REFERENCES

Bohr, N. "Constitution of Atoms and Molecules I, II, III" Phil. Mag., 26 (1913) : 1, 476, 857

Brillouin, L. Compt. Rend. 185 (1926) : 24

Brooks, H. "Cohesive Energy of Alkali Metals" Phys. Rev., 91 (1953): 1027

"Quantum Theory of Cohesion" Nuovo. cimento, 7 suppl. (1958): 165

Brooks, H., and Ham, F.S. "Energy Bands in Solids: The Quantum Defect Method" Phys. Rev., 112 (1958): 421

Demidovich, B.P., and Maron, I.A. Computational Mathematics. Moscow : Mir Publisher, 1976

Dumrongsak, M. Effective Potential of Sodium atom. Master's thesis, Chulalongkorn University (unpublished), 1976

Goldstein, H., Classical Mechanics. Massachusetts : Addison Wesley, 1959

Gorin, E. "The Theoretical Constitution of Metallic Potassium" Physik. Z. Sowjetunion 9 (1936): 328

Gradshteyn, I.S., and Ryzhik, I.M. Table of Integrals, Series and Products. New York : Academic Press, 1965

Ham, F.S. "The Quantum Defect Method" Solid State Physics 1 (1955): 127

"Expansions of the Irregular Coulomb Function" Quart. Appl. Math. 15 (1957) : 31

Hartree, D.R. Numerical Analysis. Oxford : Oxford Univ. Press, 1955

The Calculation of Atomic Structures. New York : Wiley, 1957

Jaffe, C., and Reinhardt, W.P. "Semiclassical Theory of Quantum Defects: Alkali Rydberg States" J. Chem. Phys. 66 No.3 (1977), 1285

Keller,J.B. "Corrected Bohr-Sommerfeld Quantum Conditions for Non-separable Systems" Annals of Physics 4 (1958) : 180

Kittel,C. Introduction to Solid State Physics 4th ed. New York : Wiley, 1971

Kohn,W.,and Rostoker,N. "Solution of the Schrödinger equation in Periodic Lattice with Application to Metallic Lithium" Phys.Rev. 94 (1954) : 1411

Kuhn,T.S. "An Application of the W.K.B.Method to the Cohesive Energy of Monovalent Metals" Phys.Rev. 79 (1950) : 515

"A General Solution of the Confluent Hypergeometric Equation: Analytic and Numerical Development" Quart.Appl.Math. 9(1951):1

Kuhn,T.S.,and Van Vleck,J.H. "A Simplified Method of Computing the Cohesive Energies of Monovalent Metals"Phys.Rev.79(1950): 382

Kramers,H.A. Z.Physik 39 (1926) : 828

Langer,R.E. "On the Connection Formulas and the Solution of the Wave Equation" Phys.Rev. 51 (1937) : 669

Maslov,V.P. Theorie des Perturbations et Methods Asymptotiques. Paris: Dunod Gauthier-Villars, 1972

Mathews,P.M.,and Venkatesan,K. A Textbook of Quantum Mechanics. New Delhi : Tata Mcgraw Hill, 1976

Park,D. Introduction to the Quantum Theory. New York : Mcgraw Hill,1964

Prokofjew,W.K. "Berechnung des Zahlen der Dispersionszentren des Natriums" Z.Physik 58 (1929) : 255

Schiff,L.I. Quantum Mechanics New York : McGraw Hill , 1955

Seitz,F. "The Theoretical Constitution of Metallic Lithium" Phys.Rev. 47 (1953) : 400

Slater,J.C. "One-Electron Energies of Atoms, Molecules and Solids" Phys.Rev. 98 (1955) : 1039

Sommerfeld,A. Atomic Structure and Spectral Lines. London : Methuen, 1928

Wannier,G. "Energy Eigenvalues for the Coulomb Potential with Cut-off I" Phys.Rev. 64 (1943) : 358

Whittaker,E.T.,and Watson,G.N. A Course in Modern Analysis. 4th ed. London and New York : Cambridge Univ.Press, 1952

Wigner,E.,and Seitz,F. "On the Constitution of Metallic Sodium I,II" Phys.Rev. 43 (1933): 804 ; ibid. 46 (1934) : 509

Ziman,J.M. Principle of the Theory of Solids. 2nd ed. London and New York : Cambridge Univ.Press, 1972

APPENDIX A

INTEGRATION PROCEDURE SUGGESTED BY SOMMERFELD

a) The integral of the form

$$J_1 = \oint \sqrt{A + \frac{2B}{r} - \frac{C}{r^2}}\, dr \qquad (A.1)$$

has two branch points, r_{min} and r_{max}, which have real positive values A, B, and C in eq.(A.1) are some constants.

