CHAPTER II

THEORY OF POWER AND RMS MEASUREMENTS

2.1 Introduction

This chapter lays the foundation necessary for the understanding of the power in AC circuits. AC power calculation by using an analog multiplier, true RMS measurement, and RMS converter based on steepest descent method are discussed.

2.2 Power in an AC Circuit (2)

In Fig. 2-1 if the RMS values of the voltage drop across the load Z and the current through the load are E and I, respectively, then the load power P can be calculated as follows.

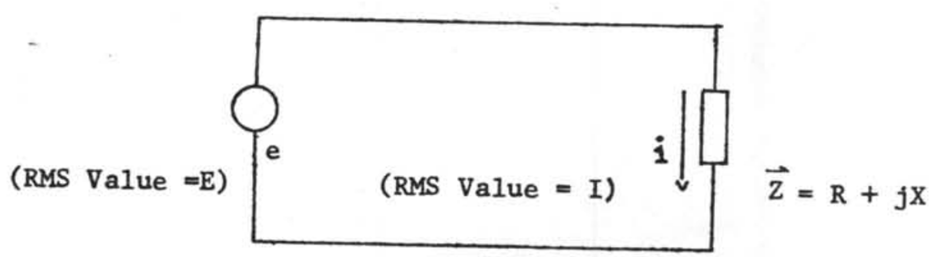


Fig. 2-1 Basic circuit

For a DC circuit, the RMS values are equal to the average values and

$$\text{DC power} \quad P = E.I \qquad [W] \quad (2\text{-}1)$$

For a single phase AC circuit

$$\text{effective power} \quad P = E.I \cos \phi \quad [W] \quad (2\text{-}2)$$

reactive power $Q = E.I \sin \phi$ [Var] (2-3)

apparant power $Ps = E.I$ [VA] (2-4)

where ϕ is the phase angle between the voltage and the current,

$\cos \phi$ is the power factor,

and $\sin \phi$ is the reactive factor.

The power can be measured by two methods, one direct and the other indirect. In direct method a conventional wattmeter or an analog system that finds the average of the instantaneous power is used. In indirect method we measure the general values on the right side of equation (2-1), (2-2), (2-3) or (2-4) and then calculate the power.

In this thesis the direct method will be used and in the following we will discuss this method only.

For the AC single phase in Fig. 2-1 the load impedance can be expressed as

$$\vec{Z} = R + jX \quad (2\text{-}5)$$

where R is the resistance and X is the reactance.

With the assumption of sinusoidal voltage and current,

load voltage $e = \sqrt{2}\, E. \sin \omega t$ (2-6)

load current $i = \sqrt{2}\, I. \sin (\omega t - \phi)$ (2-7) where E, I are the RMS values.

$$\omega = 2\pi f \qquad (2\text{-}8)$$

where f is the frequency

By using equations (2-6) and (2-7), we obtain

instantaneous power $P = e.i$

$= 2\, E.I\, \text{Sin}\, \omega t\, .\, \text{Sin}\, (\omega t - \phi)$

$= E.I.\, [\text{Cos}\, (\omega t - \omega t + \phi) - \text{Cos}\, (\omega t + \omega t - \phi)]$

