CHAPTER III

INCORPORATING RISK ATTITUDES

In this chapter the theoretical risk preference theory is introduced to overcome some limitations of conventional decision analysis techniques that already mentioned from the previous chapter. Its concept and related issues applied in this study are described in detail.

Preference theory approach

3.1 The concept of preference theory

Most formal decision analysis toolkits applied in the investment appraisal decision making under conditions of risk and uncertainty, for example the EMV concept described before, assume that a decision maker has, or ought to has, a consistent attitude toward risk and uncertainty (Macmillan, 2000). The underlying assumption is that a decision maker will choose the highest expected values by "playing the averages" on all options, regardless of the potential negative consequences that might result. As Hammond (1967) and Swalm (1966) observed, few executives adopt such an attitude toward risk and uncertainty when making important investment decisions. Rather, decision makers have specific attitudes and feelings about money, which depend on the amounts of money, their personal risk preferences, and any immediate and/or long term objectives they may have.

In 1944 two Princeton University mathematicians, John von Neumann and Oskar Morgenstern developed the mathematical theory to describe a decision maker's attitude toward money in a quantitative sense. They started with eight axioms which they considered to be the basic fundamental logic of rational decision making. These axioms can be paraphrased in the following statement (Bailey *et al.*, 2000):

> "Decision makers are generally risk averse and dislike incurring a loss of $X to a greater degree than they enjoy making a profit of the same amount. As a result, they will tend to accept a greater risk to avoid a loss than make a gain of the same amount. Also, they derive greater pleasure from an increase in

profit from a small investment (from \$X to \$X+1) than they would from the same profit increase from a large investment (from \$10 to \$10X+1)"

They proved that if a decision maker accepts the eight axioms described above as the basis of rational decision, it is possible to describe his attitudes about money in a simple function or curve. This curve called a "utility function" (or utility curve or risk preference function and also preference curve) which specifies the individual's preferences for various monetary payoff and costs, and it automatically encodes the individual's attitude toward risk.

3.2 The utility function

The utility function or risk preference function is introduced to represent a decision maker's attitude about money and also represent attitude toward risk. It is a plot between a utility (preference/pleasure) in Y-axis versus wealth (money) in X-axis. There are many types of utility curves which depend on individual risk characteristic. Different utility shaped curves would denote different types of a decision maker.

Many writers have categorized decision makers according to the shape of their preference curves. In general, there are three main types of decision makers: risk averters, average players and risk seekers. These three types of decision makers can denote with preference curves shown in Figure 3.1.

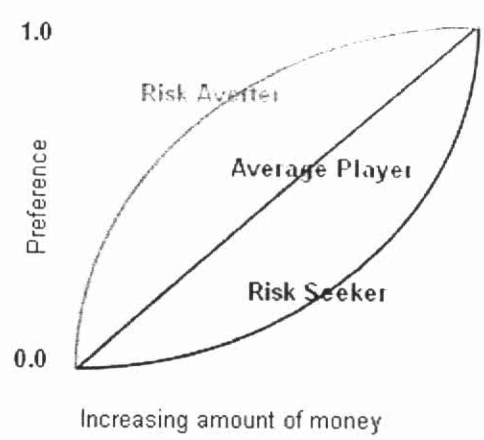

Figure 3.1: Typical risk preference curves (Macmillan, 2000).

A concave-upward curve represents a risk-seeking attitude (risk seekers), a concave-downward curve represents risk-aversion (risk averters, risk avoiders, conservative) and a straight line represents risk neutral (or EMV players or average players).

A risk averter prefers to invest in a venture having a perceived high chance of success to a second venture having a low chance of success, even if the expected value of the second venture is clearly superior. An EMV player is the person who always selects the alternatives with the highest EMV, regardless of the associated risk so he is considered as risk neutral. A risk seeker is the person who behaves in the opposite way of the attitude of risk-averse decision makers do. This type of person is usually considered as the gambler.

