CHAPTER I

INTRODUCTION

Let a group $G = \bigcup_{\alpha \in I} G_\alpha$ be the irredundant union of its
subgroups G_α (as define in Preliminaries). For convenience,
let $\mathcal{N}(I)$ be the cardinality of I. The main purpose of this
thesis is to characterize such groups. In Chapter III, we
study the general case and obtain many interesting results
as well as the case $\mathcal{N}(I) = 2$, where no group can be an
irredundant union of its two subgroups. In case $\mathcal{N}(I) = 3$,
it is shown that a group G is an irredundant union of three
of its subgroups if and only if it can be mapped homomorphically
onto the abelian group $C_2 \times C_2$, the Klein 4-group, where
C_n denotes the cyclic group of order n. Moreover, in Chapter V,
we also generalize the Klein 4-group, by letting V_n be the
abelian group such that

$$V_n = C_2 \times C_2 \times \cdots \times C_2 \text{ (n times)},$$

and study the groups which can be mapped homomorphically
onto the abelian group V_n.

We also consider fields which are irredundant unions
of their subfields in Chapter VI.

Finally, we study relationships between direct sums
and irredundant unions of groups in Chapter VII.