CHAPTER III

POWER MEASUREMENT BY FEEDBACK TIME DIVISION MULTIPLIER AND RMS CONVERTER BASED ON STEEPEST DESCENT METHOD

3.1 Introduction

In this chapter a discussion will be made on the two main topics. First, various methods of multiplication will be explained briefly and then followed by the description of power measurement by feedback time division method. Second, we will briefly describe the RMS-to-dc converter which uses thermal techniqnes and explain in detail the computing technique which is based on steepest descent method.

3.2 Analog Multiplier

It can be seen from chapter II that a successful measurement of the power comes from the multiplier. The six most common solid-state types are, logarithmic, quarter-square, current ratioing, variable transconductance, triangle averaging and feedback time division method. There are other techniques for multiplying, but these six methods are the most suitable for all-solid-state instrumentation.

3.2.1 Logarithmic multiplier [3]

In Fig. 3-1 the log and anti-log amplifier amplifier techniques [3] are used in this circuit. It is only necessary to take the log of each input, sum these inputs, and then take the anti-log of the sum. The result is the product of the two imputs. In terms of the variable shown in Fig. 3-1

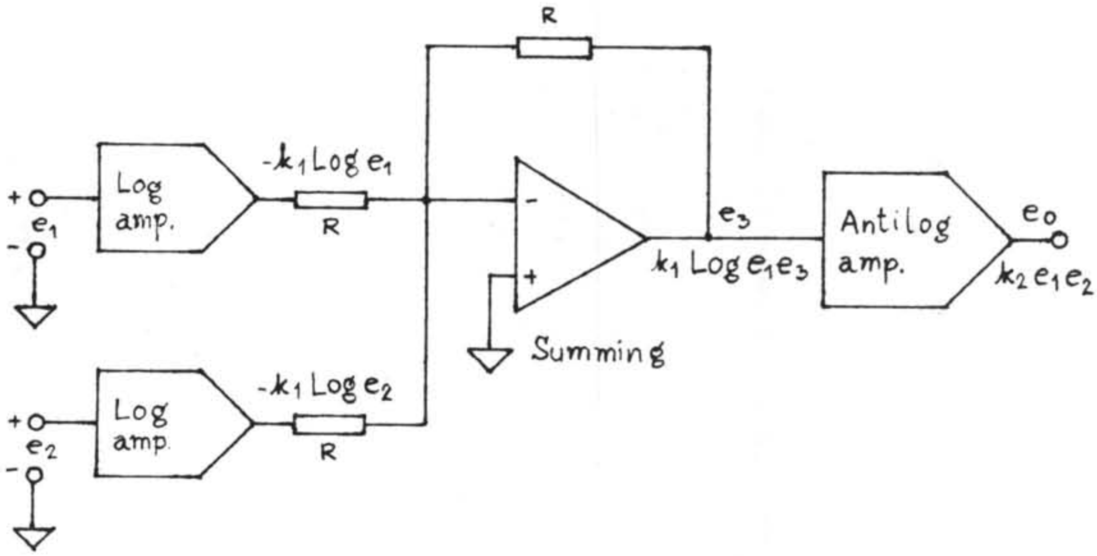


Fig. 3-1 Logarithmic multiplier

$$e3 = k1\ (\ln e1 + \ln e2) = k1\ \ln e1e2 \quad (3\text{-}1)$$

$$e0 = k2\ \ln^{-1} \frac{e3}{k1} = = k2e1e2 \quad (3\text{-}2)$$

The logarithmic technique of multiplication, of course, is useful only for unipolar input, or one-quadrant operation. And unfortunately, it suffers from rather strong temperature sensitivity. It is difficult to achieve better than 1 percent overall accuracy even for a moderate temperature range. Because of its basic simplicity, however,the logarithmic method may be attractive where accuracies of 1 to 5 percent are satisfactory and where careful temperature compensation is not required.

3.2.2 Quarter-square multiplier[3]

The quarter-square multiplier makes use of the equation

$$\frac{(X+Y)^2-(X-Y)^2}{4} = \frac{(X^2-X^2)+(Y^2-Y^2)+2XY+2XY}{4} = XY \quad (3-3)$$

to obtain the product. The squared terms are usually obtained through the use of special diode function generator, using the piecewise linear techniques.[3] In Fig. 3-2 e1 and e2 are X and Y in equation (3-3) respectively.

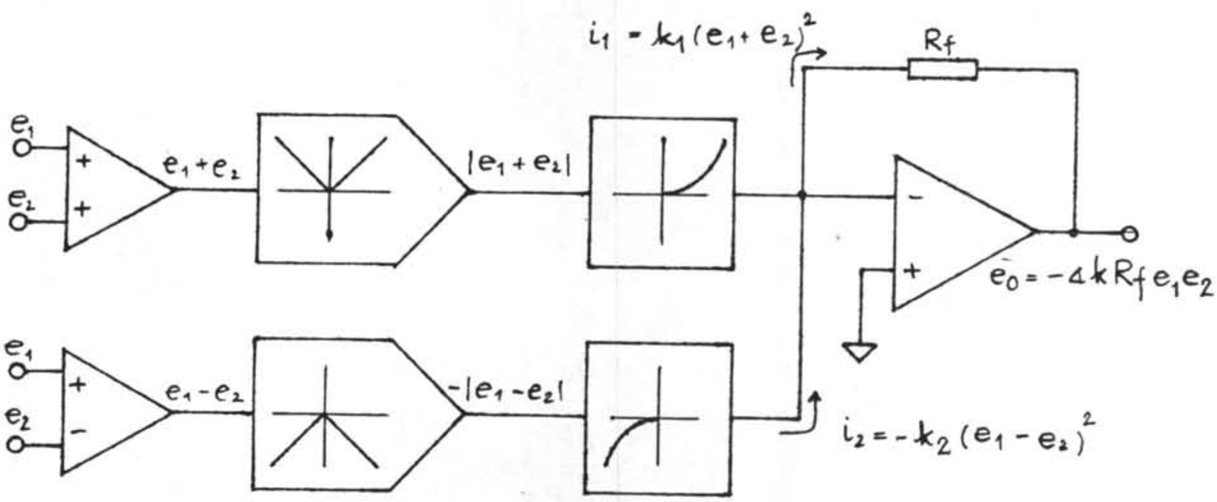
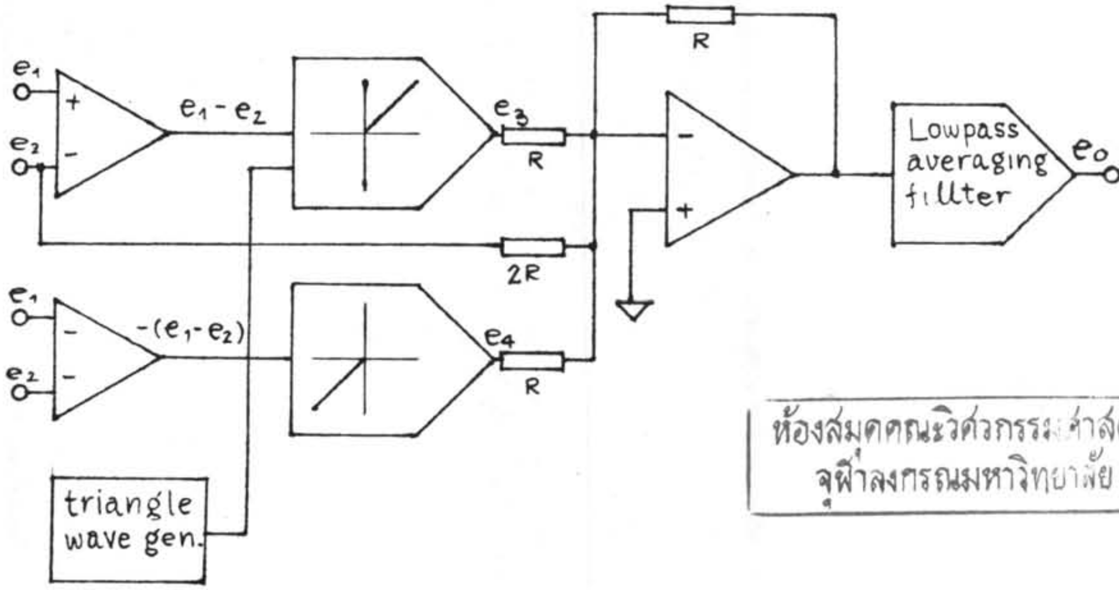


