

CHAPTER 0

INTRODUCTION

In this thesis, we study basic properties of finite fields and some systems of equations over a finite field. Particularly, we find the number of solutions of the equation

$$a = a_1x_1^2 + \dots + a_tx_t^2,$$

where a, a_1, \dots, a_t are elements of a finite field F , and use it in helping to determine the number of solutions of the following system of equations :

$$a = a_1x_1^2 + \dots + a_tx_t^2 \quad (a_1 \dots a_t \neq 0),$$

$$b = b_1x_1 + \dots + b_tx_t,$$

where $a, a_1, \dots, a_t, b, b_1, \dots, b_t$ are elements of F . Finally, we apply our results to some problems in geometry.

In chapter I, we recall some relevant notions from group theory.

In chapter II, we collect some properties of finite field which are needed for the remainder of the thesis.

In chapter III, the main purpose is to determine the number of solutions in a finite field F of the equation

$$a = a_1x_1^2 + \dots + a_tx_t^2,$$

where a_1, \dots, a_t are non-zero elements in F and $a \in F$.

In chapter IV, we introduce some elementary applications of exponential sum and use them to evaluate the number $N_{s,t}(a,b)$ of solutions in a finite field F of the system of equations,

$$a = a_1x_1^2 + \dots + a_tx_t^2 \quad (a_1 \dots a_t \neq 0),$$

$$b = b_1x_1 + \dots + b_tx_t,$$

where a, b, a_i, b_i ($1 \leq i \leq t$) are elements of F . Furthermore, we use the results from chapter III in helping to determine the number $N_{s,t}(a,b)$.

In the last chapter, we solve some problems in geometry by using the results in previous chapter.

