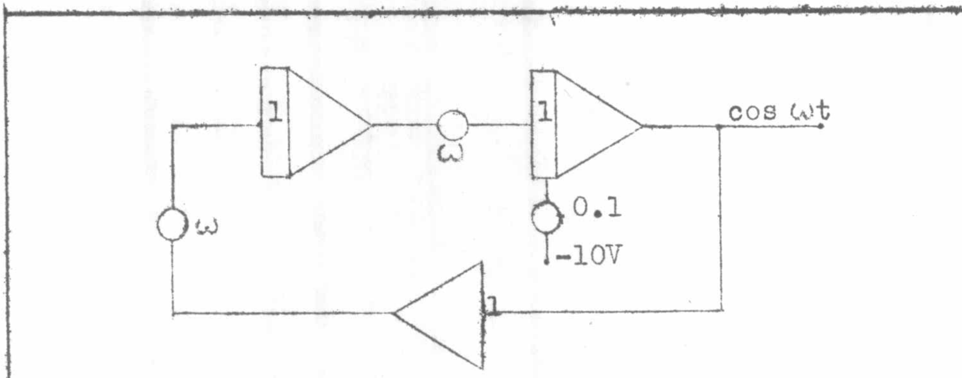
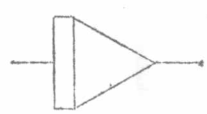
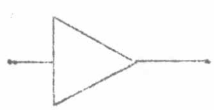



Fig.3a The block diagram for generating $\cos \omega t$ .



the summing integrator

where  represents the summing amplifier

the coefficient potentiometer

Each terms of cosine function ,$2\cos(2\pi z_n t)$, can be generated by this - set of block diagram where the coefficient potentiometer $\omega$ corresponds to $2\pi z_n$ and the initial condition is changed to 2V.

The complete block diagram of the array factor equation required one more summing amplifier to sum up the N+1 terms, as shown below :

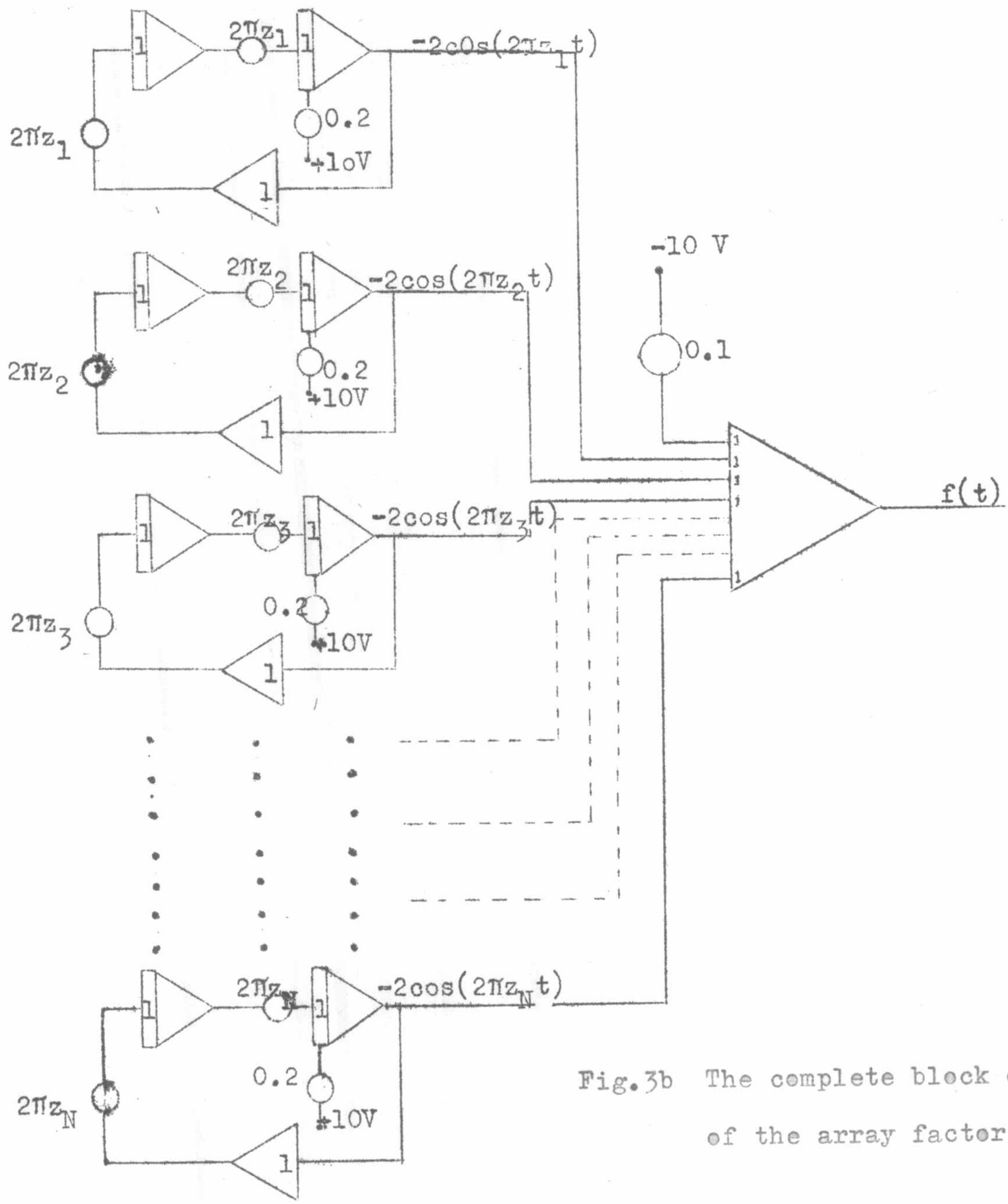



Fig.3b The complete block diagram of the array factor.

$$f(t) = 1 + 2\sum_{n=1}^{N} \cos(2\pi z_n t)$$

$(\ t = u = \sin\theta = 0 \rightarrow 1\ )$

## Example

The element locations of nonuniformly spaced array designed for the pattern of -25 dB sidelobe level, from the Taylor's line source, by density tapering method ( see appendix ) are as follow :

Total number of elements = 2N + 1 = 7

$$z_0 = 0 \quad \text{(center element)}$$
$$z_1 = 0.765\ \lambda$$
$$z_2 = 1.755\ \lambda$$
$$z_3 = 3.15\ \lambda$$

The array factor can be written as follow :

$$f(u) = 1 + 2\left[\cos(2\pi 0.765u) + \cos(2\pi 1.755u) + \cos(2\pi 3.15u)\right]$$
$$= 1 + 2\cos(4.81u) + 2\cos(11.02u) + 2\cos(19.8u)$$

This equation is programmed to an analog computer by following steps .

Scaling

In computation with an analog computer, the independent and dependent variables are replaced by time and voltage respectively. The range of computing voltage must be allowed for each type of analog computer and it is necessary that variations in the voltage of dependent variables be within this range .

The maximum computing voltage of the above equation occurs when u = 0 and equals to 7 volts. If the YEW Type Analog Computer shown in Fig. 4a is employed for this computation , requirement for the amplitude scaling is unnecessary since the computer range of computation voltage is -10 V through 10 V .

Time scaling

Although this equation has given voltage which is within the range of 10V, it is necessary to do time scaling because of the - frequency response of the computing element and the necessity of recording the solution .

The analog computer equation may be written as :

$$f(t) = 1 + 2\cos 0.481t + 2\cos 1.102t + 2\cos 1.98t$$

where $t$ = 10u is the computer time after the time scaling

number 10 = time scale factor.

Block digram

The computing equation is illustrated by the block diagram as follow :