Fig. 3-2 Quarter-square multiplier

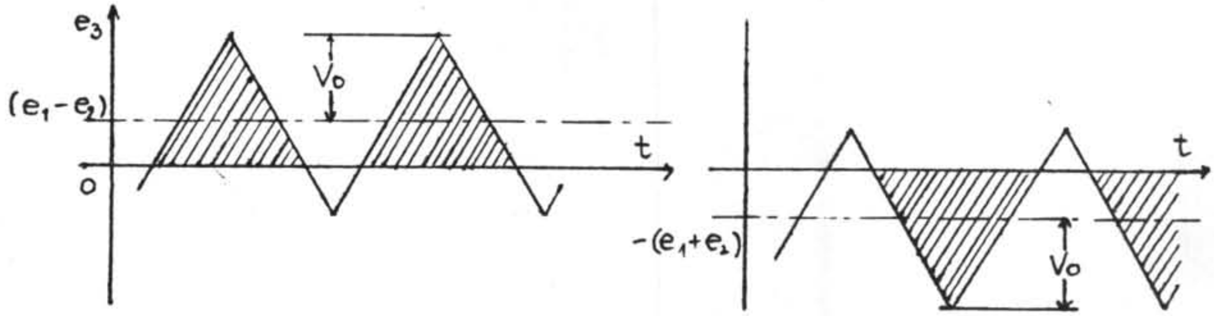
This method of multiplication is useful over a wide frequency range, which is its most attractive feature. Its principal disadvantages are the complexity, cost, and the fact that the maximum error voltage, although small as a pecentage of full scale, may exist at low input levels.

3.2.3 Triangle-averaging multiplier (3)

As illustrated in Fig. 3-3. The voltage e3 is the half-wave rectified sum of the triangle wave and e1-e2. Only the positive part of the waveform is retained, and this is time-averaged by a low-pass averaging filter.



a) Triangle-averaging multiplier



b) Multiplier waveforms

Fig. 3-3 Illustration of the triangle-averaging multiplier.

The resulting average value is

$$\bar{e}3 = \frac{1}{2}\left(\frac{1}{2}+\frac{e1-e2}{2Vo}\right)(Vo+e1-e2) \qquad (3\text{-}4)$$

Similarly,

$$\bar{e}4 = -\frac{1}{2}\left(\frac{1}{2}+\frac{e1+e2}{2Vo}\right)(Vo+e1+e2) \qquad (3\text{-}5)$$

The sum of the two voltages is

$$\bar{e}3+\bar{e}4 = -\frac{e2}{2} - \frac{e1e2}{Vo} \qquad (3\text{-}6)$$

If the $\frac{e2}{2}$ term is removed in a summing amplifier, the resulting voltage is the desired product. The frequency response of such multipliers is necessarily quite restricted because of the low-pass averaging filter at the output. The linearity of the triangle wave and the sharpness of the peaks of the waveform are the principal limitation on the accuracy of this method of multiplication.

3.2.4 Variable transconductance multiplier[3]

Perhaps the simplest multiplication technique is the variable trans-conductance method illustrated in Fig. 3-4. This method depends upon the current through the matched pair of transistors being proportional to one of the input signal.

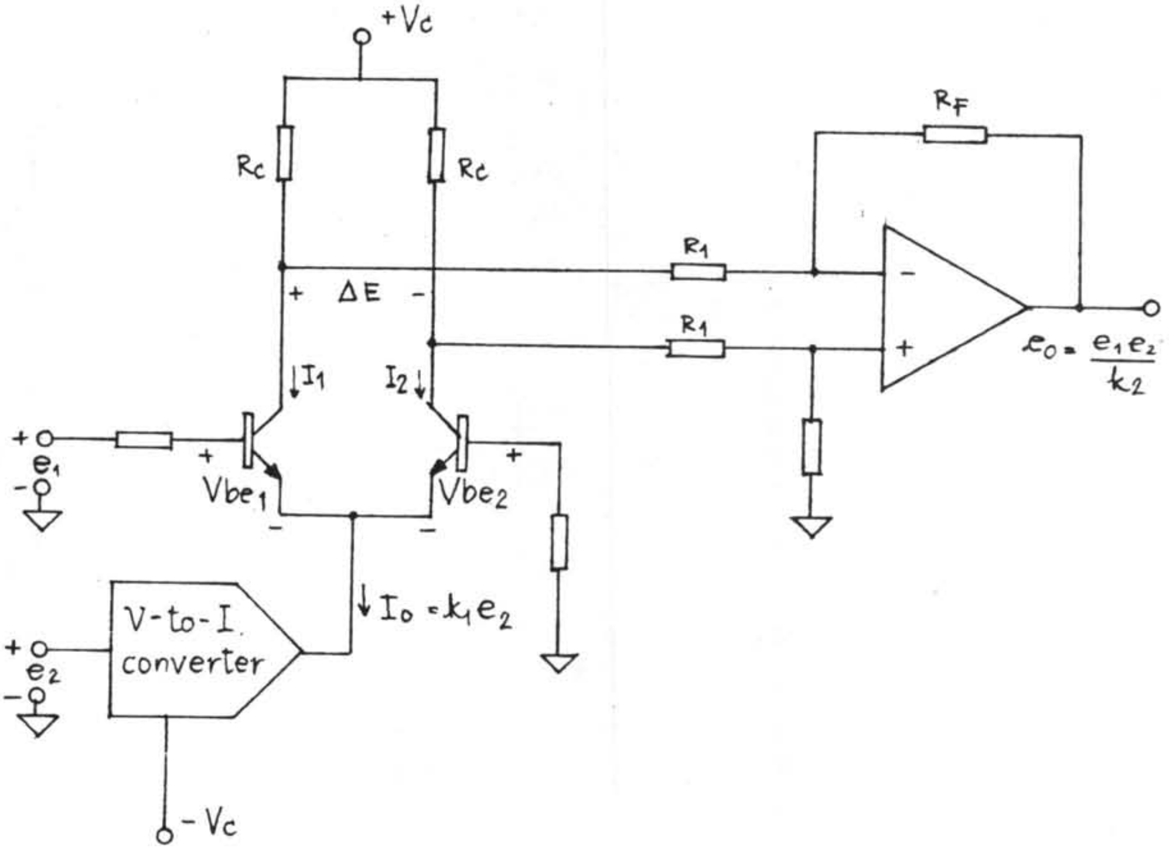


Fig. 3-4 Variable transconductance multiplier

Assuming that the transistors are a perfectly matched pair, the differential collector current can be shown to be proportional to the product of e1 and e2. The result is derived as follows

$$I1 = Ise\,\frac{q\ Vbe1}{kT}\ ,\quad I2 = Ise\,\frac{q\ Vbe2}{kT} \qquad (3\text{-}7)$$