Utility theory provides a means to incorporate a decision maker's specific attitudes or preferences about money and risk into a quantitative decision parameter. The parameter is EU, and it is considered a more realistic measure of value than EMV. The EU is computed by multiplying the preference values with their probability of occurrence to arrive at *expected preference value* or *EU* for a decisions alternative in the same way as the EMV calculated. Mathematically, the EU is given by

$$EU = \sum_{i=1}^{N} p_i U(x_i) \tag{3.1}$$

where p_i = probability of the outcome (fraction)

$U(x_i)$ = preference or utility value (unit less)

x_i = dimension of monetary value (currency units)

3.3 Functional form of risk aversion

Generally, people would display risk neutral or linear preference when the exposure to gains or losses are not large relative to the investor's total assets or in other word, the investor has unlimited source of available budget. In that case, it is valid to assume a linear preference or using expected value criteria. However, in reality the investor usually has a limited source of capital, and in the case that the stakes are high, relative to the size of the total portfolio of the investor, real preference functions are often considered nonlinear.

Extensive studies have provided strong evidence that the majority of decision makers are risk averse to some degree, so the concave downwards preference curves are the most commonly observed in practice (Macmillan, 2000). This risk aversion which encoded by the concave curve as in Figure 3.1 can be expressed in the form of mathematical formula as well. There are many mathematical forms representing the risk aversion concave curve. Some of the commonly used mathematical relationships representing the concave risk aversion are as follow:

Quadratic Utility Function which is given by the following equation.

$$U(x) = a + bx - cx^2$$

Logarithmic utility Function which is given by the following equations.

$$U(x) = a\log(b + x) + c$$

$$U(x) = \frac{1}{a}\ln(b + x) - c$$

Exponential Utility Function: Many forms of the exponential equations appear in the literature, though some of them are mathematically the same. The various forms of exponential utility curves are defined by the following equations.

$$U(x) = a + be^{-x/RT}$$

$$U(x) = a + b(1 - e^{-x/RT})$$

$$U(x) = a - be^{-x/RT}$$

$$U(x) = ae^{bx}$$

All of the above expressions would have the same general concave shaped and represent a risk-averse preference.

Among those utility functional forms, the exponential utility function is the famous and dominant function in both theoretical and applied work in the areas of decision theory and finance. The useful well known mathematical form utilized in literature and also in this study is of the form (Newendorp and Schuyler, 2000):

$$U(x_i) = RT(1 - e^{-x_i WI/RT}) \tag{3.2}$$

where $U(x_i)$ = utility value (unit less)

x_i = dimension of monetary value (currency unit)

RT = risk tolerance value (currency unit)

WI = working interest (%)

The theoretical basis for the exponential utility function is a condition called *constant risk aversion.* A term called risk aversion (RA) is defined as:

$$RA = -\frac{U''(x)}{U'(x)}$$

where U"(x) and U'(x) is the second and first derivative of U(x) respectively.

The RA function measures the degree of aversion of a person to uncertainty in a utility function. The numerator, second derivative of U(x), measures the curvature (rate of change of slope) of the utility curve. When the exponential utility function is derived, it will give a constant RA. The derivation of exponential utility function in equation 3.2 is demonstrated as follow.

$$U'(x) = RT(RT\,e^{-x/RT})$$

$$U'(x) = RT^2\,e^{-x/RT}$$

$$U''(x) = RT^2\left(e^{-x/RT}\left(\frac{-1}{RT}\right)\right)$$

$$U''(x) = -RT\,e^{-x/RT}$$

$$-\frac{U''(x)}{U'(x)} = -\frac{(-RT\,e^{-x/RT})}{RT^2\,e^{-x/RT}}$$

$$-\frac{U''(x)}{U'(x)} = \frac{1}{RT} = RA$$

The above derivation shows that the RA is constant for the exponential utility function, regardless of the total wealth level of the decision maker. This condition holds if it is true that whenever all possible outcomes of any uncertain alternative are changed by the same specified amount, the decision maker's certainty equivalent for the alternative also changes by that same amount. An individual displays constant risk aversion if the risk premium (which will be discussed in the next topic) for a gamble does not depend on the initial amount of wealth held by the decision maker. Intuitively, the idea is that a constantly risk-averse person would be just as anxious about taking a bet regardless of the amount of money available. The RT in the above equation called risk tolerance parameter. It is a measure of a degree of risk aversion of a decision maker or degree of curvature of the exponential utility curve. As RT increase, the decision maker becomes more risk tolerance, and the exponential utility curve becomes flat.

