CHAPTER III

STRESS CONCENTRATION AROUND ELLIPTIC HOLES

Elliptic Hole in Uniformly Stressed Plate

A problem of an elliptic hole placed in uniformly stressed plate was seperately studied by Kolosoff[11] and Inglis[3] with the aid of complex variables. The same method was applied to several two - dimensional problems of elasticity by Stevenson.[12]

Consider an infinite plate, as shown in Fig. 3-1, in a state of uniform tension S disturbed by an elliptic hole of semi - axes a and b, which is free from stress. The solution to this problem was presented by Timoshenko and Goodier,[1] being a combination of Kolosoff's and Stevenson's methods by introducing two complex functions and elliptic coordinates (ξ, η). The stress at the edge of the hole, $(\sigma_\eta)_H$, can be written in the form

$$(\sigma_\eta)_H = S \frac{\sinh 2\xi_0 + \cos 2\beta - e^{2\xi_0} \cos 2(\beta - \eta)}{\cosh 2\xi_0 - \cos 2\eta} \qquad (23)$$

where β is the angle of inclination of the hole.

η is the angular position in elliptic coordinates which is equal to zero on the X - axis.

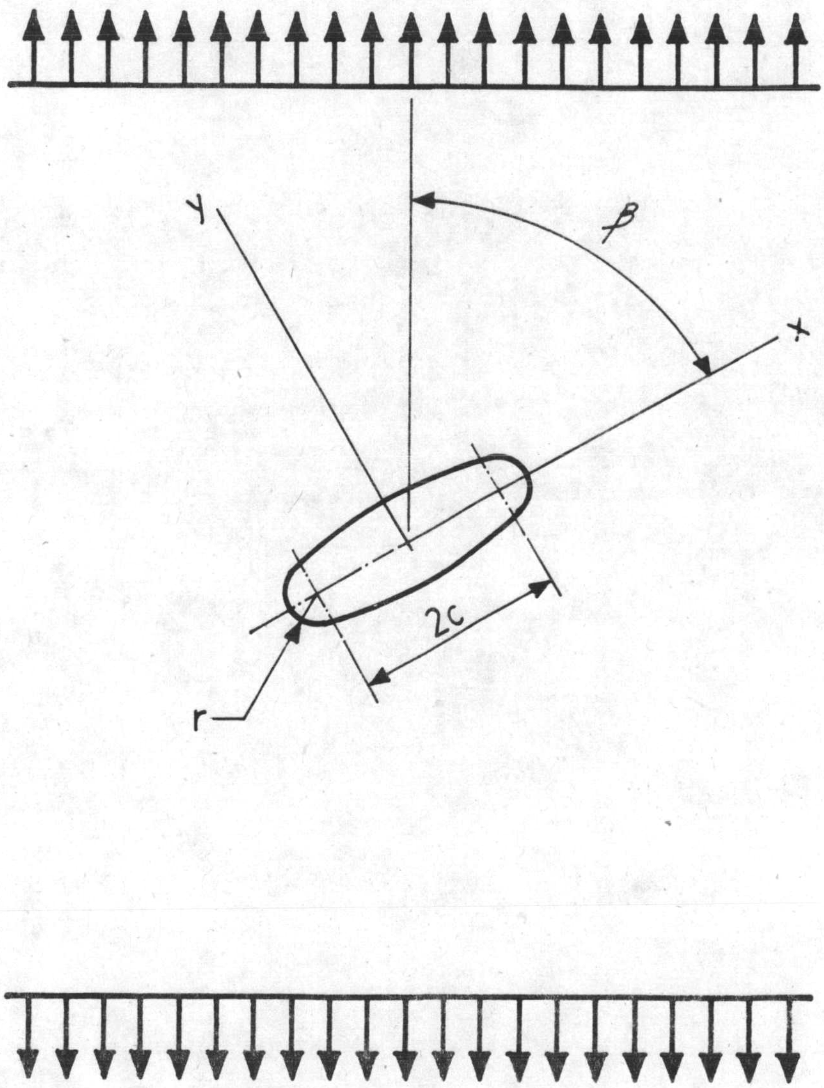

FIG. 3–1 EILLIPTIC HOLE IN UNIFORMLY STRESSED PLATE

ξ_0 is the constant which relates to the geometry of elliptic hole by the following equations:

$$\left.\begin{aligned} \sinh 2\xi_0 &= 2ab/c^2 \\ \cosh 2\xi_0 &= (a^2 + b^2)/c^2 \\ c^2 &= a^2 - b^2 \end{aligned}\right\} \qquad (24)$$

where c is the semi - focal length of the elliptic hole.