Where Ise is a single-ended output current

Vbe is the base-emitter voltage

from equation (3-7) $$\frac{\Delta I1}{\Delta Vbe1} = \frac{q}{kT}\,Ise \qquad (3\text{-}8)$$

and $$Io = I1+I2 = 2Ise \frac{q\ Vbe1}{kT} \tag{3-9}$$

then, $$\Delta I1 = \frac{q}{2kT}\ Io\Delta Vbe1\ ,\quad \Delta I2 = \frac{q}{2kT}\ Io\Delta Vbe2 \tag{3-10}$$

$$\Delta I1-\Delta I2 = \frac{q}{2kT}\ Io\ (\Delta Vbe1-\Delta Vbe2) \tag{3-11}$$

$$= \frac{q}{2kT}\ Io.e1$$

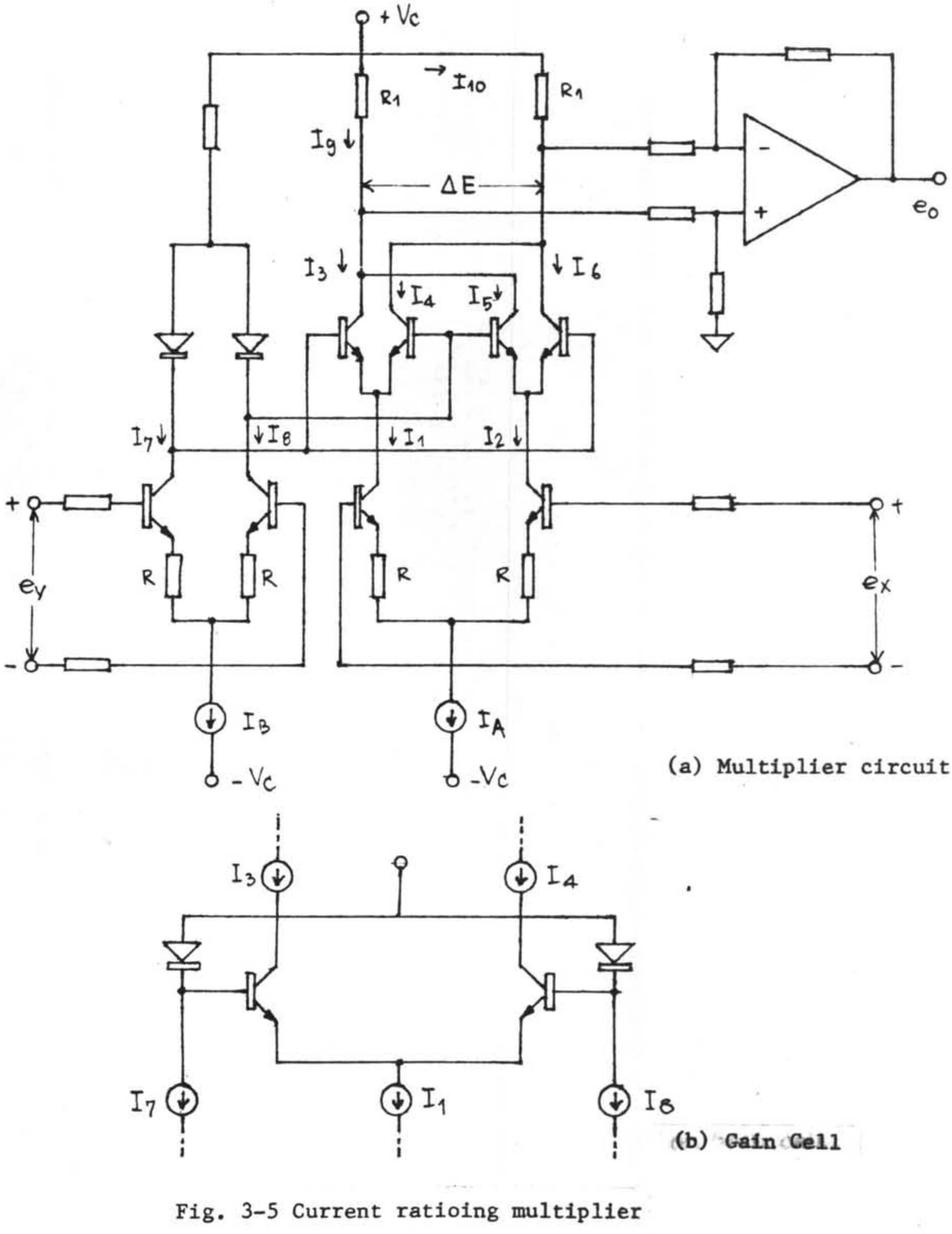
$$\Delta E = R\ (\Delta I1-\Delta I2) = Rc\ \frac{q}{2kT}\ k1e2e1 \tag{3-12}$$

$$e_o = \frac{RF}{R1}\ Rc\ \frac{q}{kT}\ k1e1e2 = \frac{e1e2}{k2} \tag{3-13}$$

Because of the extreme temperature sensitivity of this method of multiplication, it is of limited usefulness. Both the scale factor and the DC level will tend to drift, the latter because of unavoidable mismatch between the multiplying transistors. The linearity is also rather poor and AC feedthrough is appreciable.

3.2.5 Current ratioing multiplier[3]

One realization of the current ratioing multiplier is shown in Fig. 3-5a. The heart of this multiplier is the gain cell shown in Fig. 3-5b.



(a) Multiplier circuit

(b) Gain Cell

Fig. 3-5 Current ratioing multiplier

In the multiplier circuit in Fig. 3-5a, the gain cell concept is used to enforce the conditions

$$\frac{I4}{I3} = \frac{I8}{I7} \text{ and } \frac{I6}{I5} = \frac{I7}{I8} \tag{3-14}$$

Other necessary relations are

$$I1 = I3+I4 \quad , \quad I9 = I3+I5 \tag{3-15}$$

$$I2 = I5+I6 \quad , \quad I10 = I6+I4 \tag{3-16}$$

$$I1+I2 = IA \quad , \quad I7+I8 = IB \tag{3-17}$$

$$e_x = R\ (I1-I2) \quad , \quad e_y = R\ (I8-I7) \tag{3-18}$$

Combining the above equations and using a considerable amount of simple algebra, we obtain the relationship for ΔE as

$$\Delta E = R1\ (I9-I10) = \frac{(-e_y/R)(+e_x/R).R1}{IB} \tag{3-19}$$

With constant IB and proper scaling, the output voltage is

$$Eo = \frac{(e_x1 - e_x2)(e_y1 - e_y2)}{10}$$

Accurate multiplication requires that the transistor used be dynamically matched, a requirement that makes monolithic construction attractive for this type of multiplier.

