Chapter V

DISCUSSION AND CONCLUSION

1. From the characteristics of tracks, a group of particles is expected to be protons and this expectation is confirmed by the existence of masses relative to the mass of the particles expected to be protons.

2. This method of mass measurement cannot resolve the mass of muon and pion. The results of the present experiment are shown in comparison with the previous work on the measurement of mass in this range on the histograms in Fig VIII.

3. Although 80% of cosmic rays at sea level is supposed to be muon, very few muon tracks have been found in the emulsions. Probably muon is the energetic and penetrating particle.

4. From Tables V and VII showing the abundance of particles and their mean energies in each thickness of lead absorber, one can infer that there are nuclear interactions between cosmic rays and lead nuclei because the data show that the numbers and mean energies of the particles do not decrease as the thickness of lead absorber increases. The particles produced in lead are probably protons, deuterons and the particles of mass about $500m_e$ because the mean energies of these particles increase as the thickness of lead absorber. Other kinds of particles are absorbed by lead absorber since their mean energies appear to decrease as the thickness of lead

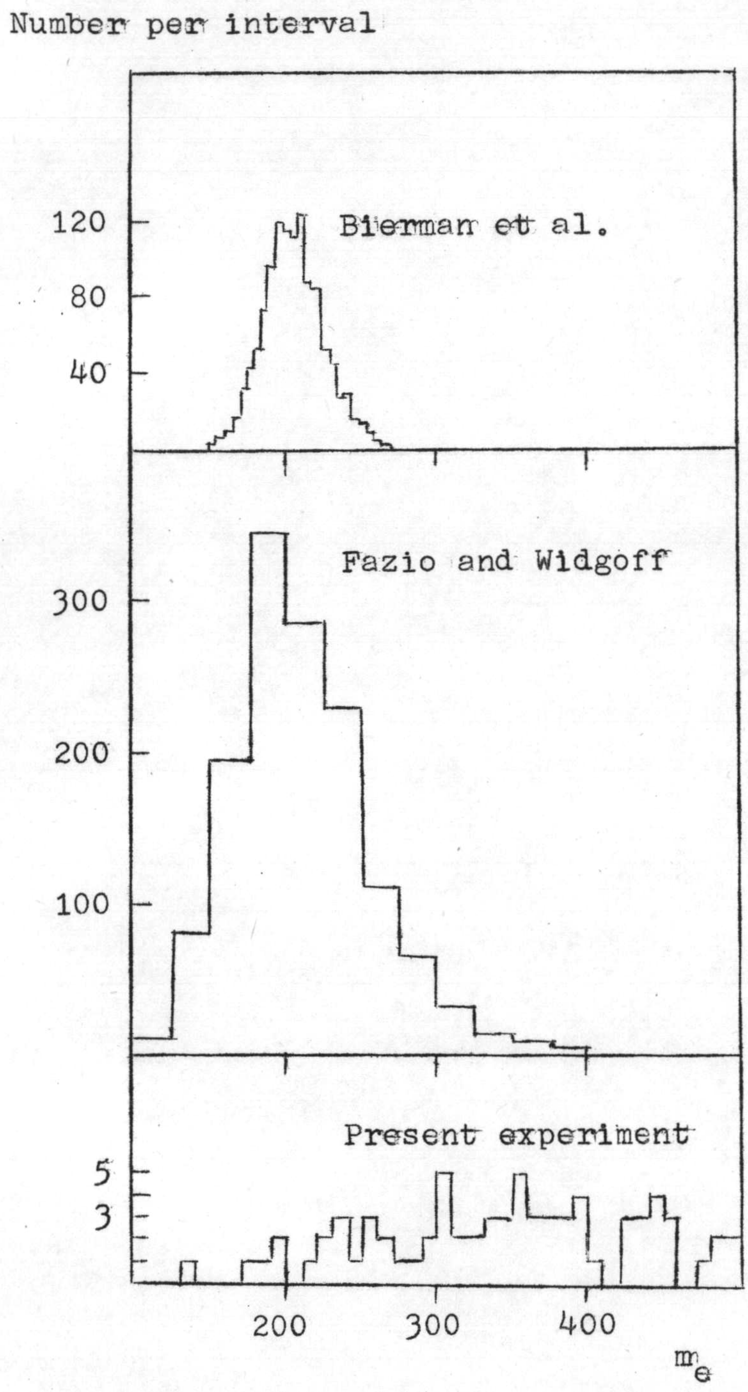

Fig VIII. Comparing the measurements of pion and muon masses in emulsions.

absorber increases. Tritons are found only in emulsions with 1 and 2 cm lead absorbers and they are absent in emulsions with 0 and 3 cm lead absorbers. The mean energies of tritons in emulsions with 1 cm lead absorber is higher than with 2 cm lead absorber. It is suggested that the further study on the production of triton in lead absorber should be made.