Stress Concentration of a Transverse Elliptic Hole

Consider eq. (23) when the tension S is at right angles to the major axis of the elliptic hole; $\beta = \pi/2$. The stress at the edge of the elliptic hole can be indicated by

$$(\sigma_\eta)_H = S\,\frac{\sinh 2\xi_0 + e^{2\xi_0}\cos 2\eta - 1}{\cosh 2\xi_0 - \cos 2\eta} \qquad (25)$$

It is evident that the elliptic hole is free from stress, that is normal stresses and shear stresses are actually zero. Consequently, the stress components at any points on the edge of the elliptic hole can be merely given by eq. (25). This equation states that the stress at the hole varies with η and the greatest value of $(\sigma_\eta)_H$ occurs at the ends of the major axis, $\cos 2\eta = 1$, which eq. (25) is reduced to

$$(\sigma_0)_H = S\left(1 + \frac{2a}{b}\right) \qquad (26)$$

But $a = c + r$; thus $b = \sqrt{2cr + r^2}$ where r is the radius of curvature at the ends of the elliptic hole, then, eq. (26) becomes

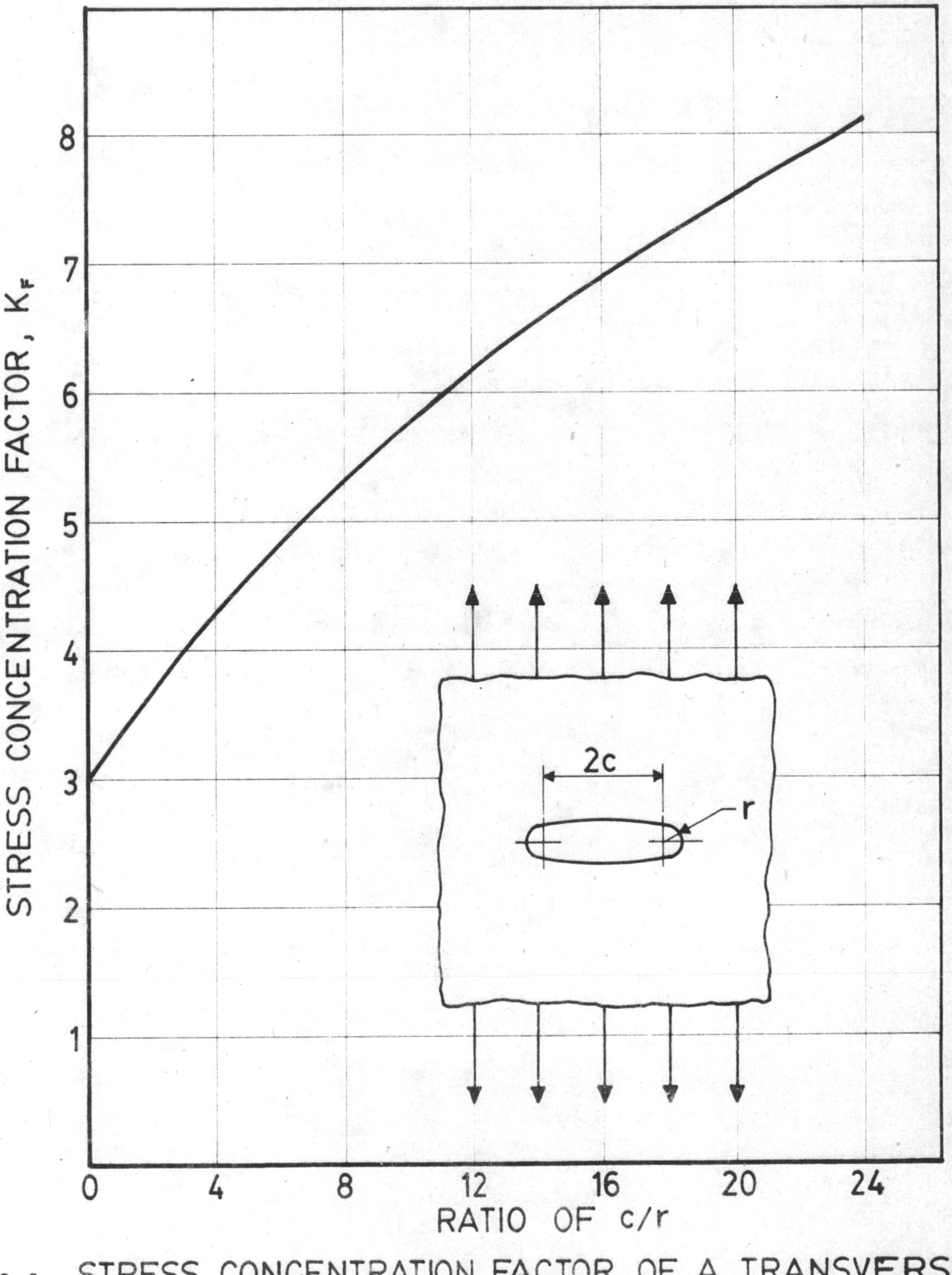

FIG. 3–2 STRESS CONCENTRATION FACTOR OF A TRANSVERSE ELLIPTIC HOLE

$$(\sigma_0)_H = S\left[1 + 2\,\frac{\frac{c}{r} + 1}{\sqrt{2\frac{c}{r} + 1}}\right] \tag{27}$$