From the previous paragraphs 3.2.1 to 3.2.5, it can be seen that every method has its weak points. These weak points lead to problems of multiplication accuracy. Some methods are too complicated, sensitive to temperature or frequency characteristic, while the others require critical components such as matched pair transistors which are rarely found in our local market. Therefore the author chooses the other method called feedback time division multiplier, which is not sensitive to surrounding conditions and for which no critical components are required.

3.3 Feedback Time Division Multiplier[7][8]

The feedback time division multiplier is a multiplying circuit based on the fact that the area of an electrical pulse is equal to the product of the pulse width and the pulse height.

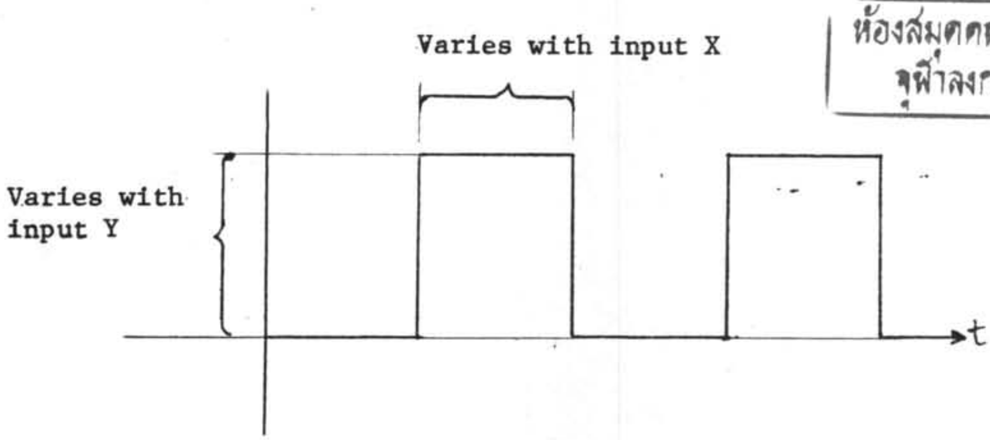


Fig. 3-6 The concept of feedback time division multiplier

The idea is to vary the pulse width of a square wave by input X and feedback the resulting output to sample the other input Y. After this sampled electrical pulse is averaged, we receive a DC voltage which varies as the product of X and Y. A pulse width modulation (PWM) circuit serves the function of varying the pulse width and sends the pulse to the multiplier circuit.

The detail of this type of multiplier will be described in the next two subsections.

3.3.1 Pulse width modulation circuit

In PWM circuit a DC or slowly varying input voltage is converted into a signal having a pulse width which is directly proportional to the input level.

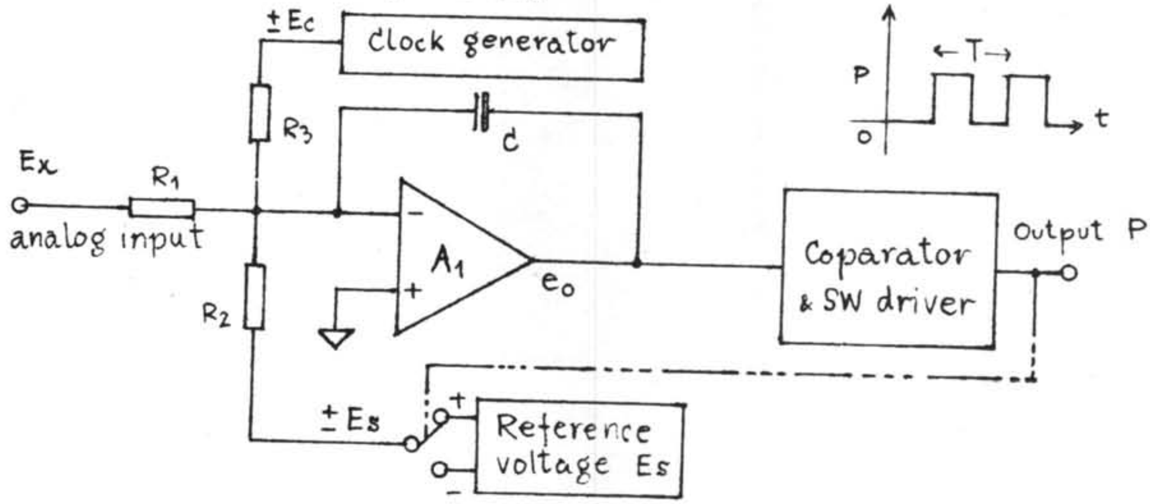


Fig. 3-7 Pulse width modulation circuit

As shown in Fig. 3-7, the circuit is consists of

(1) Input voltage Ex.

(2) $\pm$ Es Reference voltage supply : The polarity of the reference voltage is switched over alternately by an electronic switch S1 actuated by switching output P of the comparator.

(3) $\pm$ Ec Square wave clock voltage supply : The clock voltage $\pm$ Ec drives the system at repeated intervals T. Its waveform is so determined that its mean value will be zero within a cycle.

(4) Summing integrator : The operational amplifier A works as a summing integrator. The output voltage is referred to as eo.

(5) Comparator : The output eo of the summing integrator is compared with the zero level. The output pulse signal is referred to as P. The input voltage Ex, reference voltage $\pm$ Es, and clock voltage $\pm$ Ec are applied to the summing integrator. The output eo from the integrator is compared with the zero level by the comparator. The output pulse signal P actuates the switch S1 so that a negative feedback of $\pm$ Es will be supplied to the input side of the integrator when eo > 0, and -Es when eo < 0. The duration of time in which the switch S1 is turned to +Es or -Es side, that is, the duration in which +Es or -Es is applied to the input side of the integrator, is determined by the input voltage level Ex. A balanced condition is obtained by offsetting the main value within the whole relevant cycle with Ex.

Figure 3-8a shows each input waveform separately.

Figure 3-8b shows geometrically, the composite figure of these waveforms, which cannot be observed in reality.

Figure 3-8c is the output waveform from the summing integrator Everytime the output waveform intersects the zero level, the comparator turns over the voltage Es to positive or negative polarity.

Figure 3-8d shows the output pulse signal P from the comparator.

