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A MULTI-ITEM TWO-ECHELON INVENTORY PROBLEM UNDER JOINT REPLENISHMENT POLICY

Miss Varaporn Pukcarnon

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By	Miss Varaporn Pukcarnon
Field of Study	Industrial Engineering
Thesis Advisor	Assistant Professor Paveena Chaovalitwongse, Ph.D.
Thesis Co-Advisor	Naragain Phumchusri, Ph.D.

Accepted by the Faculty of Engineering, Chulalongkorn University in Partial Fulfillment of the Requirements for the Doctoral Degree

....Dean of the Faculty of Engineering

(Professor Bundhit Eua-arporn, Ph.D.)

THESIS COMMITTEE

....Chairman

(Assistant Professor Manop Reodecha, Ph.D.)

....Thesis Advisor

(Assistant Professor Paveena Chaovalitwongse, Ph.D.)

....Thesis Co-Advisor

(Naragain Phumchusri, Ph.D.)

....Examiner

(Assistant Professor Wipawee Tharmmaphornphilas, Ph.D.)

....Examiner

(Assistant Professor Seeronk Prichanont, Ph.D.)

....External Examiner

(Assistant Professor Rein Boondiskulchok, D.Eng.)

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วิทยานิพนธ์ฉบับนี้ศึกษาปัญหาพัสดุคงคลัง แบบสองระดับชั้นและพัสดุหลายชนิดภายใต้นโยบายการเติมเต็มร่วมกันแบบสามารถจัดหาได้ ประกอบด้วยศูนย์กระจายสินค้าหนึ่งแห่งและมีหลายสาขาที่ความต้องการของลูกค้าไม่แน่นอน มีการเติมเต็มคลังแบบต่อเนื่อง มีระยะเวลานำในการส่งมอบ และมีการกำหนดระดับการให้บริการเป้าหมายเป็นเงื่อนไขของระบบ การวิจัยแบ่งเป็น 3 ระยะ คือ ระยะที่ 1 – ระบบที่มีพัสดุหนึ่งชนิดและระยะเวลานำเป็นศูนย์ ระยะที่ 2 – ระบบที่มีพัสดุหนึ่งชนิดและมีระยะเวลานำในการส่งมอบ และระยะที่ 3 – ระบบที่มีพัสดุหลายชนิดและมีระยะเวลานำในการส่งมอบ แต่ละระยะประกอบด้วยจำนวนตัวแปรการตัดสินใจและปัจจัยที่เกี่ยวข้องแตกต่างกัน เนื่องจากความซับซ้อนของระบบ งานวิจัยจึงอาศัยวิธีการจำลองสถานการณ์ด้วยคอมพิวเตอร์ เพื่อศึกษาการกำหนดนโยบายพัสดุคงคลัง ซึ่งสามารถแสดงถึงอิทธิพลของปัจจัยที่เกี่ยวข้องและลักษณะคำตอบของระบบ วิธีการค้นหาคำตอบแบบฮิวริสติกได้รับการพัฒนาขึ้นสำหรับแต่ละระยะของการศึกษา โดยอาศัยเทคนิคการแบ่งส่วนย่อย (Decomposition Technique) วิธีการวนซ้ำ (Iterative Procedure) และการค้นหาคำตอบแบบหนึ่งมิติ ที่เรียกว่า การค้นหาแบบโกลเด้นเซคชั่น (Golden Section Search) เพื่อหาค่านโยบายพัสดุคงคลังที่เหมาะสมได้ สำหรับระยะที่ 1 และ 2 สมรรถนะของฮิวริสติกที่พัฒนาขึ้นสามารถวัดได้โดยการเปรียบเทียบกับนโยบายที่ดีที่สุดที่ให้ค่าต้นทุนรวมทั้งระบบเฉลี่ยต่อหน่วยเวลาต่ำที่สุด นโยบายที่ดีที่สุดสามารถหาได้โดยใช้วิธีการจำลองสถานการณ์ด้วยคอมพิวเตอร์ ผ่านขั้นตอนการค้นหาอย่างเป็นระบบ ได้แก่ การค้นหาตัวแปรนำเข้า และการยืนยันความถูกต้องของผลลัพธ์ จากผลการทำลอง ฮิวริสติกที่พัฒนาขึ้นใช้เวลาในการค้นหาคำตอบได้เร็วกว่ามาก เมื่อเปรียบเทียบกับวิธีการจำลองสถานการณ์ด้วยคอมพิวเตอร์ โดยนโยบายที่ได้จากฮิวริสติกให้ค่าต้นทุนรวมทั้งระบบต่อหน่วยเวลาสูงกว่าค่าต้นทุนรวมจากนโยบายที่ดีที่สุดเพียง 1.54% และ 1.20% โดยเฉลี่ยสำหรับระยะที่ 1 และ 2 ตามลำดับ สำหรับระยะที่ 3 นั้น งานวิจัยนี้ได้ศึกษาวิเคราะห์เปรียบเทียบฮิวริสติกที่พัฒนาขึ้น เพื่อระบุว่าสถานการณ์แบบใดเหมาะสมกับฮิวริสติกแต่ละแบบ

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VARAPORN PUKCARNON: A MULTI-ITEM TWO-ECHELON INVENTORY PROBLEM UNDER JOINT REPLENISHMENT POLICY. ADVISOR: ASST. PROF. PAVEENA CHAOVALITWONGSE, Ph.D., CO-ADVISOR: NARAGAIN PHUMCHUSRI, Ph.D., 203 pp.

This dissertation studies a multi-item two-echelon inventory problem under a joint replenishment policy called “the can-order policy”. The system is composed of one warehouse and multiple retailers facing stochastic demand, and all locations are replenished continuously. This research considers lead time and target service level as system constraints. The research is conducted in three phases: phase I – a single-item system with zero lead time, phase II – a single-item system with non-zero lead time, and phase III – a multi-item system with non-zero lead time. Each phase contains different number of decision variables and relevant factors. Due to the system complications, computer simulation is initially utilized for inventory policy setting. It provides insights of inventory policy setting: the effects of relevant factors and the solution characteristics. Heuristic approaches are developed to solve the problem for each phase. The proposed heuristics are based on decomposition approach, iterative procedure, and one-dimensional search called golden section search to determine the appropriate inventory policy setting. For phase I and II, the proposed heuristics’ performance is measured against the best-known solution providing the minimum average total system-wide cost. The best-known solution can be determined by computer simulation with systematic procedures: input determination and output validation. From the experimental results, the proposed heuristics can obtain the appropriate policy much faster than computer simulation with the average cost gap at 1.54% for phase I and 1.20% for phase II, respectively. For phase III, this research provides comparative analysis of the proposed heuristics to identify which situation is suitable for each heuristic.

Department:	Industrial Engineering	Student's Signature	....
Field of Study:	Industrial Engineering	Advisor's Signature	....
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CHAPTER I
INTRODUCTION

1.1 General Background

The growing trend of supply chain management (SCM) and supply coordination has been paid more attention since the actions of one member in the chain can influence the profitability of all others in the chain. Firms are focusing on competing as part of a supply chain against other supply chains instead of a single firm against other individual firms. Much research has been done to help companies improve their SCM. The best solutions are obtained by using global information and centralized control because the decisions are made with visibility to the entire system using information for all locations. However, these solutions require cooperation and coordination across multiple parties within operations, across functions, and in some cases, across firms. An effective strategy of centralized control using global information includes Vendor Managed Inventory (VMI), which is a specific type of Outsourcing Inventory Management (OIM) [1-5]. Its importance has been growing because there are several researches and case studies verifying that can help control inventory cost and improve internal performance. Moreover the capabilities of external sources are growing, so outsourcing becomes an increasingly attractive option [2-13]. The vendor has the liberty of controlling the downstream re-supply decisions. Consequently, VMI offers ample opportunities for synchronizing inventory and outbound transportation decisions. In some VMI applications, the vendor not only manages the retail inventory but also owns it, e.g. Procter & Gamble and Wal-Mart, or even in the healthcare industry the vendor owns some inventories in the hospitals' warehouses and manages them as a single firm. So, centralized control has also been used for managing all inventories in the chain to minimize the total system-wide cost or maximize operational performance. By this reason, our research mainly focuses on the centralized control strategy for managing overall inventories in the system.

The dissertation considers one-warehouse n-retailer inventory system (OWNR) which is a general pattern of two-echelon supply chain. We consider not only the vendor and buyer coordination but also the internal supply coordination. Such system confronts the uncertainty of demand in the reality. Centralized control strategy can reduce demand variation because of the visibility of the entire system. However, this strategy will be successful if and only if the update on information technology is considered. The growing trend in the spinning world is the information technology innovation; consequently, many companies apply the inventory planning program into their system to automatically linking the information within a company and also linking to their stakeholders in the chains. By this fascinating opportunity, continuous replenishment has been paid more attention in order for customer responsiveness. This can not only reduce their buffer stocks but also improve the entire system's performance. Hence, our research concerns OWNR under uncertainty of demand and continuous replenishment.

There are a number of researches in both practical and academic aspects to find the effective approaches for managing the entire inventories of OWNR (See e.g. Silver, Pyke, and Peterson [1], Kelle et al. [4], and Williams and Tokar [12] which demonstrate many researches of multi-echelon inventory management). Previous researches in the multi-echelon inventory system can be divided into two streams [14]: one concentrates on developing cost efficient replenishment policies by minimizing the total system-wide cost, and the other proposes price adjustment strategies which benefit both parties in the chains. However, the focus of this dissertation is generally on the problem dealing with joint optimization of both echelons' inventory policies to minimize the total system-wide cost.

Many supply chains such as healthcare industry and retail industry need to face the uncertainty of demand. Consequently, the stochastic demand is considered to represent the realistic situation. Of course, this raises several new issues and creates extreme modeling complexities in a two-echelon inventory situation. The researches with stochastic demand on two-echelon inventory problem have been intensively developed into a single-item two-echelon inventory problem. A number of researches on OWNR with single commodity have been conducted under either continuous or periodic replenishment. They proposed mathematical models and solution approaches for setting an appropriate inventory policy. Most of previous works studied two major types of the inventory policies: fixed-interval order-up-to polices and stock-based batch-ordering policies, on different conditions and relevant parameters. Further details can be seen in the reviews of Schneider, Rnks, and Kelle [15], Axsäter, Graves, and de Kok [16], and Wang, Choi, and Cheng [17]. Focusing on continuous replenishment, most researches manage multiple retailers by individual ordering decision. Factually, multiple retailers can coordinate their ordering decision to share the ordering cost[1] when an order is triggered. It creates an opportunity of reducing the total system-wide cost. We found that there have been a few works concerning this cost-saving opportunity in their ordering decisions.

Regarding coordinated ordering decision on OWNR with single commodity, most literatures applied joint replenishment problem (JRP) to OWNR due to the similarity of cost functions and solution procedures [18, 19]. JRP is originally developed for a multi-product single-location inventory problem with the replenishment coordination of a group of items jointly ordered from the same supplier. Under continuous replenishment and stochastic demand, there are many joint replenishment policies developed on multi-item single-location inventory problem. These policies can be classified into two major streams: the can-order policy and others [3, 19, 20]. For two-echelon system, the existing joint replenishment policies from multi-item single-location inventory problem were extended into OWNR on different structures. We summarize some structures as the following literatures under continuous replenishment and stochastic demand.

Cheung and Lee [21] employed a joint replenishment policy at the retailers and a traditional reorder point-fixed order quantity policy at the warehouse. The structure was composed of the holding costs at both echelons: the shared ordering cost and the penalty cost at the retailers, and target service level at the warehouse. Özkaya [22] extended four joint replenishment policies at the retailers and a traditional reorder point-based stock policy at the warehouse. Özkaya [22] converted the penalty cost into target service level occurred only at the retailers. Gou et al. [14] applied a joint replenishment policy where the retailers utilize the can-order policy and the warehouse takes a reorder point-based stock policy. However, Gou et al. [14] studied OWNR under zero lead time, so there were only holding costs and ordering costs taken into consideration. There have been other researches considering the holding cost only one echelon, such as Özkaya, Gürler, and Berk [19], Cetinkaya and Lee [23], and Gürbüz, Moinzadeh, and Zhou [24]. In addition, Axsäter and Zhang [25] developed a joint replenishment policy without concerning the shared ordering cost at the retailers. They focused on a trade-off between the holding costs and the penalty costs instead. Thus far, a few researches have

[1] Generally, the ordering cost includes administrative costs, material handling costs, and transportation costs.

concerned coordinated ordering decision under stochastic demand and continuous replenishment by considering all relevant costs on both echelons, i.e. the holding costs, the ordering costs, and the penalty costs (or in terms of service levels). We realize that all relevant costs on both echelons should be considered together to determine the inventory policy parameters for all stores in the system. Hence, it is interesting to further study the coordinated ordering decision for such structure to determine a solution approach for inventory policy setting.

Previously, we mentioned only a single-item two-echelon inventory problem; however, there are some other cases that multiple products should be considered simultaneously as appeared in the realistic situation. For a multi-item two-echelon inventory problem on stochastic demand and continuous replenishment, there have been a small number of researches. Mostly, the existing literatures were carried out on partial cost component or joint constraints. The researches considering partial cost components mean that it does not include all inventory costs[2] in the system, e.g. cross-docking system, inventory-transportation problem. According to the literatures with joint constraints, they included, such as, capacity constraints, budget constraints, aggregate time-based service level constraints. Further details about multi-item two-echelon inventory problem can be seen in e.g. Cohen et al. [26], Hopp, Zhang, and Spearman [27], Qu, Bookbinder, and Iyogun [28], Sindhuchao [29], Al-Rifai and Rosetti [30], Topan, BayIndIr, and Tan [31, 32], Zhou, Chen, and Ge [33].

Regarding a few of literatures studied on the shared ordering costs among retailers/items, it is interesting to apply joint replenishment policy into OWNR under stochastic demand and continuous replenishment. Then, the system including all inventory costs should be more taken into consideration in order to determine the inventory policy parameters which are suitable for all stores in the system. Furthermore, multi-item model should be concerned, since the model could more reduce the total system-wide cost from item joint replenishment not only at the retailer echelon but also at the warehouse echelon. Hence, it is desirable to develop an efficient joint replenishment policy for Multi-Item Two-Echelon Inventory Problem with stochastic demand and continuous replenishment for the general purpose of the system-wide cost optimization.

[2] All inventory costs are the holding costs and the ordering costs at both echelons with either the penalty costs or service levels as needed.

1.2 Example Industry

This dissertation is generalized for any industry which matches the considered system. However, this section exemplifies a specific industry to show the real situation. Since the research problem originally surveyed in the healthcare industry, the following content will specify such industry. The research can be applied into many parts of healthcare industry for pharmaceuticals and medical supplies management such as hospital's internal chain (central storeroom and multiple departments), hospital network (central warehouse and multiple hospitals), and drug store chain (central warehouse and multiple drug stores). The survey on healthcare industry was conducted by two approaches: firstly, interviewing healthcare organizations' staffs and other related stakeholders; and secondly, surveying literatures relating to healthcare industry and operations.

In the interview process, we visited various healthcare organizations according to administration system and size of organization (measured from number of hospital beds): three private hospitals with 300, 400 and 600 beds, and two public hospitals with 300 and 800 beds. Interviewees comprise doctors, nurses, pharmacists, and inventory planners in order to cover all main human resources in pharmaceuticals and medical supplies management. For a survey on literatures, there are a wide range of literatures about healthcare supply chain and operations, for example, Kim [2], Kelle et al. [4], Woosley [5], Freudenheim [6], Jarrett [7], Rivard-Royer, Landry, and Beaulieu [8], Nicholson, Vakharia, and Erenguc [9], Moschuris and Kondylis [10], Foxx, Bunn, and McCay. [13], Dellaert and van de Poel [34], Totrakool [35], Rattanasin [36], Belson [37], Tongrod [38], Rudeejaroensakul [39], Arshinder, Kanda, and Deshmukh [40], as well as the information from Drugs and Medical Supplies Information Center (DMSIC), and The Government Pharmaceutical Organization (GPO), Thailand.

Thailand's healthcare supply chain, like other countries, consists of various stakeholders. Figure I-1 which is adapted from Rivard-Royer et al. [8] demonstrates the stakeholders at upstream and downstream levels, and also extensively focuses on hospital's internal supply chain to illustrate multi-echelon inventory system.