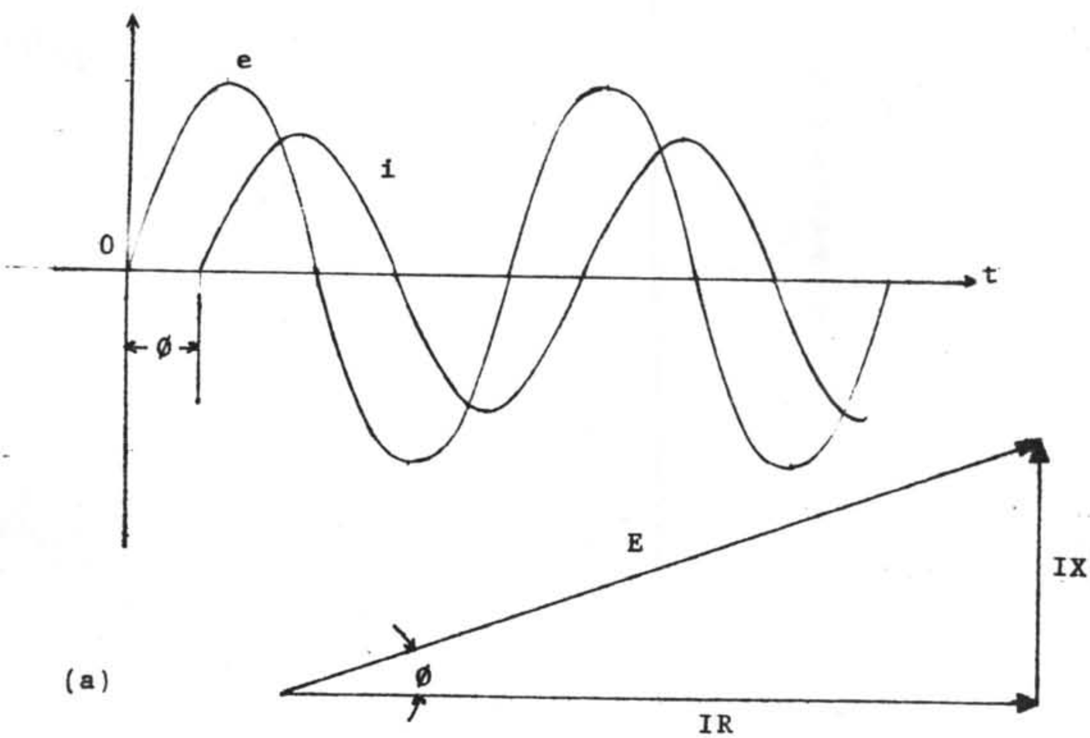
$= E.I.\, [\text{Cos}\, \phi - \text{Cos}\, (2\omega t - \phi)]$

$= E.I.\, [\text{Cos}\, \phi - \text{Cos}\, \phi\, \text{Cos}\, 2\omega t - \text{Sin}\, \phi\, \text{Sin}\, 2\omega t] \qquad (2\text{-}9)$

$= E.I\, \text{Cos}\, \phi - E.I\, \text{Cos}\, \phi\, \text{Cos}\, 2\omega t - E.I\, \text{Sin}\, \phi\, \text{Sin}\, 2\omega t \qquad (2\text{-}10)$

$= P1 + P2 + P3$

The waveforms of P1, P2, and P3 are shown in Figs. 2-2b and 2-2c



(a)

