CHAPTER II

THEORETICAL CONSIDERATION

A composite action of brick infilled reinforced concrete frame is a highly indeterminate structural problem. An interaction behaviour between infill and the frame depends on their stiffness and strength. A proposed method to analyze the infilled frame will be introduced in this reasearch as an approximate analysis by the equivalent strut theory. Therefore, it is necessary to search for a reasonable model of equivalent strut by examine the resulting stresses in the infill.

2.1 Contact Length and Relative Stiffness Parameters.

When an infilled frame is subjected to lateral load, the load is transmitted from the frame to the infill through regions so called contact length. Smith (17, 18, 19, 20, 21) stated that diagonal stiffness and strength of an infilled panel depend not only on its dimensions and physical Properties but also on the contact length with its surrounding frame.

According to a differential equation for beams on elastic foundation

$$E_f I \frac{\partial^4 y}{\partial x^4} + ky = P \qquad (2.1)$$

where P is the applied load of beams on elastic foundation

$E_f I$ is the stiffness of beam (as column of frame)

k is the stiffness of foundation (as brick infill)

The complementary solution of Eq. (2.1) is

$$y = e^{\lambda x}(C_1 \cos\lambda x + C_2 \sin\lambda x) + e^{-\lambda x}(C_3 \cos\lambda x + C_4 \sin\lambda x) \quad (2.2)$$

where $\lambda = \sqrt[4]{k/4E_f I}$

λ is so called the relative stiffness in which λh is non-dimensional parameter expressing the relative stiffness of the frame to the brick infill. Main Stone (12) showed that the infill stiffness correspond to column was ;

$$k = E_w t_w \sin 2\theta/h' \quad (2.3)$$

Subsituting Eq. (2.3) into λ , then

$$\lambda = \sqrt[4]{E_w t_w \sin 2\theta/4E_f Ih'} \quad (2.4)$$

where :

E_w = Young's modulus of the brick infill

E_f = Young's modulus of the frame

t_w = infill thickness

I = moment of inertia of the column section

h' = brick infill height

h = column height

θ = slope of the diagonal line of brick panel with respect to the horizontal

The contact length, C, is given approximately by the equation (17, 18, 19, 20, 21)

$$\frac{C}{h} = \frac{\pi}{2\lambda h} \tag{2.5}$$

When the contact length is known then it is possible to make a series of stress analysis by mean of diagonal compressive force distributed over the contact length. In doing so, however, it is necessary to assume the shape of the force distribution between the frame and the infill along the length of contact and also to ignore the occurrence of slip across the interface, according to Smith's research work (17,18,19,20,21) that the shape of triangular may be used as distribution force on the infill over the contact length, C, against the column and a half-span against beam.

Consider the infill in Fig. 1, a triangular distribution is assumed to act on the infill over the contact length against the column PC and RG and over the beam, CQ and GS respectively,

Let P_1 = total vertical force act on length CQ

P_2 = total horizontal force act on length CP

$\bar{\theta}$ = angle of the resultant R with respect to horizontal force

then $$P_1 = R \sin \bar{\theta}$$
$$P_2 = R \cos \bar{\theta} \qquad (2.6)$$

By calculating the resultant force of each triangular distribution passing through one-third point, to give balanced couples acting on the infill then ;

$$P_1 (L' - \frac{2}{3} (\frac{L'}{2})) = P_2 (h' - \frac{2}{3} C)$$

$$P_1 \frac{2}{3} L' = P_2 h' (1 - \frac{2}{3} \frac{C}{h'})$$

$$\frac{P_1}{P_2} = \frac{3}{2} \frac{h'}{L'} (1 - \frac{2}{3} \frac{C}{h'}) \qquad (2.7)$$

Subsituting Eq. (2.6) into Eq. (2.7), yield

$$\tan \bar{\theta} = \frac{3}{2} \frac{h'}{L'} (1 - \frac{2}{3} \frac{C}{h'})$$

$$\bar{\theta} = (\frac{3}{2} \frac{h'}{L'} (1 - \frac{2}{3} \frac{C}{h'})) \tan^{-1} \qquad (2.8)$$

A load, R is assumed to be 1000 unit acts on a panel and then total vertical and horizontal forces can be determined by substituting Eq. (2.8) into Eq. (2.6). The values for various size of the panels are tabulated in Table 1 and Table 2 and these forces will be input to the computer program.