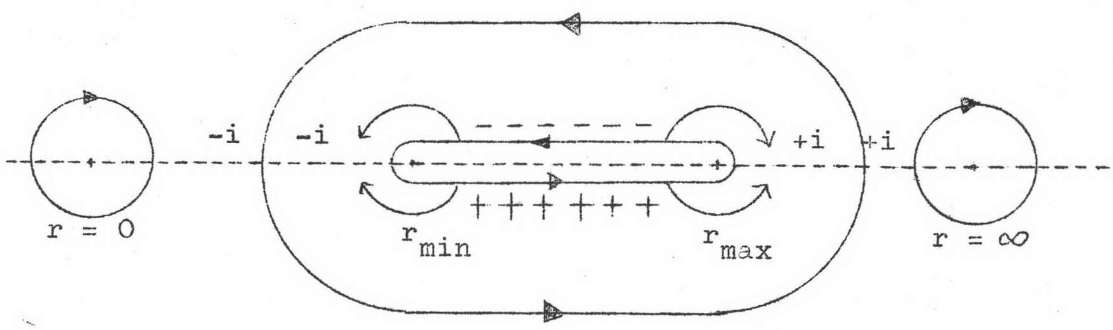


Fig. A.1

The r plane is supposed to be slit up between r_{min} and r_{max} and represent the upper sheet of a two-surface Riemann sheet. On account of the positive character of phase integral, the sign of the square root is taken to be positive for a positive dr (section below the slit) and negative for a negative dr (section above the slit). The square root on the real axis of the r plane outside the slit is imaginary and indeed positive and imaginary for $r > r_{max}$, negative and imaginary for $0 < r < r_{min}$.

The poles of the integrand are at r=0 and $r=\infty$. Since the sign in front of the square root in the integrand must be negative for the region along the real axis below r_{min}, the integrand is represent as $-\sqrt{A+\frac{2B}{r}-\frac{C}{r^2}}$. The integrand is expanded and it is found that the residue at the point r=0 is $-\sqrt{-C}$. Since the square root on the real axis above r_{max} is positive, the integral becomes $-\int\sqrt{A+2Bs-Cs^2}\,\frac{ds}{s^2}$ by setting $r = 1/s$. By expanding the integrand we find the residue at $r = \infty$ is $-B/\sqrt{A}$. By applying the Residue Theorem we obtain

$$J_1 = -2\pi i\left(-\sqrt{-C}-B/\sqrt{A}\right). \qquad (A.2)$$

We see that J_1 will be real for $C>0$ and $A<0$. If we start with the integrand $\sqrt{A+\frac{2B}{r}+\frac{C}{r^2}}$ we will obtain the total result

$$J_1 = -2\pi i\left(\sqrt{C}-B/\sqrt{A}\right) \qquad (A.3)$$

which is real if $A<0$ and $C<0$.

b) The integral of the form

$$J_2 = \oint\sqrt{A+\frac{2B}{r}+\frac{C}{r^2}+\frac{D_1}{r^3}+\frac{D}{r^4}}\,dr \qquad (A.4)$$

has the same two branch points as J_1. Therefore we can still use the integration path given in Fig.A.1. D_1 and D in eq.(A.4) are constants.