5. The particles of mass 480-580m_e were found only in emulsions with lead absorbers as shown in Table V. The measured mass in the emulsions with 1, 2 and 3 cm lead absorbers are $543 \pm 22m_e$, $527 \pm 46m_e$ and $538 \pm 44m_e$ respectively. All of these particles are expected to be of the same kind. In 1954, Wilson using data from emulsions of Bristol group, proposed the relationship of the nuclear product and the existence of ζ meson of mass $535 \pm 35m_e$. This particle should have the scheme of decay: $\zeta^{\pm} \longrightarrow \pi^{\pm} + \pi^{o} + 1$ MeV. He also predicted that ζ meson is a very short lived particle and is a decay product of strange particles. From present data, the decay process as proposed by Wilson is not observed. All particles of mass about 500m_e are found to stop in emulsion without decaying into charged particles. Some probably are captured by the atoms of emulsion and some perhaps decay to be electrons and neutral particles and one cannot detect them because the Ilford K2 emulsion is not electron sensitive.

It is primarily the work of Alikhanian et al. which aroused so much interest in the particles of mass about 500m_e.

The results of Alikhanian et al. and many workers after them are shown in Table XII and a comparison between the present experimental result and some previous data is shown in Fig IX. Many workers do not believe in the existence of particles of mass about $500m_e$ because they are very rarely found, however a remarkable amount is observed here. Ever since Alikhanian time, such particles are believed to be produced in the atmosphere and not in the absorber, but here these particles are found only in emulsions with lead absorbers. Moreover these particles are probably produced by the interaction of lead nuclei with cosmic rays.

6. The particles originating in the emulsions are expected to be produced by collisions of neutral particles with the nuclei of the emulsions. Because there is no such evidence found in the emulsion without lead absorber, this can be explained as neutron productions by protons or fast muons in lead. This is to be expected because the result indicates that the most abundant particle is proton, meanwhile the theory suggests that muon is the most abundant at sea level. E.B. Hughes et al.[1] and M.A. Meyer et al.[2] also arrived at the same result as this experiment.

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[1] E.B. Hughes et al., "Neutron Production by Cosmic Ray Protons in Lead," Proceeding of the Physical Society, 83(1964), 239-251.

[2] M.A. Meyer et al., "The Production of Neutrons by Fast Cosmic Ray Muons," Proceeding of the Physical Society, 83(1964), 253-258.

Table XII. Measurements of mass $500m_e$ in cosmic rays.

Experimenters	Technique	Altitude	Particle mass (m_e)	Absorber	Upper limet abundance relative to muons
Alikhanian et al.	magnetic-spectrometer and cloud chambers	3250 m.	500 ± 50	lead	0.5%
Keuffel et al.	counters	4700 ft.	550	lead	lesser than 0.05%
Peyrou and Hendel*	cloud chambers	3250 m.	500		0.3%
Bombay*	emulsions	3400 m.	380-900		0.1%
Hinks*	counters	sea level	500		0.05%
Fazio and Widgoff*	emulsions	sea level	400-900	iron	0.04%
Beirman et al.	emulsions	10600 ft.	500	copper	0.09%
Conversi et al.	counters	2550 m.	550	no	0.02%
McDiarmid	counters	sea level	550	no	≤ 0.02%
Hendel et al.	cloud chambers	3250 m. and sea level	500	no	0.3% 0.2%
present experiment	emulsions	sea level	480-580	lead	19.3% relative to proton

* From table of Fazio and Widgoff.