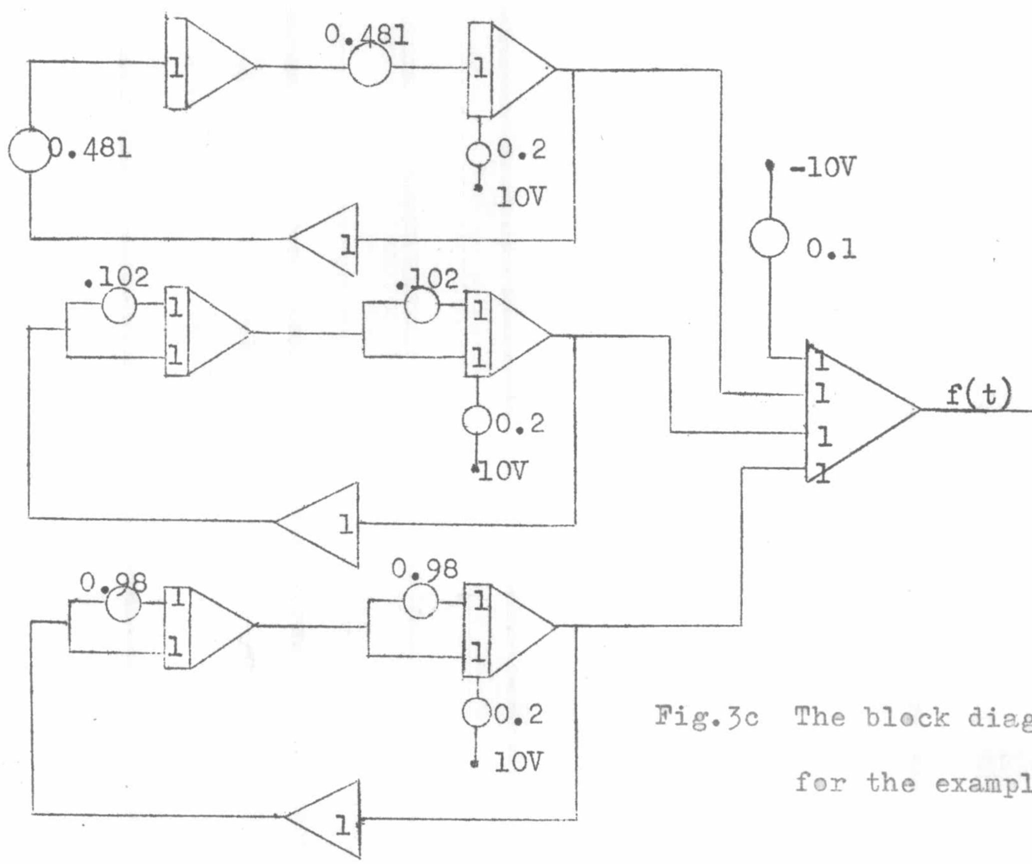



Fig.3c The block diagram for the example

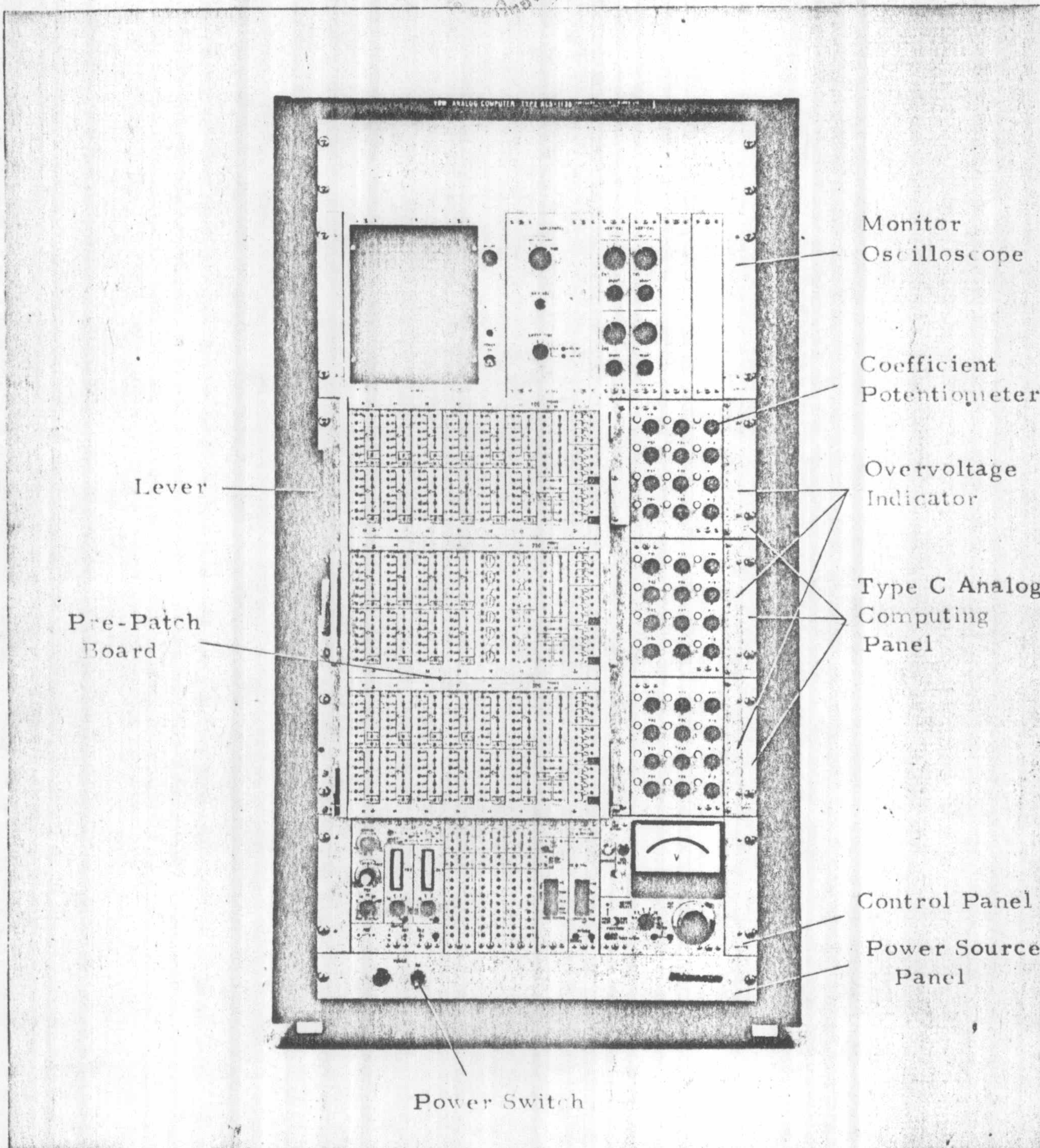



**Fig. 4a Analog coputer employed for the calculation.**

Patching

Patch cords are used in connecting computing elements according to the block diagram. Fig.4b illustrates this procedure for the block diagram in page 17 by using YEW TYPE 3304 ANALOG COMPUTER .

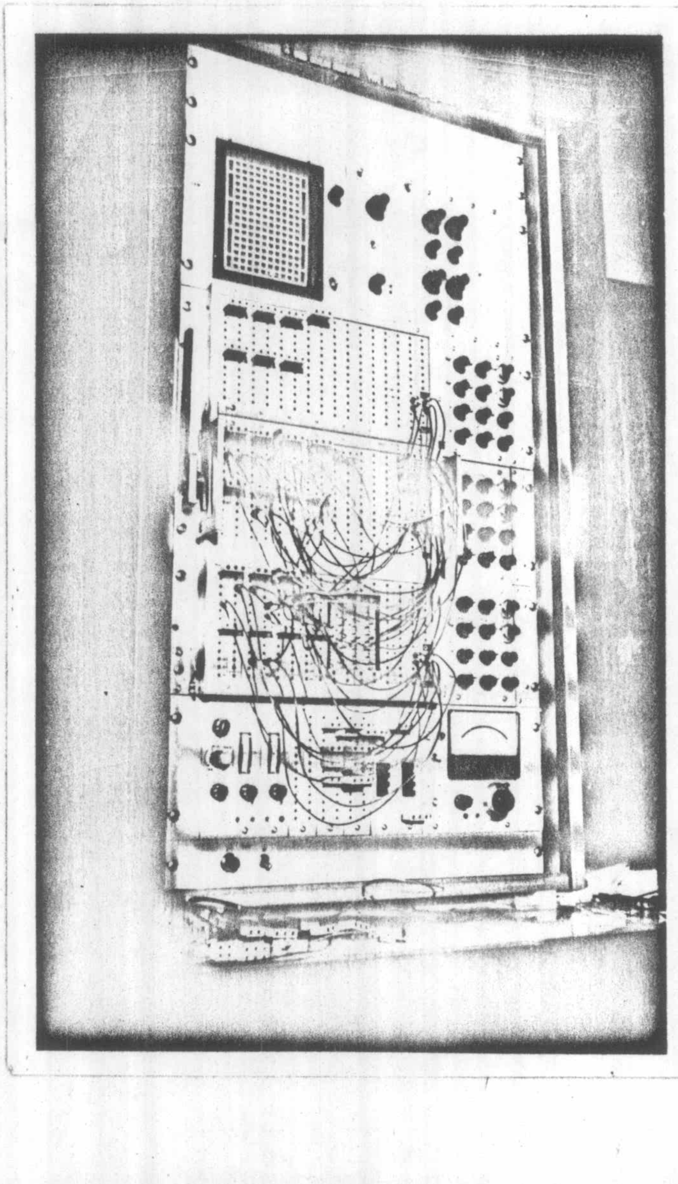


Fig.4b Connection of the computing units

## Solution Recording by High Speed Computation

After the setting of the coefficient potentiometer is completed the calculation is already to be operated. Introduce the solution - voltages into the monitor oscilloscope and the solution waveforms can be observed . For convenience,it is suggested to select the high speed computation.Fig.5 illustrates the result .

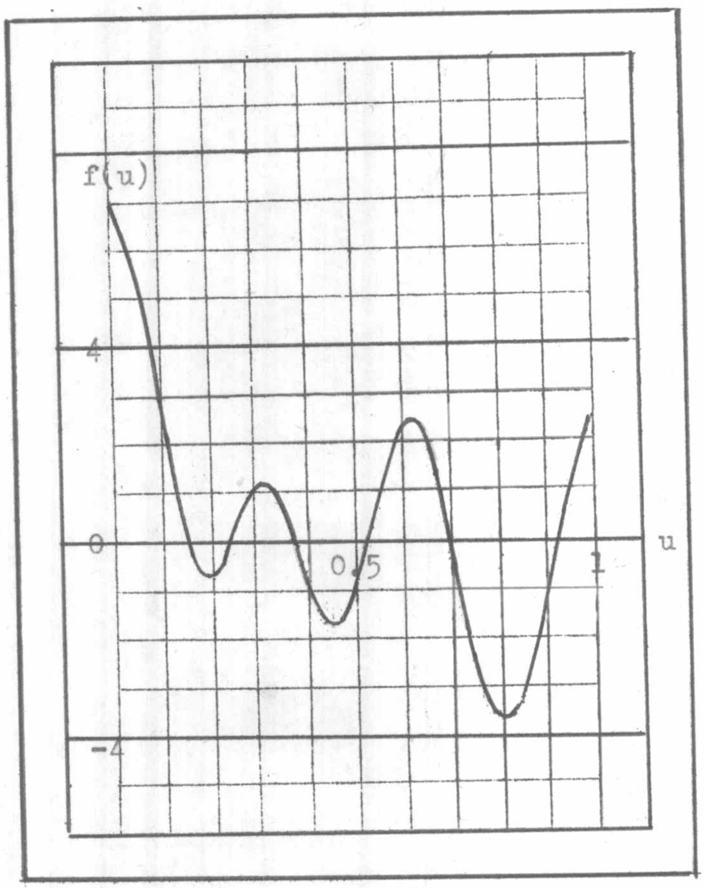



Fig.5

The visual curve of a directional pattern as seen from oscilloscope of an analog com puter.

Following the foregoing procedures various patterns of nonuni formly spaced arrays can be examined. Table.1 provides the initial values of coefficient potentiometers to be set in perturbation meth od.Fig.6 -12 show the characteristics of nonuniformly spaced arrays obtained from these settings .