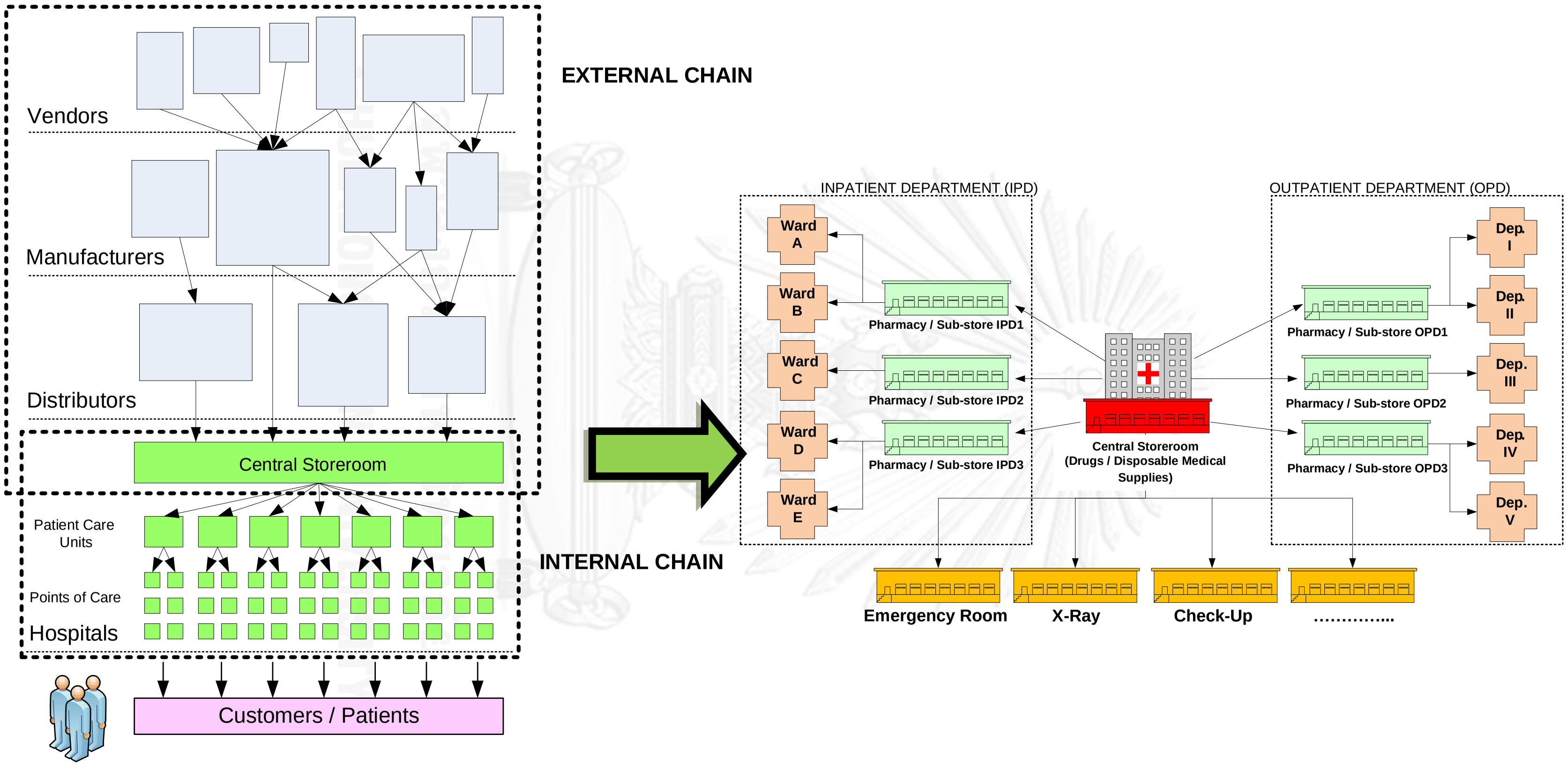


Figure I-1 Healthcare supply chain

Healthcare supply chain is formed into an arborescent distribution system which is a multi-echelon system that each location receives input from an immediate predecessor and supplies one or more immediate successors.

For an external chain, there are three partners at the upstream level: vendors, manufacturers, and distributors. Vendors take responsibility for procuring and providing items to their successors: manufacturers, distributors, or hospitals. Manufacturers who produce pharmaceuticals or medical supplies and either distribute their products directly to the hospitals or outsource this activity to distributor companies. Subsequently, distributors supply items to the hospitals according to stock replenishment system. Some companies perform as vendor, manufacturer, and distributor at the same time. Each partner plans and controls their inventories to supply the customers either separately or coordinately along the chain. The dissertation defines "supplier" for a general term of vendor, manufacturer, and distributor who directly supplies items to the hospitals.

With regard to an internal chain, the surveyed hospitals have commonly three internal echelons: central storeroom (CS), patient care units (CU), and points of care (PC). CS is in charge of purchasing all items from the suppliers, setting inventory policies for all stores' items in the hospital, planning, controlling, and monitoring its own inventories, and replenishing required stocks at CUs. CU is pharmacy or medical supplies substore located in a department or a region (group of departments). A hospital has many CUs depending on, for example, the area of hospital including size and layout, hospital specialization, administrative system. Inventory planners at each CU are responsible for monitoring and controlling its own inventories under CS's policy, issuing the order to CS when any item is needed, and dispensing to PCs or directly to patients when receiving request from doctors or nurses. PC including ward, clinic, and laboratory directly services the customers or patients. It is supplied by CU and keeps some small stocks. However, some units such as emergency room, X-ray department, and check-up department have their own inventories provided by CS and service patients as a PC; therefore, some stores perform as both CU and PC at the same time.

Currently, there is not only hospital's internal chain (central storeroom and multiple CUs), but also hospital network comprising central warehouse and multiple hospitals. An outsourced distributor manages its inventory and hospitals' inventory simultaneously. Some networks manage at CS level; the others manage at CU level without CS. The distributor in hospital network owns all inventories in the network as a single firm. Some hospital networks implement the VMI system to enhanced material handling efficiency through a growing trend of information technology, i.e. online procurement system and the real-time information sharing. The improved information sharing throughout the supply chain provides more timely and accurate inventory data resulted in better demand forecasts and materials management.

For the inventory policy setting, healthcare services have implemented both types of inventory reviews: continuous review and periodic review. Each type is considered depending on item types, demands, suppliers, replenishment and distribution operations, and resource constraints. According to the information technology, inventories are mostly reviewed continuously. The computer system facilitates to monitor inventory level all the times and automatically notifies when the inventory level is at or below reorder point, then items are ordered and delivered at just the right time. Meanwhile, period review has been used in the system which has strictly resource constraints (i.e. planners, transporters, budgets, information). Mostly, healthcare inventory management has commonly adopted “par level” policy which is special feature only in healthcare. There are two kinds of par levels. The minimum par level is equivalent to the reorder point and the maximum par level is equivalent to the order-up-to level (or base stock). Each kind of par levels can be used separately or together such as

- (s,S) policy where s represents the reorder point or the minimum par level and S represents the based stock or the maximum par level.
- (r,Q) policy where r represents the reorder point or the minimum par level and Q represents the fixed order quantity.
- (R,S) policy where R represents the length of review period and S represents the based stock or the maximum par level.

The (s,S) policy is the most popular approach for planning and controlling most of pharmaceutical and medical supplies inventories. Presently, several hospitals employ a continuous review (s,S) inventory control policy. When inventory level for an item at a CU reaches a predetermined minimum level s, an order is automatically generated and transmitted directly to the supplier. The supplier, in turn, ships the amount necessary to refill to the maximum quantity S. Depending on the specific circumstances, materials can be either sent to CS for repacking and distribution or sent directly to CUs, which bypasses CS entirely. The central warehouse at the supplier also employs a continuous review (s,S) inventory control policy to fast react in the replenishment process and reduce the inventory level comparing to periodic stocking.

Pharmaceuticals and medical supplies are life-threatening products. They need more restriction and condition for holding inventories than other products. A variety of products is a complex issue as more than 2,000 specific items are controlled under various policies and constraints to serve customers' satisfaction, employees' efficiency and cost minimization. However, 40% – 60% of inventories are high-demand items which are forecasted based on usage statistics. They are planned and controlled as the same as general merchandizing items in other industries. In practical situation according to continuous review, each item is individually reviewed. Inventory replenishments are not considered jointly even though items are ordered from the same supplier (i.e. generally, a supplier sells more than one product to a hospital). Many times inventory managers and pharmacists find that they have to place many orders for different items to the same supplier more than once a week. Order frequency reflects the ordering costs not only charged at the hospitals but also added up to the supplier. Therefore, under VMI system, total system-wide cost is considered to compromise the holding costs and the ordering costs at both echelons. On another hand, joint ordering should be operated to reduce ordering costs, number of orders, and employees' workloads [5, 9, 34].

At the downstream level, customers or patients are the last in supply chain; they are served by the hospitals and their demands have shaped the system. In healthcare demand is uncertainty; therefore, stochastic demand is better considered to represent realistic healthcare demand. Patients' demands are derived from item usage at all points of care (the first echelon) whose stocks will be replenished by their respective immediate predecessors (the upper echelons). In a traditional system which supplier's inventories and hospitals' inventories are managed separately as multiple firms, a supplier accounts for hospitals' demands from their purchase orders without supply coordination. On the other hand, under the VMI system the supplier considers hospitals' demands from the usage at hospitals' stores instead of the traditional system to reduce bullwhip effect where the orders' variability is amplified in each echelon of the supply chain: from retailer to distributor, from distributor to manufacturer and from the manufacturer to the suppliers [41]. Moreover, the VMI system can increase the accuracy of forecasted demand. In the healthcare industry demand variation is one of the important characteristics which influence the inventory levels. High demand variation from uncertainty of customer arrival makes planner build up the stock to support this circumstance to prevent shortage.

In replenishment process, lead time and service level are the important factors influencing to the inventory level at both echelons. Traditionally, supplier's lead time of distributing products to CS is uncertain according to product availability, document processing and financial activities, distribution schedule, and resource constraints. Based on surveyed hospitals, lead time varies from 3 to 14 days for usual order and three hours to one day for emergency order. However, according to VMI the supplier can reduce and specify more certain lead time. In addition, lead time of hospital's internal chain is less than lead time of the supplier-hospital chain or sometimes it can be negligible since the distance between CS and CUs are not significant. In practical situation, there is a possibility of stock out but the backlog must be replenished as soon as possible (emergency case). Thus, target service level (TSL) is a key performance indicator required at higher rate than other industries. Many organizations in the supply chain use TSL instead of the penalty cost as this cost cannot simply formulate.

Generally, inventory costs consist of three components: holding cost, ordering cost, and penalty cost. However, as mentioned above, penalty cost is transformed into service level instead. Holding cost is the cost of keeping and maintaining a stock of goods in storage. Healthcare industry encounters a huge of holding cost, since many hospitals hold excessive stocks to prevent an occurrence of backlog reflecting to patients' perspective. Ordering cost is separated to two types: fixed ordering cost and additional ordering cost. Fixed ordering cost includes administrative costs, material handling costs, and transportation costs. It occurs once an order is triggered and does not depend on the number of items (or locations) in the order. Meanwhile, additional ordering cost depends on the number of involved items (or locations) in that order, for example, additional operations cost for managing different items, additional transportation cost relating to distance or other charges. However, some hospitals do not concern additional ordering cost since it is difficult to identify in detail. All relevant inventory costs are traded off to determine the inventory policy setting to serve TSL.

In conclusion, healthcare industry is an example industry managing several different products stored in their group of warehouses as well as customer demands are uncertainty. Under OWNR, they can apply the continuous review to monitor all inventory levels as the real-time and fast react. Joint replenishment is able to apply for multi-item multi-location inventory system with concerning lead time and target service level. However, the inventory planning and control process needs to encounter more complication of the system characteristics in order to set the best inventory policy parameters for coordinated supply chain.

1.3 Statement of Problem

This section is separated to two sub-sections: inventory policy selection and problem description. Since section 1.1 (general background) and 1.2 (example industry) provide inventory policies in general and propose JRP but not yet identified which inventory policy will be studied in the dissertation. Due to the fact that there are various inventory policies under JRP, inventory policy selection will be analyzed before describing the research problem in detail. Then, problem description is stated following the selected inventory policy. It also shows the mechanism of such inventory policy for multi-item two-echelon inventory system.

1.3.1 Inventory policy selection

Recall that JRP or joint replenishment problem is originally developed for the multi-product single-location inventory problem by coordinating the replenishment of a group of items that are jointly ordered from the same supplier. Focusing on stochastic demand and continuous replenishment, there are four main inventory policies proposed under JRP as follows: (Let j denote the item j stored in a location)

(1) The can-order (s_j, c_j, S_j) policy [42]

When the inventory position (on hand + on order – amount backlogged) of any item drops to or below its must-order level s_j an order is placed to bring its inventory level to base stock S_j and for all items $j \neq k$ with the inventory below can-order level c_k, inventory levels are also replenished to S_k.

(2) The (Q, S_j) policy [43]

Aggregate consumption of all items is monitored and when it reaches a certain level Q, all items are replenished to their order-up-to level S_j.

(3) The $Q(s_j, S_j)$ policy [44, 45]

Aggregate consumption is continuously reviewed whereas the inventory level of item j are only reviewed when aggregated consumption of all items reaches or exceeds a certain level Q. Any item has its inventory level less than s_j, its inventory level is brought up to its order-up-to level S_j.

(4) The $Q(S_j,T)$ policy [19]

It is a hybrid inventory policy between continuous replenishment and periodic replenishment. When the aggregate demand since last replenishment reaches Q units or the time elapsed since last replenishment reaches T, all items are replenished up to their order-up-to level S_j.

Each policy has the advantages and disadvantages on different situations. Considering the example industry, there are a number of items stored in each store and high service level is required. Practically all policies can be applied into the system; however, the can-order (s_j,c_j,S_j) policy seems to be more practical by the reasons that

- It is straightforward and appealing to one's common sense [46]
- The study of Gou et al. [14] demonstrated that the can-order (s_j,c_j,S_j) policy can save the total system-wide cost on OWNR about 5-20% as comparing with the independent controlled (s_j,S_j) policy at the retailers. Additionally, Özkaya [22] studied the special can-order (s_j,S_j-1,S_j) policy where the can-order level c_j equals to S_j-1. The result showed that the total system-wide cost can be saved up to 30% depending on relevant factors.
- Özkaya [22] also showed that the special can-order policy increases cost-saving when higher number of retailers in the system and/or higher target service level. These situations are substantially consistent with the example industry.
- From the example industry survey, compatibility of the can-order policy with the current computer software for inventory management is practically preferable because the computer software includes the can-order policy into the system as an option. The software defines two levels for reorder policy as demonstrated in the can-order policy, although it has never been used in reality.

Academically, all considered inventory policies are mostly compared on the test beds [44, 47] which all parameters are identical for all items under single-location consideration. The results compared by Özkaya et al. [19] are depicted in the summary table.

Table I-1: Summary results of the comparison of the continuous joint inventory policies

Instances	Relevant Factors			Summary Results
	Major ordering cost	Unit holding cost	Unit penalty cost	
1	Low	Low	Low	The can-order (s_j, c_j, S_j) policy performs better than the others
2	High	Low	Low	The $Q(s_j, S_j)$ policy outperforms
3	Low	High	High	The can-order (s_j, c_j, S_j) policy performs better than the others
4	High	High	High	The $Q(S_j, T)$ policy slightly outperforms and followed by the can-order (s_j, c_j, S_j) policy and the $Q(s_j, S_j)$ policy respectively.

The (Q, S_j) policy is not raised in the table because it is beaten by the other policies. According to the table, the can-order policy is interesting since it outperforms the other policies in many instances. However, the can-order (s_j, c_j, S_j) policy analyzed in Özkaya et al. [19] is developed under the approximate mathematical model on the assumption that joint replenishment is Poisson distributed. On the other hand, van Eijs [48] using the exact mathematical model showed that the can-order (s_j, c_j, S_j) policy performs well in the case of high major ordering cost when using the special can-order $(s_j, S_j - 1, S_j)$ policy. Therefore, the can-order policy in instances 2 and instance 4 is likely to perform better result than the study of Özkaya et al. [19]. Comparing the can-order (s_j, c_j, S_j) policy with the periodic joint replenishment, the $P(s_j, S_j)$ policy is an outstanding periodic joint replenishment policy where the inventory level of all items are reviewed once every P time units and each item with the inventory level below s_j is replenished up to level S_j. The comparative result showed the similar pattern as Table I-1. For instance 2, the $P(s_j, S_j)$ policy is slightly better than the can-order (s_j, c_j, S_j) policy. However, the can-order (s_j, c_j, S_j) policy is a continuous review replenishment policy and therefore can react faster to new information than the periodic replenishment policies, the can-order (s_j, c_j, S_j) policy should intuitively perform better than the periodic replenishment policies.