3.3.1 Risk tolerance

In the preference theory approach, the RT value has a considerable effect on the valuation of a risky investment. It is a measure of how much risk a decision maker can tolerate. Risk tolerance can also go under terms such as risk preference, risk aversion, risk attitude. It is defined as the sum of money at which the decision makers will be indifferent between a 50:50 chance of winning that sum and losing half of that sum. In order to estimate RT, a way to determine its value is shown for example with the interpretation of the following gamble:

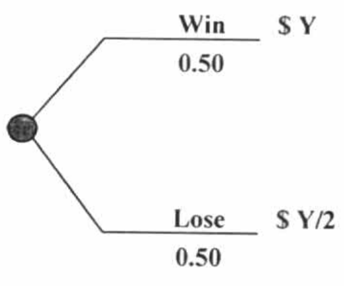

Figure 3.2: The reference gamble for assessing the risk tolerance value.

In this game, would the investor be willing to take this game if Y were $100, $200, $35,000? In term of an investment, how is the investor willing to risk ($Y/2) having a 50% chance of tripping the money (winning $Y and keeping $Y/2)? What is the point where the risk becomes intolerable? The largest value of $Y for which you would prefer to take this gamble rather than not take it is approximately equal to your RT.

The larger the value of RT, the less risk averse the decision maker is. A person or company with a large value of RT is more willing to take risks than those with a smaller value of RT.

3.3.2 Certainty equivalent and risk premium

One important term used in preference theory refers to *certainty equivalent (CE)*. It is defined as "the amount of money that is equivalent in your mind to a given situation that involves uncertainty" (Clemen, 1991). The certainty equivalent valuation aids the decision maker in selecting the approximate level of participation consistent with the firm's risk propensity. The concept of certainty equivalent is easily explained by the following example:

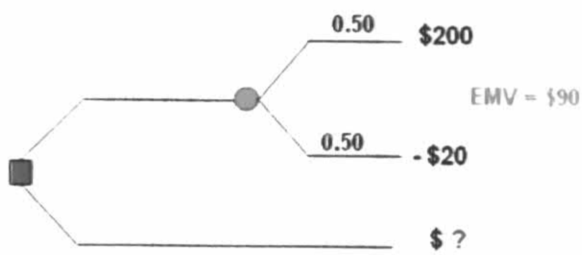

Figure 3.3: An example of a risky project for assessing certainty equivalent.

If an investor owns an above risky project and asked to sell this risky project, the investor decides that at least he will sell his position for example for $60. That is a sure thing and no risk is involved. That value ($60) is his certainty equivalent for that game. The gamble must be equivalent in his mind to a certain $60. If he unable to sell this project at this price ($60) then he prefer to keep it. The mathematical expression for the certainty equivalent, based on an exponential utility function, is shown by Cozzolino (1977)

$$CE = -RT \ln\left(\sum_{i=1}^{n} p_i \, e^{-x_i * WI/RT}\right) \qquad (3.3)$$

where RT = risk tolerance value (currency unit)

p_i = probability of outcome *i* (fraction)

x_i = the value of outcome *i* (currency unit)

n	=	total number of possible outcomes
WI	=	working interest (%)

If certainty equivalent is known for the alternatives in a decision, then it is easy to find the most preferred alternative. It is the one with the highest certainty equivalent if we are considering profit (Kirkwood, 1991). This is the property of risk averter who makes a decision by maximizing CE while an average player makes a decision by maximizing EMV. The certainty equivalent is also known as the risk adjusted value (RAV) which is the term studied in many works (Cozzolino, 1978; Mackay, 1995; Lima and Suslick, 2005).