Excluding all powers higher than D_1 , D_1^2 and D we get upon expanding the integrand

$$\sqrt{A+\frac{2B}{r}+\frac{C}{r^2}+\frac{D_1}{r^3}+\frac{D_1}{r^4}} = \sqrt{A+\frac{2B}{r}+\frac{C}{r^2}} + \frac{1}{2}\left(A+\frac{2B}{r}+\frac{C}{r^2}\right)^{-1/2}\left(\frac{D_1}{r^3}+\frac{D}{r^4}\right)$$
$$-\frac{1}{8}\left(A+\frac{2B}{r}+\frac{C}{r^2}\right)^{-3/2}\frac{D_1^2}{r^6} \quad .$$

The integral becomes

$$J_2 = J_1 + \frac{D_1}{2}J_3 + \frac{D}{2}J_4 - \frac{D_1^2}{8}J_5 \qquad \text{(A.5)}$$

where $$J_3 = \oint\left(A+\frac{2B}{r}+\frac{C}{r^2}\right)^{-1/2}\frac{dr}{r^3} \quad , \qquad \text{(A.6)}$$

$$J_4 = \oint\left(A+\frac{2B}{r}+\frac{C}{r^2}\right)^{-1/2}\frac{dr}{r^4} \quad , \qquad \text{(A.7)}$$

and $$J_5 = \oint\left(A+\frac{2B}{r}+\frac{C}{r^2}\right)^{-1/2}\frac{dr}{r^6} \quad . \qquad \text{(A.8)}$$

These integrals can be performed by the same procedure as that applied to J_1 . The results are

$$J_3 = 2\pi i\,\frac{B}{\sqrt{C}} \quad , \qquad \text{(A.9)}$$

$$J_4 = \frac{\pi i}{\sqrt{C}}\left(\frac{A}{C}-\frac{3B^2}{C^2}\right) \quad , \qquad \text{(A.10)}$$

and $$J_5 = \frac{3\pi i}{C\sqrt{C}}\left(\frac{A}{C}-\frac{5B^2}{C^2}\right) \quad . \qquad \text{(A.11)}$$

Eq.(A.5) now becomes

$$J_2 = -2\pi i\left\{\sqrt{C} - \frac{B}{\sqrt{A}} - \frac{1}{2}\frac{B}{C\sqrt{C}}\left(D_1 - \frac{3}{2}\frac{DB}{C} + \frac{15}{8}\frac{D_1^2 B}{C^2}\right)\right.$$
$$\left. - \frac{1}{4}\frac{A}{C\sqrt{C}}\left(D - \frac{3}{4}\frac{D_1^2}{C}\right)\right\} . \qquad (A.12)$$

For $D_1 = 0$, we have

$$J_2 = -2\pi i\left\{\sqrt{C} - \frac{B}{\sqrt{A}} + \frac{3}{4}\frac{DB^2}{C^2\sqrt{C}} - \frac{1}{4}\frac{AD}{C\sqrt{C}}\right\} . \qquad (A.13)$$

If A and C themselves are negative, we can rewritten eq.(A.13) to be

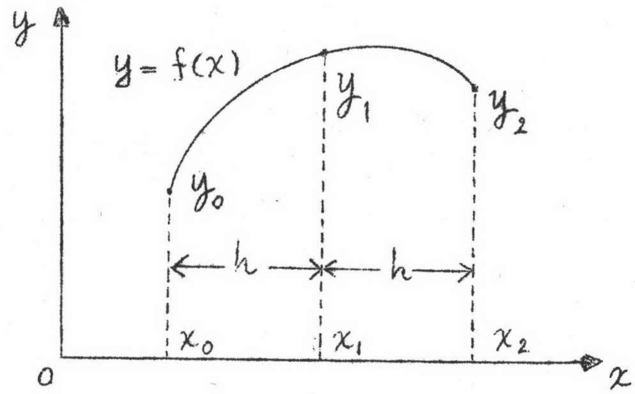
$$J_2 = 2\pi\left\{\frac{B}{\sqrt{-A}} - \sqrt{-C} + \frac{D}{4C\sqrt{-C}}\left(\frac{3}{C}B^2 - A\right)\right\} . \qquad (A.14)$$

APPENDIX B

SIMPSON'S INTEGRATION FORMULA

The following Simpson's integration formula is adapted from Demidovich and Maron (1976).