In conclusion, even though the can-order (s_j, c_j, S_j) policy is not the best policy in every situation, it performs well in the important circumstances relating to the example industry (e.g. high service level, high number of retailers/items). The can-order (s_j, c_j, S_j) policy does not perform bad itself but depends on the heuristic approach to determine the appropriate inventory policy setting [48]. Hence, this dissertation focuses on the can-order (s_j, c_j, S_j) policy which is an important class of joint replenishment policy. Later section will combine the can-order (s_j, c_j, S_j) policy into OWNR, as well as describe the research problem with such policy relating to the example industry with two-echelon inventory system.

1.3.2 Problem description

This dissertation considers inventory policy parameter setting under joint replenishment policy called the continuous can-order (s_j, c_j, S_j) policy in the complicated system consisting of one warehouse and multiple retailers. It is an arborescent distribution system or a well-known one warehouse n-retailer distribution system. A warehouse and multiple retailers are cooperated as a single firm to concern total system-wide cost under global information and centralized control. So, inventory planner is in charge of planning and controlling overall inventories of all locations in the system under certain circumstances to minimize the total system-wide cost. Planner needs to determine the inventory policy parameters for all items in all locations to usually plan and control them under this predetermined setting.

1) <u>System structure</u>

A warehouse is placed at the upper echelon called “warehouse echelon”. It holds inventories for supplying all retailers’ orders. Inventories at the warehouse are assumed to replenish by an outside supplier whose ample stock is not considered in the problem. Although this assumption seems quite unrealistic since normally the warehouse orders several items from various suppliers, the problem only specifies the group of items supplied by the same outside supplier (e.g. in healthcare industry, pharmaceuticals and medical supplies are often ordered from the same manufacturer or vendor). The warehouse distributes required items to retailers within the same lot (no-splitting lot) to reduce replenishment frequency which directly reflects to reduce ordering costs as well. In this problem, it is supposed that vehicle capacity is uncapacitated to sufficiently supply all required items in an order. Multiple retailers are placed at the lower echelon called "retailer echelon" and located in proximity. They have their own stores to keep multiple items supplied by the warehouse. Each retailer holds inventories for serving customer demands which are uncertainty but it can represent by the mean, thus customer demands are defined as stochastic demand. This characteristic makes the problem more realistic than considering with deterministic in the current situation that customer requirements can be easily changed all the time.

Figure I-2 illustrates the structure of the multi-item two-echelon inventory system. Information flows from the retailer echelon to the warehouse echelon, whereas material flows from the warehouse echelon to the retailer echelon. The warehouse gets information from retailers, aggregates all information, create replenishment plan, and distributes the required items to the retailers. According to the two-echelon arborescent distribution system, there is a location set composed of $n+1$ locations; one location of warehouse and n locations of retailers. Define that index i represents location i where i = 0 for the warehouse and $i \in N$, N = {1, 2, ..., n} for the retailers. Considering the multi-item inventory system, such system comprises an item set with m items. Let index j denote item j in the system, so that $j \in M$, M = {1, 2, ..., m}. Thus, the whole system is composed of multiple location-items indexed by ij representing item j at location i. Totally, the system has $(n+1)\times m$ location-items. Customer demands come from the end customers at the retailers. In the dissertation, we assume that customer demands are identical Poisson distributed with rate λ_{ij}. Using Poisson process properties facilitates the study of the complicated system as found in many researches on joint replenishment policies. See e.g. a review of joint replenishment policies by Khouja and Goyal [20].

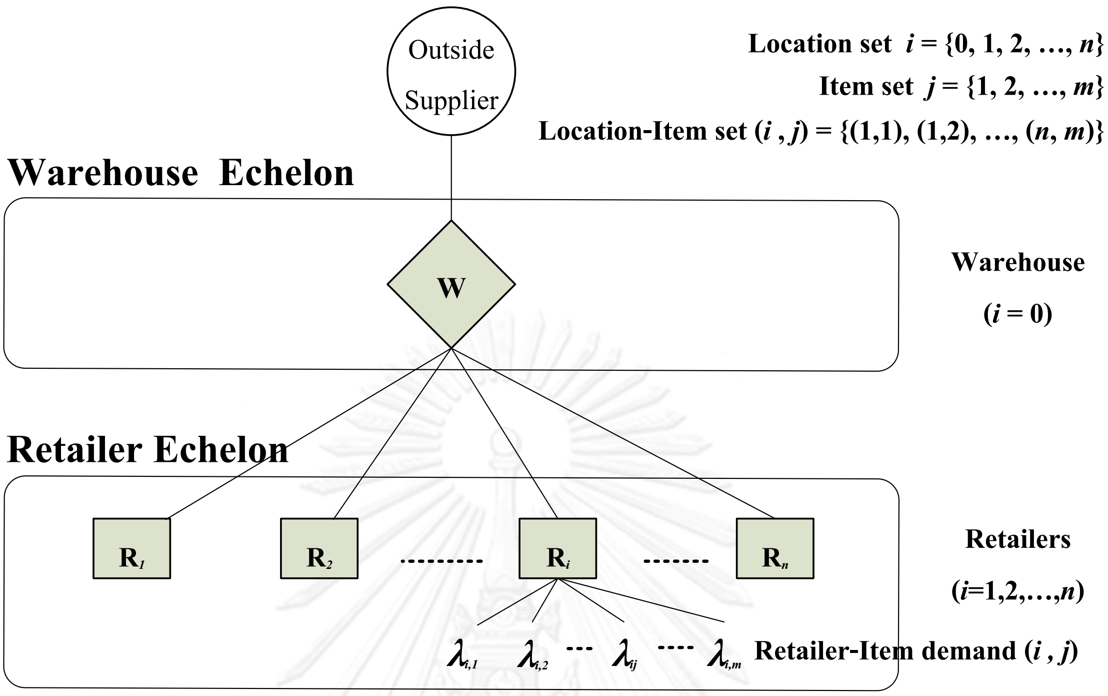


Figure I-2 Multi-item two-echelon inventory system

2) <u>Inventory policy</u>

Inventory planner is in charge of planning and controlling overall inventories of all locations in the system to minimize the total system-wide cost. At the beginning of considered period (e.g. year, three months, month, twice weeks, week), planner needs to determine the inventory policy parameters for all items in all locations to usually plan and control them under the predetermined setting. Planner uses input data for making decision, e.g. number of retailer-items considered in the system, cost components, forecasted retailer-item demand characteristics, location-items' lead times, and target service levels. Planner needs to tradeoff between the relevant costs at both echelons to minimize total system-wide cost. Then, daily operations are executed with continuous review by utilizing the predetermined inventory policy setting.

"Inventory position" is used for ordering decision. This quantity includes the outstanding orders that have not yet arrived and backorders which units have been demanded but not yet delivered [49]. Thus,

Inventory position = stock on hand + outstanding orders – backorders

The can-order (s_{ij}, c_{ij}, S_{ij}) policy is selected to apply into the considered system and used at all locations (i.e. warehouse and retailers). For each location, it has two reorder points: the must-order level for location-item ij represented by s_{ij} providing normal replenishment, and the can-order level c_{ij} making special replenishment. For retailer echelon (retailer $i \in N$, N = {1, 2, ..., n}), an order will be triggered to create normal replenishment when the inventory position of any retailer-item drops to or below its must-order level s_{ij}. Then, other retailer-items in the system will be also included by this order if their inventory position is at or below its can-order level c_{ij}; a special replenishment is occurred. All the involved retailer-items' inventory will be fulfilled from the warehouse to their own order-up-to level S_{ij}. Summarily, at the retailer echelon coordinated ordering decision can be occurred among retailer-items. Figure I-3 shows an example of joint ordering model which an order is replenished from the warehouse. Suppose that there are four retailers and three items.

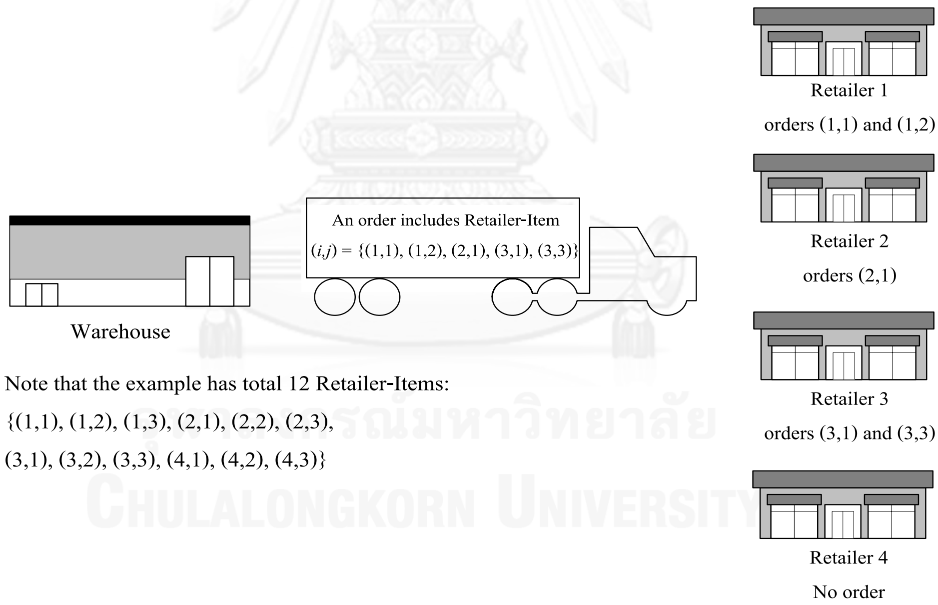


Figure I-3 Example of the joint ordering model at the retailer echelon

For the warehouse, it also employs the can-order $(s_{(0,j)}, c_{(0,j)}, S_{(0,j)})$ policy using for coordinating multiple items at single location i = 0. Warehouse will issue an order when the inventory position of any item reaches its must-order level $s_{(0,j)}$. Meanwhile if other items' inventory position reach their can-order level $c_{(0,j)}$; they will be also included in the order sent to the outside supplier who sells a group of items. All the involved items' inventory will be fulfilled to their own order-up-to level $S_{(0,j)}$. Therefore, at warehouse echelon coordinated ordering decision can be occurred among items. Factually, Fig.I-3 can represent joint ordering model at the warehouse as well by adapting to multi-item single location model. Since there are two levels of order cycle in the system, we differentiate between order cycle at retailer echelon and order cycle at warehouse echelon by defining "dispatch cycle" and "replenishment cycle" for retailer echelon and warehouse echelon, respectively.

An example of the can-order (s_{ij}, c_{ij}, S_{ij}) policy is shown in Fig.I-4 and Fig.I-.5 to express the inventory process of the can-order policy for OWNR. The example sets the policy for the warehouse and the retailers. In this example, it is assumed that lead time is zero at both echelons and shortage is not allowed. There are two retailers and two items, so four retailer-items are considered as defined index (i, j) = {(1,1), (1,2), (2,1), (2,2)}.

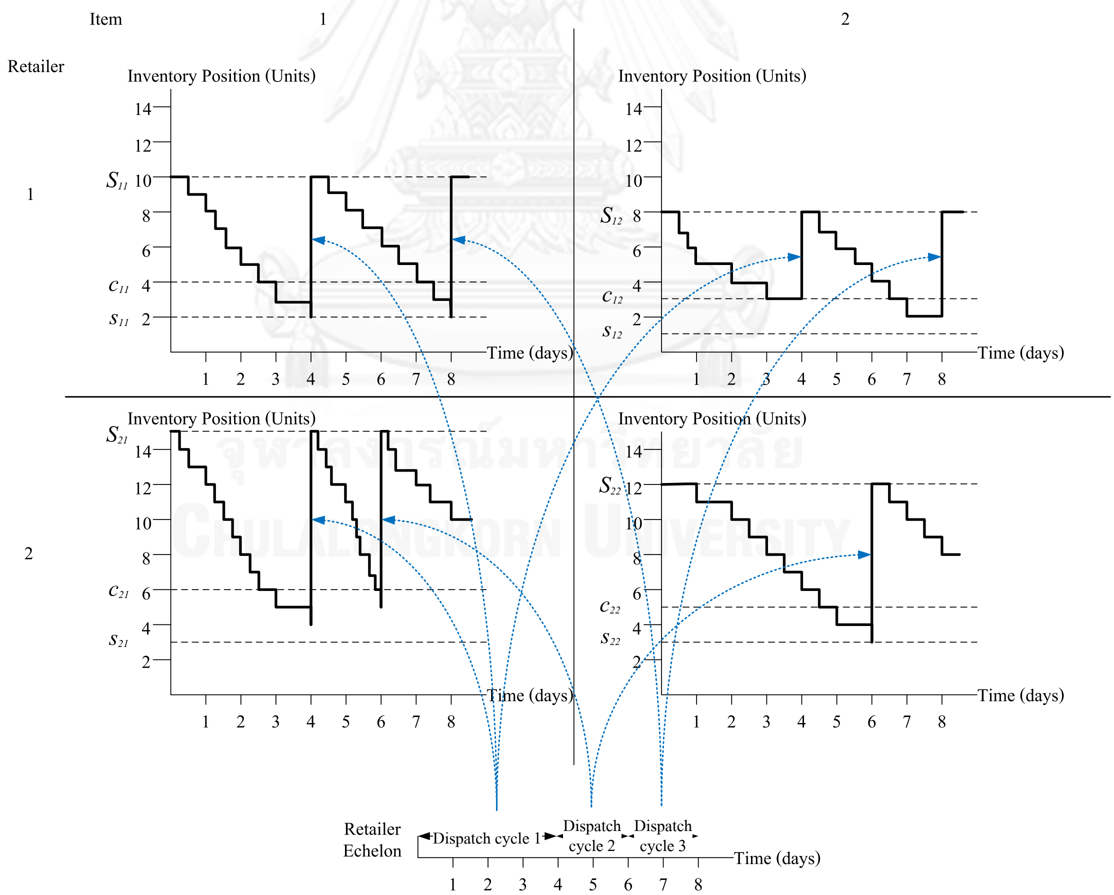


Figure I-4 Example of the inventory process: Retailer echelon

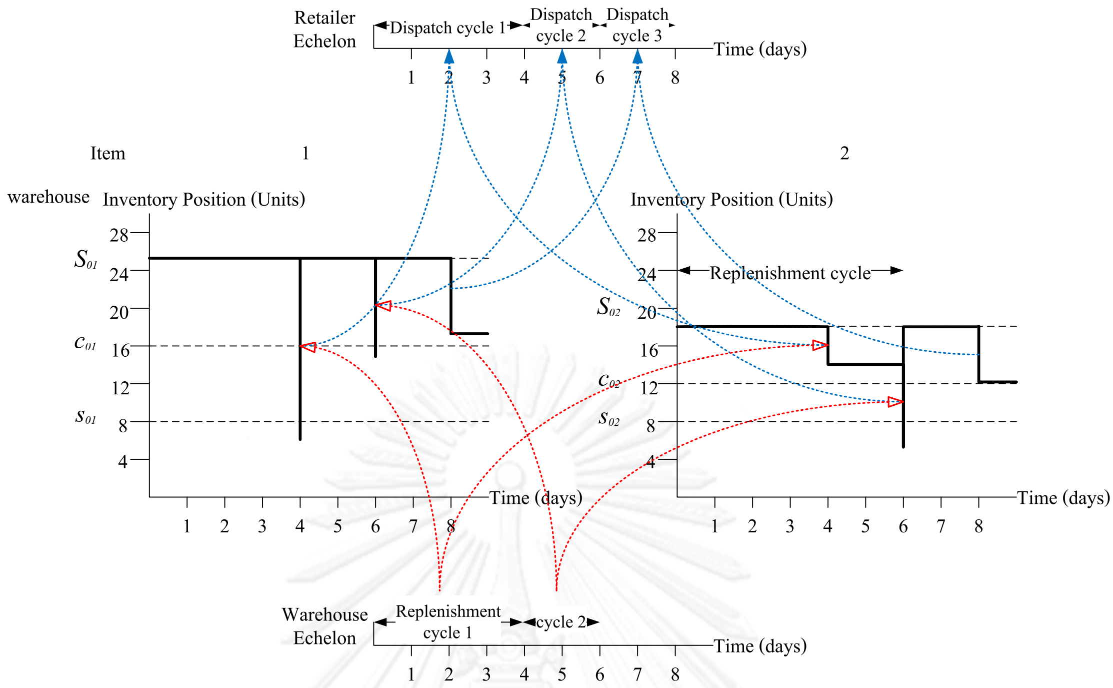


Figure I-5 Example of the inventory process: Warehouse echelon

At each location-item, inventory position continuously reduces when demand is arrived and increase when order is triggered. To explain the example, let $\bar{I}_{ij}$ and $\underline{I}_{ij}$ represents the inventory position before demand arrival and after demand arrival of item j at location i, respectively. Let Q_{ij} denote the dispatch (or replenishment) quantity of item j at location i.