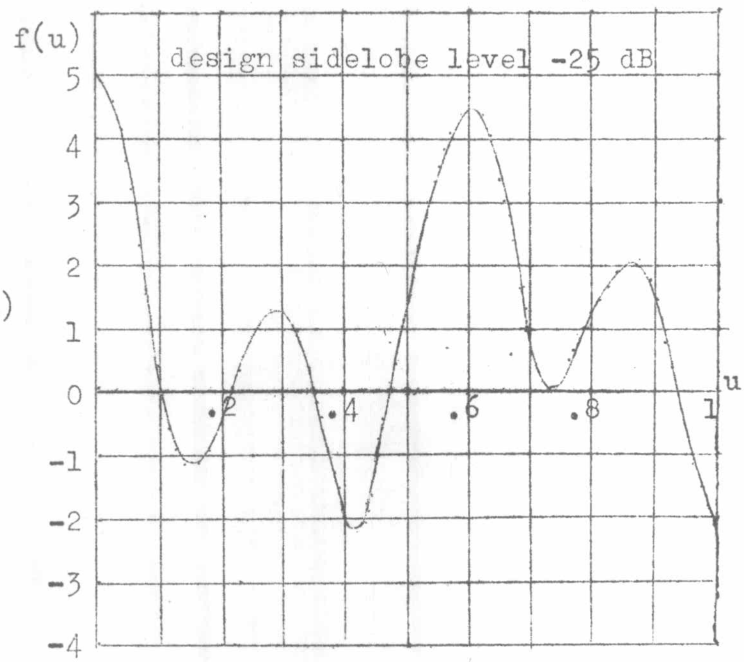
Table.1 The initial values of coefficient potentiometer settings .

| Number of elements | Design sidelobe level(dB) | Coefficient Potentiometer Settings | | | Time scale factor |
|---|---|---|---|---|---|
| | | $2\pi z_1$ | $2\pi z_2$ | $2\pi z_3$ | |
| 4 | 15 | 0.23 | 0.72 | | 2π |
| | 20 | 0.21 | 0.65 | | |
| | 25 | 0.185 | 0.585 | | |
| | 30 | 0.16 | 0.54 | | |
| | 35 | 0.145 | 0.5 | | |
| | 40 | 0.142 | 0.47 | | |
| 5 | 15 | 0.365 | 0.78 | | 2π |
| | 20 | 0.33 | 0.705 | | |
| | 25 | 0.29 | 0.64 | | |
| | 30 | 0.25 | 0.59 | | |
| | 35 | 0.23 | 0.55 | | |
| | 40 | 0.21 | 0.52 | | |
| 6 | 15 | 0.15 | 0.46 | 0.82 | 2π |
| | 20 | 0.14 | 0.41 | 0.75 | |
| | 25 | 0.13 | 0.36 | 0.685 | |
| | 30 | 0.11 | 0.315 | 0.635 | |
| | 35 | 0.1 | 0.29 | 0.58 | |
| | 40 | 0.095 | 0.27 | 0.56 | |

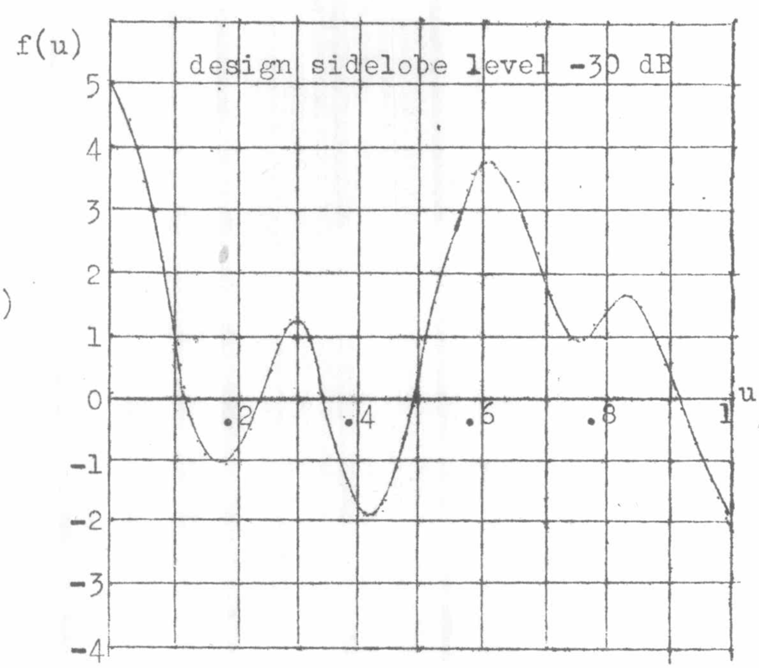
| Number of elements | Design sidelobe level(dB) | Coefficient Potentiometers Settings | | | |
|---|---|---|---|---|---|
| | | $2\pi z_1$ | $2\pi z_2$ | $2\pi z_3$ | $2\pi z_4$ |
| 7 | 15 | 0.25 | 0.53 | 0.84 | |
| | 20 | 0.23 | 0.47 | 0.78 | |
| | 25 | 0.205 | 0.415 | 0.71 | |
| | 30 | 0.175 | 0.42 | 0.7 | |
| | 35 | 0.16 | 0.34 | 0.625 | |
| | 40 | 0.15 | 0.32 | 0.59 | |
| 8 | 15 | 0.115 | 0.34 | 0.59 | 0.865 |
| | 20 | 0.115 | 0.31 | 0.52 | 0.8 |
| | 25 | 0.1 | 0.27 | 0.46 | 0.735 |
| | 30 | 0.085 | 0.23 | 0.415 | 0.69 |
| | 35 | 0.08 | 0.21 | 0.385 | 0.65 |
| | 40 | 0.075 | 0.2 | 0.36 | 0.62 |
| 9 | 15 | 0.2 | 0.415 | 0.65 | 0.88 |
| | 20 | 0.19 | 0.37 | 0.575 | 0.83 |
| | 25 | 0.18 | 0.33 | 0.515 | 0.76 |
| | 30 | 0.145 | 0.29 | 0.47 | 0.72 |
| | 35 | 0.135 | 0.26 | 0.43 | 0.68 |
| | 40 | 0.125 | 0.24 | 0.4 | 0.65 |

| Number of elements | Design sidelobe level (dB) | Coefficient Potentiometer Settings | | | | | |
|---|---|---|---|---|---|---|---|
| | | $2\pi z_1$ | $2\pi z_2$ | $2\pi z_3$ | $2\pi z_4$ | $2\pi z_5$ | $2\pi z_6$ |
| 10 | 15 | 0.09 | 0.27 | 0.465 | 0.675 | 0.89 | |
| | 20 | 0.08 | 0.25 | 0.415 | 0.6 | 0.84 | |
| | 25 | 0.072 | 0.22 | 0.365 | 0.535 | 0.77 | |
| | 30 | 0.065 | 0.19 | 0.32 | 0.485 | 0.73 | |
| | 35 | 0.063 | 0.17 | 0.29 | 0.45 | 0.7 | |
| | 40 | 0.062 | 0.16 | 0.275 | 0.425 | 0.66 | |
| 11 | 15 | 0.16 | 0.315 | 0.5 | 0.7 | 0.9 | |
| | 20 | 0.15 | 0.29 | 0.45 | 0.625 | 0.86 | |
| | 25 | 0.135 | 0.255 | 0.395 | 0.56 | 0.8 | |
| | 30 | 0.12 | 0.22 | 0.35 | 0.51 | 0.755 | |
| | 35 | 0.11 | 0.2 | 0.32 | 0.475 | 0.72 | |
| | 40 | 0.1 | 0.19 | 0.3 | 0.45 | 0.685 | |
| 12 | 15 | 0.07 | 0.2 | 0.36 | 0.545 | 0.72 | 0.915 |
| | 20 | 0.065 | 0.19 | 0.335 | 0.48 | 0.65 | 0.87 |
| | 25 | 0.06 | 0.17 | 0.29 | 0.43 | 0.58 | 0.81 |
| | 30 | 0.06 | 0.145 | 0.25 | 0.38 | 0.53 | 0.77 |
| | 35 | 0.058 | 0.135 | 0.23 | 0.35 | 0.495 | 0.74 |
| | 40 | 0.055 | 0.13 | 0.215 | 0.33 | 0.465 | 0.7 |

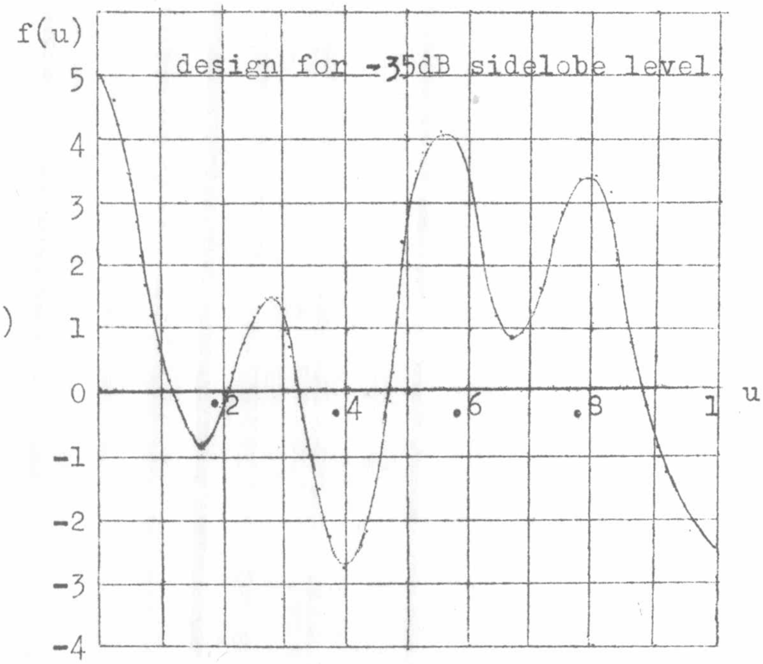
(a)


f(u)
design sidelobe level -25 dB
5
4
3
2
1
0
-1
-2
-3
-4
u
.2
.4
.6
.8
1

(b)


f(u)
design sidelobe level -30 dB
5
4
3
2
1
0
-1
-2
-3
-4
u
.2
.4
.6
.8
1

(c)




(d)