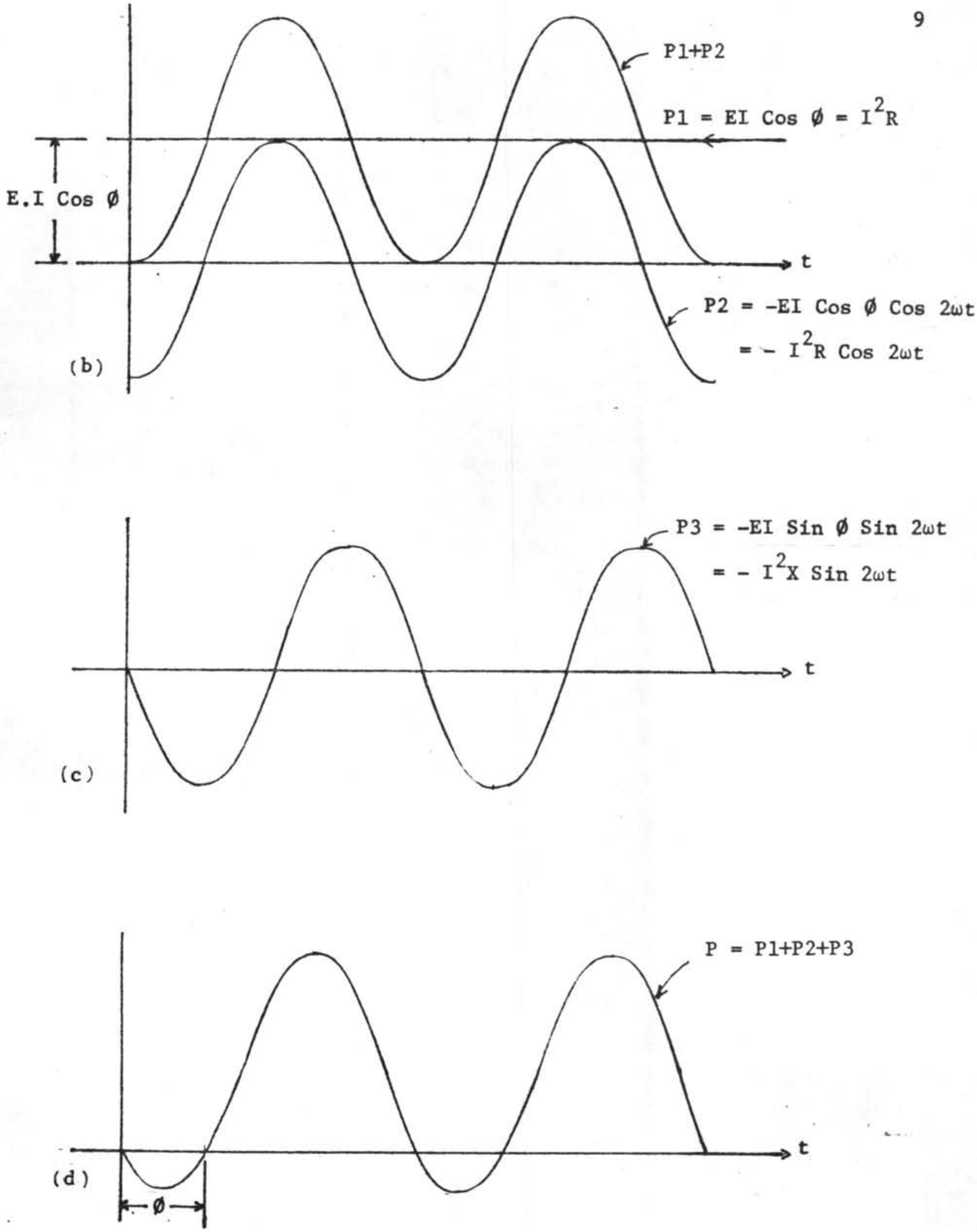


Fig.2-2 Instantaneous value of AC power

Considering the relation between e and i in Fig. 2-1, we can show that

$$I = \frac{E}{\sqrt{R^2 + X^2}} \quad (2-11)$$

and

$$\cos \phi = \frac{R}{\sqrt{R^2 + X^2}} \quad (2-12)$$

then

$$P1 = E.I \cos \phi = I^2 R \quad (2-13)$$

$$P2 = -E.I \cos \phi \cos 2\omega t = -I^2 R \cos 2\omega t \quad (2-14)$$

$$P3 = -E.I \sin \phi \sin 2\omega t = -I^2 X \sin 2\omega t \quad (2-15)$$

It can be seen that P1 is a constant value while P2 is in a sinusoidal form with a frequency of twice that of the voltage and the current. The maximum value of P2 is equal to P1. P1 and P2 are the powers that are consumed by the load. P3 is similar to P2 but it maximum value is equal to E.I sin Ø.

Composite value of P1, P2, and P3 are shown in Fig. 2-2d.

In an AC circuit the instantaneous ppwer varies continuously as the voltage and the current go through a cycle of values. Usually the cyclic variation of power has a period so short that it can be followed only by special instruments such as oscillographs. However, we are not interested in the instantaneous power, except where transient phenomena are being studied, but in its time average

$$P = \frac{1}{T} \int_0^T e.i \, dt$$

Since the average power multiplied by time measures energy transfer over an interval of steady state condition, we will therefore confine our discussion to the measurement of average power. The smallest interval that will concern us is the period of the AC signal, since the average power for one cycle is the same as for any integral number of cycles under steady state condition. If the voltage and the current are both sinusoidal, the average power over a cycle may be easily derived. From our difinition of average power we have

$$Pav = \frac{1}{T} \int_{o}^{T} e.i \, dt$$

$$= \frac{1}{2\pi} \int_{o}^{2\pi} Em \, Sin \, \omega t \; . \; Im \, Sin \, (\omega t - \phi) \, d\omega t$$

where Em and Im are the maximum values of voltage and current, ϕ is the phase angle by which the current lags behind the voltage.