Fig.B.1



The three-point integration as shown above is found to be

$$\int_{x_0}^{x_2} f(x)\,dx = \frac{h}{3}(y_0 + 4y_1 + y_2) \tag{B.1}$$

and the remainder

$$R = -\frac{h^5}{90} f^{iv}(\xi), \quad x_0 < \xi < x_2 \tag{B.2}$$

The proper integration result will be obtained if the interval h tends to zero. We will divide the integration limits to be n intervals as shown. n must be an even integer because we want to apply the integration formula obtained in eq.(B.1) on each interval pair. We have

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}(y_0 + 4y_1 + y_2) + \frac{h}{3}(y_2 + 4y_3 + y_4) + \cdots\cdots + \frac{h}{3}(y_{2n-2} + 4y_{2n-1} + y_n)$$

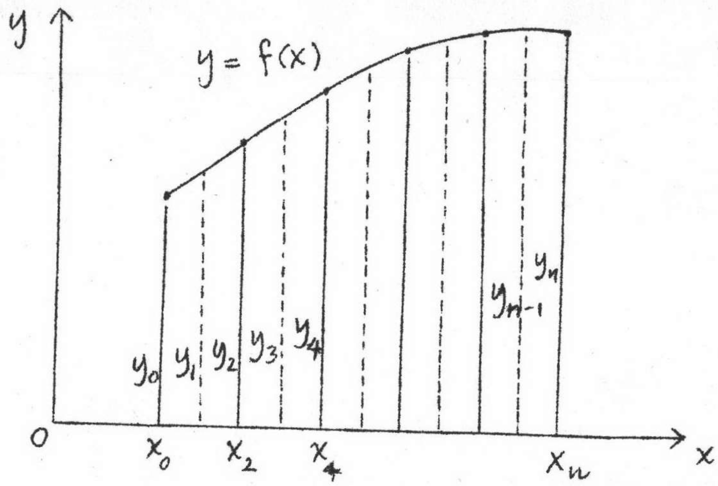


Fig.B.2

$$\int_{x_0}^{x_n} f(x)dx = \frac{h}{3}\left[y_0 + y_n + 4(y_1 + y_3 + \cdots y_{n-1}) + 2(y_2 + y_4 + \cdots y_{n-2}),\right] \quad \text{(B.3)}$$

Since we are concerned with an integral of the form

$$\int_{\rho_i}^{\rho_j} \left[\frac{\sqrt{(\alpha - \epsilon_{ne}/2)\rho^2 + \beta\rho + (\gamma - \frac{1}{2}(\ell + \frac{1}{2})^2) - \delta/\rho^2}}{\rho}\right] d\rho ,$$

a program for this calculation is made. The program is showm in table B.1 and is appropriate to a calculator model HP-97. The user instruction are given in table B.2 .

The program can be adapted to the integration from ρ_{min} to ρ_j. In this case , the value of $\Delta\rho$ is negative, thus some steps in the program must be changed. The steps 014 and 015 must be interchanged and an ABS key must be inserted between steps 044 and 045. The value of h must be set to be negative and thus we can use ρ_{min} instead of ρ_{max}. We are now able to calculate the integration by using the user instructions in table B.2 .

```
001  *LBLA      21 11
002   RCL9      36 09
003      3         03
004      ÷        -24
005   STOB      35 12
006   GSBe   23 16 15
007    SPC      16-11
008   PRTX        -14
009      x        -35
010   STO4      35 04
011  *LBL1      21 01
012   RCL9      36 09
013   ST+3   35-55 03
014   RCL5      36 05
015   RCL3      36 03
016   X>Y?      16-34
017   GTO2      22 02
018    CLX        -51
019   GSBe   23 16 15
020   RCLB      36 12
021      x        -35
022   STOE      35 15
023      4         04
024      x        -35
025   ST+4   35-55 04
026   RCL3      36 03
027   ST+3   35-55 03
028   GSBe   23 16 15
029   RCLB      36 12
030      x        -35
031   STOE      35 15
032      2         02
033      x        -35
034   ST+4   35-55 04
035   GTO1      22 01
036  *LBL2      21 02
037   RCL9      36 09
038   ST-3   35-45 03
039   GSBe   23 16 15
040    SPC      16-11
041   PRTX        -14
042   RCLE      36 15
043   ST-4   35-45 04
044   RCL4      36 04
045    SPC      16-11
046   PRTX        -14
047    R/S         51
048  *LBLe   21 16 15
049   RCLA      36 11
050   RCL3      36 03
051     X²         53
052      x        -35
053   RCL7      36 07
054   RCL3      36 03
055      x        -35
056      +        -55
057   RCLC      36 13
058      +        -55
059   RCLD      36 14
060   RCL3      36 03
061     X²         53
062      ÷        -24
063      +        -55
064     √X         54
065   RCL3      36 03
066      ÷        -24
067    RTN         24
068  *LBLa   21 16 11
069   RCL6      36 06
070   RCL0      36 00
071      2         02
072      ÷        -24
073      -        -45
074   STOA      35 11
075   RCL2      36 02
076      .        -62
077      5         05
078      +        -55
079     X²         53
080      2         02
081      ÷        -24
082    CHS        -22
083   RCL8      36 08
084      +        -55
085   STOC      35 13
086    RTN         24
087  *LBLB      21 12
088   RCL4      36 04
089    SPC      16-11
090   PRTX        -14
091    RTN         24
092  *LBLD      21 14
093   RCLI      36 46
094   ST+3   35-55 03
095   GSBe   23 16 15
096   GTOD      22 14
097    RTN         24
098  *LBLE      21 15
099   RCLI      36 46
100   ST-3   35-45 03
101   RCL3      36 03
102   STO5      35 05
103    R/S         51
```