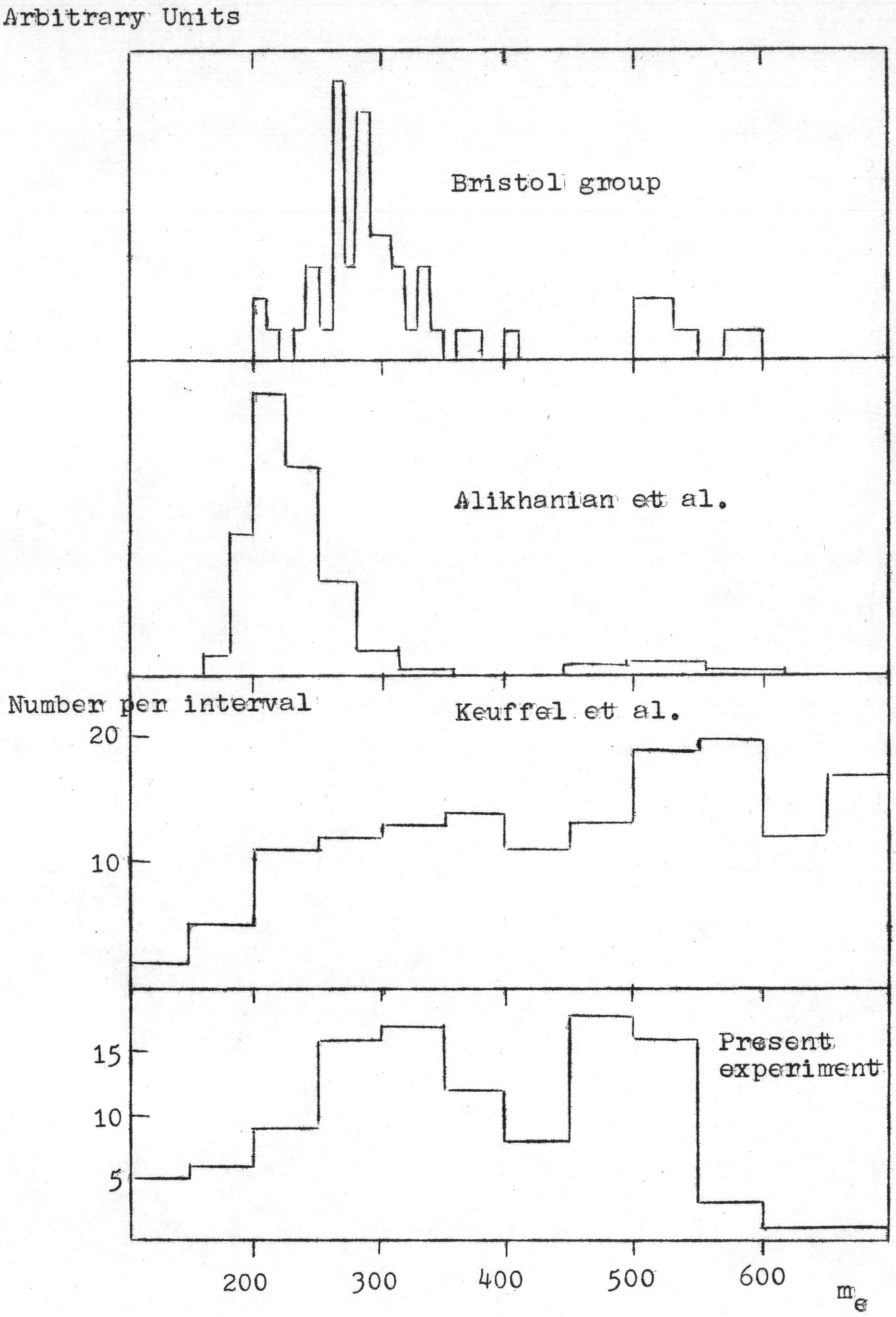

Fig IX. The comparison of experiments obtaining the particles of mass about $500m_e$.
The present experiment is taken from the total data.

APPENDIX

Appendix I

Gaussian Error Distribution

The Gaussian distribution function is defined as

$$P(x,\langle x\rangle,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp -\tfrac{1}{2}\left(\frac{x-\langle x\rangle}{\sigma}\right)^2$$

where $\langle x\rangle$ is mean value of x,

σ is standard deviation defined as

$$\sigma^2 = \langle (x_i - \langle x\rangle)^2\rangle = \langle x^2\rangle - \langle x\rangle^2.$$

The width of Gaussian distribution is characterized by the half-width Γ. This is defined as the range of x between the values at which the probability is half the maximum value.

$$P(\langle x\rangle \pm \tfrac{1}{2}\Gamma, \langle x\rangle, \sigma) = \tfrac{1}{2}P(\langle x\rangle, \langle x\rangle, \sigma).$$

This gives $\Gamma = 2.354\sigma$.

Appendix II

Method of Least Squares

Determining Best Fit Straight Line

Assuming the probability distribution of measurement is Gaussian, the probability of n measurements obtaining quantities $x_1, x_2, \ldots, x_n$, is

$$P(x_1, \langle x \rangle, \sigma) P(x_2, \langle x \rangle, \sigma) \ldots P(x_n, \langle x \rangle, \sigma)$$

$$= \frac{1}{\sigma^n (2\pi)^{n/2}} \exp \left[-\frac{1}{2\sigma^2} \sum_{i=1}^{n} (x_i - \langle x \rangle)^2 \right]$$

For this probability to be a maximum requires $\sum_{i=1}^{n} (x_i - \langle x \rangle)^2$ minimum. This is called Legendre's principle of least square which implies that the mean value is the most probable value.

Suppose n pairs of measurements are believed to be connected by a linear relationship of the form

$$y = mx + c.$$

By postulating that the variable x is axact, for a pair of measurement (x_i, y_i), $y_i - mx_i - c = v_i$ will not in general be zero because of errors in measuring y_i. From the principle of least squares, one can determine m and c by making $\sum_{i=1}^{n} v_i^2 = $ minimum or

$$\sum_{i=1}^{n} (y_i - mx_i - c)^2 = \text{minimum.}$$

For this minimum, differentiating partially with respect to m and c gives two equations:

$$\sum_{i=1}^{n} x_i(y_i - mx_i - c) = 0,$$

$$\sum_{i=1}^{n} (y_i - mx_i - c) = 0.$$

These two equations may be written as

$$\sum x_i y_i - m\sum x_i^2 - c\sum x_i = 0,$$
$$\sum y_i - m\sum x_i - cn = 0.$$

Solving the above equations for m and c gives

$$m = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - (\sum x_i)^2},$$

$$c = \frac{\sum x_i \sum x_i y_i - \sum y_i \sum x_i^2}{(\sum x_i)^2 - n\sum x_i^2}.$$