Since, $Sin\,(\omega t-\phi) = Sin\,\omega t\;Cos\,\phi - Cos\,\omega t\;Sin\,\phi$, we can write

$$Pav = \frac{Em\;.\;Im}{2\pi}\left[\int_{o}^{2\pi} Sin^2\omega t\;Cos\,\phi\;d\omega t - \int_{o}^{2\pi} Sin\,\omega t\;Cos\,\omega t\;Sin\,\phi\;d\omega t\right]$$

$$= \frac{Em\;.\;Im}{2\pi}\left[\left(\frac{\omega t}{2} - \frac{Sin\,2\omega t}{4}\right)\Big|_{o}^{2\pi} .\;Cos\,\phi - \frac{(Sin^2\omega t)}{2}\Big|_{o}^{2\pi}\;.\;Sin\,\phi\right]$$

$$= \frac{Em\;.\;Im}{2}\;.\;Cos\,\phi$$

With our assumption of sinusoidal voltage and current, their RMS values are

$$E = \frac{Em}{\sqrt{2}}$$

and

$$I = \frac{Im}{\sqrt{2}}$$

If these values are substituted in the equation above, then

$$Pav = E.I \cos \emptyset \qquad (2\text{-}16)$$

Under these conditions we could, if E, I and Ø were determined, take the indicated product as a measure of the average power.

From Fig. 2-2 b it can be seen that the power that is consumed by the load is the average value of P1 + P2 in one cycle and is equal to P1. The average power in equation (2-16) is called effective power (or real power).

Thus, effective power $Pav = E.I \cos \emptyset = I^2R \quad [\omega]$ (2-17)

The product of E and I (E.I) is called apparent power. The usefulness of the apparent power in calculating effective power depends on Cos Ø . Cos Ø is the coefficient value which is determined by the resistance and the reactance of the load

$$\emptyset = \tan^{-1} (\frac{X}{R}) \qquad (2\text{-}18)$$

For P3, its average value in one cycle is equal to zero. This power is supplied to a reactance load. A purely reactive load will accumulate and then return all of the power to the supply.

Since P3 cannot be consumed by the load, we call it the reactive power.