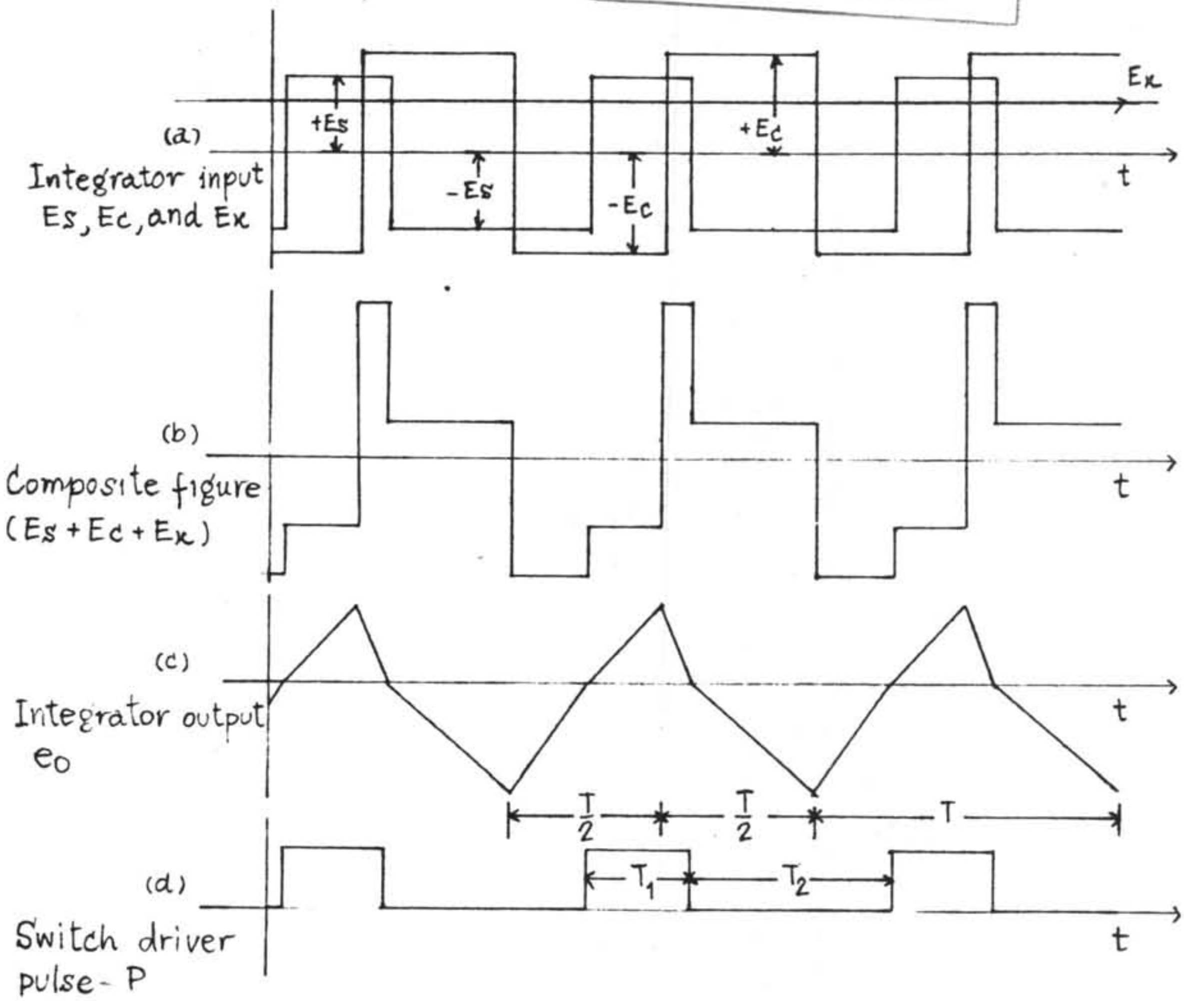


Fig. 3-8 Es, Ec, and Ex waveforms of PWM

If the duration in which + Es is fed back is T1, and the duration in which -Es is fed back is T2, as shown in Fig. 3-8, provided that the system is balanced, the following formula holds.

$$\left(\frac{Ex}{CR1} + \frac{Es}{CR2}\right)T1 = -\left(\frac{Ex}{CR1} - \frac{Es}{CR2}\right)T2 \tag{3-20}$$

$$\frac{Ex}{CR1}(T1+T2) = \frac{Es}{CR2}(T2-T1) \tag{3-21}$$

then

$$Ex = Es.\ \frac{R1}{R2}\ \frac{(T2-T1)}{(T2+T1)} \qquad (3\text{-}22)$$

Where C, R1, and R2 are the capacitance and resistance shown in Fig. 3-7.

T1+T2 = T represents a cycle of the clock signal voltage $\pm$ Ec and it kept constant. Therefore the difference of the pulse widths of Es, i.e., (T2-T1), is directly proportional to the input voltage level Ex. From this, Ex can be converted into a digital form by counting the time (T2-T1).

3.3.2 Feedback time division multiplier circuit

This multiplier circuit makes use of a principle that the area of a pulse is the product of the pulse width and the pulse height. The circuit is employed in the power measuring circuit and RMS value measuring circuit.

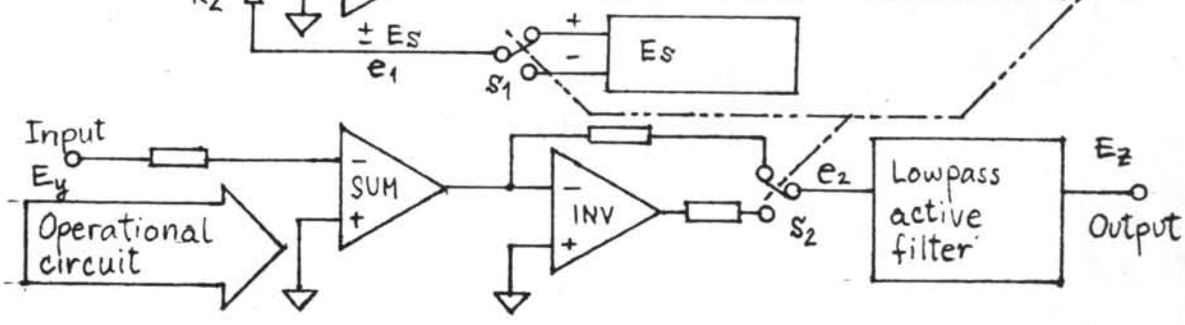


Fig. 3-9 Feedback time division multiplier circuit.

As shown in Fig. 3-9, the multiplier circuit is roughly divided into two parts as follows.

(a) Pulse-width modulation circuit : Pulse having a width dircetly proportional to the input voltage Ex of one side is obtained.

(b) Operational circuit : The width and height of the said pulse are proportional to the input voltage Ey of the other side, and the output is smoothed by passing through an active filter circuit.

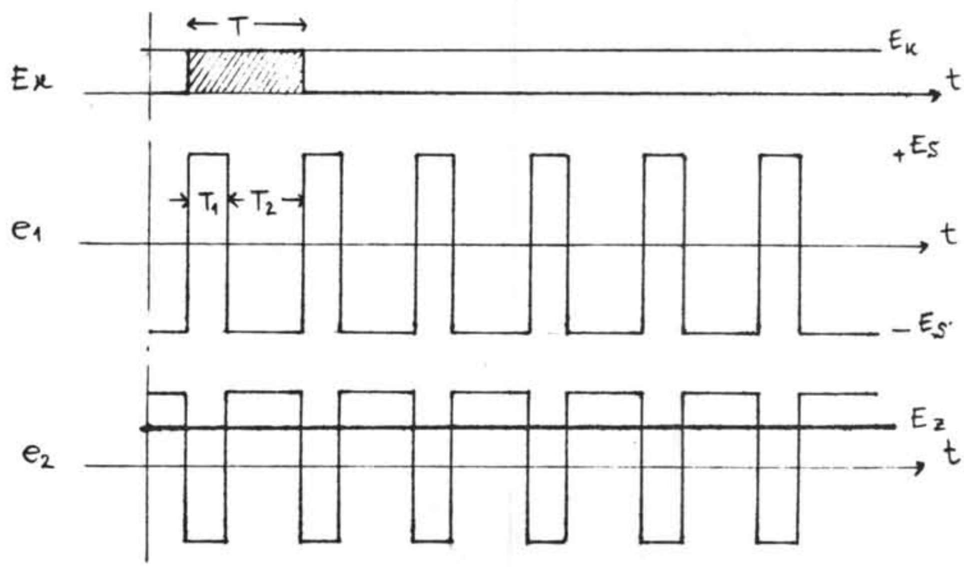


Fig. 3-10 Waveform of Ex, e1, and e2

Figures 3-9 and 3-10 show the operating principle diagram of the multiplier circuit and its waveforms.