<u>The replenishment policies at retailer echelon</u>

- At least one retailer-item that $\underline{I}_{ij} \le s_{ij}$, the order will be triggered
- The other retailer-items that $s_{ij} < \underline{I}_{ij} \le c_{ij}$ will be included in the same order
- Thus, the dispatch quantity of item j at retailer $i \in N$ is equal to $Q_{ij} = S_{ij} - \underline{I}_{ij}$
- The total dispatch quantity of item j sent to warehouse is equal to $\sum_{i \in N} Q_{ij}$

<u>The replenishment policies at warehouse echelon</u>

- Inventory position of item j at the warehouse $\underline{I}_{(0,j)} = \bar{I}_{(0,j)} - \sum_{i \in N} Q_{ij}$
- At least one item that $\underline{I}_{(0,j)} \le s_{(0,j)}$, an order will be sent to an outside supplier
- The other items that $s_{(0,j)} < \underline{I}_{(0,j)} \le c_{(0,j)}$ will be included in the same order
- The replenishment quantity for item j, $Q_{(0,j)} = S_{(0,j)} - \underline{I}_{(0,j)}$

From the example,

Until the end of the 4th day:

<u>At retailer echelon</u>

Retailer-item (1,1): $\underline{I}_{(1,1)} = 2 \le s_{(1,1)}$, order is triggered (normal replenishment) with $Q_{(1,1)} = S_{(1,1)} - \underline{I}_{(1,1)} =$ 8 units.

Retailer-item (2,1): $s_{(2,1)} < \underline{I}_{(2,1)} = 4 \le c_{(2,1)}$, retailer-item is included by this order (special replenishment) with $Q_{(2,1)} = S_{(2,1)} - \underline{I}_{(2,1)} = 11$ units.

Retailer-item (1,2): $s_{(1,2)} < \underline{I}_{(1,2)} = 3 \le c_{(1,2)}$, retailer-item is included by this order (special replenishment) with $Q_{(1,2)} = S_{(1,2)} - \underline{I}_{(1,2)} = 5$ units

Retailer-item (2,2): $c_{(2,2)} < \underline{I}_{(2,2)} = 6$, retailer-item is not included by this order.

Total dispatch quantity of item 1 sent to the warehouse $\sum_{i \in N} Q_{(i,1)}$ = 19 units and total dispatch quantity of item 2 $\sum_{i \in N} Q_{(i,2)}$ = 5 units.

<u>At warehouse echelon</u>

Warehouse-item (0,1): $\underline{I}_{(0,1)} = 6 \le s_{(0,1)}$, order is triggered (normal replenishment) with $Q_{(0,1)} = S_{(0,1)} - \underline{I}_{(0,1)} =$ 19 units.

Warehouse-item (0,2): $c_{(0,2)} < \underline{I}_{(0,2)} = 14$, this item is not included by this order.

Then, there is only replenishment quantity of item 1 sent to the outside supplier with 19 units.

Until the end of the 6th day:

<u>At retailer echelon</u>

Retailer-item (2,2): $\underline{I}_{(2,2)} = 3 \le s_{(2,2)}$, order is triggered (normal replenishment) with $Q_{(2,2)} = S_{(2,2)} - \underline{I}_{(2,2)} =$ 9 units.

Retailer-item (2,1): $s_{(2,1)} < \underline{I}_{(2,1)} = 5 \le c_{(2,1)}$, retailer-item is included by this order (special replenishment) with $Q_{(2,1)} = S_{(2,1)} - \underline{I}_{(2,1)} = 10$ units.

Retailer-item (1,1): $c_{(1,1)} < \underline{I}_{(1,1)} = 6$, retailer-item is not included by this order.

Retailer-item (1,2): $c_{(1,2)} < \underline{I}_{(1,2)} = 4$, retailer-item is not included by this order.

Total dispatch quantity of item 1 is 10 units and of item 2 is 9 units, respectively.

<u>At warehouse echelon</u>

Warehouse-item (0,2): $\underline{I}_{(0,2)} = 5 \le s_{(0,2)}$, order is triggered (normal replenishment) with $Q_{(0,2)} = S_{(0,2)} - \underline{I}_{(0,2)} =$ 13 units.

Warehouse-item (0,1): $s_{(0,1)} < \underline{I}_{(0,1)} = 15 \le c_{(0,1)}$, the item is included by this replenishment (special replenishment) with $Q_{(0,1)} = S_{(0,1)} - \underline{I}_{(0,1)} = 10$ units.

Then, replenishment quantity of item 1 and item 2 are 10 and 13 units, respectively.

By this inventory process, when special replenishment is occurred there is a residual stock [48] which is a stock left above the must-order level s_{ij} at the order-triggered point. For example, at the end of the 4th day retailer-item (2,1) and retailer-item (1,2) happen the residual stocks since they are reordered before reaching their own must-order level. If this situation frequently happens, such two retailer-items have to hold more stock than the expectation. Therefore, setting the appropriate inventory policy at both echelons is an important procedure concerning a trade-off between all relevant inventory costs to balance between order frequency and inventory amount, and eventually to minimize the total-system wide cost.

3) ***Relevant inventory costs***

Relevant inventory costs in the system are composed of holding costs and ordering costs; meanwhile penalty costs are estimated to service level which will be described later. Relevant inventory costs are demonstrated by echelon as follows:

At retailer echelon

1) *Holding cost of retailer-item ij*

The holding cost occurs at each retailer-item having physical stock. The holding cost over the time period at retailer-item ij (HC_{ij}), can be determined from the unit holding cost (h_{ij}) and the accumulated inventory over the time period (INV_{ij}). The total holding cost at retailer echelon is a summation of all retailer-items' holding cost.

2) *Retailer echelon's ordering cost*

It is composed of two types of ordering cost [20]: major ordering cost and minor ordering cost.

Major ordering cost is the fixed cost occurring once an order is triggered. This cost includes administrative costs, material handling costs, and transportation costs not depended on the number of retailer-items in the order. So, the retailer-items in the system can share the major ordering cost together for replenishing in one round trip. The total major ordering cost over the time period at retailer echelon (MJ_r) is the retailers' major ordering cost per an order (K_r) multiplied by the number of dispatch cycle (ND_r).

Minor ordering cost is an additional cost of each retailer-item when replenishing their inventories, such as additional transportation cost relating to distance or other charges, additional operations cost for managing different items. This cost depends on the number of involved retailer-items in that order. The total minor ordering cost over the time period (MN_r) is accumulated from the involved retailer-items in each order multiplied by its minor ordering cost of retailer-item ij (κ_{ij}) over the time period.

<u>At warehouse echelon</u>

1) *Holding cost of item j at the warehouse*

Similar to retailer echelon, the holding cost occurs at each item with physical stock. The warehouse's holding cost over the time period for item j ($HC_{(0,j)}$) can be calculated from the unit holding cost ($h_{(0,j)}$) and the accumulated inventory over the time period ($INV_{(0,j)}$). The total holding cost at warehouse echelon is a summation of all items' holding cost.

2) *Warehouse echelon's ordering cost*

According to multiple items, warehouse echelon has the same cost structure as retailer echelon composed of two types of ordering cost: major ordering cost and minor ordering cost. Major ordering cost is the fixed cost occurring once replenishment is occurred. It does not depended on the number of items in the replenishment. The involved items in the replenishment can share the major ordering cost in one round trip. The total major ordering cost over the time period at warehouse echelon (MJ_w) is the warehouse' major ordering cost per an order (K_w) multiplied by the number of replenishment cycle (NR_w).

Minor ordering cost is an additional cost for managing different items. This cost depends on the number of involved items in that order. The total minor ordering cost over the time period (MN_w) is accumulated from the involved items in each order multiplied by its minor ordering cost of item j ($\kappa_{(0,j)}$) over the time period.

The concept of the can-order policy is balancing among reduced major ordering costs, varied minor ordering costs, and increased holding costs. Reduced major ordering cost occurs if special replenishment is included in an order.

On the other hand, from special replenishment there is a residual stock. Then, the involved location-items have to hold more stock increasing the holding cost. Meanwhile, the minor ordering costs can be either reduced or increased depending on order frequency at each location-item. Hence, we have to consolidate all relevant costs to determine the appropriate inventory policy setting under the total system-wide cost minimization.

4) ***Lead time***

For the example shown in Fig.I-4 and Fig.I-5, lead time is negligible. If lead time is considered, the problem is more complicated because lead time will affect the inventory policy setting at all location-items. Generally, lead time is defined that the duration from the moment an order is placed to the warehouse (outside supplier) until the moment the order is received by the retailers (warehouse). The problem assumes constant lead time for each location-item (L_{ij}). According to two-echelon system, the supplier can reduce and specify more certain lead time as our assumption.

5) ***Target service level***

Under stochastic conditions it is unavoidable that in some periods the inventory on hand is not sufficient to deliver the complete demand and, as a consequence, that part of the demand is filled only after an inventory-related waiting time. The amount of late deliveries can be influenced through the penalty costs. Unfortunately, these costs are difficult to quantify in practice, hence, "Fill Rate" widely used in industrial practice [22, 50] is a measurement of service level to quantify the logistical performance. It is a quantity-oriented performance measure describing the proportion of total demand within a reference period delivered without delay from stock on hand. Normally, service is measured only at the lowest echelon since in a multi-echelon system a stockout at one of the higher echelons has only a secondary effect on service. Thus, service level will be considered only at the lowest echelon to avoid unnecessary duplication of safety stock. For the problem, service level is considered as a system constraint defined that is target service level ($TSL_{ij},\ i \in N$). Consequently, all retailer-items must concern this constraint for setting their inventory policy.

1.3.3 Problem discussion

The can-order (s_{ij}, c_{ij}, S_{ij}) policy for OWNR initiates three main complications of the research problem to determine the appropriate inventory policy setting as follows:

(1) Uncertainty of reorder epoch and order quantity at both echelons

Since continuous replenishment at both echelons makes the problem encounter the uncertainty of reorder epoch. So, normal replenishment and special replenishment also create non-constant order quantities. For retailer echelon, each involved retailer-item's order quantity in an order can be varied from $S_{ij} - c_{ij}$ to $S_{ij} - s_{ij}$. Thus, the total dispatch quantity of item j issued to the warehouse can also be varied from $\min\{S_{ij} - s_{ij}; \forall i, \forall j\}$ to $\sum_{i \in N} \sum_{j \in M} (S_{ij} - s_{ij}) - (n \times m - 1)$. By this circumstance, the warehouse echelon encounters the uncertainty of lot-size demands. Hence, setting the inventory policies at all location-items directly affect each other.

(2) Time synchronization

Typically, the problem on OWNR faces time synchronization between warehouse echelon and retailer echelon. Transaction at each echelon also influences each other, so it needs to be consistent. For example, reorder epoch at retailer echelon affects inventory position and reorder epoch at warehouse echelon, then inventory on hand at warehouse also affects an outstanding order arrival to retailer echelon. According to the continuous replenishment with uncertainty of demands at both echelons, it makes this problem more complicated.

(3) Interaction among location-items in each echelon

Interaction among location-items is an important problem since a location-item's inventory policy setting affects the probability of special replenishment for other location-items. Therefore, changing inventory policy of just one location-item has an effect to the whole system.

In conclusion, our research problem focuses on the can-order (s_{ij}, c_{ij}, S_{ij}) policy for OWNR composing of one warehouse and multiple retailers with multiple items. Assuming that customer demands are Poisson distributed. Coordinated ordering decision within any echelon can be occurred according to such policy. There are two types of cost components considered in the system: the holding costs at all location-items and the ordering costs at both echelons. All relevant inventory costs are traded off to minimize the total system-wide cost. Lead time and target service level (or target fill rate) are included. The system creates the complications of research problem that encounters the uncertainty of reorder epoch and order quantity at echelons, time synchronization, and interaction among location-items in each echelon. Hence, the dissertation will explore the considered system and significantly fulfill knowledge in the area of the inventory control and supply chain.

1.4 Dissertation Objective

The objective of this dissertation is to develop the stochastic joint replenishment model and the solution approach for determining the best values of inventory policy parameters under the continuous can-order policy. The dissertation focuses on a multi-item two-echelon inventory problem structured as a warehouse n-retailer inventory system by considering the appropriate total system-wide cost.

1.5 Dissertation Scope

1) System structure and planning control: The study focuses on a multi-item two-echelon inventory problem, known as one-warehouse n-retailer system. Inventory policy parameters are determined under the can-order (s_{ij}, c_{ij}, S_{ij}) policy which is considered at both echelons. Planning horizon is infinite and the objective function is to minimize the expected long-run total system-wide cost. Planner is responsible for inventory planning and control over both echelons considered as a single firm.

2) Coordinated ordering decision: At retailer echelon, ordering decision can be jointly worked together for multiple retailer-items. Meanwhile, warehouse echelon can have coordinated ordering decision among various items.

3) Replenishment process: Warehouse placed at the upper echelon holds inventories for supplying all retailers' orders. Warehouse's inventories are replenished by an outside supplier whose ample warehouse is not considered in the problem. Multiple retailers placed at the lower echelon have their own stores to keep multiple items supplied by the warehouse.

4) Distribution process: It is supposed that vehicle capacity is uncapacitated at both echelons to sufficiently supply all required items in an order. Multiple retailers are located in a close proximity to distribute all complete order in one round trip with a constant lead time. By this assumption,

- No-split lot is allowed. This can simplify the problem not to concern the allocation problem.
- Routing for distribution is not included in the problem.
- Transshipment between retailers is not allowed in order to consider only customer demands for each retailer.

5) Relevant inventory costs: They are composed of holding costs and ordering costs at both echelons. The research utilizes target service level (fill rate) instead of penalty costs as the system's constraint at retailer echelon.

6) Demand consideration: Each retailer holds inventories for serving customer demands. They are defined as stochastic demand represented by the stationary mean. The research assumes customer demands with Poisson distribution. Other probability distributions are not included in the study.

7) Item characteristics: The considered items are merchandizing items, and their shelf-life is longer than dispatch (or replenishment) cycle, so the expiration can be ignorable. The correlation of product formulary is not concerned; on another hand, individual item's demand is independent of the other items' demands.

8) Research methodology: The dissertation excludes the implementation phase into the industry and all inputs are based on the simulated data which is randomly generated.

1.6 Dissertation Contribution

1) <u>Practical contribution</u>

The multi-item two-echelon inventory system is considered with global information and centralized control; the decisions are made with visibility to the entire system using information for all locations through the cooperation and coordination across multiple parties or across firms. This system significantly provides cost reduction and service quality improvement for all stakeholders. Additionally, one-warehouse n-retailer inventory system is considered to represent general distribution in a supply chain, not limited to a serial inventory system.

The multi-item two-echelon inventory problem is studied under taking the can-order (s_{ij}, c_{ij}, S_{ij}) policy into consideration. This policy is a kind of joint replenishment policies with stochastic demand and continuous replenishment. Comparing with the traditional inventory policies which are independent control on multi-echelon system (e.g. the (s, S) policy, the (R, S) policy, and the (r, Q) policy), using the can-order policy may lead to substantial cost savings owning to the shared major ordering among location-items in the system. Moreover, as the can-order policy is a continuous review replenishment policy, it can react faster to new information than the periodic replenishment policies and it should intuitively perform better than the periodic replenishment policies as found in the previous researches.