2.3 AC Power Calculation

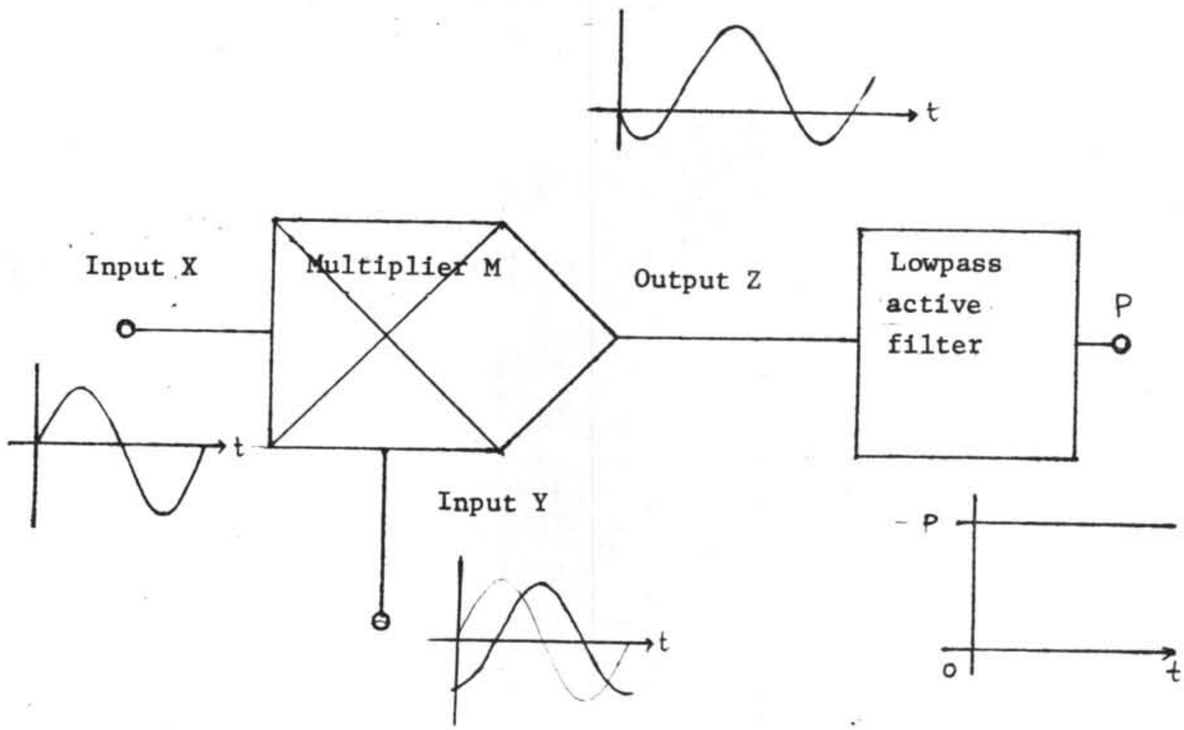


Fig. 2-3 A direct method to measure AC power

For the direct method, the AC power is found from the product of the instantaneous AC current and voltage. This can be done by an analog multiplier.

In Fig. 2-3, which shows a direct method of power measurement, the output of the multiplier is in form of instantaneous power. From equation (2-16) the effective power is the average value of the instantaneous power. If the output of the multiplier is applied to a low - pass active filter circuit, it becomes the DC voltage that corresponds to the average power. By using a DC voltmeter (which may be of digital type) to measure this voltage, we will obtain the value of average power.

It can be seen that a successful measurement of power, either by a direct or indirect method, comes from the multiplier. The six most common solid-state types of multiplier are logarithmic, quarter square, current ratioing, variable transconductance, triangle averaging, and feedback time division method. There are other techniques for multiplying, but these six methods are the most suitable for all-solid-state instrumentation. Together, they span a wide spectrum of accuracy, speed, and cost. In the next chapter we will discuss the feedbock time division method, which is used in this thesis.

2.4 RMS Measurements

Except in special cases, such as insulation testing or certain magnetic measurements, where it really is essential to measure peak or average values, the most useful way to describe an AC signal is by means of its RMS value. This value is the one that a DC voltage would need in order to transfer the same energy as the AC signal in a given period of time. The amount of energy ΔE dissipated in a resistor of value R when a voltage of value V is impressed upon it for a period of time Δt is given by

$$\Delta E = \frac{V^2}{R} \Delta t \tag{2-19}$$

For voltages that vary with time, we can rewrite equation (2-19) as

$$dE = \frac{V^2}{R} dt \qquad (2\text{-}20)$$

The total amount of energy developed as heat in a resistor between t = 0 to t = T is then

$$E = \frac{1}{R} \int_0^T V^2 \, dt \qquad (2\text{-}21)$$

If the effective or RMS value of a voltage V is defined as the DC voltage that would produce the same amount of heat in a certain time, then Vdc = Vrms and

$$\int_0^T V^2 \, dt = V^2 rms \, . \, T \qquad (2\text{-}22)$$

or

$$Vrms = \left[\frac{1}{T} \int_0^T V^2 \, dt\right]^{\frac{1}{2}} \qquad (2\text{-}23)$$

For a sine wave in which V = Vp Sin ωt and ω = 2πf, the RMS value can be calculated as follows.