2.2 Finite Element Method

Calculation of principal stresses and displacements has been conducted for each specific panel configuration and this is done by a plane stress finite element analysis adapted for required informations. A computer program using finite element method developed by R.S. Sandhu(34) is employed in this study. The purpose of this program is to determine deformations and stresses in two-dimensional structure of arbitary shape. The principle of the method is assumed to be linear isotropic elasticity. Three types of elements are incorporated in the program ; one-dimensional constant strain element, constant strain trianglar element and four-point isoparametric quadrilateral element. The computation procedure in the program is shown in Fig. 2 and only four-point rectangular elements with constant thickness are used in this study. Three sizes with different span length to height ratio are considered.

The first size, length and height of the infill are equal, they are 80 cm. This infill will be divided into 64 elements, 81 nodes and 10 x 10 cm. element size.

The second one where length to height ratio is 1.5. The panel is 80 cm. in height and 120 cm. in length and it is divided into 96 elements, 117 nodes and 10x 10 cm. element.

The third infill is 80 cm. high and 160 cm. long ; the length to height ratio is 2.0. The infill consist of 128 elements, 153 nodes and also 10 x 10 cm. element.

Lateral loads are supperimposed as an equivalent triangular distribution load as shown in Fig. 3 to Fig. 7 and the contact lengths are shown in Table 1 and 2. Both geometry, boundary condition along the panel as well as the material properties were input to the computer by card reader and the problem is then solved by mean of computer IBM-360. The computer out-put are nodals displacements, normal stresses, shear stresses, principal stresses and angle of principal stress. The principal stresses of infills with various length to height ratio and contact lengths are ploted as contour stresses shown in Fig. 9 through Fig. 11. Shapes and patterns of the principal compressive stress contours indicate diagonal brace while the principal tensile stress contours tend to have diagonal tension in the direction which is perpendicular to the diagonal brace. This result confirmed an idea of the strut analogy to replace the infill.

2.3 Effective Width of Strut

The stress contours in Sect. 2.2 have proved that the infill behaves as a diagonal strut and the analogus as shown in Fig. 13, the infill may be replaced by an equivalent struts. The equivalent strut area can by compatibility of the load corner displacements. From the computer output, the corner displacement can be transformed to strut direction and yield.

$$\Delta_e = \Delta_w \cos (\theta - \theta') \qquad (2.9)$$

where :

Δ_w = resultant displacement at loaded corner of the infill (as defined in Fig. 3 through Fig. 5)

θ' = angle between the resultant displacement with respect to horizontal displacement at loaded corner.

θ = angle between diagonal line with respect to horizontal direction

Then the equivalent strut area can be written as

$$A_w = \frac{R \cos(\bar{\theta} - \theta) d}{\Delta_e E_w} \tag{2.10}$$

where :

R = the applied diagonal load

d = infill diagonal length

E_w = modulus of elasticity of the infill

$\bar{\theta}$ = angle of the diagonal load with respect to horizontal direction

The numerical results of Eq. (2.9) and Eq. (2.10) are tabulated in Table 4 and Table 5. The results from these tables indicate that the relationship between the effective width and the relative stiffness to be linear in the log-log scale (12), then the equivalent strut area in terms of effective width to diagonal length can be expressed in terms of effective width and relative stiffness as ;

$$\frac{W}{d} = A(\lambda h)^B \tag{2.11}$$

where ;

A and B are constant

W is effective width of equivalent strut

d is diagonal length of the infill

λ is characteristic of the infilled frame

h is story height

Assuming that c/h' is approximately equal to c/h and then the expression in Eq. (2.5) can be rewritten as :

$$\frac{c}{h'} = \frac{\pi}{2\lambda h} \quad (2.12)$$

then the relationship between w/d and λh can be determined by least square method. Principle of least square is to minimize the sum of the squares of the deviations of true value and approximate value.