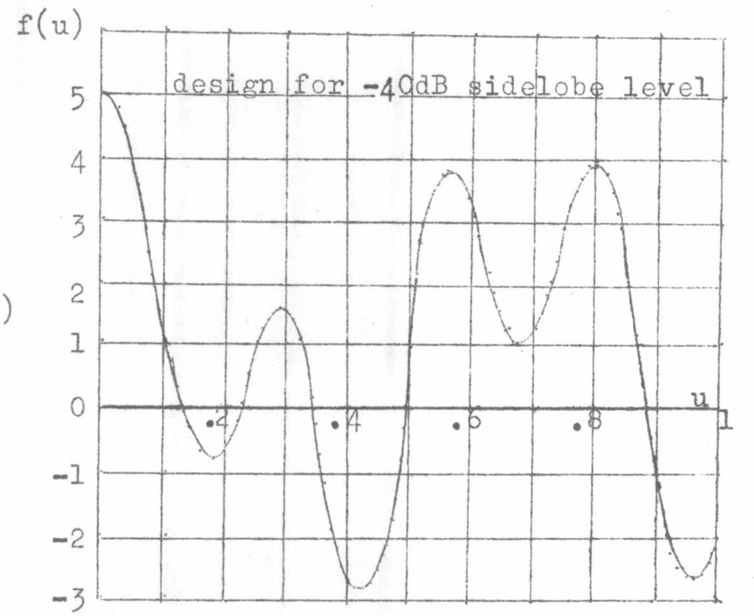
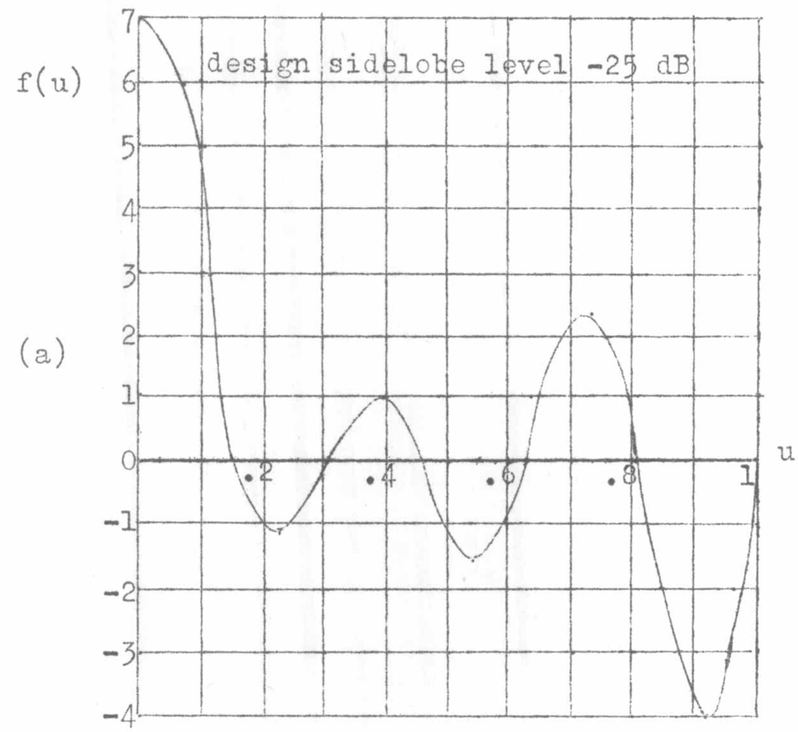
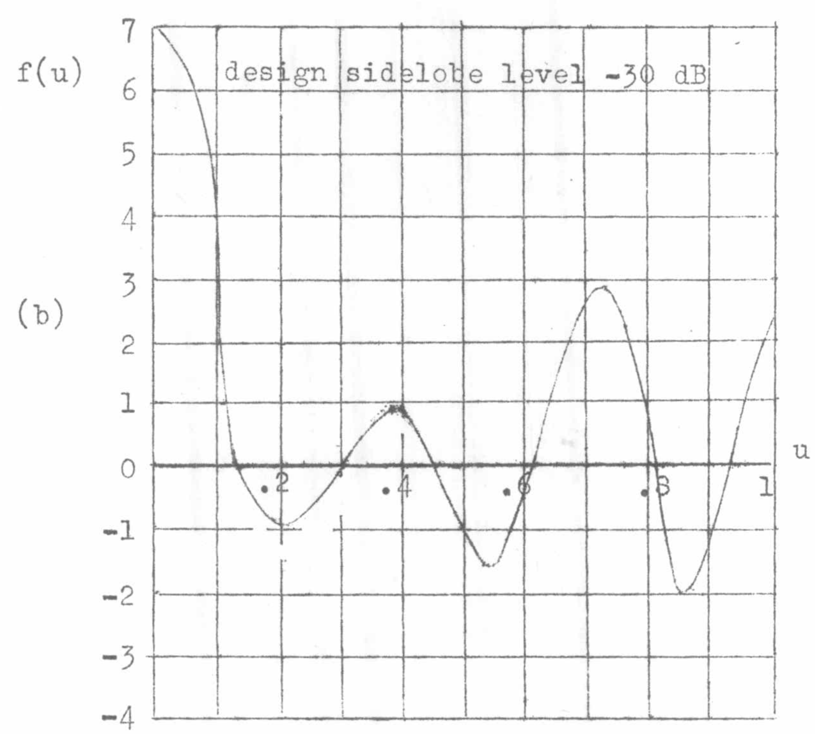



Fig. 6 Patterns of 5 element nonuniformly spaced arrays designed for various sidelobe levels ( the minimum spacing between adjacent elements is 0.5 $\lambda$ )

(a)


f(u)
design sidelobe level -25 dB
7
6
5
4
3
2
1
0
-1
-2
-3
-4
.2
.4
.6
.8
1
u

(b)


f(u)
design sidelobe level -30 dB
7
6
5
4
3
2
1
0
-1
-2
-3
-4
.2
.4
.6
.8
1
u

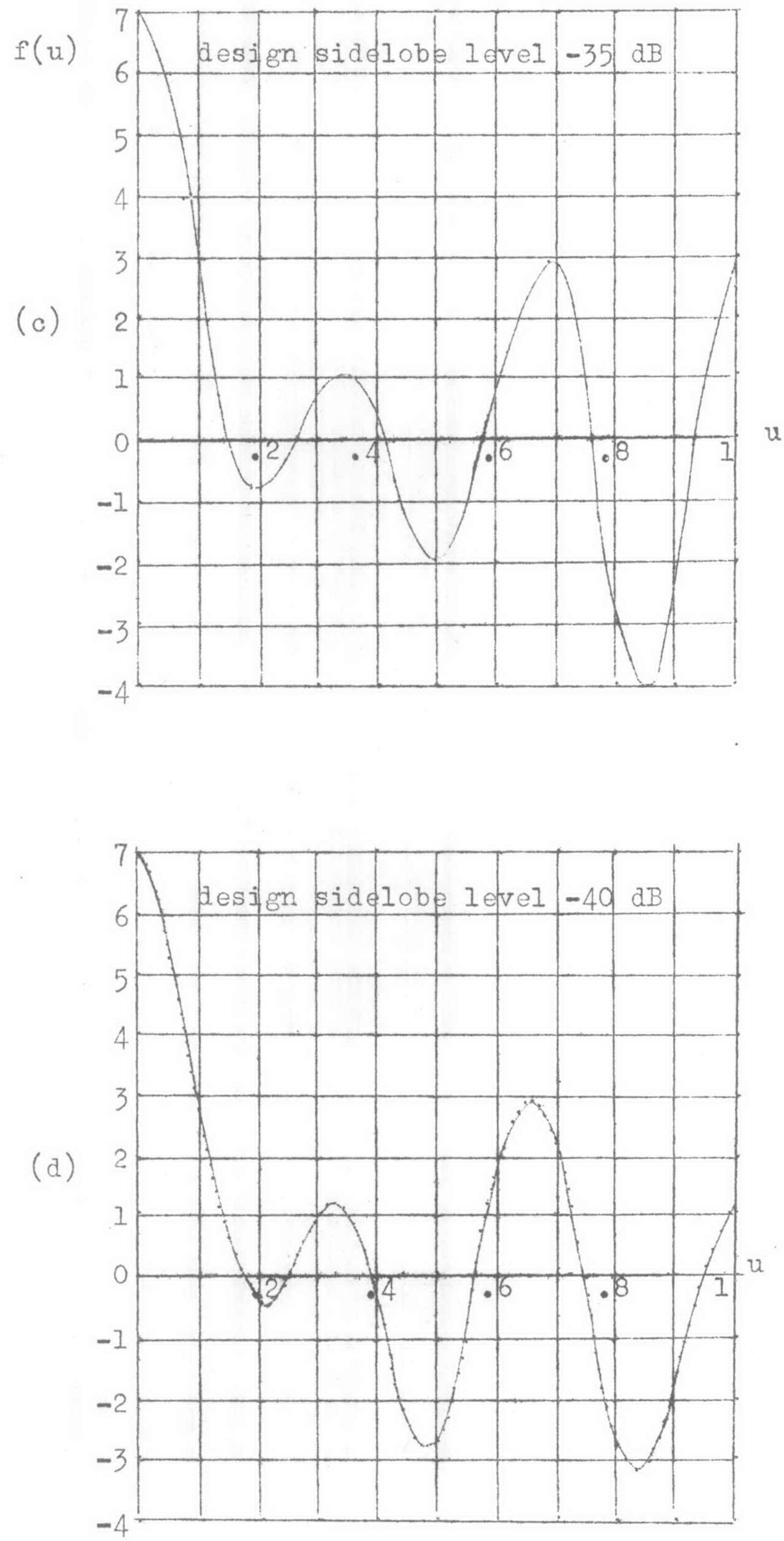
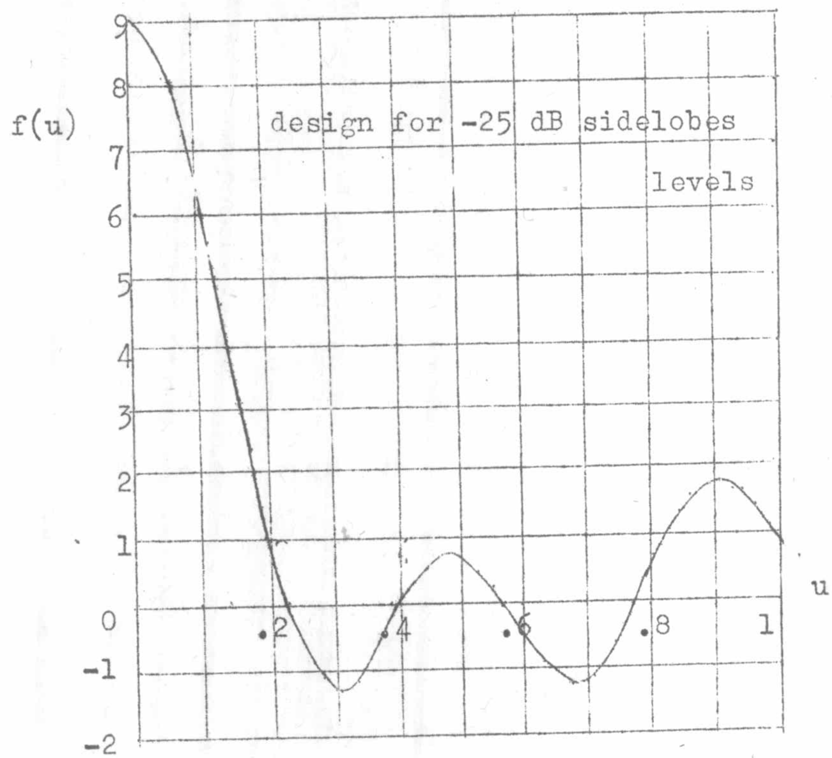
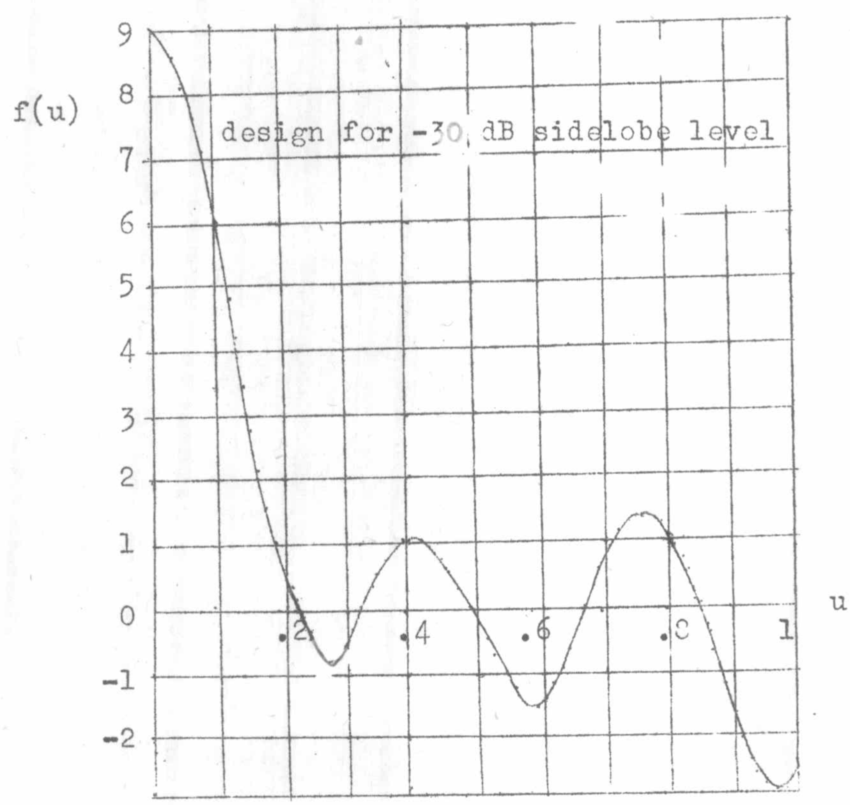