Inventory planner needs to control a number of items stored in both echelons. The dissertation will facilitate them to jointly determine the best inventory policy parameters for continuous replenishment under stochastic demand by employing the can-order policy for the general item case. As realistic situation has been concerned to develop the problem, the application of the dissertation can provide significant advantages into many industries. Moreover, interaction between multiple retailers in a close proximity is also considered to share retailer echelon's ordering cost with a single round trip. The inventory total system-wide cost could be more saved relating to a number of retailer-items instead of considering multiple items only in a retailer.

2) <u>Academic contribution</u>

As most of previous researches have conducted on single-item two-echelon inventory system under traditional inventory policies, the dissertation can extend knowledge of inventory control with joint replenishment policy called the can-order (s_{ij}, c_{ij}, S_{ij}) policy. This policy has not been profoundly studied on OWNR. Thus, the dissertation will explore insights of the inventory policy setting in widely various conditions on single-item two-echelon inventory system. Moreover, a new solution approach will be proposed to determine the appropriate inventory policy setting on the considered system.

Additionally, a few of researches have been carried out on multi-item two-echelon inventory system. They studied multiple items under different conditions, i.e. periodic joint replenishment, integration of inventory and transportation problem with periodic joint replenishment, cross-docking inventory system, and other system constraints without joint replenishment consideration. The dissertation can fulfill research gap on another area of multi-item inventory control with joint replenishment policy named the can-order (s_{ij}, c_{ij}, S_{ij}) policy at both echelons under continuous replenishment and stochastic customer demand. This fulfillment also takes both echelons' stocks into consideration which differentiates from other researches on continuous replenishment for the supply chain. The entire chain is considered to determine all location-items' inventory policy setting for total system-wide cost minimization. Due to the system complication, decision variables between two echelons and among location-items are strong related and very difficult to find the (near) optimal solutions. Another new solution approach will be proposed for managing multiple items on OWNR with coordinated ordering decision. This facilitates inventory planner or related positions to understand and to determine the appropriate inventory policy setting.

With the existing literatures on the can-order policy, decomposition technique for breaking the multi-item models into the single-item models and iterative algorithm for solving such models are widely utilized to determine the inventory policy setting. The important challenge is an integration of the existing formulation and heuristics into OWNR. This will provide the significant contribution to the multi-item two-echelon inventory problem. In addition, another challenge is how to simplify the complicated system, but yet obtain the appropriate inventory setting.

The dissertation is expected to be a basis for other researches on joint replenishment policies. Since we study the insights of the can-order (s_{ij}, c_{ij}, S_{ij}) policy on OWNR and also provides the solution approaches for various situations. This knowledge is a valuable contribution to the field of inventory control and supply chain management.

1.7 Dissertation Methodology

To obtain insight of the can-order (s_{ij}, c_{ij}, S_{ij}) policy on OWNR, the dissertation methodology is divided into three phases as the following figure:

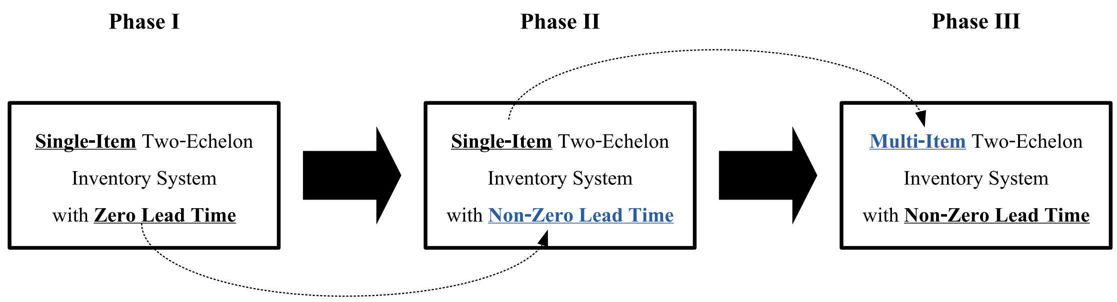


Figure I-6 Three phases for dissertation methodology

The first phase (Phase I) is the basic model for the can-order policy on OWNR. The system just has an interaction among retailers without joint ordering decision at warehouse echelon. The objective of this phase is to gain the insight of such policy on OWNR, and then to develop the heuristic approach for determining the appropriate inventory policy setting. There are three relevant factors considered, i.e. cost components, demand rates, and number of retailers. According to single item and zero lead time, only decision variables c_i, S_i, S_0 are considered with $2n+1$ variables. This simplifies the can-order policy on OWNR which will be a basic knowledge for the next phase.

The second phase (Phase II) is an extension of the basic model. The complication is added by non-zero lead time and service level constraint. Research remains taking single item into consideration to study an interaction among retailers without joint ordering decision at the warehouse echelon. The objective of this phase is to study inventory policy characteristics with the conditional relevant factors, i.e. lead time and target service level, as well as to develop the heuristic approach consistent with such characteristics provided. Relating to single-item consideration, decision variables are s_i, c_i, S_i, s_0, S_0 with $3n+2$ variables. More complexity of the model is contributed to the research.

Lastly, the third phase (Phase III) is the widest system for OWNR. Coordinated ordering decisions are concerned at both echelons. The warehouse's items are jointly replenished. Thus, decision variables are s_{ij}, c_{ij}, S_{ij} with $3m(n+1)$ variables. The ultimate objective of the research is provided by the most complication of the system. All valuable findings from phase I and II enable this phase to develop the heuristic approaches.

Based on the research process, all phases are carried out by the following methods as depicted in Fig. I-7.

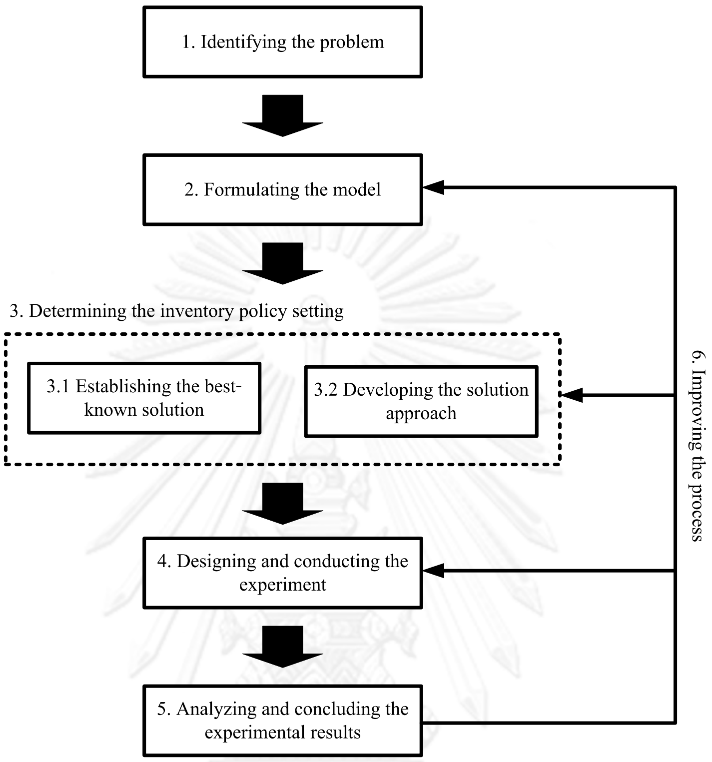


Figure I-7 Research process

1) ***Identifying the problem***: According to three phases as mentioned above, each phase deals with different problem. All considered problem should be clarified in order for conducting the research in later steps.

2) ***Formulating the model***: It is certain that different models are provided to serve three phases. In the dissertation, exact model and approximation model are combined to represent the system and simplified to determine the inventory policy setting. Due to the system complication, computer simulation becomes the most important tool in the dissertation. Hence, simulation model has the great importance on the research process.

3) ***Determining the inventory policy setting***: There are two sub-processes classified. Firstly, establishing the best-known solution utilizes computer simulation under multiple replications, and secondly, developing heuristic approach applies decomposition technique, iterative algorithm, and one-dimensional search for non-derivative function. Decomposition technique and iterative procedure can be applied

to break multiple locations into single location and to recurrently find the minimum solution as far as the best solution has been found. Both techniques have been intensively used in stochastic joint replenishment problem [34, 48, 51-56]. One-dimensional search called "golden section search" is a simple and efficient method for finding extremum of a unimodal function [48, 57, 58].

4) ***Designing and conducting the experiment***: All relevant factors are considered in the experimental design. The dissertation concerns various situations to study the effect of the can-order policy and the proposed heuristic approaches including their performance. Additionally, a lot of experiments are conducted to validate the best solution.

5) ***Analyzing and concluding the experimental results***: This step is a general process of research methodology. All findings in the experiments will be analyzed and discussed in order to explicate the can-order policy's characteristic and to evaluate the proposed heuristic approaches' performance.

6) ***Improving the process***: The improvement process is provided for better solution approaches. The feedback from the 5^{th} step leads to get back to the following activities: revising or simplifying model formulation, modifying the current solution approach or proposing the new one, redesigning the experiment to study in further details explicating the unclear circumstances.

This section gives an introduction of dissertation methodology. A great depth of research process will be explained in each chapter since three phases of dissertation methodology are established in the different contexts.

1.8 Dissertation Organization

There are six chapters organized in this research. Chapter I (Introduction) has already been mentioned above, and then the overview of the other chapters can be described as follows. The main contents are addressed following three phases of dissertation methodology:

- **Chapter II – Literature Review**: This chapter reviews previous researches and explains the inventory theory relating to the dissertation. Main knowledge is associated with joint replenishment problem, two-echelon inventory problem, and modeling and solution approaches. All literatures are discussed to identify research gap, raise their useful methodologies and results.

- **Chapter III – The Can-Order Policy for Single-Item Two-Echelon Inventory System with Zero Lead Time**: This chapter presents the insight of the can-order policy on the uncomplicated system including the effect of the can-order policy, comparative analysis with other policies, and inventory policy characteristics. Research methodology for this phase is also provided in detail to study the can-order policy, to determine the best-known solution, and to measure the solution's performance. All insights lead to develop the solution approaches. The experiment is designed and processed to study the heuristic's performance in various situations. Then, discussion on the problem is demonstrated.
- **Chapter IV – The Can-Order Policy for Single-Item Two-Echelon Inventory System with Non-Zero Lead Time**: Extending from phase I, this chapter describes the system characteristic and its complexity. The insight of the can-order policy is also provided as chapter III but not the same relevant factors (i.e. lead time and target service level). This chapter explains research methodology and proposes the heuristic approaches, as well as an improvement of the heuristic approach is demonstrated to reduce cost gap between the heuristic's solution and the best-known solution. The experiment with its result is analyzed and discussed.
- **Chapter V – The Can-Order Policy for Multi-Item Two-Echelon Inventory System with Non-Zero Lead Time**: This chapter combines all findings from phase I and phase II to extend the knowledge for controlling multiple items. Problem description is identified with the classification of inventory policy setting. The development of heuristic approaches conforms to the core context. Finally, comparative study of the classification is conducted in various situations.
- **Chapter VI – Conclusions**: It is the summary of the research along with three phases of research methodology, as well as further researches are recommended for the future improvement.

CHAPTER II
LITERATURE REVIEW

This chapter reviews previous researches and explains the inventory theory relating to the dissertation. The research considers two major areas of problem: joint replenishment problem and multi-echelon inventory problem, as well as interesting solution approaches are addressed. A review of joint replenishment problem demonstrates various joint inventory policies, specifically the can-order policy considered herein to continuously manage multiple items and other interesting policies. For the multi-echelon inventory problem, the literatures are divided into the single-item problem and the multi-item problem under both serial and arborescent system, like OWNR, to explore various inventory policies. We also present the interesting modeling and solution approaches for the problems.

2.1 Joint Replenishment Problem

Joint replenishment problem (JRP) is the multi-product inventory problem of coordinating the replenishment of a group of items that may be jointly ordered from the same supplier. The objective of JRP is generally to minimize the total cost whilst satisfying demand. The total cost is mainly composed of two parts: the holding cost and the ordering cost [20].

- **The holding cost** is the cost of holding inventory including the cost of capital tied up in inventory, taxes, and insurance.
- **The ordering cost** is the cost of preparing and receiving an order, the cost of material handling and transportation. When placing the order to the supplier for a number of different items, two components of the ordering cost are occurred:
    - **The major ordering cost** which is independent of the number of different items in the order.
    - **The minor ordering cost** which depends on the number of different items in the order.

The common decision on JRP is to determine the optimal quantities (generally relating to when and how much to order) for items ordered from the same supplier by trading off between the holding cost and the ordering cost. Using group replenishment may lead to substantial cost savings because of the major ordering cost shared among items in the group. Many algorithms have been proposed to find quality solutions for the JRP. Typically, there are three types of replenishment pattern: type 1 – each item is independently ordered; type 2 – all items are jointly ordered in the same lot (or at the same cycle time); type 3 – each lot contains a selected subset of items (not every order contains every item). Figure II-1 illustrates the replenishment pattern of each type. In the figure, an example contains three items (A, B, and C) in the system.

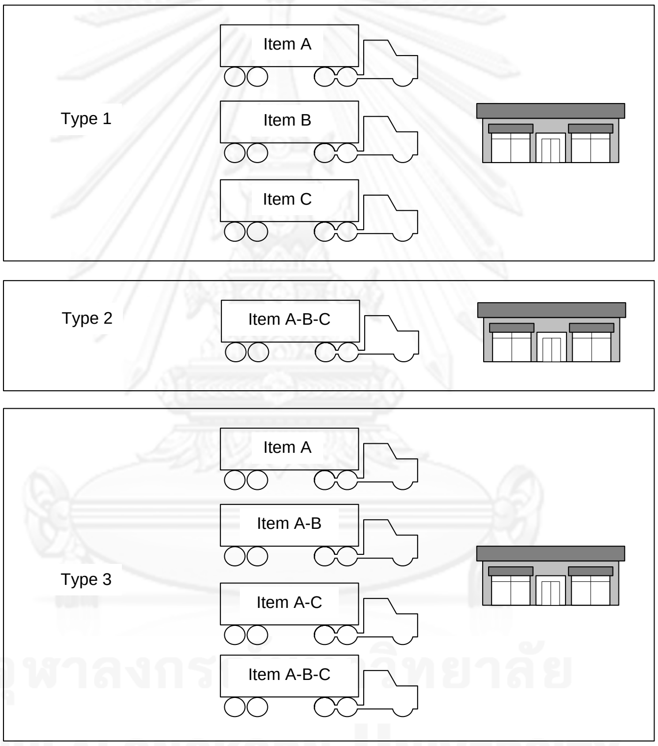


Figure II-1 Three types of replenishment pattern

According to grouping multiple items, strategy to solving the JRP can be classified into two types: A direct grouping strategy (DGS) and an indirect grouping strategy (IGS). Under DGS, items are partitioned into a predetermined number of sets and the items within each set are jointly replenished with the same cycle time. DGS is consistent with replenishment pattern type 2. IGS could be defined that not every order contains every items as consistent with replenishment pattern type 3.

The review is divided according to types of demand: deterministic demand and stochastic demand. Most of review are examined JRP with stochastic demand since it directly deals with the dissertation problem. Meanwhile the following review also gives a consideration on JRP with deterministic demand, though it indirectly relates with the dissertation. Hence, the review of deterministic part aims at providing an understanding of the basic concept and the approaches for finding quality solutions.

The review on JRP with deterministic demand focuses on the classic joint replenishment problem (CJRP) which is similar to the economic order quantity (EOQ) including deterministic and uniform demand, no shortages allowed, no quantity discounts, and holding cost is linear. For DGS, CJRP can be solved under EOQ with the same cycle time for all items. For IGS, the cycle time for every item is an integer multiple k_j of the order cycle time T. Thus, the cycle time for item j is $T_j = k_j T$ and the order quantity for item j is $Q_j = T_j D_j = k_j T D_j$ where D_j is demand per unit time for item j. The policy defined by the basic cycle time and a set of multipliers are known as the cyclic policy. Arkin, Joneja, and Roundy [59] provided a proof that the CJRP is an NP-hard problem. van Eijs, Heuts, and Kleijnen [60] compared the solutions of DGS and IGS for a set of randomly generated problems. The authors identified two factors that are important in determining the relative performance of the two strategies. The first factor is the ratio of the major ordering cost to the average minor ordering cost (called "the ordering cost ratio" for the entire of the dissertation), and the second factor is the number of items. The results indicated that IGS outperforms DGS but the differences are small. For values of the ordering cost ratio above 75, the IGS and DGS become the same because only a single group is created.