Then the values of constant A and B in Eq. (2.11) can be solved and rewritten for various sized infill as :

a) $L'/h' = 2.0$

$$w/d = 0.477\ (\lambda h)^{-0.361} \quad (2.13a)$$

b) $L'/h' = 1.5$

$$w/d = 0.617\ (\lambda h)^{-0.360} \quad (2.13b)$$

c) $L'/h' = 1.0$

$$w/d = 0.734\ (\lambda h)^{-0.354} \quad (2.13c)$$

2.4 Structural Stability Consideration

The earlier investigations (22, 26) have been studied the strength of boundary frame that can sufficiently restrain the exterior load so that failure would occur in an infill first. According to Meli (22), reinforced concrete column must be designed so as to with stand only one-half of the lateral load. Similarly, Masahide tommii(26) suggested that the cross section area of reinforced concrete column and beam of the bounding frame should be larger than $St_w/2$, where S is the lesser of clear height or clear span length and t_w is the thickness of the infill. Commentary of Japanese code (26) recommened that width and depth of columns and beams of the bounding frame should be larger than twice of the wall thickness. Masahide (26) also suggested that the width and depth of the columns and beams should be larger than $St_w/3$.

Avoid the effect of structural instability or bucking of an infill, the lateral stability must be checked to make sure that none of such problem would occurs before the strut is formed. Timoshenko and Gere (28) stated that in order to eliminate the possibility of buckling on the web under service condition, the critical stress (σ_{cr}) must be larger than maximum bending stress in the web. To simplify the problem of lateral stability the brick infill may consider as a cantilever shear wall. Then Timoshenko formulae can be introduced as :

$$P_{cv} = \frac{4.013\sqrt{E_w I_\eta GJ}}{(h')^2}\left(1 - \frac{a}{h'}\sqrt{\frac{EI_\eta}{GJ}}\right) \quad (2.14)$$

where

$$J = L't_w^3/3$$

$$G = E_w/2(1 + \nu)$$

$$I_\eta = L't_3w/12 \quad (2.15)$$

$$I_\xi = twL'^3/12$$

$$a = L'/2$$

Hence the critical stress of Eq. (2.14) for = 0.2

$$\sigma_{cr} = \frac{2.59\ E_w t_w^2}{h'L'}\left(1 - 0.387\ \frac{L'}{h'}\right) \quad (2.16)$$

From this equation, length to height of the wall will be limited to about 2.58 where critical stress equal to zero.

For non-reinforced brick masonry, the allowable stresses has been suggested (33) as the following :

Flexural compressive and tension, $f_m = 0.32\ f'_w$

axial tension stress , $f_t = 0.10\ f'_w$

where f'_w is the compressive strength of the brick panel. Material properties of the building brick had been carried out by

Tonpatankul (24) show that

Compressive strength , $f'_w = 41.55 \ kg/cm^2$.

Modulus of elasticity, $E_w = 7.05 \times 10^3 \ kg/cm^2$.

Modulus of rupture, $f_r = 5.97 \ kg/cm^2$.

According to these allowable stresses and material properties, the limittation of height to thickness of an infill to avoid stability can be 29.0 , 21.0 and 18.0 for the panel length to height ratio 1, 1.5 and 2.0 repectively, these result indicated that for story height around 3.00 m the thickness should be more than 10.50 cm,14.0 cm. and 16.0 cm. for span length 3.00 m, 4.50 m and 6.00 m repectively.