Registers

0	ϵ_{nl}	1	-	2	ℓ	3	ρ_i
4	Int.result	5	ρ_j	6	α	7	β
8	γ	9	h	A	$A=\alpha-\epsilon_{nl}/2$	B	$h/3$
C	$C=\gamma-\frac{1}{2}(\ell+\frac{1}{2})^2$	D	δ	E	cal.memory	I	$\Delta\rho$

Table B.1 Integration Program

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 of program			
2	Load input data	ϵ_{nl}	STO 0	ϵ_{nl}
		l	STO 2	l
		ρ_i	STO 3	ρ_i
	[$h = (\rho_j - \rho_i)/n$	ρ_j	STO 5	ρ_j
	n = even integer]	α	STO 6	α
		β	STO 7	β
		γ	STO 8	γ
		h	STO 9	h
		δ	STO D	δ
3	Start the calculation		f a	C
4	If ρ_j is not a ρ_{max} and is known, go to step 8			
5	Solve for ρ_{max}			
	5.1 find possible $\rho_j < \rho_{max}$	$\Delta\rho_j$	STO I	$\Delta\rho_j$
			D	error
	(now ρ is greater than ρ_{max})		CLX	
	5.2 set ρ_j to be $\rho_j - \Delta\rho_j$ automatically		E	
6	Adjust ρ_j by setting a new suitable $\Delta\rho_j$ as fine as you need and repeat step 5 again			
7	Set ρ_i and h	ρ_i	STO 3	ρ_i
		h	STO 9	h
8	Integration		A	$f(\rho_i)$
				$f(\rho_j)$
				Int.result

Table B.2 User instructions

for Simpson's integration formula

APPENDIX C

DETERMINATION OF PARAMETERS

The parameters are determined from the quantization condition and the matching conditions by solving the following equations simultaneously.

$$n-\ell-\frac{1}{2} = \frac{\sqrt{2}}{\pi}\left[\left\{F_2 - G_2(\rho_j)\right\} + \left\{G_1(\rho_j) - G_1(\rho_i)\right\} + \left\{G_0(\rho_i) + F_0\right\}\right], \quad (C.1)$$

$$\beta_{m+1} = 2(\alpha_m - \alpha_{m+1})\rho_i + \beta_m , \quad (C.2)$$

$$\gamma_{m+1} = (\alpha_{m+1} - \alpha_m)\rho_i^2 + \gamma_m . \quad (C.3)$$

All terms in the above eqations have been defined in chapter IV . It is noted that if there exists two integrals in the quantization expression, the terms in the middle curly brackets in eq.(C.1) will disappear and the index "2" changes to "1" automatically.

The program designed for the case without polarization is given in table C.1 and the user instructions are given in table C.2 and C.3 .

