CHAPTER II

THEORETICAL CONSIDERATION

2.1 Euler formula

Consider a perfectly elastic slender column of constant cross-section, ideal column, and initially perfectly straight. The column in figure (1) is pin ended and subjected to longitudinal force P. At slow rate of increment of load P to a certain critical value the column will undergo laterally deflected as shown. This critical load P is called Euler load. The deflection will occur in the plane of minimum flexural rigidity, EI.

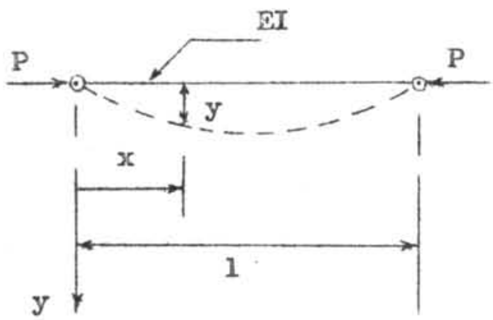

Fig. 1

The governing differential equation is:

$$EI \frac{d^2y}{d x^2} = -P.y \qquad \text{.............(1)}$$

The lowest critical load is

$$P_e = \frac{\pi^2 EI}{1^2} \qquad \text{.............(2)}$$

Therefore the stress at critical load is

$$\sigma_e = \frac{\pi^2 E}{(1/r)^2} \qquad \text{.............(3)}$$

Or in term of least dimension, d

$$\sigma_e = \frac{\pi^2 E}{12(1/d)^2} \qquad \text{.............(4)}$$

The first mode shape is

$$y = B \sin \frac{\pi x}{1} \qquad \text{.............(5)}$$

2.2 Perry Robertson formula

Most of timber columns are containing initial curvature due to seasoning. To determine the strength of such columns Perry and Robertson had included these effects in the derivation of formula.

Assume the initial curvature as shown in figure (2) for an initial mid span deviation of magnitude 'a'. The initial curvature can be represented by:

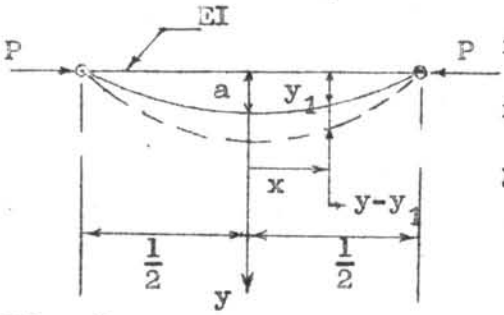

Fig. 2

$$y_1 = a \cos \frac{\pi x}{l} \quad \ldots\ldots\ldots\ldots(6)$$

The governing differential equation is:

$$EI \frac{d^2(y-y_1)}{dx^2} = -P.y \quad \ldots\ldots\ldots\ldots(7)$$

Hence

$$y = \frac{(\pi/l)^2}{(\pi/l)^2 - P/EI} . \ a \cos \frac{\pi x}{l} \quad \ldots.(8)$$

Maximum y when x = 0 :

$$y = \frac{P_e}{P_e - P} . \ a \quad \ldots\ldots\ldots\ldots(9)$$

Therefore maximum bending moment:

$$M = P.a.\frac{P_e}{P_e - P} \quad \ldots\ldots\ldots\ldots(10)$$

Maximum stress

$$= \frac{P}{A} \pm \frac{P.a.c}{I} . \frac{P_e}{P_e - P}$$

Define axial stress $\frac{P}{A} = \sigma_o$

Hence maximum stress

$$\sigma = \sigma_o\left(1 \pm \frac{a.c.}{r^2} \frac{P_e}{P_e - P}\right) \quad \ldots\ldots\ldots\ldots(11)$$

Perry[1] has found out that the effect of unavoidable of eccentricity due to load P on a peffectly straight column is equivalent to an initial curvature of column axis with mid length deviation of about 1.2 a. To combine the two effects equation (11) becomes:

$$\sigma = \sigma_o\left(1 \pm \frac{\sigma_e}{\sigma_e - \sigma_o} \cdot \frac{a'c}{r^2}\right) \ldots\ldots\ldots (12)$$

Where $a' = (a + 1.2\ a)$

And denote $n = \frac{a'c}{r^2}$

for possitive sign only is considered

$$\sigma = \sigma_o\left(1 + \frac{\sigma_e}{\sigma_e - \sigma_o} \cdot n\right)$$

Hence

$$\sigma_o = \frac{\sigma + (1+n)\sigma_e - \sqrt{\left\{\sigma_e(1+n) + \sigma\right\}^2 - 4\sigma \cdot \sigma_e}}{2} \ldots (13)$$

2.3 Fourth-Power Parabolic formula

The well known and widely used column formula in the United States of America, is an empirical formula which assumes that from the elastic limit to the point of maximum stress the strength of timber column varies according to the corresponding stress-strain relationship of the material within this range. It is also assumed that the stress-strain curve for timber is smooth from the limit of proportionality up to maximum compressive stress. Upon these assumptions the parabolic curve with its vertex at zero slenderness ratio at point of maximum stress and tangential to the Euler curve at the elastic limit represents the column strength suitably. The general form of this parabolic equation is:

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1 O.P. JAIN and B.K. JAIN, THEORY AND ANALYSIS OF STRUCTURES, VOLUME 1, NEM CHAND & BROS: ROORKEE (U.P.) 1967.

$$\frac{P}{A} = \sigma\left\{1 - \left(\frac{\sigma-\sigma_1}{\sigma}\right)\left(\frac{l/d}{l_1/d_1}\right)\left(\frac{2\sigma_1}{\sigma-\sigma_1}\right)\right\} \quad \text{........(14)}$$

Where $\frac{P}{A}$ = allowable axial stress

σ = maximum compressive stress

σ_1 = elastic limit stress

l_1/d_1 = slenderness ratio at the stress of σ_1

If the elastic limit stress, σ_1, is taken as four-fifths of the maximum compressive stress of timber, the parabolic equation becomes

$$\frac{P}{A} = \sigma\left\{1 - \frac{1}{5}\left(\frac{l/d}{l_1/d_1}\right)^8\right\}$$

When the elastic stress, σ_1, is taken as two-thirds of the maximum compressive stress of timber the equation becomes a fourth power parabolic formula:

$$\frac{P}{A} = \sigma\left\{1 - \frac{1}{3}\left(\frac{l/d}{l_1/d_1}\right)^4\right\} \quad \text{.............(15)}$$

Equation (15) is generally used to compute the strength of timber columns of intermediate class, i.e. slenderness ratios of 11 to l_1/d_1.

2.4 Spaced column formula

There are many factors affecting the strength of spaced columns. These factors are type of connectors, location of end spacer blocks and the successive spacing of spacer blocks. To determine the strength of spaced columns, the formulae can be adapted from Euler equation. These empirical formulae for determining safe load are: End condition type 'a' which has the end spacer blocks located at $\frac{1}{20}$ from the end of column

$$\text{Allowable stress } \frac{P}{A} = \frac{0.75\ E}{(1/d)^2} \qquad \text{..............(16)}$$

End condition type 'b' which has the end spacer blocks located at $\frac{1}{10}$ from the end of column

$$\text{Allowable stress } \frac{P}{A} = \frac{0.90\ E}{(1/d)^2} \qquad \text{..............(17)}$$