2.5 Proposed Analysis Method of Infilled Frame.

The composite action of a brick infilled frame is a complex indeterminate structural problem. The structural behaviour of this composite structure depends on the stiffness and strength of the infill and the frame. The proposed method will separate the structure into two parts ; plain frame and brick infill. For an analysis,

Plain frame:

The portal frame with fixed base from analysis, the result is

$$H_A = H_D = \frac{P}{2} \tag{2.17}$$

$$V_A = -V_D = \frac{3Ph^2 I_1}{L(6I_1 h + I_2 L)} \tag{2.18}$$

$$M_{AB} = M_{DC} = \frac{Ph(3I_1h + I_2L)}{2(6I_1h + I_2L)} \quad (2.19)$$

$$M_{BA} = M_{CD} = \frac{3Ph^2I_1}{2(6I_1h + I_2L)} \quad (2.20)$$

$$\Delta_{BC} = \frac{Ph^3(3I_1h + 2I_2L)}{12E_fI_2(6I_1h+I_2L)} \quad (2.21)$$

Brick infill:

When an infilled frame is subjected to horizontal load, it will tend to deflect and separate as shown in Fig. 15 (a) and Fig. 15 (b). Thus behavior has confirmed that the infill can be replaced by an equivalent diagonal strut. The equivalent frame shown in Fig. 16, which has a strut BD with an area of A_w, may be assumed to represent the infilled frame. In this composite structure, the diagonal strut BD is subjected to direct stresses only, and the failure of the infill is assumed to occur at the instant of compressive failure of the equivalent strut BD, the force and deformation can be written as

$$F = fA_w \quad (2.22)$$

$$\Delta_E = \frac{eL}{\cos\gamma} \quad (2.23)$$

where

F = the force in the strut due to lateral load P.

A_w = cross section area of the strut.

f = the compressive stress in the strut due to lateral load P.

e = compressive strain in the strut.

E_w = modulus of elasticity of the equivalent strut (brick infill).

Δ_E = shortening of the strut.

For the horizontal displacement of the structure as shown in Fig. 16

$$\Delta = \frac{\Delta_E}{\cos\gamma} \tag{2.24}$$

Substituting Δ_E in Eq. (2.24) by Eq. (2.23) then obtain

$$\Delta = \frac{eL}{\cos^2\gamma} \tag{2.25}$$

from Eq. (2.21)

$$\Delta = (P - F\cos\gamma)\,\frac{h^3(3I_1h + 2I_2L)}{12E_fI_2(6I_1h + I_2L)} \tag{2.26}$$

where

E_f = modulus of elasticity of the frame.

Substituting the values of F and Δ from Eq. (2.22) and Eq. (2.24) into Eq. (2.26) to obtain :

$$P = \frac{12eLE_f I_2 (6I_1 h + I_2 L)}{h^3 \cos^2\gamma \, (3I_1 h + 2I_2 L)} + A_w f \cos\gamma \qquad (2.27)$$

the lateral load, P, in Eq. (2.27) is divided by the displacement, Δ, in Eq. (2.25) to obtain the lateral stiffness and by applying Hook's law, $e = f/E_w$, then the lateral stiffness can be expressed as ;

$$\frac{P}{\Delta} = \frac{12E_f I_2 (6I_1 h + I_2 L)}{h^3 (3I_1 h + 2I_2 L)} + \frac{A_w E_w \cos^3\gamma}{L} \qquad (2.28)$$

The ultimate load of the infilled frame is considered to be linear behaviour and fails in compression when the strain exceeds the ultimate strain.

$$P_u = \frac{12e'_w LE_f I_2 (6I_1 h + I_2 L)}{h^3 \cos^2\gamma (3I_1 h + 2I_2 L)} + A_w f'_w \cos\gamma \qquad (2.29)$$

where

P_u = Ultimate load considered as compression failure

e'_w = Ultimate strain of an infill

f'_w = Ultimate compressive stress of an infill