The operation of the feedback time division multiplier circuit is the same as the pulse-width modulation circuit described in the previous subsection. It generates a pulse of $\pm$ Es having a pulse width proportional to the input voltage Ex as given below.

$$Ex = - Es \cdot \frac{R1}{R2} \frac{(T2-T1)}{(T2+T1)} \qquad (3\text{-}23)$$

The other input signal Ey is switched with S2, interlocked with S1, giving a mean output

$$Ez = Ey \frac{(T2-T1)}{(T2+T1)} \qquad (3\text{-}24)$$

Thus, the output waveform has a pulse width proportional to input signal Ex, and pulse height to input signal Ey. If (T2-T1)/(T2+T1) is eliminated from equations (3-23) and (3-24), we obtain

$$Ez = - \frac{R2}{R1} \cdot \frac{ExEy}{Es} \qquad (3\text{-}25)$$

Hence, the output that is proportional to the product Ex.Ey is obtained.

3.4 Power Measurement by Feedback Time Division Multiplier method

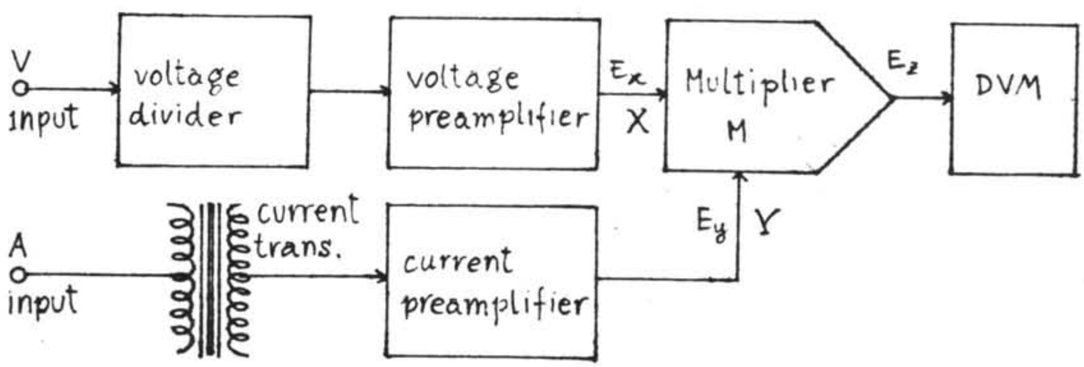


Fig. 3-11 Power measuring circuit

In this method of measuring power, which is shown diagramatically in Fig. 3-11, the input voltage to be measured (V-input) is converted into a voltage Ex by the V-preamplifier and applied to the input terminal X of the multiplier. The input current to be measured (A-input) is converted into a voltage Ey by the A-preamplifier, and applied to the input terminal Y of the multiplier.

The above mentioned Ex and Ey are subjected to a multiplication by the multiplier in the manner described in the previous paragraph, and converted into a DC output voltage Ez which is directly proportional to the wattage. By using a DC voltmeter (which may be of digital type) to measure this voltage, we will obtain the value of average power.

3.5 RMS Converter

The RMS value of a signal is generally the most meaningful since it is an indicator of the energy content of the signal without regard to its waveform. A number of operational amplifier circuit techniques indicate the RMS signal level by converting the signal to a corresponding DC voltage. These techniques include analog computing methods and thermal approaches, which use the heating value of a signal as a measure of its energy content.

3.5.1 Thermal techniques[4]

The traditional method for extracting RMS values of a waveform is to amplify it, apply it to a resistance, and then use a thermocouple to measure the temperature rise of the resistance. This method is shown in Fig. 3-12 below.

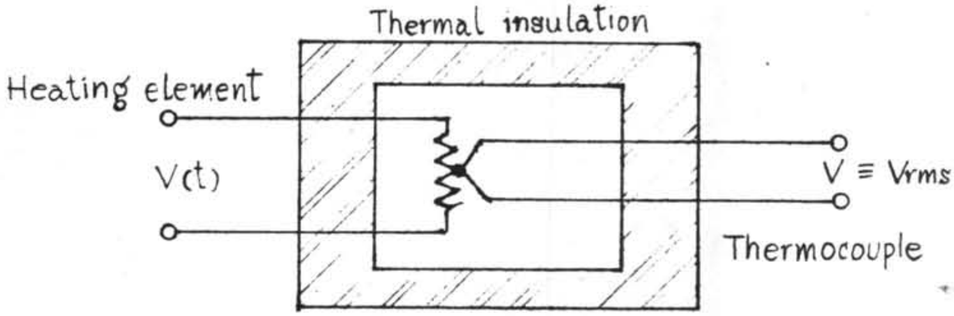


Fig. 3-12 Thermal response. The traditional method for extracting RMS value.

The DC output is proportional to the RMS value of the input. The problems inherent in this approach are many and well known. For instance, the thermocouple is sensitive to over load and therefore must frequently be replaced. Thermocouples are also nonlinear, and the instrument's measuring circuits must compensate for this fact. The output voltage is also very low, in the order of tens of milivolts, so an accurate, high resolution DC voltmeter is required to obtain high accuracies.

A newer thermal method makes use of two resistors plus feedback. The input voltage heats up on resistor, and a DC voltage brings the other to the same temperature. A pair of sensing transistors in a control voltage is, by definition, equal to the RMS volue of the input voltage.

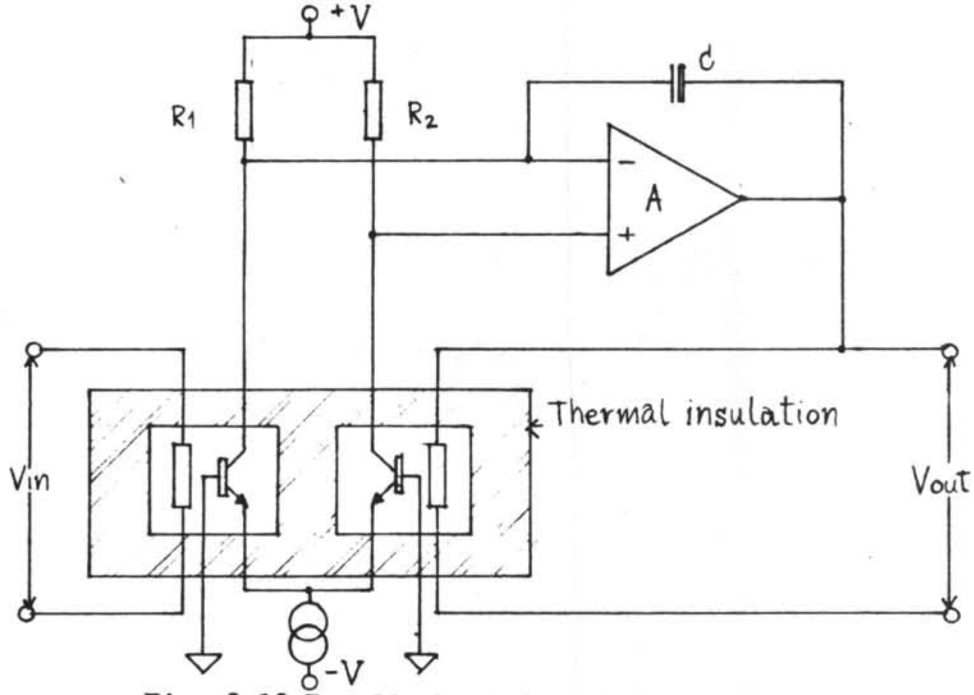


Fig. 3-13 Equally heated method.