$$Vrms = \left[\frac{Vp^2}{T} \int_0^T Sin^2 \, \omega t \, dt\right]^{\frac{1}{2}}$$

$$= Vp \left[\frac{1}{T} \int_0^T (\frac{1}{2} - \frac{1}{2} Cos \, 2\omega t) \, dt\right]^{\frac{1}{2}}$$

$$= Vp \left[\frac{1}{T} (\frac{1}{2}t - \frac{1}{2} (\frac{1}{2\omega} Sin \, 2\omega t) \Big|_0^T\right]^{\frac{1}{2}}$$

$$= Vp \left[\frac{1}{T} \cdot \frac{1}{2} T\right]^{\frac{1}{2}}$$

$$= Vp \left[\frac{1}{2}\right]^{\frac{1}{2}}$$

$$= 0.707\ Vp \qquad (2\text{-}25)$$

Calculating the RMS value of a wave form that is more complex than sine wave is usually posible but can be tedious. For electrical signals, a mumber of circuit techniques indicates the RMS signal level by converting the signal to a corresponding DC voltage. These techniques include analog computing methods and thermal approaches, which use the heating value of a signal as a measure of its energy content. In the next chapter discussion will be made on the RMS converter based on steepest descent method, which is one of analog computing method. This method is used in this thesis for measured the RMS value of an AC voltage and current.

2.5 How to Measure AC Signal Accuracy[6]

The only true measure of the power capability or heating value of a waveform is its root mean square value. It is the only precise description of a signal's power and therefore the only quantity that permits a direct, accurate comparison between the effects of DC and AC signals, regardless of waveshape.

The ability to make RMS measurements directly is becoming increasingly important because of the growing need to quantify nonsinusoidal waveshape accuracy. When a signal is a sine wave, or close enough to one, its peak or average value can be measured and it RMS value extracted from that figure by a simple multiplication.