Kaspi and Rosenblatt [61] proposed a simple heuristic algorithm (called RAND) by computing k equally spaced values of the fundamental cycle T within its lower bound and upper bound $[T_{\min}, T_{\max}]$. Then, Goyal and Deshmukh [62] introduced a new lower bound on $T_{\min}$ which reduces the range of T. Hariga [63] developed two heuristics for solving CJRP. Both procedures relax the order frequency in which the multipliers need not to be integer number. Ben-Daya and Hariga [64] conducted a numerical experiment to test the performance of Hariga [63]'s heuristic against Goyal and Deshmukh [62]. Hariga's algorithm gives lower total cost for 86.9% of 24,000 randomly generated problems. In addition, Hariga's algorithm is 21 times faster for

10-product problems and 40 times faster for the 20-product problems. Viswanathan [65] proposed an algorithm which iteratively improves the bounds on T. The performance of the proposed algorithm relative to Goyal's algorithm improves as the problem size increases. However, Goyal's algorithm is faster when the major ordering cost is small. More details relating to other solution approaches can be seen in Khouja, Michalewicz, and Satoskarl [66], Lee and Yao [67], and Olsen [68].

Summarily, the basic concept of the JRP with deterministic demand is the use of the base period T with integer multipliers for determining each order cycle of item j. The important relevant factors are the ratio of the major ordering cost to the average minor ordering cost, and the number of items. From this review of deterministic part, we raise the basic concepts and approaches for finding quality solutions in order to comprehend simple part of JRP. Then, the next issue continues to the main part of the dissertation which is more complicated.

Focusing on JRP under stochastic demand (SJRP) which customer demand is stationary in the mean, a general application of SJRP has been developed on multi-item single-location inventory system. The objective is to minimize the expected total cost per unit time. The optimal joint replenishment policy can theoretically be found by solving a huge Markov decision model. However, the size of the state and the decision space grow exponentially with the number of different items, it seems intractable to solve the model for obtaining the optimal solution. Ignall [69] solved the problem for two items and found that the optimal policy is in general unfortunately not a simple policy. Instead of focusing on the optimal policy, the literatures on the SJRP proposed the joint replenishment policies and heuristic approaches to determine the appropriate inventory policy setting. SJRP can be classified into two major streams based on the type of policy class under consideration as follows [3, 19, 20]:

2.1.1 Can-order policies

The general concept of the can-order policy is usually applied in a continuous review system as originally suggested by Balintfy [42]. Balintfy provided an initial insight into the problem with a queuing-based approach assuming no lead time and identical items. When the inventory position of any item drops to or below its must-order level s_j an order is placed to bring its inventory level to order-up-to level S_j. For other items $k \neq j$ with the inventory below the can-order level c_k, its inventory level is replenished to S_k. This is also known as a (s_j, c_j, S_j) policy in common. So, it is composed of two reorder points: the must-order level occurring normal replenishment, and the can-order level occurring special replenishment. Special replenishment is an opportunity of a discount replenishment which an item is faced with when another item reaches its must-order level and places an order.

Silver [70] analyzed a special case with $c_j = S_j$ - 1 and s_j = 0 in a two-item inventory system facing identical Poisson demands and zero lead time. Under the assumption that shortages are not allowed, Silver proved the can-order policy is always better than independent control. An exact analysis has been possible for this special case because the inventory levels of both items provide regeneration points at the order instances and, hence, the renewal reward theorem is applicable. However, the same approach cannot be used for the general case. Therefore, different approximate models and solution methods have been proposed later on.

An approximation technique proposed by Silver [51] is to decompose the m-item problem into m single-item problems facing Poisson demands and Poisson special replenishment opportunities. Assuming this process of discount opportunities is independent of item j, the multi-item inventory problem can be solved by successive iterations. The same decomposition technique has later been extended to compound Poisson demand by Thompstone and Silver [71] and Silver [52]. The popular method for computing the can-order policy referred in many comparisons is of Federgruen et al. [53]. They modeled the can-order policy as a semi-Markov decision problem with compound Poisson demands, and positive lead times. Poisson special replenishment opportunity was assumed as Silver [51]. They decomposed the multi-item model into single-item problems and used a policy-iteration algorithm to solve for the best values of the control policy parameters. Policy-iteration algorithm searches for solutions of the single-item model and then extends this solution, to the multi-item case.

Another approximation technique was proposed by Love [46] using the basis of single-item economic order quantity on deterministic model to determine the initial individual order cycle time of item j with respect to its own minor ordering cost. The concept of periodic replenishment was applied to determine integer multiple of the minimum order cycle time. Then, each item can be determined decision variables (s_j, c_j, S_j) by a closed-form formula obtained from an approximation model. Comparing to Silver [51], the numerical result showed that Love [46]'s approach obtains lower total cost.

Zheng [54] proved that if the discount opportunity process is Poisson then the can-order policy is optimal. After m single-item problems are solved, the rate at which discount opportunities are generated is calculated and used in the next iteration. The procedure stops when the best policies are unchanged. On the other hand, van Eijs [48] and Schultz and Johansen [55] illustrated the assumption of a Poisson arrival process for the special replenishment opportunities could lead to poor performance of the can-order policies. Instead, they proposed using Erlang distributions. The best values of the policy parameters are obtained through policy iteration. van Eijs [48] also suggested a can-order policy where the can-order level c_j is always equal to S_j - 1 when the major ordering cost is high compared with the average of the minor ordering costs. For such a policy, whenever an item places an order, all other items join the order. He minimized the holding and ordering costs subject to a service level constraint. With decomposition technique and iterative procedure, the best policy can be determined.

Melchiors [56] provided an improvement to the can-order policy using a compensation approach, where an item placing an order receives compensation from other items benefitting from the order opportunity, to improve the previous approximations of the can-order policy for Poisson special replenishment opportunity. The single-item model and decomposition procedure were developed. Melchiors observed that the can-order policy obtained from Federgruen et al. [53] gave a poor performance with high ordering cost ratio, as the same result of van Eijs [48]. The results showed that in cases of low ordering cost ratio the best can-order policy outperforms the periodic replenishment policy proposed by Viswanathan [44]. For higher ordering cost ratio, such periodic replenishment policy gains the lowest cost, but the difference is very small. The example clearly illustrated that the Federgruen's can-order policy is far from the optimal can-order policy. However, at higher ordering cost ratio, the can-order policy can be solved under ($s_j, S_j - 1, S_j$) policy as suggested by van Eijs [48]. Another conclusion from the results was that the periodic replenishment policy proposed by Viswanathan [44] should be used on the problem where demand variation is low, but the can-order policy should be used when demand variation is high. At low demand variation, the periodic replenishment policy can reduce the holding cost by eliminating the small uncertainty of not knowing exactly when the next order is placed. Meanwhile, at high demand variation where the time between two consecutive replenishments is more unpredictable, the reaction time is much more significant and using the can-order policy would be better off.

Kayiş, Bilgiç, and Karabulut [72] proposed a semi-Markov decision model for the can-order policy under two-item inventory system. Their main objective is to describe the whole system without decomposing it into two single-item inventory systems. Since the dimension of the state space is larger than the single-item inventory problem, the problem is solved using an enumerative approach. The comparative result showed that the policy iteration algorithm of Federgruen et al. [53] does not always converge to the best can-order policy.

The can-order policy has also been applied in periodic replenishment. Dellaert and van de Poel [34]. They derived a simple inventory (R, s_j, c_j, S_j) model. All items in the group are reviewed periodically at every R period. They extended an EOQ model to (R, s_j, c_j, S_j) model, in which the values of the control parameters are determined in a simplistic manner. After this approach was implemented in the hospital over a year, it resulted in substantial gains, such as improved service levels, reductions in supplier orders, smaller total inventory levels and holding costs, and eventually lower system costs. Later, the compensation approach was extended by Johansen and Melchiors [73] but on the periodic review system by approximating the discount opportunities by a Bernoulli process with outcome 1 if a discount order opportunity occurs and 0 otherwise. The performance of the extended compensation can-order policy was compared to the periodic replenishment policy of Viswanathan [44]. The periodic can-order policy is advantageous on cost saving around 15% for the problem with high demand variation. Interesting issue is that the periodic replenishment policy of Viswanathan [44] and the new policy provide indifferent results for the problem with low demand variation.

Additionally, the literatures on the can-order policy were extended in more complicated system. Duyn Schouten, Eijs, and Heuts [74] conducted a research on a framework of the can-order policies with quantity discounts. Liu and Yuan [75] studied the can-order policy for a two-item system with correlated Poisson demands. Tsai, Tsai, and Huang [76] proposed an association clustering algorithm to group multiple items based on the can-order policy. Nagasawa et al. [77] applied genetic algorithm (GA) to set the optimal can-order level of many items on a given (s_j, S_j). The knowledge of the can-order policy is likely to be enhanced and taken into more consideration.

2.1.2 Other policies

Besides the can-order policies, various joint replenishment policies were introduced in both periodic and continuous replenishment. The basic joint replenishment policies on periodic replenishment were developed by Atkin and Iyogun [47]. They proposed two periodic replenishment policies: (R_j, T) policy and $MP(R_j, T)$ policy. The (R_j, T) policy is defined that the same review interval T is used for all items and each item is brought up to order-up-to level R_j at review period. Then, the $MP(R_j, T)$ policy is a modified periodic review policy which items belonging to a base set are brought up to their R_j at every review interval T, while other items are brought up the their level R_j at every $k_j T$ time units.

Pantumsinchai [43] developed the continuous review (Q, S_j) policy originally introduced by Renberg and Planche [78]. Under the (Q, S_j) policy, aggregate consumption of all items is monitored and when it reaches a certain level Q, all items are replenished to their order-up-to level S_j. Comparing the (Q, S_j) policy to the $MP(R_j, T)$ policy and the can-order policy obtained by Federgruen et al. [53], the (Q, S_j) policy performs well when high major ordering cost.

Viswanathan [44] introduced the $P(s_j, S_j)$ policy. It is a periodic reprenishment policy in which inventory level of all items are reviewed once every T time units and an independent (s_j, S_j) policy is applied. Each item with inventory level below s_j is replenished up to the order-up-to level S_j. The $P(s_j, S_j)$ policy was compared with the can-order policy obtained by Federgruen et al. [53], the $MP(R_j, T)$ policy, and the (Q, S_i) policy. These policies were tested on the data sets used by Atkin and Iyogun [47] and on some additional problems. The results indicated that the $P(s_j, S_j)$ policy performs best overall with only a slight improvement over the $MP(R_j, T)$ policy. Cachon [79] proposed another periodic replenishment policy called the $(Q, S_j \mid T)$ policy which combines the (Q, S_i) policy to periodic replenishment. The system is reviewed every T time units, and item j is ordered up to level S_j if accumulated demands of all items reach at least Q units.

Nielson and Larson [45] studied the continuous $Q(s_j, S_j)$ policy. Aggregate consumption is continuously reviewed, whereas item inventory levels are only reviewed when aggregated consumption of all items reaches or exceeds a certain level Q, each item with inventory level less than s_j is replenished up to S_j. They used Markov decision theory to develop an analytical solution procedure under Poisson demand process. Numerical tests indicated that the $Q(s_j, S_j)$ policy outperforms the $P(s_j, S_j)$ policy and the (Q, S_j) policy.

Özkaya et al. [19] introduced a new control policy denoted (Q, S_j, T) policy which is a hybrid of the continuous replenishment (Q, S_j) policy, and the periodic replenishment (R_j, T) policy. The (Q, S_j, T) policy combines features of both periodic and continuous replenishment policies. Inventory positions are monitored continuously and when the aggregate demands since last replenishment reaches Q units or the time elapsed since last replenishment reaches T, all items are replenished up to S_j. The new policy identified overall average performance better than other existing policies, i.e. the $P(s_j, S_j)$ policy, the (Q, S_j) policy, the $Q(s_j, S_j)$ policy, the can-order policy by Federgruen et al. [53], and the can-order policy by Melchiors [56].

Mustafa Tanrikulu, Şen, and Alp [80] proposed the (s_j, Q) policy. A replenishment order of constant size Q is triggered when the inventory position of any item drops to its reorder point s_j. The replenishment order is allocated to multiple items so that the inventory positions are equalized as much as possible. A numerical study showed that the (s_j, Q) policy outperforms the (Q, S_j) policy when high backorder cost and small lead time.

Roushdy et al. [81] suggested the (R_j, s_k, S_k) policy. Item j is defined that any item has the shortest order cycle among all items. Item j is continuously replenished by triggering an order when its inventory position reaches the re-order level R_j and order quantity is equal to Q_j. The other items $k \neq j$ are periodically reviewed with the same interval as item j and are included in the same order as item j if their inventory positions reach s_k denoted the re-order level for any item $k \neq j$. Their inventories are ordered up to the order-up-to level S_k. An inventory cost formula is similar to Axsäter [82] used to evaluate an approximate Poisson cost function. Iterative method is applied to determine the best solution. They compared the proposed policy with the independent (R_j, Q_j) policy and the $MP(R_j, T)$ policy. The proposed policy outperforms for every instance in the experiment except when the major ordering cost is zero.

The above review of the existing policies showed various policies and their mechanisms proposed until the present time. A lot of researches primarily focused on the can-order policy which is the basis of the coordinated replenishment decision, because it is straightforward and appealing to one's common sense for practical use. Mostly the can-order policies were developed on the approximate models, except the special can-order policies on a given $c_j = S_j$ - 1 used the exact models. Many heuristics were developed to determine the best can-order policy instead of finding the optimal solution. However, general determination of the can-order policies needs to deal with $3N$ control policy parameters for N - item setting. Therefore, other policies were proposed to reduce the complication with respect to the control policy parameters as follows:

- The (R_j, T) policy, the (Q, S_j) policy, and the (s_j, Q) policy with N + 1 control policy parameters;
- The $(Q, S_j | T)$ policy and the (Q, S_j, T) policy with N + 2 control policy parameters;
- The (R_j, s_k, S_k) policy with $2N$ control policy parameters;
- The $MP(R_j, T)$ policy, the $P(s_j, S_j)$ policy, the $Q(s_j, S_j)$ policy with $2N$ + 1 control policy parameters.

However, from the comparative results on the existing literatures an outstanding policy has beaten others in all situations do not appear. Therefore, selecting the policy to be studied on OWNR depends on the dissertation's consideration (as mentioned in section 1.3). Even though the can-order policy is not the best policy in every situation, it performs well in the important circumstances relating to the example industry. Another important issue is that the can-order policy does not perform bad itself but depends on the heuristic approach to determine the appropriate inventory policy setting. As found in comparative results among the can-order policies developed by Federgruen et al. [53], van Eijs [48], and Melchiors [56], the experimental results indicated that the can-order policy can be more improved as long as the approximate model is closer to the actual model. Hence, recent researches have concentrated on an improvement of the can-order policy setting.

2.2 Multi-Echelon Inventory Problem

More complex inventory systems are so called multi-echelon inventory systems. An echelon is referred to a single level in a supply chain. Multi-echelon inventory system is classified into three structures: the serial system (linear chain), the arborescent system (diverging chain), and the assembly system (converging chain) [1, 3, 83].

The serial system (linear chain) is the simplest system. All stocking points follow the same path or route as showed in Fig.II-2. This may occur in an environment with only sequential working. Usually, they are considered as a part of more complex chains. A serial system contains two or more stocking points coupled. For instance, a serial system where the first inventory holds the stock of a sub-assembly and the second inventory holds the final parts. The second inventory can be considered as a customer of inventory one.

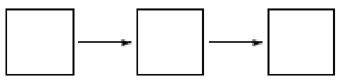


Figure II-2 The serial system

The arborescent system (diverging chain) is that each stocking point has one predecessor as depicted in Fig.II-3. A typical situation in practice is when a central warehouse supplies goods to several retailers. In other situation, the system occurs in factories where raw materials are cut into various part types and where semi-manufactured items are made into various end products. With regard to a typical system which contains one warehouse and multiple retailers, the warehouse can perform as either a stocking point or a cross-docking point. Stocking point means that there is a physical inventory kept in the warehouse. On the other hand, cross-docking point means that the warehouse is a hub for unloading materials from an incoming vehicle and loading these materials directly to outbound vehicle. So, the warehouse has a little or no storage in between process.