This type of RMS detector must be assembled into a hybrid package. Here, the input resistor and transistor are put on one chip mounted in the same package as the resistor transistor pair of the control circuit. To insure that the integrated circuits are as similar as possible, two adjacent circuits from the same silicon slice are selected during manufacture. They are not seperated until they are mounted on a small glass plate that serves as thermal insulation.

It can be seen the thermal techniques are very complicated and also difficult to process it.

3.5.2 Computing techniques[4]

By means of analog computation a signal can be electrically proceeded through the mathematical operations required to derive its RMS value. In the general instrumentation application it is desirable to detect the RMS value of any arbitrary waveform, and the average reading circuit introduces significant errors when used with waveforms other than its desired waveform. Even if only one type of waveform is to be measured, distorting components will disturb precise RMS measurements. For these reasons it is often desirable to use a true RMS reading circuit.

One example of computing techniques is to compute the true RMS value of a signal which the mathematically operation defined in the following expression.

$$E\ (rms) = \sqrt{\frac{1}{T}\int_0^T e(t)^2\,dt} = \sqrt{\overline{e(t)^2}} \qquad (3\text{-}26)$$

From this expression it can be seen that the RMS computation involves squaring the signal, averaging this result, and then taking the square root of this average as shows in Fig. 3-14.

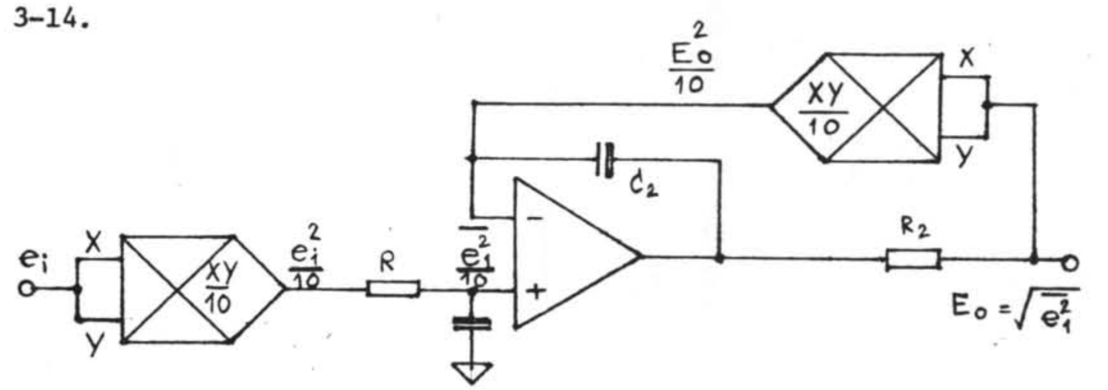


Fig. 3-14 Straightforward analog computation.

An accuracy of this conversion is directly limited by the errors of the multipliers and the amplifier. However, according to the expression in Eq.(3-26) we require two blocks of multiplier as shown in Fig. 3-14 and the error trends to be high.

3.6 RMS Converter Based on Steepest Descent Method

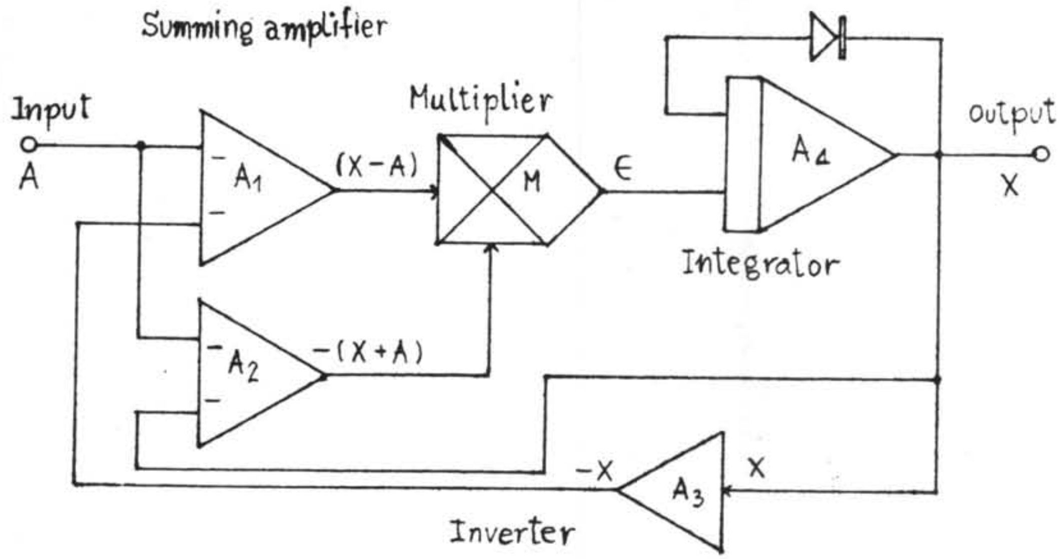


Fig. 3-15 RMS converter based on steepest descent method

The measuring circuit of RMS value shown in Fig. 3-15 is an RMS converter, based on the steepest descent method employing only one multiplier circuit, an integrator, an inverter, and two summing amplifiers. First, the negative voltage sum of input A and output X, −(X+A) and the difference voltage between A and X, (X-A), are applied to multiplier and then the two inputs are multiplied. If the output voltage of the multiplier is represented in ϵ, then

$$\epsilon = (X+A)(X-A) = (X^2-A^2) \tag{3-27}$$

Suppose that the input A is a distorted wave including a third harmonic, the following formula holds.

$$A = E1\ \mathrm{Sin}\omega t + E3\ \mathrm{Sin}\ (3\ \omega t + \emptyset) + \ldots \qquad (3\text{-}28)$$

And A^2 is expressed by the following formula

$$A^2 = \frac{E^2 1 + E^2 3}{2} - \left[\frac{E^2 1}{2} \mathrm{Cos}\ 2\omega t + \frac{E^2 3}{2} \mathrm{Cos}\ (6\omega t + \emptyset) - E1E3\ \mathrm{Cos}\ (2\omega t + \emptyset) + E1E3\ \mathrm{Cos}\ (4\omega t + \emptyset) \right] \qquad (3\text{-}29)$$

Therefore, output ϵ of the multiplier can be written as

$$\epsilon = \left[\chi^2 - \left(\frac{E^2 1 + E^2 3}{2}\right) \right] + \epsilon ac = \epsilon dc + \epsilon ac \qquad (3\text{-}30)$$

Where

$$\epsilon dc = \chi^2 - \frac{(E^2 1 + E^2 3)}{2} \qquad (3\text{-}31)$$

And ϵac is the AC component of the output. When large time constant is selected for the integrator, this AC component is attenuated until it becomes zero. That is, the integrator output include only DC component. If the integrator output $\chi > 0$ and $\epsilon dc > 0$, the integrator output is reduced and fed back to multiplier, then ϵdc is attenuated to zero and output χ becomes constant value. Constrastingly, if $\epsilon dc < 0$, output χ is increased and ϵdc also becomes zero. In this case, output voltage χ also becomes stable at the point given by the following equation.

$$\chi^2 - \left(\frac{E^2 1 + E^2 3}{2}\right) = 0$$

Hence

$$X = \sqrt{\frac{E^2{}_1 + E^2{}_3}{2}} \tag{3-32}$$

Consequently, the output is the RMS value of the distorted input wave voltage A. This solution also applies to any distorted waveshape which include all harmonics.