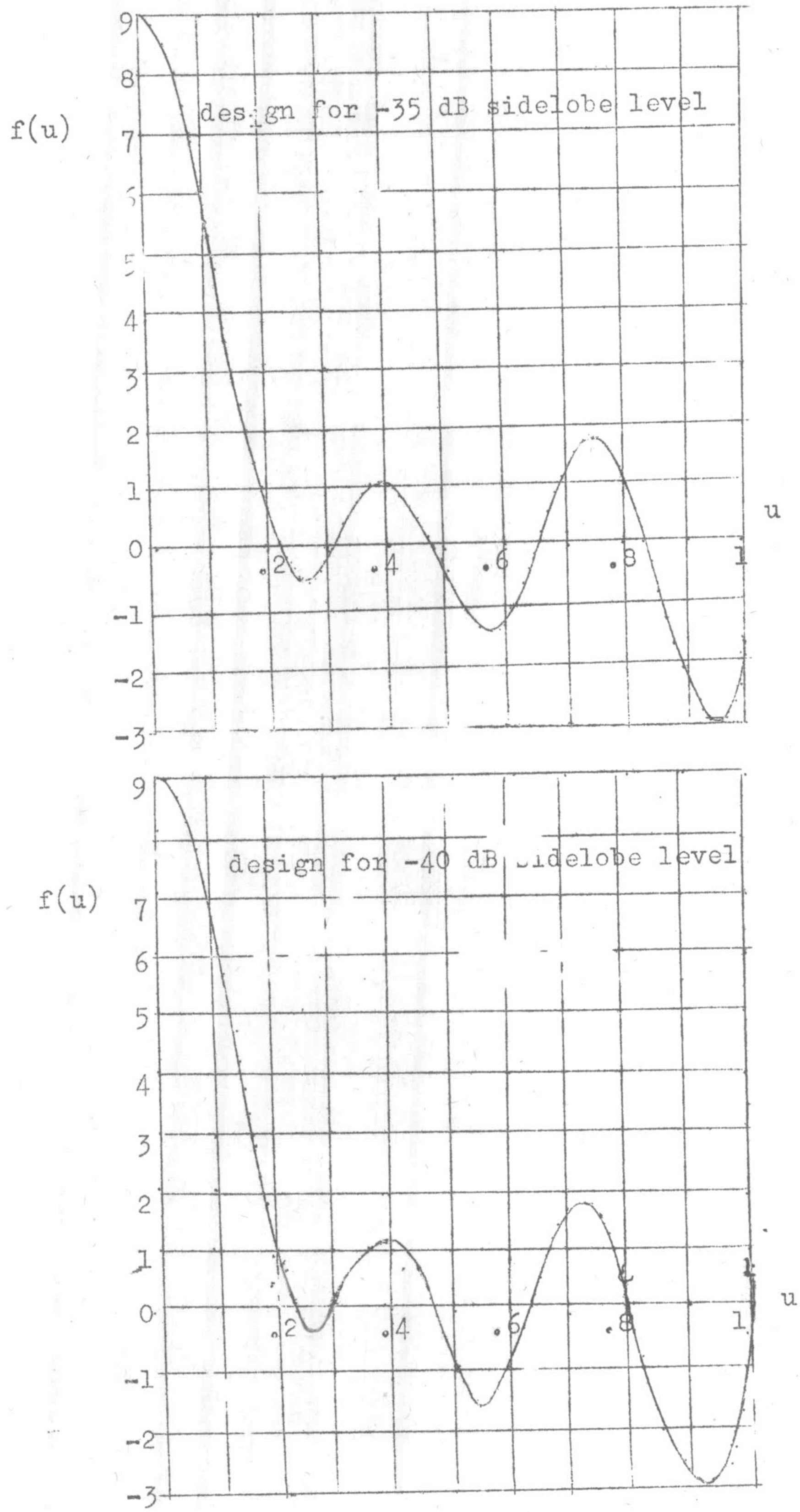

Fig. 7 Patterns of 7 element nonuniformly spaced arrays


f(u)
design for -25 dB sidelobes levels
9
8
7
6
5
4
3
2
1
0
-1
-2
.2
.4
.6
.8
1
u

(a)


f(u)
design for -30 dB sidelobe level
9
8
7
6
5
4
3
2
1
0
-1
-2
.2
.4
.6
.8
1
u

(b)




(c)

(d)

Fig.6 Patterns of 9 elements nonuniformly spaced arrays designed from Taylor line source. ( $d_{min}$= 0.5$\lambda$ )

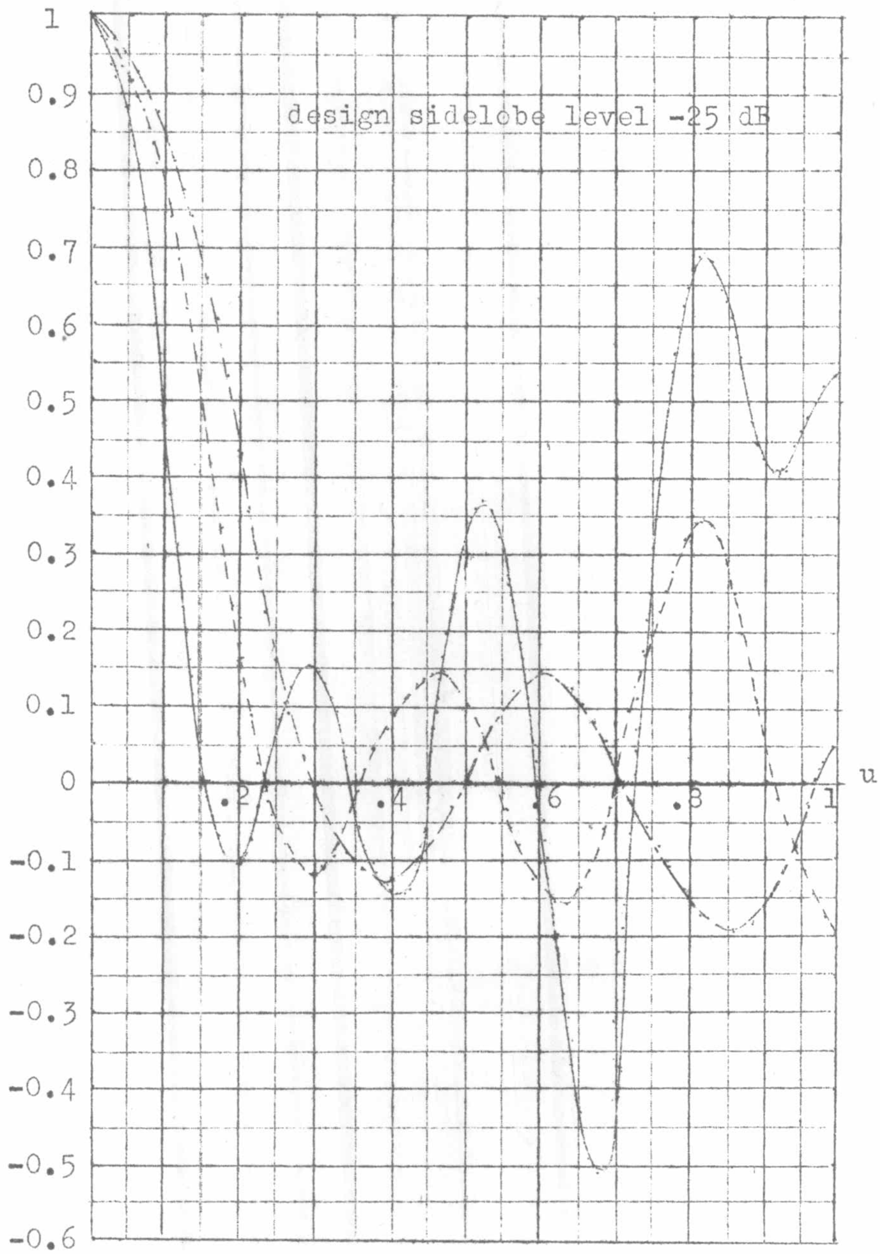



Fig. 9 The comparison between the patterns of 7 element nonuniformly spaced arrays which the total lengths are 3.5 , 5 and 8 wavelengths .