001	*LBLA	21 11
002	RCL6	36 06
003	RCL8	36 08
004	2	02
005	÷	-24
006	-	-45
007	STOA	35 11
008	RCL3	36 03
009	X²	53
010	RCL6	36 06
011	RCL4	36 04
012	-	-45
013	×	-35
014	RCL1	36 01
015	+	-55
016	STO8	35 08
017	RCL2	36 02
018	.	-62
019	5	05
020	+	-55
021	X²	53
022	2	02
023	÷	-24
024	-	-45
025	STOE	35 15
026	RCL4	36 04
027	RCL6	36 06
028	-	-45
029	2	02
030	×	-35
031	RCL3	36 03
032	×	-35
033	RCL5	36 05
034	+	-55
035	STO7	35 07
036	2	02
037	÷	-24
038	RCLA	36 11
039	CHS	-22
040	√X	54
041	÷	-24
042	STOB	35 12
043	RCLE	36 15
044	CHS	-22
045	√X	54
046	-	-45
047	Pi	16-24
048	×	-35
049	2	02
050	÷	-24
051	RTN	24
052	*LBLA	21 11
053	2	02
054	RCLA	36 11
055	×	-35
056	RCL3	36 03
057	×	-35
058	RCL7	36 07
059	+	-55
060	GSBa	23 16 11
061	÷	-24
062	RAD	16-22
063	SIN⁻¹	16 41
064	RCLB	36 12
065	×	-35
066	STOI	35 46
067	RCL7	36 07
068	RCL3	36 03
069	×	-35
070	2	02
071	RCLE	36 15
072	×	-35
073	+	-55
074	RCL3	36 03
075	÷	-24
076	GSBa	23 16 11
077	÷	-24
078	RAD	16-22
079	SIN⁻¹	16 41
080	RCLE	36 15
081	CHS	-22
082	√X	54
083	×	-35
084	RCLI	36 46
085	+	-55
086	RTN	24
087	*LBLC	21 13
088	RCLC	36 13
089	+	-55
090	STOC	35 13
091	RTN	24
092	*LBLD	21 14
093	RCLD	36 14
094	+	-55
095	STOD	35 14
096	RTN	24
097	*LBLE	21 15
098	RCLD	36 14
099	RCLC	36 13
100	+	-55
101	2	02
102	√X	54
103	×	-35
104	Pi	16-24
105	÷	-24
106	RTN	24
107	0	00
108	STOD	35 14
109	RCL9	36 09
110	ST+6	35-55 06
111	GTOA	22 11
112	*LBLa	21 16 11
113	RCL7	36 07
114	X²	53
115	4	04
116	RCLA	36 11
117	×	-35
118	RCLE	36 15
119	×	-35
120	-	-45
121	√X	54
122	RTN	24
123	*LBLb	21 16 12
124	RCL3	36 03
125	X²	53
126	RCLA	36 11
127	×	-35
128	RCL3	36 03
129	RCL7	36 07
130	×	-35
131	+	-55
132	RCLE	36 15
133	+	-55
134	√X	54
135	RTN	24
136	R/S	51

Registers

0	ϵ_{nl}	1	γ_0	2	ℓ	3	ρ_i
44	α_0	5	β_0	6	α_1	7	β_1
8	γ_1	9	$\Delta\alpha_1$	A	A	B	$B/2\sqrt{A}$
C	SUM C	D	SUM D	E	d	I	

Table C.1 Parameter determination program

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	This program may be employed to calculate ρ_{min} of $Q(\rho)$ of any state			
1	Load side 1 and side 2 of program			
2	Calculate ρ_{min} of an f state			
	2.1 Load input data	ϵ_{nl}	STO 0	ϵ_{nl}
		$\alpha_0 = 0$	STO 4	α_0
		$\beta_0 = 1$	STO 5	β_0
		$\gamma_0 = 0$	STO 1	γ_0
		$\ell = 3$	STO 2	ℓ
		$\alpha_1 = 0$	STO 6	α_1
	2.2 Start the calculation		A	c
	2.3 Calculate ρ_{min}		f a	Δ
			RCL 7	
			- 2	
			÷ RCL	
			A ÷	ρ_{min}
3	Calculate ρ_{min} of a d state			
	3.1 Reset ϵ_{nl}, ℓ and α_1 (known) by repeating some steps in 2.1			
	3.2 Since $Q_0(\rho)$ have been determined, we can set ρ_i	ρ_i	STO 3	ρ_i
	3.3 Repeat step 2.2 and 2.3			
4	Calculate ρ_{min} of a p state : The procedure is the same as that of a d state except α_0, β_0, and γ_0 must be changed to α_1, β_1, and γ_1 respectively			