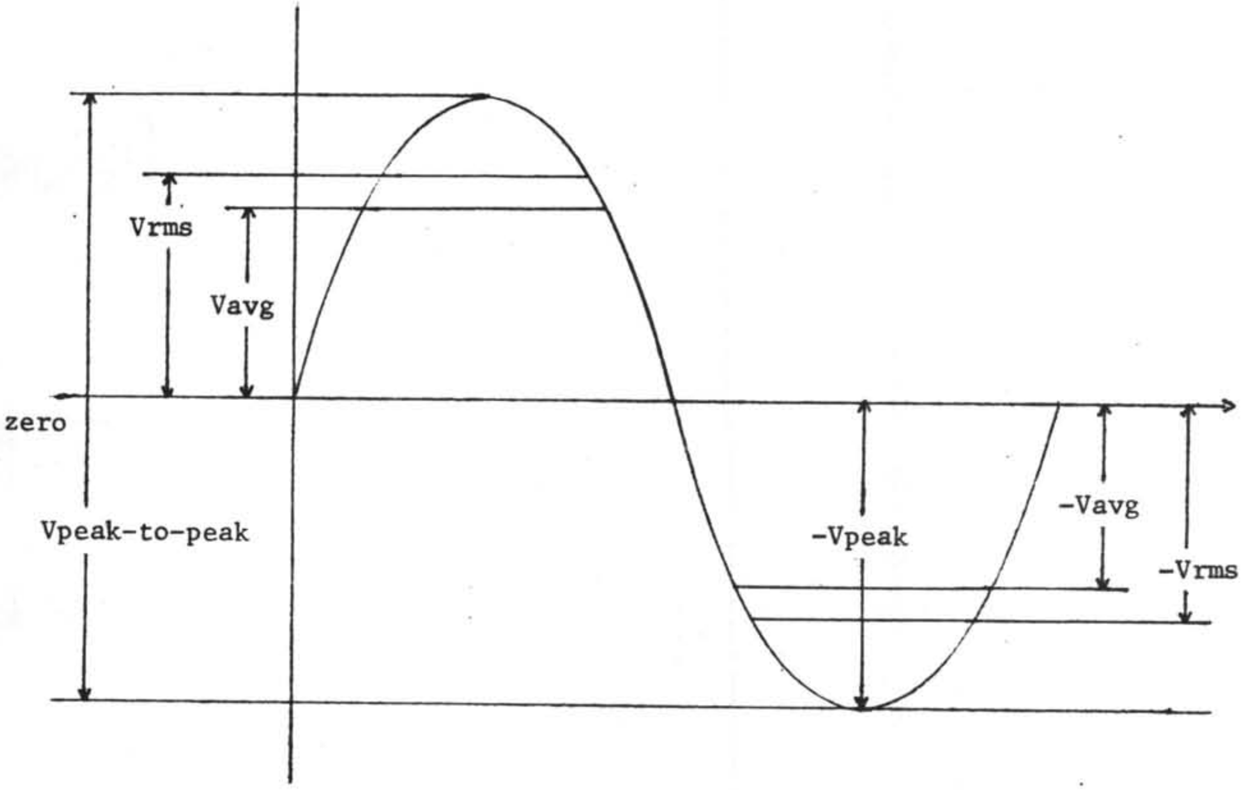


Fig. 2-4 Perfect sine wave

The average, peak and RMS values of a pure sine wave are related constants. So one can be calculated from another.(See Table 2-1)

To derive these

Multiply these	Avg	Peak	Pk-to-Pk	RMS
Avg	1.000	1.572	3.141	1.111
Peak	0.636	1.000	2.000	0.707
Pk-to-Pk	0.318	0.500	1.000	0.353
RMS	0.899	1.414	2.828	1.000

Table 2-1 Table of multiplier constants of a pure sine wave

But often, the waveform is a chopped or otherwise distorted sine wave, a random, noise-like signal, or otherwise nonsinusoidal, such as a square, pulse, or triangular waveform. Consequently, in power measurements involving thyristors or other chopper-type controls in telecommunication and audio wave testing, and in digital circuit measurements, actual RMS values alone yield useful information.

Such information can be translated into lower manufacturing costs. For example, assume that a piece of equipment must work properly even when the line voltage varies $\pm$ 10% from its normal value. In production testing of such equipment, an AC voltmeter usually measures the output of an autotransformer that can vary the power line voltage seen by the unit under test. But the measurement itself suffers a degree of uncertainty from the combined effects of basic meter error and waveform distortion, and this uncertainty may reasonably be estimated at 5% half of the allowable line voltage variation. This means that the equipment must be designed to withstand deviations of more than $\pm$ 15% to allow for a voltmeter reading that is 5% low when the real deviation is $\pm$ 10%. Such overdesign can be very expensive.

2.6 Source of Error (6)

If the 5% uncertainty level seem high, consider how much error in measuring distorted sine waves with an RMS-calibrated average-responding voltmeter adds to basic meter error (figure 2-5). When only 5% of the signal is third harmonics (n=3), the additional error caused by harmonic distortion is already more than 1.5%, and this analysis does not take into account the effects of higher-order harmonics or variations in the phase angles between the different frequency components.