—— - —— - —— - —— 3.5 wavelengths

- - - - - - - - - - - - 5.0 wavelengths

—— - —— - —— - - —— 8.0 wavelengths

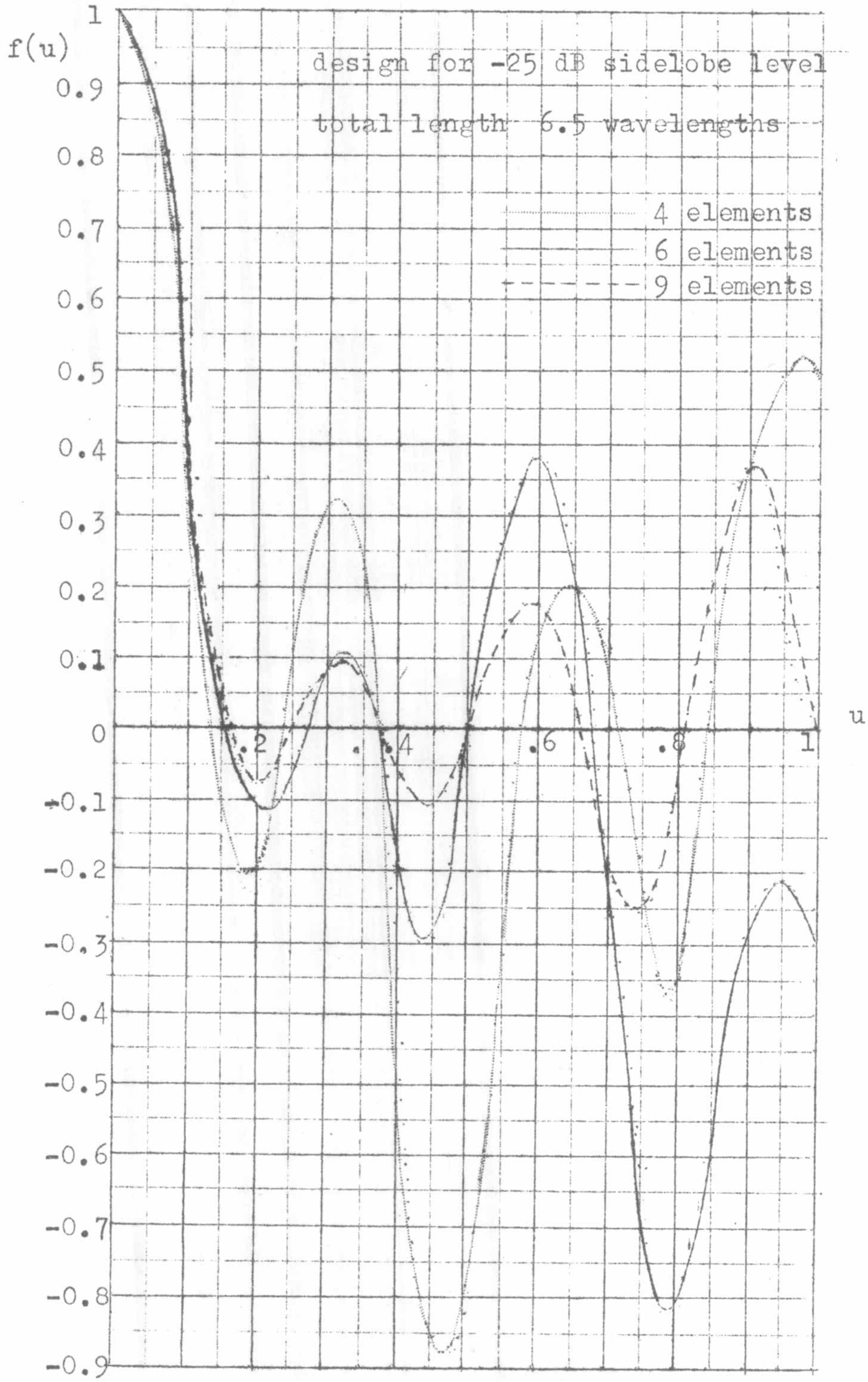



Fig. 10 The comparison between the patterns of 4,6 and 9 element nonuniformly spaced arrays occupying the same total length

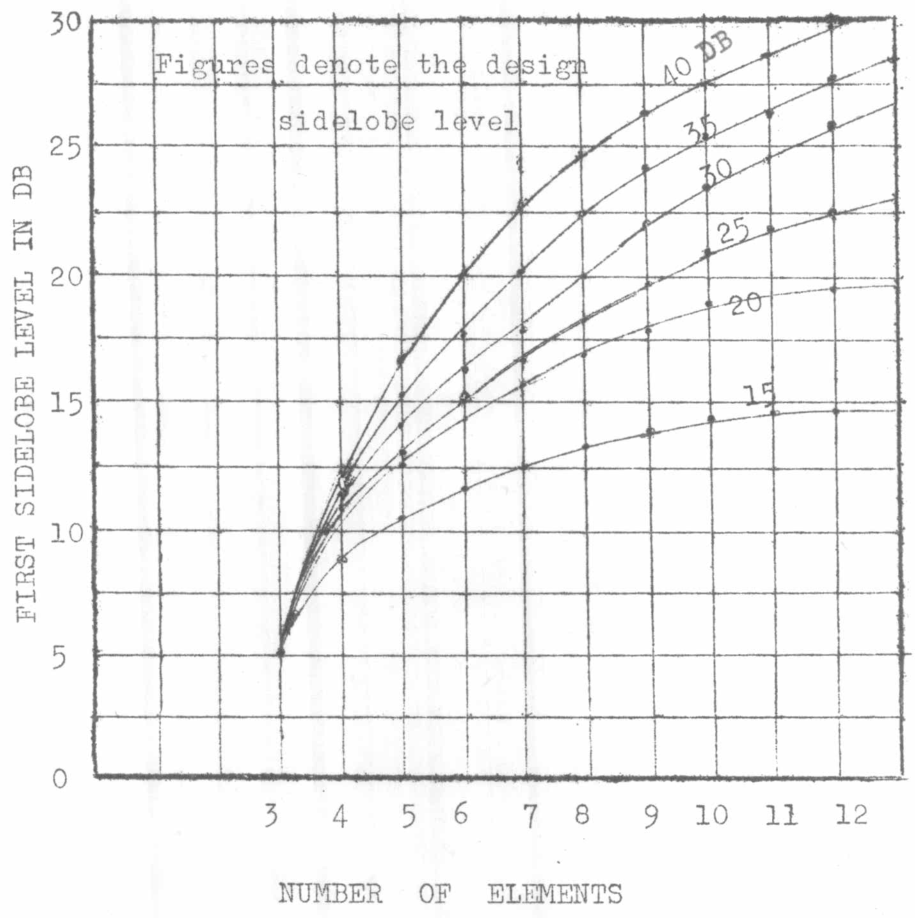



Fig. 11 Sidelobe characteristics of nonuniformly spaced arrays designed from the one parameter Taylor's line source.

The sidelobe characteristics of nonuniformly spaced arrays, so designed are examined and are plotted in Fig. 11 in which the relationships between the design sidelobe level and the achieved first sidelobe level are described.

It is noted that the smaller the number of elements the poorer the approximation.

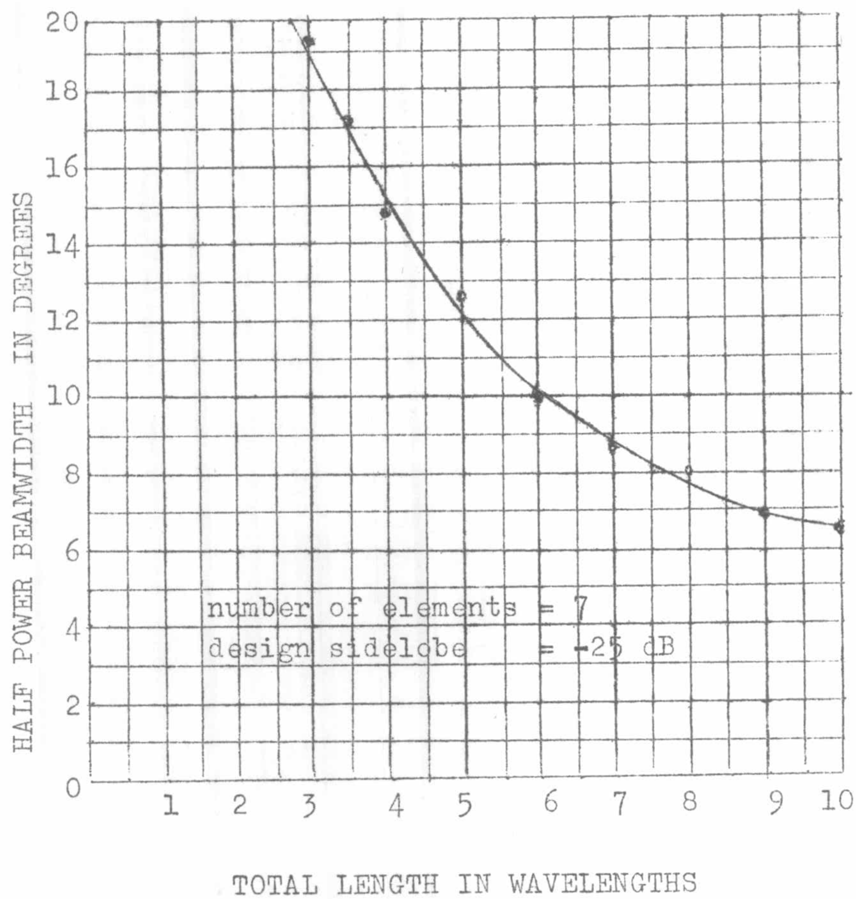



Fig. 12 The half-power beamwidth characteristics of nonuniformly spaced arrays designed from the one parameter Taylor's line source .