Table C.2 Instructions for calculating ρ_{min}

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	Determination of parameters			
1	Load side 1 and side 2 of program			
2	Calculate $F_0+G_0(\rho_i)$ for any state			
	2.1 Load input data by repeating step 2.1 in table C.2			
	2.2 Load a selected ρ_i	ρ_i	STO 3	ρ_i
	2.3 Operate and store the functions in register C		A C	F_0
			B C	G_0
3	If there exists only two integrals, go to step 5.2 and use the index "1" instead of "2".			
4	Calculate $G_1(\rho_i)$ and $G_1(\rho_j)$			
	4.1 Set a new (known) α_1	α_1	STO 6	α_1
	4.2 Calculate $G_1(\rho_i)$ and deduct the register C		A B	$G_1(\rho_i)$
			CHS C	$-G_1(\rho_i)$
	4.3 Calculate $G_1(\rho_j)$ and sum within the register C	ρ_j	STO 3	
			B C	$G_1(\rho_j)$
5	Calculate $F_2-G_2(\rho_j)$ and determine α_2			
	5.1 Replace zero-index parameters by 1-index parameters		RCL 6	α_1
			STO 4	α_1
			RCL 7	β_1
			STO 5	β_1
			RCL 8	γ_1
			STO 1	γ_1

(Continued)

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
	5.2 Assume a value of α_2	α_2	STO 4	α_2
	5.3 Calculate F_2 and store in register D		A D	F_2
	5.4 Calculate $G_2(\rho_j)$ and deduct the register D		B CHS	
			D	$-G_2(\rho_j)$
6	Testing			
	6.1 Calculate the right hand side of eq.(C.1)		E	RHS
	6.2 If RHS $> n-\ell-\frac{1}{2}$, set $\Delta\alpha_2 > 0$	$\Delta\alpha_2$	STO 9	$\Delta\alpha_2$
	6.3 Repeat step 5 (register D is clear automatically)		R/S	
7	Step 6 is repeated again and again till the right hand side is equal to $n-\ell-\frac{1}{2}$ as near as you need.			
8	After getting a satisfaction we recall the results from the registers		RCL 6	α_2
			RCL 7	β_2
			RCL 8	γ_2

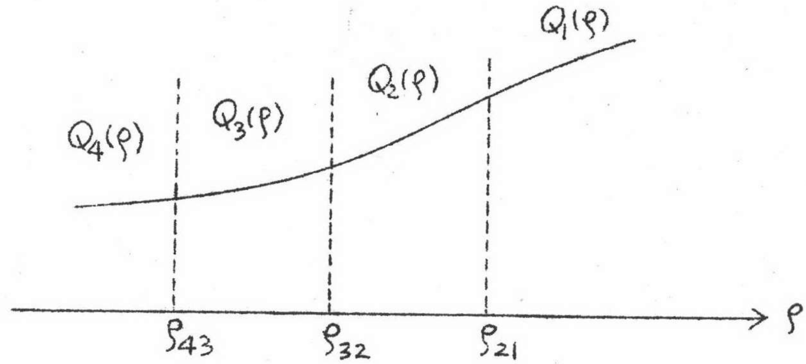
TableC.3 Instructions for determining parameters

APPENDIX D

DETERMINATION OF MATCHING FUNCTIONS

If the functions $Q_3(\rho)$ and $Q_2(\rho)$ are created to match the function $Q_4(\rho)$ with the function $Q_1(\rho)$ at ρ_{43} , ρ_{32} and ρ_{21} respectively, we must solve six matching conditions simultaneously. These equations are given in eq.(C.1) and eq.(C.2) where the running number m now runs from 1 to 3 .

Fig.C.1



The program and the user instructions are given in table D.1 and table D.2 respectively. It is noted that the program can be employed to match any parabolic functions with the same matching conditions.