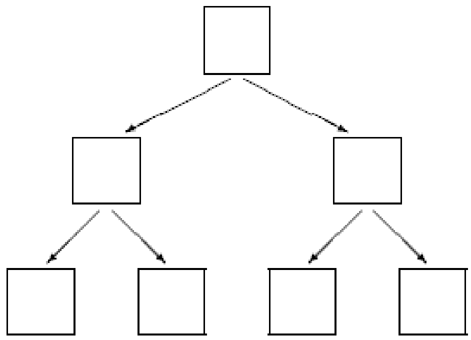


Figure II-3 The arborescent system

The assembly system (converging chain) is the opposite of a general distribution system. Each stocking point has one immediate successor. It occurs in factories where parts are assembled or in distribution from various factories to a single distribution center.

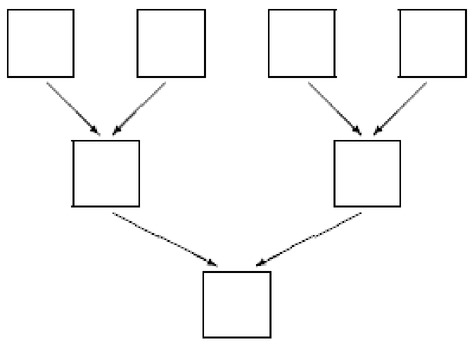


Figure II-4 The assembly system

General systems in a supply chain can of course be of more complex nature and be a combination of different systems described above.

Two useful dimensions for inventory management in multi-echelon inventory problem are the visibility of information and the control of echelon [1].

- Relating to the visibility of information, local information and global information are identified. Local information implies that each stocking point knows demand in the form of the orders arriving from its immediate successor(s). Meanwhile, global information is an open of information through the supply chain where the planner has visibility of all information of all stocking points in the system.

- The control of echelon is defined in terms of centralized versus decentralized control. Centralized control implies that decisions are made for the entire system to jointly optimize the advantage for all echelons. It is often identified with push system, because a central planner pushes stock to the locations that need it. Contrarily, decentralized control means that decisions are made independently by separate stocking points. It is associated with pull system because independent planners pull stock from their predecessor(s).

The best solution is likely to use global information and centralized control verified by the success of Vendor Managed Inventory (VMI). The VMI system has heavily increased from the early 2000s, since the collaboration between internal and external firms is a significant key to improving a firm's customer service [12]. The VMI system is a specific type of outsourcing inventory management (OIM). Outsourcing is a contractual agreement between the customer and one or more suppliers to provide services or processes that the customer is currently providing internally [84]. This logical approach has become attraction when 1) outside providers can produce needed products (services) more efficiently than internal departments or 2) outside providers can produce desired products (services) at a higher level of quality than an organization [85]. The growing importance of this strategy has emerged for many organizations, because several researches and case studies verified that OIM can help control inventory cost and improve internal performance influencing customer satisfaction and perception. Moreover, the capabilities of external sources are growing. Hence, outsourcing becomes an increasingly attractive option [2-11, 13, 86]. We recommend Arshinder et al. [87]'s work which is a review on supply chain coordination in aspect of mechanisms, managing uncertainty and research directions. Their work enables the reader to comprehend the overview and trend of supply chain coordination.

The dissertation problem considers centralized control of OWNR, which is a general inventory system in supply chain, to minimize the total system-wide cost. In the next part of review, researcher restricts only the literatures on two-echelon serial and divergent inventory systems. Note that the serial system is studied herein as it is a small part of the divergent inventory system. According to the existing literatures, most of the ordering policies are developed around two major policy classes: installation stock policies and echelon stock policies [22, 50, 88].

- **Installation stock policy**: ordering decisions at each location are based on its own installation inventory position, which is equal to sum of its physical stock and on order minus the backlog. This policy acts as a common-sense approach to control overall inventories in the system. It employs together with the nested policy. The nested policy is an ordering policy for the warehouse (upper stream) where an order is triggered at the warehouse if and only if any retailer reaches its reorder point. By this policy, the warehouse cannot trigger an order at any other time of no demand arriving at the retailer echelon.
- **Echelon stock policy**: the information of the down-stream locations is taken into account. Ordering decisions at each location are based on the echelon inventory position defined as sum of installation inventory positions at the location and all its down-stream locations. In the opposition to installation stock policy, the warehouse can trigger an order at any other time of no demand arriving at the retailer echelon. This pre-ordering decision expects that the retailers' waiting time from insufficient stock at the warehouse would reduce.

Axsäter and Juntti [88] compared two policies in both deterministic demand and stochastic demand. Even though in case of deterministic demand echelon stock policy dominates installation stock policy, in case of stochastic demand either installation stock or echelon stock policies may be advantageous depending on the structure of the inventory system. Cost difference between two policies is about 5%. Echelon stock policy seems to dominate installation stock policy for long warehouse's lead times, while the opposite is true for short warehouse's lead times. However, Axsäter and Juntti [88] stated that when ratio of the replenishment quantity at the warehouse to the dispatch quantity at the retailers (called the Q_w / Q_r ratio) is not positive integer value, echelon stock policies could not be applied. The reason is that the warehouse echelon stock inventory position is decreasing continuously with the retailer demands, non-integer ratio of Q_w / Q_r cannot be duplicated by an echelon stock policy. Consequently, our dissertation cannot apply echelon stock policy due to the uncertainty of dispatch quantity issued at the retailer echelon and non-integer ratio of Q_w / Q_r. Therefore, we mainly focus on installation stock policy to develop our joint replenishment models.

Regarding our problem, we emphasize in both single-item and multi-item models following three phases of dissertation methodology. Hence, we review the interesting literatures according to such two models as follows:

2.2.1 Single-item models

According to the dissertation problem, the review in this section focuses on stochastic demand, which raises several new issues and creates extreme modeling complexities in a multi-echelon inventory situation. Not only the arborescent system is considered, but the serial system is examined as well since it usually is a part of more complex chains. As a matter of the fact that there have been a number of researches on single-item multi-echelon inventory problem conducted under either continuous or periodic replenishment. They proposed mathematical models and solution approaches for setting an appropriate inventory policy. Most of previous works studied two major types of the inventory policies: Fixed-interval order-up-to polices and Stock-based batch-ordering policies, on different conditions and relevant parameters. Further details can be seen in the reviews of Schneider et al. [15], Axsäter et al. [16], and Wang et al. [17] In our dissertation, we are interested in both order-up-to (base-stock) control policies and batch-ordering policies. For order-up-to polices which are related to the can-order policy employing an order-up-to level S_{ij}, we consider them in both periodic and continuous replenishments. The following review is aimed at identifying various common inventory policies applied into single-item two-echelon inventory system.

1) The order-up-to control policies

They are used in both periodic and continuous replenishment in different policy parameters. The most common order-up-to policies are:

- The (R,S) policy (some literatures use the (S,T) policy) where all locations' inventory position are reviewed at the same period R (or T) and they replenish inventories to reach their respective order-up-to levels S.

- The (s,S) policy is a traditional order-up-to policy where each location reviews its inventory position continuously when the reorder point s is reached its inventory is replenished to the order-up-to level S.
- The (R,s,S) policy is a hybrid of periodic and continuous replenishments where all locations' inventory position are reviewed at the same period R. If their inventory position reaches their respective reorder points s, they also replenish the inventories to their respective order-up-to levels S.
- Another continuous policy is called a pure base stock policy or one-for-one replenishment policy, $(S-1,S)$ policy. It is normally used for repairable items with a fixed unit of order quantity. When the warehouse receives the request from the retailer, it orders a new unit from the outside supplier.

We demonstrate some interesting literatures herein to provide insights of their works. One of the first literatures relating to the base-stock policies is the METRIC (Multi-Echelon Technique for Recoverable Items Control) model of Sherbrooke [89]. The objective function in METRIC is to minimize the expected number of backorders at retailer echelon, subject to budget constraints. The METRIC approximation assumed that the lead time at any retailer is constant at the average lead time[3], so the expected inventory levels and backorder units at the retailers can be easily evaluated. Later, METRIC is a basic model for several extensions of the order-up-to policies and the batch-ordering policies. For a more extensive review we refer Diks et al. [50] which provided the development of the METRIC model in two perspectives: repairable items and consumable items.

Federgruen and Zipkin [90] studied the (R,S) policy on OWNR with no stock at the warehouse. The warehouse places an order periodically; its order arrives after a fixed lead time and is allocated among several retailers who face normally distributed demand. So, the allocation problem was included in their study. Several approaches were proposed to approximate the dynamic program describing the problem, and then a near-optimal order policy was provided. Matta and Sinha [91] developed the two-echelon inventory problem on OWNR with stock at the warehouse. Each retailer orders from a single warehouse according to (R,S)

[3] The average lead time at any retailer is equal to its constant lead time plus the expected waiting time from the warehouse which can be determined by Little's Law.

policy, and the warehouse employs (R,s,S) policy. All locations use the same review period R. Demand is assumed to be normal distributed. The shortage cost per unit time and the procurement cost at the warehouse are applied, but the retailers' ordering cost is negligible. The cost function and algorithm are approximate based on renewal theory and queuing theory. Cetinkaya and Lee [23] provided (s,S,T) policy to the warehouse (i.e. we categorized the (s,S,T) policy as the (R,s,S) policy since both policies have their characteristics as time-based and quantity-based ordering decision). They considered VMI system with coordinating inventory and transportation decisions. Under the (s,S,T) policy, the warehouse holds small orders until an agreeable dispatch time T then dispatching to the retailers will be done. The retailers are willing to wait at the expense of waiting costs. For such a delivery policy, Cetinkaya and Lee [23] realized that larger loads could benefit the economies of scale in transportation problem.

Recently, Chu and Shen [92] and Shang and Zhou [93, 94] studied periodic base-stock policies. Chu and Shen [92] studied OWNR with the so-called power-of-two (POT) policy first introduced to stochastic demand. Shang and Zhou [93, 94] considered the integer-ratio of replenishment intervals at the warehouse and the retailers. Their numerical study suggested that the optimal policy tends to be an integer-ratio policy rather than POT policy under some conditions. According to the review on periodic base-stock policies, they were employed into various systems.

More details of the fixed-interval order-up-to policies can be obtained from, for instance, Nicholson et al. [9], Schneider et al. [15], Eppen and Schrage [95], Rogers and Tsubakitani [96], Axsäter [97], Diks and de Kok [98], Axsäter [99], Rao [100], Li [101], Wang et al. [17, 102], Wang [103], Wang and Axsäter [104].

For the continuous (s,S) policy, most research interpreted to the reorder point batch-ordering policy due to their equivalent. Hence, this kind of policies will be included in the section of batch-ordering policies.

2) The batch-ordering policies

Normally, the batch-ordering policy is able to be represented by (r,Q) where each location reviews its inventory position continuously when the reorder point r is reached order quantities Q are issued to the upper echelon.

Some interesting literatures are demonstrated herein to raise various systems and significant insights of their works.

Focusing on the serial system which is a smallest part of the supply chain, the following are a few notable examples. De Bodt and Grave [105] considered a multi-stage, serial inventory system under (r,Q) policy facing stochastic and stationary demands at the lowest echelon. The installation stock policy was employed with the nested policy. The relevant costs include the fixed ordering cost and the inventory holding cost for each echelon, and a backordering cost for the lowest echelon. The objective is to determine the inventory policy setting which obtain the minimum expected average total system-wide cost. Meanwhile, Chen and Zheng [106] first studied echelon stock (r,Q) policy in a serial system. They also extended their work into compound Poisson process [107].

Deuermeyer and Schwarz [108] presented an analytical model for the one-warehouse n-identical retailer inventory system facing stationary Poisson demand and operating under the (r,Q) policy. They developed a decomposition technique for analyzing OWNR by finding inventory policies for each retailer independently and adapting the METRIC technique. Later, Deuermeyer and Schwarz [108]'s work was examined by Svoronos and Ziphin [109] in order for more accurate approximate solutions. Axsäter is one of the most popular researchers in the field of multi-echelon inventory system as he has been developing many papers continuously and his works have been cited in over 100 papers. He carried out many researches on both the base-stock policies and the batch-ordering policies. The following are some examples of his works specifically on the batch-ordering policies. Axsäter [110] considered one warehouse and n identical retailers under the (r,Q) policy. Lead times are constant and the retailers face independent Poisson demand. Axsäter [110] showed an extension of Axsäter [111] used for batch-ordering policies. Axsäter [112] proposed a generalized model of Axsäter [110] considering two non-identical retailers.

Further details of the batch-ordering policies can be found in various systems, for example, Wang et al. [17], Axsäter [99, 113, 114], Schwarz, Deuermeyer, and Badinelli [115], Ahire and schmidt [116], Chen and Zheng [117], Tee and Rossetti [118], Hill, Seifbarghy, and Smith. [119], Jha and Shanker [120].

According to aforementioned policies, periodic based-stock policies enable the system to make coordinated ordering decision, whereas the continuous replenishment policies (including based-stock policies and batch-ordering policies) are independent ordering decision. We found that most of literatures studied the coordinated ordering decision for the continuous replenishment when employing joint replenishment policies (already mentioned in Section 2.1 on multi-item single-location inventory system). We realize that continuous joint replenishment policies are a special group of inventory policies utilized for coordinated ordering decision. Therefore, we categorize them to the third group of inventory policies, besides the based-stock policies and the batch-ordering policies. Dealing with our dissertation's consideration, the following are an extensive review on coordinated ordering decision for the continuous replenishment.

3) The continuous joint replenishment policies

A few of literatures on continuous joint replenishment policies on OWNR have been conducted. Focusing on our considered cost structure which includes the ordering costs and holding costs at both echelons, and either the penalty costs or service levels, the interesting literatures are reviewed as follows:

Cheung and Lee [21] studied the (Q,S) policy. When the cumulative demands over all retailers reach a given Q units (i.e. truckload size for all retailers in single trip), an order is placed at the warehouse to replenish the retailer to their respective order-up-to levels S. The inventory policy at the warehouse is the (r,Q) policy.

Özkaya [22] proposed analytical models and heuristic approaches for four types of joint replenishment policies at the retailers, and utilized a traditional (s,S) policy at the warehouse. Such four types of joint replenishment policies are the (Q,S) policy, the (Q,S,T) policy, the $(Q,S|T)$ policy, and the $(s,S-1,S)$ policy. The (Q,S) policy of Cheung and Lee (2002) and Özkaya (2005) was studied on different structures. The former sets target service level at the warehouse and penalty cost at the retailers, meanwhile the latter sets target service level only at the retailers. The (Q,S,T) policy is a hybrid of continuous and periodic replenishments. An order is placed at the warehouse either when the cumulative demands over all retailers reach Q units or when at least one demand arrives in T time units after the last ordering instance. The $(Q,S|T)$ policy is a periodic replenishment policy and the ordering decision arises every T time units. At the decision epoch, if at least Q demands have accumulated for the retailers since the last ordering instance, an order is placed at the warehouse. The ($s, S-1, S$) policy is a special can-order policy which an order is triggered when any retailer's inventory position reaches its must-order level s. Then other retailers in the system will be also included by this order if at least one demand arrives to each retailer. All proposed policies commonly have the retailer's order-up-to level S to which the warehouse replenishes all retailers' inventories. Özkaya [22] showed comparative results among these policies without comparing to the lower bound or the best-known solution.

Gou et al. [14] introduced a joint replenishment policy where the warehouse takes a traditional (s, S) policy and the retailers utilize the can-order (s, c, S) policy. When an order is triggered by a retailer, other retailers whose inventory position reaches its can-order level c will be included by this order as well. Even though zero lead time was assumed in their study, they cannot provide an analytical model due to the complication. Thus, computer simulation was used instead. Their result showed that about 5 to 20% of the cost can be saved as comparing with the independent (s, S) policy at the retailers. Nevertheless, they did not provide a solution approach for setting the appropriate inventory policy.

There are other researches on joint ordering decision conducted on different cost structures. Cross-docking system were carried out in Gürbüz [24] Axsäter and Zhang [25] developed joint ordering policy by not concerning the shared ordering cost.

Thus far, a few researches have concerned coordinated ordering decision under continuous replenishment and stochastic demand with considering all relevant costs on both echelons. We recognize that all relevant costs on both echelons should be considered together in order to determine the inventory policy parameters for all stores in the system as the general inventory control process. Moreover, a very few of them focused on determining the appropriate inventory policy setting especially for the can-order policy in OWNR. Hence, it is interesting to develop a heuristic approach to determine the appropriate can-order policy in OWNR so as to extend the knowledge of the can-order policy into the two-echelon inventory system.