The half-power beamwidth characteristics of nonuniformly spaced arrays so designed are also plotted in Fig. 12 in which the relationships between the achieved half-power beamwidth and the total length are described.

It is noted that the shorter the total length the wider the half-power beamwidth .

It can be seen from figures 6 to 8 that nonuniformly spaced arrays firstly obtained have the directional patterns with increasing sidelobe levels in the farout region . This will limit the operating region of the antenna.

The results shown in Fig.9,10,11 and 12 introduce the ways to perturb the element locations to achieve the desired patterns. The following items are suggested in perturbing the element position and are realized from studying on the results of Fig.9-12.

1) If decreasing in the half-power beamwidth is desired the total length must be increased,providing the increase in the values of coefficient potentiometers

2) For the directional pattern of sidelobe levels below -13.5 dB,the density of the elements in the central part must be higher than at the end of the array,

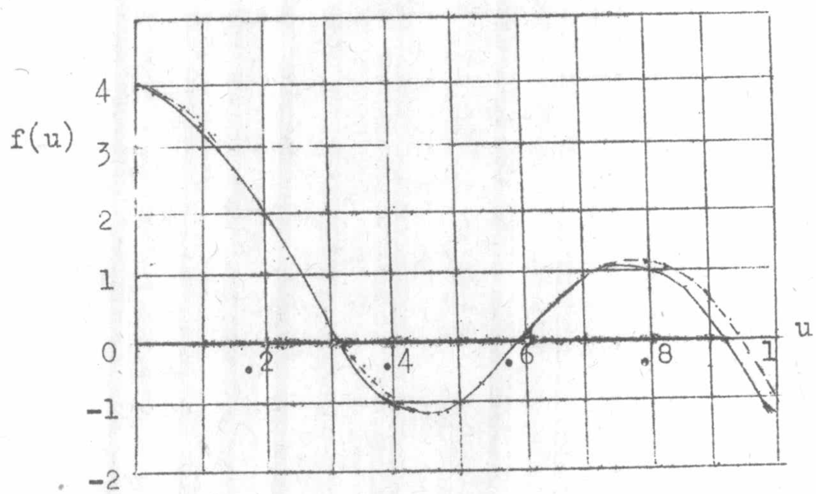
Another important effect that must be considered in determining the best element location is that the distance between an adjacent elements should not be closer than $\frac{1}{2}$ wavelengths since large values of mutual interaction between the elements would be effected.

The following results were obtained from the perturbation method done with an analog computer,in which the desired directional pattern was the type of nearly equal sidelobe levels throughout the visible range ( $u = 0 \rightarrow 1$ ).The element locations are selected to give these sidelobe levels as minimum as possible .

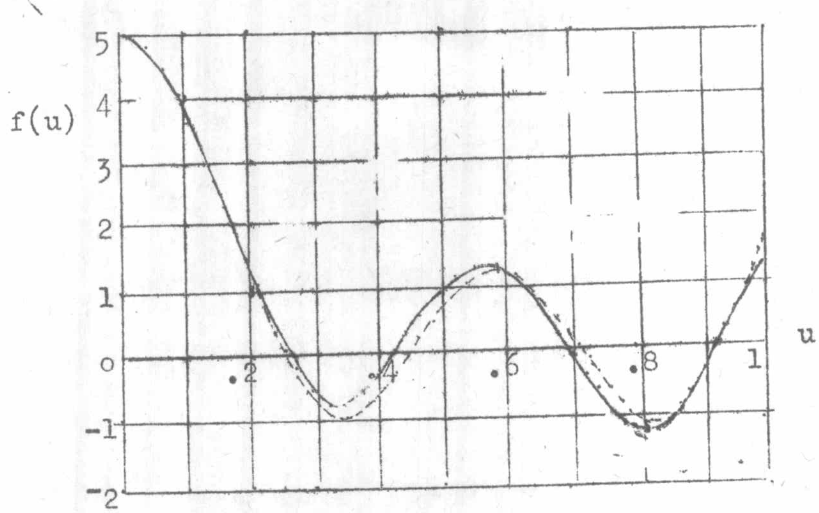
Table.2 Nonuniformly spaced array which the element locations are selected by the perturbation method to give minimum average sidelobe level .

| Number of element | Element location in wavelength | | | | | Total length | Ave element spac | Minimum ave sidelobe level | | Beam width |
|---|---|---|---|---|---|---|---|---|---|---|
| | $z_1$ | $z_2$ | $z_3$ | $z_4$ | $z_5$ | | | relative | dB | |
| 4 | 0.425 | 1.28 | | | | 2.56λ | 0.853λ | 0.26 | 11.7 | 14.9° |
| 5 | 0.835 | 1.7 | | | | 3.4 λ | 0.85 λ | 0.24 | 12.38 | 12.7° |
| 6 | 0.43 | 1.285 | 2.13 | | | 4.26 λ | 0.852λ | 0.21 | 13.46 | 10.5° |
| 7 | 0.857 | 1.74 | 2.64 | | | 5.28 λ | 0.88 λ | 0.179 | 14.92 | 8.6° |
| 8 | 0.42 | 1.3 | 2.2 | 3.12 | | 24 λ | 0.891λ | 0.176 | 15.08 | 7.5° |
| 9 | 0.88 | 1.78 | 2.7 | 3.62 | | 7.24λ | 0.905λ | 0.171 | 15.34 | 6.5° |
| 10 | 0.425 | 1.32 | 2.3 | 3.2 | 4.12 | 8.24 λ | .91 λ | 0.17 | 15.39 | 5.8° |
| 11 | 0.86 | 1.79 | 2.75 | 3.65 | 4.6 | 9.2 λ | .92 λ | 0.169 | 15.43 | 5.2° |

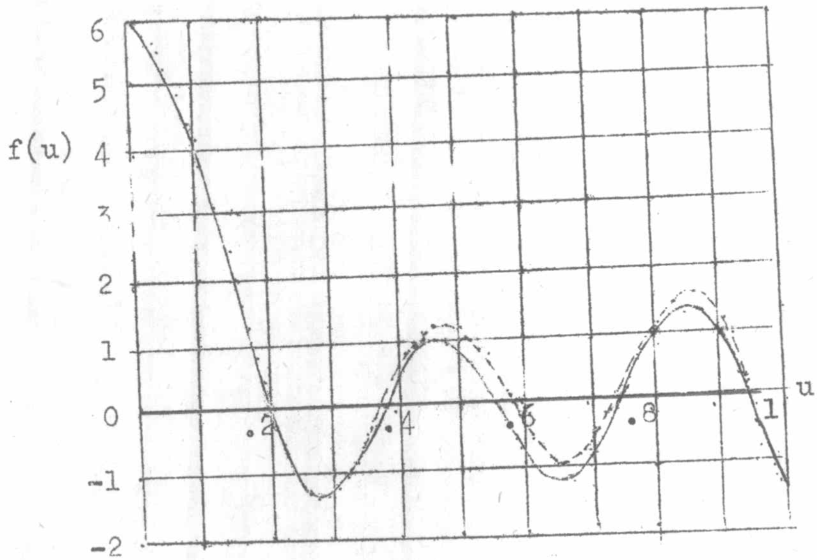
Table.2 above shows some results of nonuniformly spaced arrays designed by the perturbation method in which the element locations are selected to give the directional pattern of minimum average side lobe level.Fig.13 shows their patterns comparing with the minimum side lobe patterns obtained from the equally spaced arrays.