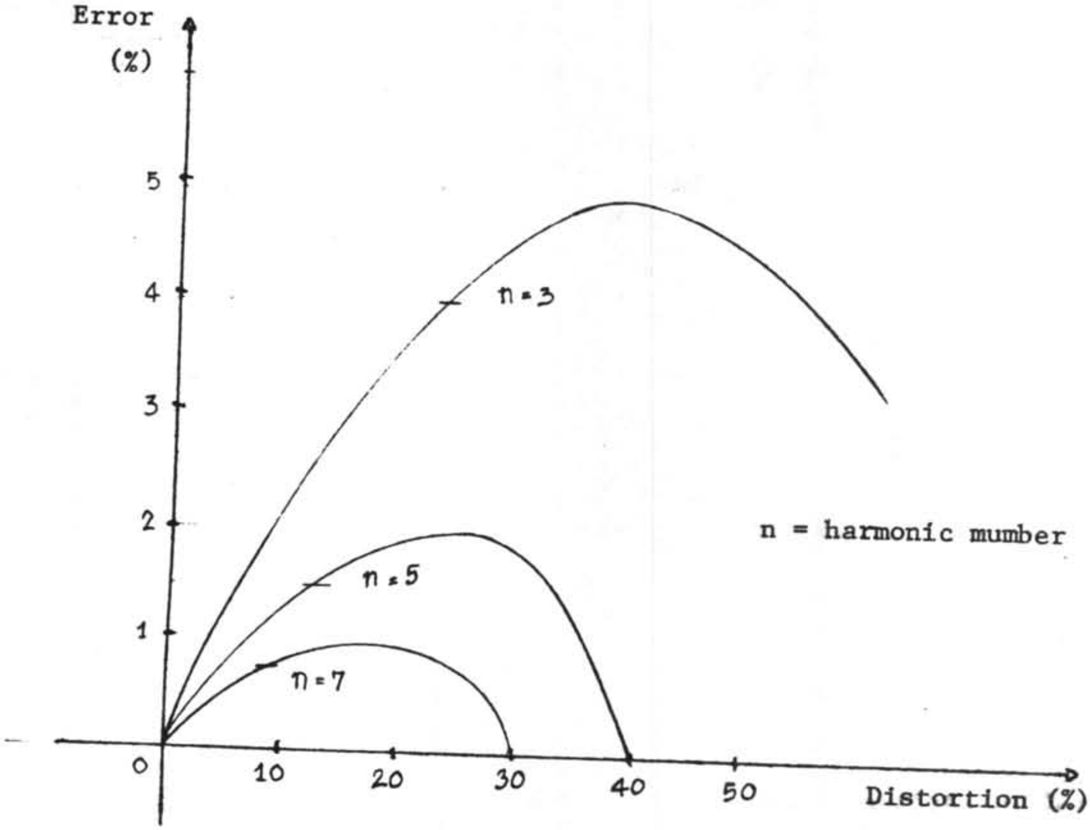


Fig. 2-5 Distorted sine wave error

An average-responding meters cannot accurately measure the RMS value of a distorted sine wave. In Fig. 2-5 the error is plotted as a function of increasing distortion for three odd-order harmonics.

Every time power goes through an iron core transformer or works into a nonideal inductive or capacitive load, some harmonics are generated. The total harmonic distortion on a power line may easily become more than 5% or 6%.

Another waveshape that can only be measured accurately by RMS techniques is the kind of switched sine wave commonly seen in power control circuits such as light dimmers. If the sine wave is switched off for 20% of each cycle ($\alpha = 0.2$ in Fig. 2.6), the average-responding meter will make a 10% error in its estimate of the RMS value, and this 10% must again be added to any other source of measurement error.

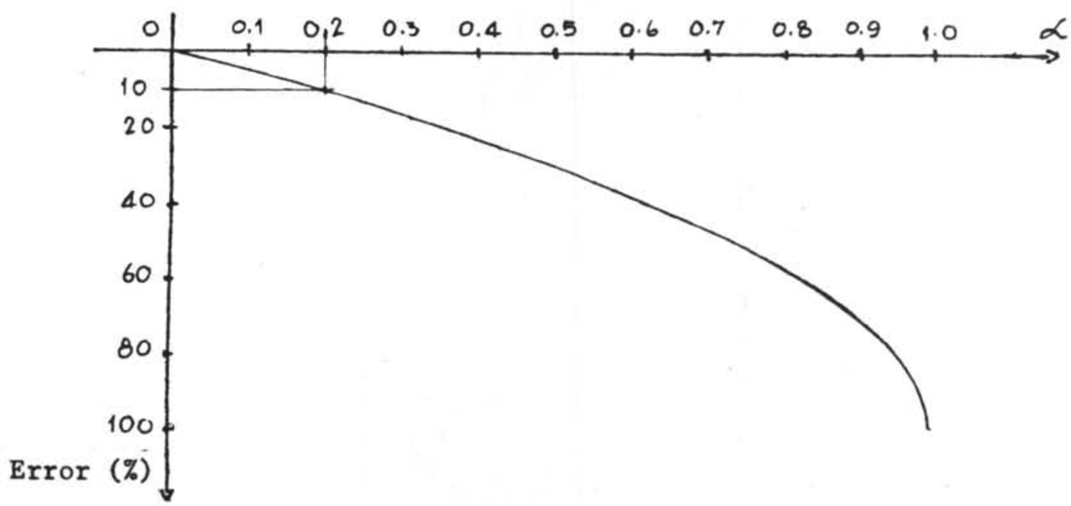


Fig. 2-6 Chopped sine wave error

When an average-responding meter is used to measure the RMS value of a sine wave that is switched off for part of it cycle, that error made by the meter can be very high.