A closely related term, *risk premium (RP)*, refers to the EMV a decision-maker is willing to give up in order to avoid the risky decision. That is the money that an investor will pay to avoid risk. It is defined as the difference between the EMV and the CE. Thus, the risk premium is positive for a risk averse decision maker, 0 for one who is risk neutral, and negative for one who is risk seeking.

$$\text{Risk Premium} = \text{EMV} - \text{CE}$$

From the previous gamble in Figure 3.3, the EMV is \$90, and the certainty equivalent for the investor is \$60, then the risk premium is

$$\text{Risk premium} = \$90 - \$60 = \$30$$

As the investor is willing to trade the gamble for \$60, he is willing to give up \$30 to avoid the risk of the gamble. Therefore the risk premium is the premium (lost opportunity) one pays to avoid risk. The higher the risk premium for an investment, the more risk averse the decision maker is. In any given situation, the CE, EMV and RP all depend on the decision maker's utility function and the probability distribution of the payoffs. Figure 3.4 shows the risk preference curve of risk aversion which also illustrates the correlation of CE, RP and EMV in the form of utility curve.

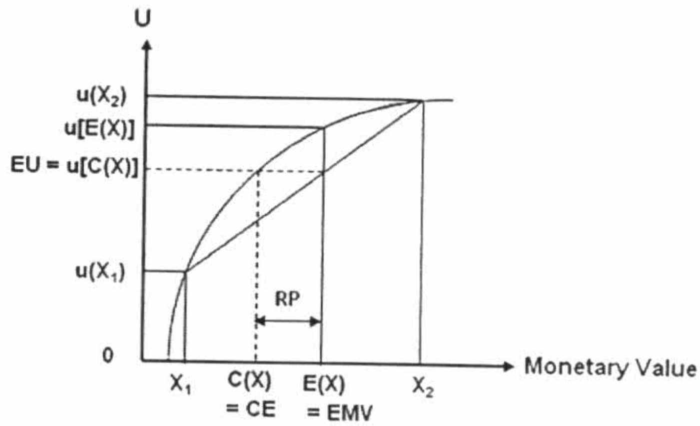

Figure 3.4: Risk aversion and certainty equivalence.

Suppose there are two lotteries, one pays E(X) with certainty and another that pays X_1 or X_2 with probabilities (*p*, 1-*p*) respectively. Reverting to von Neumann-Morgenstern notation, the utility of the first lottery is U[E(X)] = u[E(X)] as E(X) is received with certainty; the utility of the second lottery is $U(X_1, X_2; p, 1\text{-}p) = p\, u(X_1) + (1\text{-}p)\, u(X_2)$. Now the expected income in both lotteries is the same, yet it is obvious that if an agent is generally averse to risk he would *prefer* E(X) with certainty than E(X) with uncertainty, i.e. he would choose the first lottery over the second. This is what is captured in Figure 3.4 as u[E(X)] > E(X).

Another way to capture this plot is by finding a "*certainty equivalent*" allocation. In other word, if we consider a third lottery which yields the income C(X) with certainty. As is obvious from Figure 3.4, the utility of this allocation is equal to the expected utility of the random prospect, i.e. u[C(X)] = E(u) = EU. Thus, lottery C(X) with certainty is known as the *certainty equivalent* lottery, i.e. the sure thing lottery which yields the same utility as the random lottery. However, notice that the income C(X) is *less* than the expected income, C(X) < E(X). Yet we know that an agent would be indifferent between receiving C(X) with certainty and E(X) with uncertainty. This difference, which we denote RP = E(X) – C(X) is known as the risk premium.

According to expected utility theory, if we know the utility curve of a person, we can determine the preferred risk projects, which based on the risk attitude of a decision maker, by calculating the expected value (EU) in the same way as EMV concept, and then the project with the highest EU is the most desired project.

In this chapter the concept of risk preference theory is described. The following chapter presents the applications of risk preference concept by utilizing the certainty equivalent and risk tolerance values into a process of investment appraisal decision making.