Chapter 6

Conclusions

In several works [6-8], the electric potentials of weakly nonlinear composites have been only obtained up to the second order. The purpose of this research is firstly to extend the derivation for the electric potential equations and the boundary conditions based on the third-order perturbation expansion for weakly nonlinear dielectric composite materials with the electric displacement (**D**) and the electric field (**E**) relation including the fifth-order nonlinear coefficient, Eq. (2.2). These expressions have been applied to obtain the electric potentials up to the third order for weakly nonlinear dielectric composites consisting of dilute linear cylindrical inclusions randomly dispersed in a nonlinear host medium. This result of the third order potential has not been reported before we therefore confirm our result by the uniqueness theorem.

Since the higher-order effective nonlinear coefficients of composites can be obtained from the lower-order nonlinear coefficients. Therefore the general formulae for higher-order effective nonlinear coefficients have been derived by using the decoupling approximation method [9] and spectral representation theory [11,12]. In this research, the general formulae for effective nonlinear coefficients up to the ninth order, including terms up to the fifth-order nonlinear coefficient, of weakly nonlinear dielectric composites, have been derived by using the third order perturbation expansion method. The first to the fifth-order effective nonlinear coefficients, ε_e, χ_e and η_e, had been reported by Yu *et al.* [8]. We have derived the more general formulae of the seventh (δ_e) and the ninth-order (μ_e) effective nonlinear coefficients which are the new results.

In order to confirm our general formulae of the seventh and the ninth-order effective nonlinear coefficients, we have considered a simple case of nonlinear dielectric composite, consisting of dilute weakly nonlinear cylindrical inclusions randomly dispersed in a linear host medium. Our results of δ_e and μ_e using the formulae presented in this research work are the same as those obtained using the method of Gu and Yu [6] with the average electric displacement definition of the effective nonlinear coefficients. It is more concise and less complicated to use our formulae to calculate higher-order effective nonlinear coefficients. Moreover we also find that our method is more general than the method of Gu and Yu [6] because their method is valid for low inclusion packing fraction.

Finally, we also report the relative fifth and the ninth-order effective nonlinear coefficients of weakly nonlinear dielectric composite consisting of dilute nonlinear cylindrical inclusions randomly dispersed in a linear host medium. We consider the case of the inclusion which the $\mathbf{D}$ and $\mathbf{E}$ relation obey $\mathbf{D}^i = \varepsilon_i \mathbf{E}^i + \eta_i |\mathbf{E}^i|^4 \mathbf{E}^i$ with $\chi_i = 0$. In this case $\delta_e = 0$ and the relative fifth-order and the relative ninth-order coefficients, η_e/η_i and $\mu_e E_0^4/\eta_i$ are shown in Fig. 5.1 and Fig. 5.2. We conclude that the effective higher-order nonlinear coefficients cannot be ignored, especially for $\varepsilon_m/\varepsilon_i > 1$ and p_i approaches 0.1. These results show that the high-order effective nonlinear coefficients are important as pointed out by many authors for both theoretical [9,11,12] and experimental works [3,4,13,14].

Moreover, in this research, we have also applied the general formulae for effective nonlinear coefficients to a more complicated weakly nonlinear composite consisting of dilute linear cylindrical inclusions randomly dispersed in a nonlinear host medium.