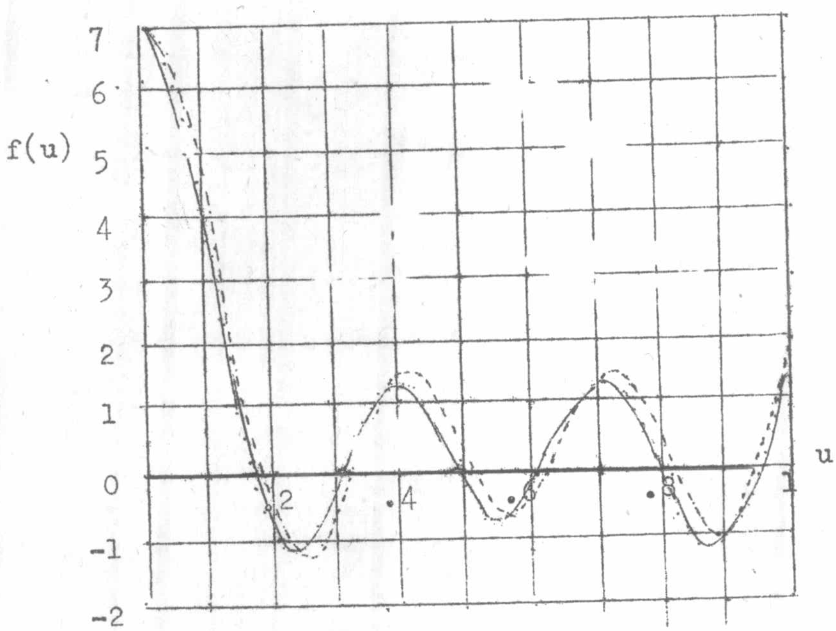

(a) pattern of 4 element array



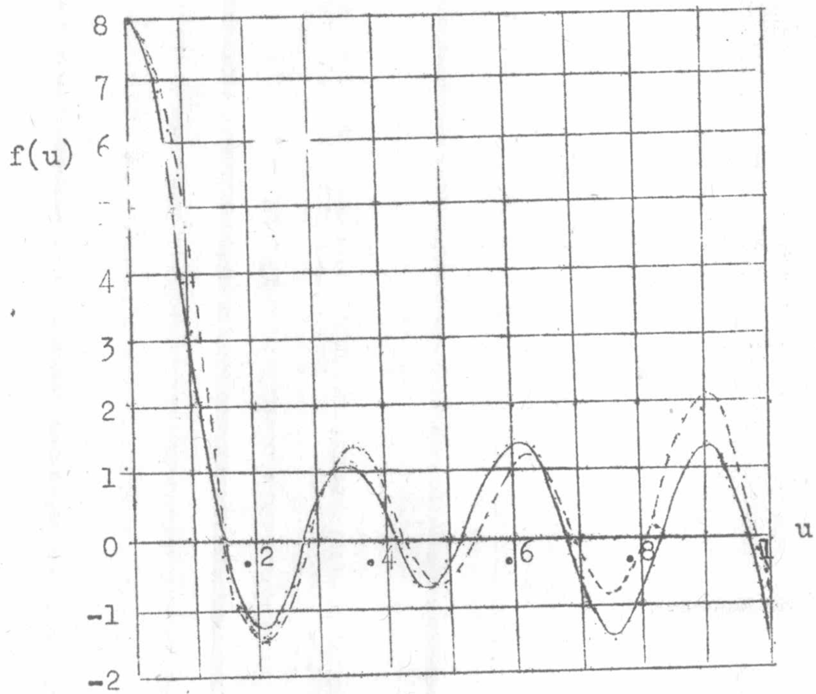

(b) pattern of 5 element array




(c) pattern of 6 element array

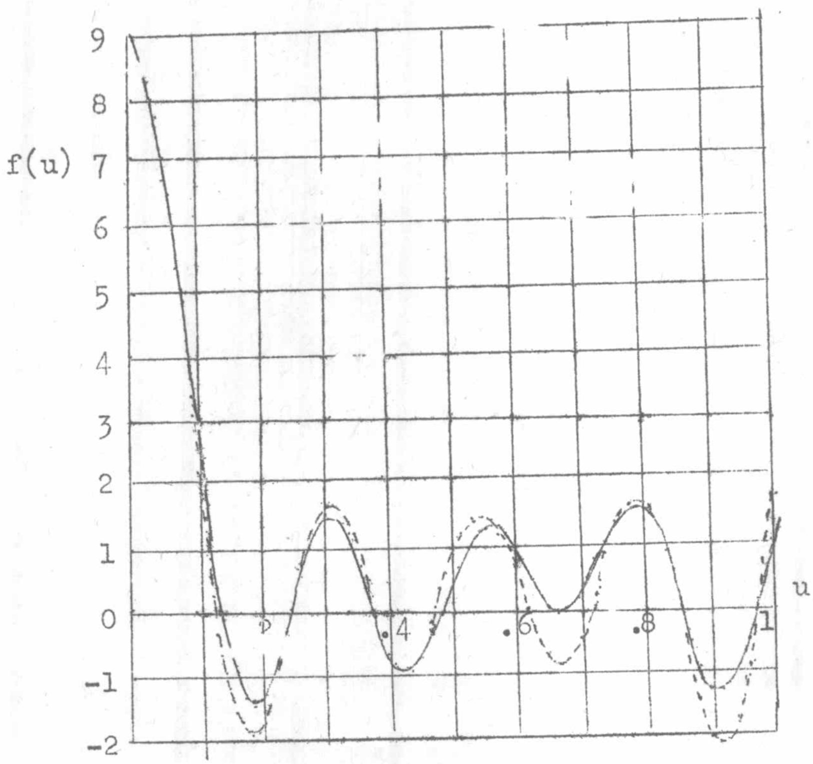



(d` pattern of 7 eleme:t array




(d)

pattern of
8 element
array




(e)

Pattern of
9 element
array

Fig.13 The minimum sidelobe level patterns for various number of elements of nonuniformly spaced arrays as achieved from the perturbation method comparing with those of equally spaced arrays ( dotted lines ).

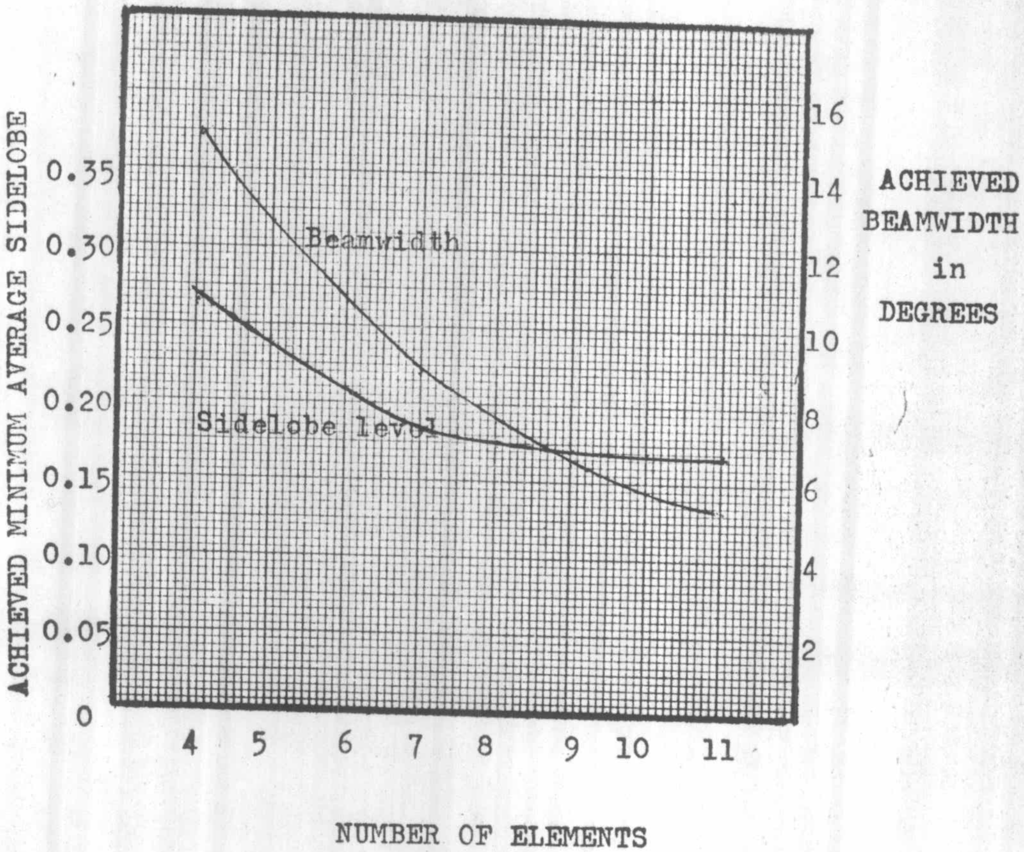



**Fig.14** **The minimum average sidelobe level and the beamwidth for various number of elements of nonuniformly spaced array achieved from the perturbation method.**

Two important characteristics of the directional patterns of nonuniformly spaced arrays shown in Fig.13 are plotted for various number of elements in Fig.14. It is seen that higher number of element provides the pattern of lower sidelobe level and narrower beamwidth .Figure 15 shows the relation between the sidelobe levels and the total length of the array for both the case of equally spaced arrays (dotted line) and unequally spaced arrays (solid line) with the element spacing selected to give the minimum average sidelobe level.

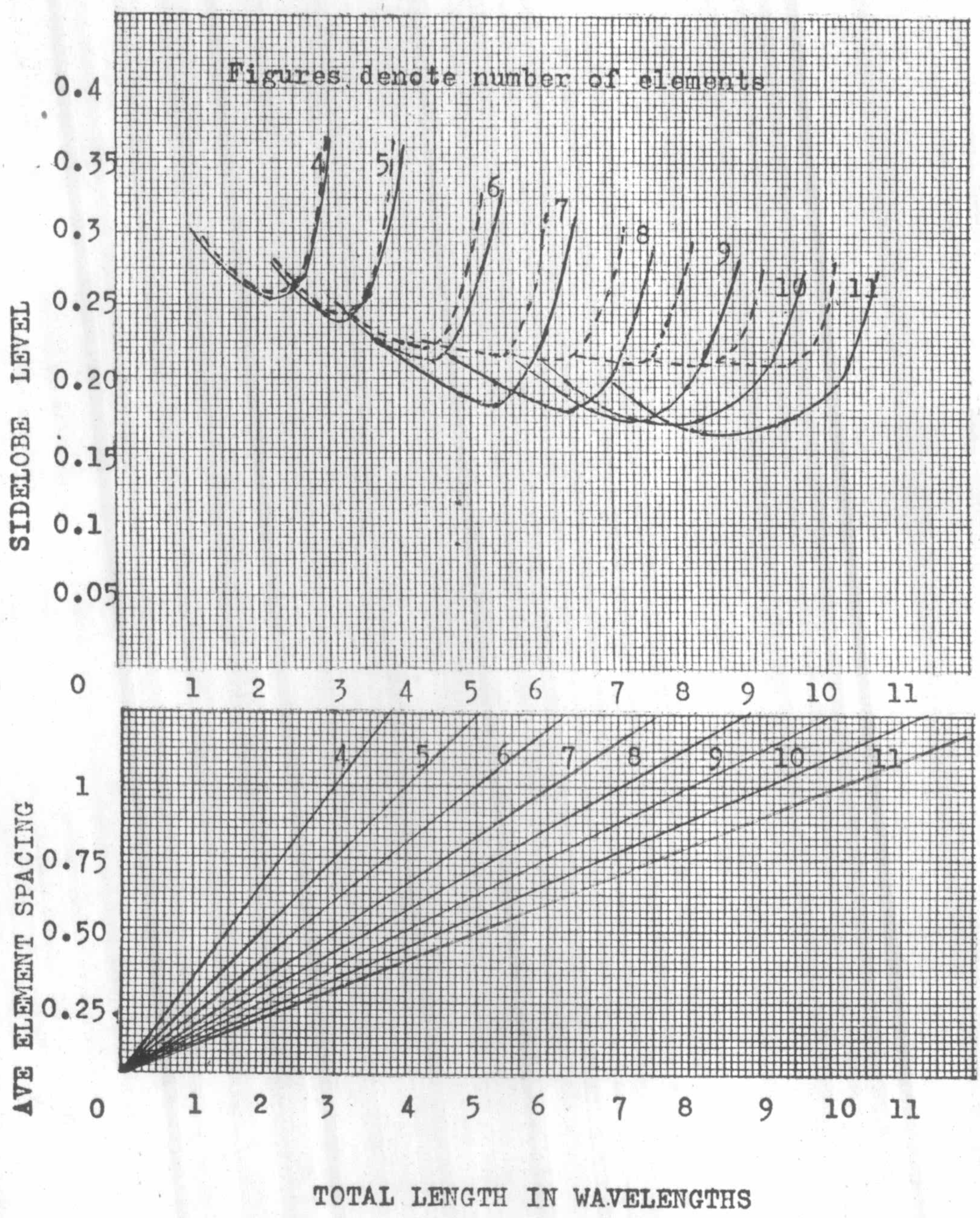



Fig.15 Sidelobe level vs Total length of an array

———————— nonuniformly spaced array

------------ equally spaced array