3.7 An RMS and Power Measuring Instrument

In this instrument, both voltage and current are converted into certain voltages, which in turn are converted, according to the purpose of measurement into RMS values, or multiplied into a power and subjected to ranging. Their output is displayed by a digital voltmeter (DVM)

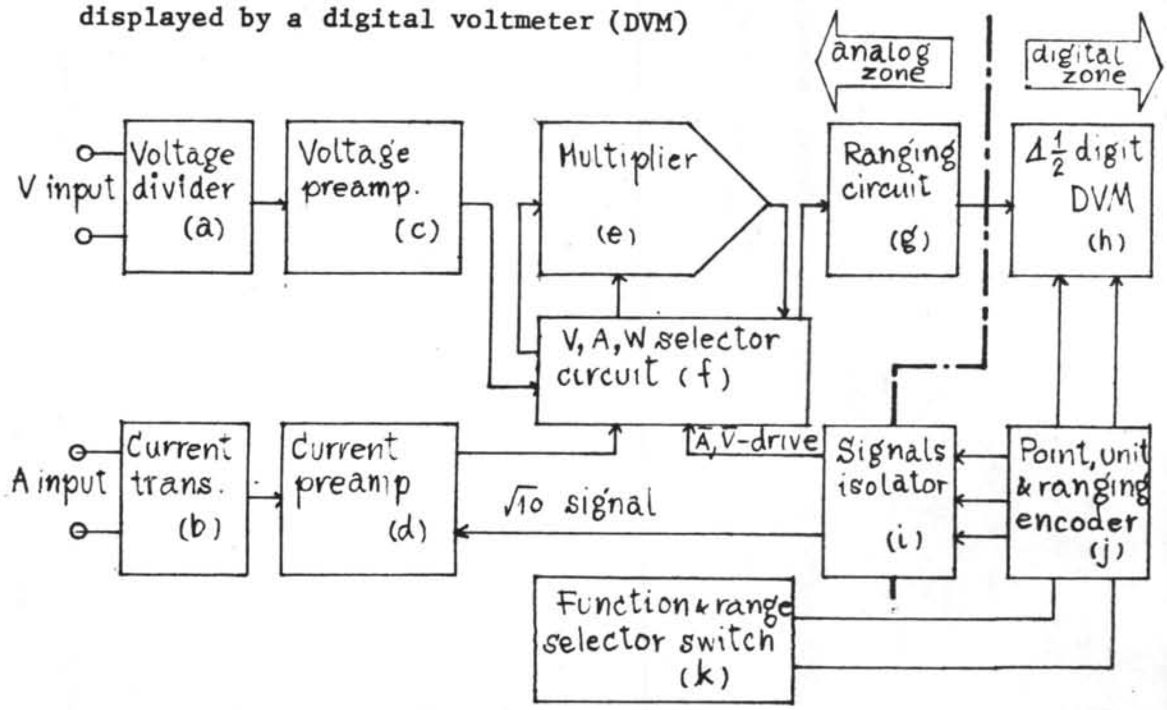


Fig. 3-16 Block diagram for whole instrument

As shown in Fig. 3-16, the input voltage to be measured is divided by the voltage divider (block a in Fig. 3-16) and sent out to V-preamplifier (block c). The V-preamplifier is a fixed gain type. The voltage dividing ratio is also fixed so that either of the following full scale output voltage (V-out) will be obtained from the amplifier by setting the V-range selector switch.

(1) 3 Vrms full scale : When range setting is a multiple of 10

(2) $3 \times 3/\sqrt{10}$ Vrms full scale : When range setting is a multiple of 3 (There are 3V, 30V, 300V and 600V)

The input current to be measured is applied to a variable input current transformer (block b), from the secondary winding of which a current of 10 mA full scale for each range is sent out to the A-preamplifier (block d). The A-preamplifier is composed of a current-to-voltage converting amplifier section and variable-gain amplifier section. The preamplifier gain is determined by an A-range selector switch setting and an output voltage (A-out) of either 3 Vrms full scale or $3 \times 3/\sqrt{10}$ Vrms full scale is obtained in the same way as the voltage side. The selection of the A-preamplifier gain is performed by the $\sqrt{10}$ control signal which is sent out from ranging encoder (block j)

The output voltage from the V-or A-preamplifier is sent out to the W/ RMS converter (block e and f).

(1) The W/ RMS converter is separately shown by a V, A, W selector (block f) and a multiplier (block e).

(2) The selection of the V, A, W operation circuit is made by the V- and A- drive control determined by the V, A, W function selector switch setting (block k).

The main signal of 3 or $3 \times 3/\sqrt{10}$ Vdc (The actual circuit is designed so that the output of multiplier is -1/3 of the product of the voltage input and the current input) from the W/ RMS converter is then applied to the ranging circuit (block g), where the main signal is discriminated for its function. In the same time the gain of the variable-gain amplifier for ranging is selected and accurate setting is made to enable direct reading on the DVM. Thus the measuring operation is completed here.

At the ranging and function encoder (block j) the several control signals are encoded. Both V- and A- Range selector switches have range signal switching sections in addition to main signal range switching sections. These V- and A- range signals and the function signal from the V, A, W function switch are directly transmitted to the encoder, where several code signals are formed from the above signals in order to be sent out to relevant circuits. The decimal point signal and unit signal are also formed within the encoder, and sent out directly to the digital circuit for display. The analog circuit operating independently from the decimal point signal.

The overall control signals and monitor signal are enumerated as follows.

1. Control signals sent out from the encoder (block j)

a) $\sqrt{10}$: To switches over the A-preamplifier gain to a multiple of $3/\sqrt{10}$

b) A-drive and V-drive : To determine the configulation of the measuring circuit for wattage and RMS voltage or current in the W/ RMS converter.

c) 2^0, 2^1, 2^2, range signal : Subjects the DC output voltage from the W/RMS converter within the ranging circuit to ranging.

2. Monitor signal sent out from the W. RMS converter (block f)

a) V-over, A-over : To energize the overrange and the V-under, A-under underrange indicator lamps.

b) V-meter, A-meter : To indicate the level of the measuring signal. Lasting, the auxiliary input V (1V), A (1V), and common terminals, are provided for adjustment, test and calibration. For the detail, refer to chapter V.