Hence, the stress concentration factor at the ends of the transverse elliptic hole can be defined by

$$K_F = \frac{(\sigma_0)_H}{S} \tag{28}$$

$$K_F = \frac{2\left[\left(\frac{c}{r}\right) + 1\right]}{\sqrt{2\left(\frac{c}{r}\right) + 1}} + 1 \tag{29}$$

The variation of K_F with c/r are plotted in Fig. 3-2.

<u>Stress at the Ends of the Elliptic Hole</u>

Substilute $\eta = 0$ into eq. (23), the result will be obtained

$$(\sigma_0)_H = S\,\frac{\sinh 2\xi_0 + (1 - e^{2\xi_0})\cos 2\beta}{\cosh 2\xi_0 - 1} \tag{30}$$

This equation says that the stress at the ends of the hole varies with S, geometry of the hole, and the angle of inclination β, so stress concentration factor at any values of β are expressed by the following equation

$$K_F = \frac{\sinh 2\xi_0}{\cosh 2\xi_0 - 1} + \frac{1 - e^{2\xi_0}}{\cosh 2\xi_0 - 1}\cos 2\beta$$

Where $\dfrac{\sinh 2\xi_0}{\cosh 2\xi_0 - 1} = \dfrac{a}{b}$ and

$$\frac{1 - e^{2\xi_0}}{\cosh 2\xi_0 - 1} = -\left(1 + \frac{a}{b}\right), \text{ then}$$

$$K_F = \frac{a}{b} - \left(1 + \frac{a}{b}\right) \cos 2\beta \qquad (31)$$

<u>Parallel - Side Slit with Semi - Circular Ends</u>

It is stated in chapter I that when three types of slit: an elliptic slit, a narrow slit with circular ends, and a parallel - side slit with semi - circular ends are transversely placed in a plate under simple tension, their stress concentration factors are considered approximately the same within some acceptable limit. If the statement holds for any values of angle of inclination β of the hole, it means, eq. (31) can be used for the case of parallel - side slits with semi - circular ends.

Thus, eq. (31) exists for a parallel - side slit with semi - circular ends if the following assumptions are satisfied

(a) The maximum intensity of stress occurs at the ends of the slit or approximately very close to the ends.

(b) A parallel - side slit with semi - circular ends having a semi - focal length c, and a radius of curvature r, is assumed to be equivalent to an elliptic hole having the same value of c and r.

Rewrite eq. (31) again in terms of c/r, results

$$K_F = \frac{\frac{c}{r} + 1}{\sqrt{2 \frac{c}{r} + 1}} - \left[1 + \frac{\frac{c}{r} + 1}{\sqrt{2 \frac{c}{r} + 1}} \right] \cos 2\beta \qquad (32)$$

Simplified Equation for Parallel - Side Slit

If the above assumptions are not accurately satisfied, the expression for the stress concentration factor must be modified in order that the geometrical effect of a parallel - side slit on the expression can be determined by an experiment. Let's consider eq. (32) again, it is inspected that for a particular value of c/r, the expression for stress concentration factors of a parallel - side slit can be written as

$$K_F = k_1 + k_2 \cos 2\beta \qquad (33)$$

where k_1 and k_2 are functions of c/r.

When the major axis of the slit makes an angles $\beta = \pi/4$ with the direction of the applied stress S, eq. (33) becomes

$$K_F = k_1$$

That is the relation between k_1 and c/r can be determined from the result of an experiment by varying values of c/r at $\beta = \pi/4$. The expression for k_2 can also be determined by the experiment but more complicated than k_1 because the experiment must be conducted by varying c/r and β.

Photoelastic Stress Determination

It is shown in chapter II that the difference of principal stresses at any points can be expressed by

$$\sigma_1 - \sigma_2 = 2F\,n \tag{34}$$

where n is the isochromatic fringe order at that point and

F is the model fringe value.

Consider the case which a slit is placed in an infinite plate loaded by uniformly distributed force of intensity S. When the edge of the slit is free from any external forces, it is the fact that the stress components at all points on the edge of the slit must be zero, except the component of which direction tangents to the boundary of the slit, that is

$$\sigma_1 = \sigma_t \qquad \sigma_2 = 0 \tag{35}$$

Substitute eq. (35) into eq. (34), yields

$$\sigma_t = 2F\,n$$

Thus, the stress concentration factor can be given by

$$K_F = \sigma_t / S$$

$$K_F = \frac{2F\,n}{S} \tag{36}$$

where n is the maximum isochromatic fringe order occuring at the edge of the slit.