001	*LBLA	21 11
002	RCL7	36 07
003	RCL6	36 06
004	-	-45
005	RCL7	36 07
006	X²	53
007	RCL6	36 06
008	X²	53
009	-	-45
010	x	-35
011	RCL6	36 06
012	RCL7	36 07
013	-	-45
014	RCL6	36 06
015	X²	53
016	RCL7	36 07
017	X²	53
018	-	-45
019	x	-35
020	-	-45
021	1/X	52
022	STOA	35 11
023	CLX	-51
024	RCL1	36 01
025	2	02
026	÷	-24
027	RCL4	36 04
028	2	02
029	÷	-24
030	-	-45
031	RCL0	36 00
032	RCL6	36 06
033	x	-35
034	+	-55
035	RCL3	36 03
036	RCL8	36 08
037	x	-35
038	-	-45
039	RCL7	36 07
040	X²	53
041	RCL6	36 06
042	X²	53
043	-	-45
044	x	-35
045	RCL2	36 02
046	RCL5	36 05
047	-	-45
048	RCL3	36 03
049	RCL8	36 08
050	X²	53
051	x	-35
052	+	-55
053	RCL0	36 00
054	RCL6	36 06
055	X²	53
056	x	-35
057	-	-45
058	RCL6	36 06
059	RCL7	36 07
060	-	-45
061	x	-35
062	-	-45
063	RCLA	36 11
064	x	-35
065	PRTX	-14
066	RCL3	36 03
067	-	-45
068	STOB	35 12
069	CHS	-22
070	2	02
071	x	-35
072	RCL8	36 08
073	x	-35
074	RCL4	36 04
075	+	-55
076	PRTX	-14
077	RCL8	36 08
078	X²	53
079	RCLB	36 12
080	x	-35
081	RCL5	36 05
082	+	-55
083	PRTX	-14
084	RCL2	36 02
085	RCL5	36 05
086	-	-45
087	RCL3	36 03
088	RCL8	36 08
089	X²	53
090	x	-35
091	+	-55
092	RCL0	36 00
093	RCL6	36 06
094	X²	53
095	x	-35
096	-	-45
097	RCL7	36 07
098	RCL8	36 08
099	-	-45
100	x	-35
101	RCL1	36 01
102	2	02
103	÷	-24
104	RCL4	36 04
105	2	02
106	÷	-24
107	-	-45
108	RCL0	36 00
109	RCL6	36 06
110	x	-35
111	+	-55
112	RCL3	36 03
113	RCL8	36 08
114	x	-35
115	-	-45
116	RCL8	36 08
117	X²	53
118	RCL7	36 07
119	X²	53
120	-	-45
121	x	-35
122	-	-45
123	RCLA	36 11
124	x	-35
125	PRTX	-14
126	CHS	-22
127	RCL0	36 00
128	+	-55
129	RCL6	36 06
130	x	-35
131	STOC	35 13
132	2	02
133	x	-35
134	RCL1	36 01
135	+	-55
136	PRTX	-14
137	RCLC	36 13
138	RCL6	36 06
139	CHS	-22
140	x	-35
141	RCL2	36 02
142	+	-55
143	PRTX	-14
144	RTN	24
145	R/S	51

Registers

0 α_4	1 β_4	2 γ_4	3 α_1	4 β_1
5 γ_1	6 ρ_{43}	7 ρ_{32}	8 ρ_{21}	9 $\alpha_2-\alpha_1$

Table D.1 Program for the matching functions

STEP	INSTRUCTIONS	INPUT DATA/UNITS	KEYS	OUTPUT DATA/UNITS
1	Load side 1 and side 2 of program			
2	Load input data	α_4	STO 0	α_4
		β_4	STO 1	β_4
		γ_4	STO 2	γ_4
		α_1	STO 3	α_1
		β_1	STO 4	β_1
		γ_1	STO 5	γ_1
		ρ_{43}	STO 6	ρ_{43}
		ρ_{32}	STO 7	ρ_{32}
		ρ_{21}	STO 8	ρ_{21}
3	Start the calculation		A	α_2
				β_2
				γ_2
				α_3
				β_3
				γ_3

Table D.2 User instructions

for calculating matching functions

VITA

Name	Mr. Wichit Sritrakool
Born	October 10 , 1953
	Meklong, Samutsongkram
Degree	B.Sc. in Physics, April, 1975
	Chulalongkorn University
	Bangkok, Thailand
Graduate Scholarship	University Development Commission
	National Education Council
	Bangkok
	June 1975 - May 1977
	Graduate School Assistantship
	Chulalongkorn University
	Bangkok
	August 1977 - November 1977