2.2.2 Multi-item models

Number of researches on multi-item multi-echelon inventory problem is much smaller than the one on a single item case. We classify the existing literatures into two groups of multi-item problem: coordinated ordering decision and joint constraint. For coordinated ordering decision, periodic joint replenishment policies have been conducted. Some works have also included integration of inventory and transportation problem. Thus far, the continuous joint replenishment policies have not yet been found to employ into multi-item multi-echelon inventory problem. It appears that there is another research gap to extend the continuous joint replenishment policies into multi-item multi-echelon inventory problem. However, in this section we raise some literatures on periodic joint replenishment policies and also exemplify some with joint constraint in order to illustrate a direction of multi-item multi-echelon inventory system. Note that this review is not limited to only two echelons.

Relating to periodic joint replenishment policies on multi-item multi-echelon inventory system, Qu et al. [28] dealt with an inbound material-collection problem. A central warehouse sends an uncapacitated vehicle to collect multiple items at geographically dispersed suppliers in a stochastic setting. They developed an integrated inventory and transportation system for joint replenishment with a modified periodic $MP(R_j,T)$ inventory policy originally proposed by Atkin and Iyogun [47]. Any item j belonging to a base set is brought up to its R_j at every review interval T, while other items are brought up the their level R_j at every k_jT time units where k_j is an integer value. Since the problem only focuses on holding stock at the warehouse echelon, so it is able to directly apply the $MP(R_j,T)$ inventory policy which was initiated for a multi-item single-location inventory problem. A heuristic decomposition method was proposed to solve the problem by separating the model into two sub-problems namely conventional inventory and vehicle routing models. This modified periodic inventory policy has been extended into Zhou et al. [33] for controlling all inventories on the multi-echelon system. An algorithm designed by Genetic Algorithm (GA) is used for solving the problem.

Sindhuchao [29] also studied an inbound material-collection problem with capacitated vehicle. The system consists of a set of geographically dispersed suppliers producing one or more non-identical items and a central warehouse stocking these items. The problem is to partition items into a number of subsets. For each item, the replenishment quantity and the replenishment interval must be determined along with the efficient route for the vehicle. Thus, each subset concerns the same replenishment interval of all items and the aggregated replenishment quantity for all items in a subset. The integrated inventory-transportation problem was formulated as a set partitioning problem and a mathematical programming approach was developed for coordinating inventory and transportation decisions. Sindhuchao [29] decomposed problem into lot sizing problem and Vehicle routing problem (VRP), then various heuristic algorithms were used to solve this problem in small problem. See more review of inventory and transportation problem considering multiple items in Moin, and Salh [121].

Other literatures have been associated with the independent ordering policies under joint constraints for multiple items. For example, Cohen et al. [26] developed a multi-echelon inventory model for the IBM network in the United States. They developed and implemented a system called "Optimizer" to determine the inventory policy setting for each part at each location employing the (s,S) policy. Joint service constraint for a product, which is composed of multiple parts, is concerned. They considered holding costs, replenishment costs, and emergency shipments. To solve the problem, they decomposed the model development into three stages; a one-part one-location model, a multi-product one-location model, and a multi-product multi-echelon model. Under decomposition, each facility is modeled under the assumption of ample supply at its supplier. Hopp et al. [27] and Al-Rifai and Rosetti [30] considered a system involving a target level on the aggregate ordering frequency to determine the (r,Q) policy parameters. Topen et al. [31, 32] considered a multi-item two-echelon inventory system in which the central warehouse operates under the (r,Q) policy, and each local warehouse implements one-for-one replenishment policy. The objective is to determine the inventory policy parameters minimizing the expected total system-wide cost subject to an aggregate mean response time constraint.

According to a few literatures studied coordinated ordering decision under the continuous replenishment and stochastic demand on OWNR, it is an open research area for the development of the can-order policies into more complex system.

2.3 Modeling and Solution Approaches

Previously, we reviewed a lot of literatures on various kinds of problem including the multiple-item single-location inventory problem, the single-item multi-echelon inventory problem, and the multi-item multi-echelon inventory problem. Then, this section emphasizes on an interesting issue about modeling and solution approaches for determining the can-order policy parameters.

Modeling the can-order policies can be classified into two approaches. The first approach is decomposition of a multi-item model into m single-item models [34, 46, 51-56, 71, 73] and the second approach considers a multi-item model [22, 48, 72]. The decomposed model has been extensively used in various literatures since it can reduce the dimension of search space. Focusing on a multi-item model, Kayiş et al. [72] concerned only two items, so it was able to formulate on a semi-Markov model without decomposition. van Eijs [48] and Özkaya [22] considered the multi-item models with the fixed value of the can-order policy c = S - 1. The dimension of search space can be reduced. Interesting that van Eijs [48] used decomposition technique into search algorithm instead, this can reduce the dimension of search space as well.

Relating to search algorithm, iterative procedure is the most common approach to determine the appropriate inventory policy parameters. Iterative procedure is adopted to improve all inventory policy parameters determined from each iteration. After an iteration is executed, some considered values are updated for using in the next iteration. Terminate condition can employ when 1) inventory policy parameters are unchanged from previous adjacent iteration, 2) the current total cost is not reduced by more than pre-specified tolerance value as comparing to the last total cost considered as a minimum cost of previous iteration, or 3) number of iterations are exceeded the setting if computational time is too long.

One-dimensional searches have been utilized together with iterative procedure. Exhaustive search (enumerative search) was typically used to search inventory policy parameters within a range of minimum and maximum values. This search seems not to enhance the heuristic algorithm in the aspect of computational time, since all possible values in the range must be considered. Advantageously, the best solution can be thoroughly determined. For two-echelon system, Özkaya [22] employed the exhaustive search into the special can-order $(s, S-1, S)$ policy.

Another interesting search algorithm is “golden section search” which is a search method for finding extremum of a unimodal function in the case of non-derivative function. It is a simple and efficient method by successively narrowing the range of search space until the desired accuracy in minimum value of the objective function is achieved. A golden ratio, which is a constant reduction factor for the size of the interval, is utilized to maintain the successive range of dynamic triples of points (i.e. upper point, middle point, and lower point). Advantageously, each successive range we only want to perform one new function evaluation. From this algorithm, we can obtain the best policy parameters provided the satisfying total cost with the saved computational time. van Eijs [48] used the golden section search to determine the best value of Δ which is equal to $S-s$. Recently, we found an interesting work of Nagasawa et al. [77]. They applied genetic algorithm (GA) to determine the can-order level on given s and S. Further details about one-dimensional search can be seen in, for example, Antoniou and Lu [122], Rios and Sahinidis [123].

2.4 Conclusion

This chapter reviewed previous researches on two major areas of problem: joint replenishment problem and multi-echelon inventory problem. We demonstrated a review of according to various joint inventory policies, specifically the can-order policies developed by several approaches and other interesting policies. Comparative analyses among these joint inventory policies were provided.

For multi-echelon inventory problem, the literatures were divided into the single-item models and the multi-item models. We identified two common types of inventory policies on the single-item models: the order-up-to (base-stock) policies and the batch-ordering policies. Additionally, continuous joint replenishment policies were raised into OWNR but there have been a few of literatures studied on this kind of system. For the multi-item models, Number of researches on multi-item multi-echelon inventory problem has been much smaller than the one on a single item case. The existing literatures could be classified into two groups of multi-item problem: coordinated ordering decision and joint constraint. For coordinated ordering decision, periodic joint replenishment policies were conducted. Some works integrated inventory and transportation problems. However, the continuous joint replenishment policies have not yet been found to employ into multi-item multi-echelon inventory problem.

In the last section, we summarized the interesting modeling and solution approaches for the can-order policies. Modeling the can-order policies could be classified into two approaches. The first approach was decomposition of a multi-item model into m single-item models and the second approach considered a multi-item model with either the fixed value of the can-order level at S - 1 or only two items. With regard to search algorithm, iterative procedure was the most common approach to determine the appropriate inventory policy parameters. One-dimensional searches for non-derivative function were utilized, such as exhaustive search and golden section search.

It is interesting that a few literatures studied coordinated ordering decision under the continuous replenishment and stochastic demand on OWNR. This is a great opportunity for the development of the can-order policies into more complex system so as to fulfill the knowledge in the area of inventory problem.

CHAPTER III

THE CAN-ORDER POLICY FOR SINGLE-ITEM TWO-ECHELON INVENTORY SYSTEM WITH ZERO LEAD TIME

This chapter is related to the 1st phase of dissertation methodology. It is the most important chapter to build up a basic knowledge of the can-order policy which is used throughout the dissertation. For phase I, we study the basic model for the can-order policy on OWNR with single item and zero-lead time consideration. The system just has an interaction among retailers without joint ordering decision at the warehouse echelon. The objective of this phase is to gain the insight of such policy on OWNR, and then to develop the heuristic approaches for determining the appropriate inventory policy setting.

3.1 Problem Description

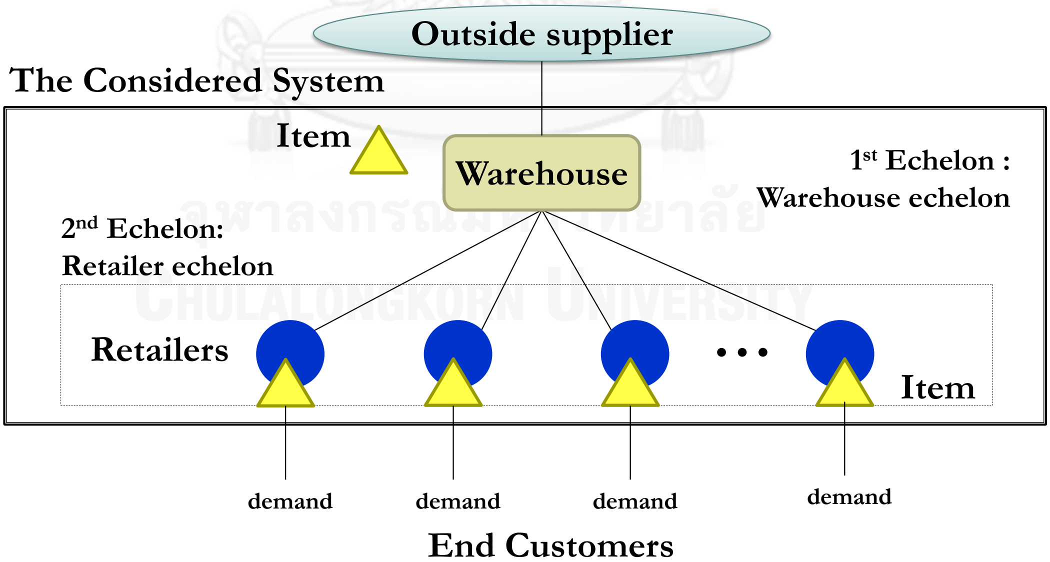


Figure III-1 Single-item two-echelon inventory system with zero lead time

The system consists of a warehouse and multiple retailers with single commodity. Let n denote number of retailers and i denote the location i = {0, 1, 2, ..., n} where the warehouse is set by i = 0 and the retailer $i \in N$, N = {1, 2, ..., n}. The warehouse is assigned in the first echelon called warehouse echelon, and all retailers are assigned in the second echelon called retailer echelon. Demands come from each retailer's customers defined as end customers. The warehouse and multiple retailers are cooperated as a single firm to concern the total system-wide cost under global information and centralized control. The warehouse is available to hold inventories for supplying all retailers' orders. Inventories at the warehouse are fulfilled by an outside supplier whose ample stock is not considered in the problem. The warehouse distributes all required quantities to the retailers in a single trip without splitting lot. It is supposed that uncapacitated vehicle is available to supply all required quantities in the order. Multiple retailers have their own inventories to serve their customer demands. Poisson demand is assumed to represent the customer demands, denoted by λ_i which is a constant mean of customer demand at retailer i.

Regarding the can-order (s_i, c_i, S_i) policy applied to the system, it has two reorder points: the must-order level s_i providing normal replenishment, and the can-order level c_i making special replenishment. Special replenishment is an opportunity of a joint replenishment which a retailer is faced with when other retailers reach their must-order levels. When the inventory position of any retailer drops to or below its must-order level s_i, an order is triggered to create normal replenishment. Then, other retailers in the system can also be included by this order if their inventory position is at or below its can-order level c_i; a special replenishment is occurred. All the involved retailers' inventories are fulfilled from the warehouse to their own order-up-to level S_i. Considering single commodity, the warehouse modifies the can-order policy to a traditional (s_0, S_0) policy by setting its can-order level equals its must-order level. The warehouse issues an order when its inventory position reaches its must-order level s_0. Then the outside supplier will replenish the warehouse's inventory to its order-up-to level S_0. The warehouse places an order to the outside supplier if and only if retailer echelon triggers an order to the warehouse. We differentiate between order cycle at retailer echelon and order cycle at warehouse echelon by defining "dispatch cycle" and "replenishment cycle" for retailer echelon and warehouse echelon, respectively.

The system considers all inventory costs at both echelons. The inventory costs consist of 1) The holding costs at the warehouse and all retailers, 2) The major ordering costs for warehouse echelon and retailer echelon, and 3) The minor ordering costs for retailer echelon. The holding cost occurs at each location having physical stock. The total holding cost over the time period at location i (HC_i) can be determined from the unit holding cost (h_i) and the accumulated inventory over the time period (INV_i). The major ordering cost is the fixed cost occurring once an order is triggered. This cost includes administrative costs, material handling costs, and transportation costs which do not depended on the number of retailers in the order. So, the retailers in the system can share the major ordering cost together for replenishing in one round trip. The total major ordering cost over the time period at retailer echelon (MJ_r) is the retailers' major ordering cost per an order (K_r) multiplied by the number of dispatch cycle (ND_r). Similarly, the total major ordering cost over the time period at warehouse echelon (MJ_w) is the multiplication of the warehouse' major ordering cost per an order (K_w) and the number of replenishment cycle (NR_w). The minor ordering cost is an additional cost of each retailer when replenishing their inventories, such as additional transportation cost relating to distance or other charges. This cost depends on the number of involved retailers in that order. The total minor ordering cost over the time period (MN_r) is accumulated from the involved retailers in each order multiplied by its minor ordering cost of retailer i (κ_i) over the time period. Prior works on coordinated ordering decision ignored this additional cost in spite of the fact that this additional cost directly affects the inventory policy setting [48, 56, 60].

The concept of the can-order policy is balancing among reduced major ordering costs, varied minor ordering costs, and increased holding costs. Reduced major ordering cost occurs if special replenishment is included in an order. On the other hand, from special replenishment there is a residual stock [48] which is a stock left above the must-order level at the order-triggered point. Then, the involved retailers have to hold more stock increasing the holding cost. Meanwhile, the minor ordering costs can be either reduced or increased depending on order frequency at each retailer. Hence, we have to consolidate all relevant costs to determine the appropriate inventory policy setting under the total system-wide cost minimization.

It is, however, difficult to deal with the problem mainly because of demand uncertainty, variation of retailers' order quantity, retailer's two-order point setting, and order time synchronization at all locations. We simplify the problem by assuming zero lead time. Retailers' order is instantly dispatched from the warehouse. All retailers' must-order levels are then equal to zero (s_i = 0, $i \in N$), since shortage at the retailers are not allowed. The warehouse's order is also replenished from the outside supplier immediately. In this case, warehouse's must-order level is equal to -1 because the warehouse is allowed to hold zero inventory level until the next replenishment will be issued. This uses the same setting as Gou et al. [14]. It can help the warehouse not to keep the excessive stock waiting for the next dispatch to retailer echelon. Therefore, decision variables are c_i, S_i and S_0. This is a simple case of the can-order policy on OWNR.

The notations and problem formulation are demonstrated as follows:

n = Number of retailers in the system
i = Index of the location i; the warehouse i = 0 and the retailer $i \in N$
T = The time period considered in the problem (time units)
s_0 = The must-order level at the warehouse (units);
(Assign s_0 = -1 from the zero-lead time assumption)
S_0 = The order-up-to level at the warehouse (units)
s_i = The must-order level at retailer i (units);
(Assign s_i = 0 from the zero-lead time assumption)
c_i = The can-order level at retailer i (units)
S_i = The order-up-to level at retailer i (units)
λ_i = Demand rate of retailer i (units/time unit)
h_0 = The unit holding cost per unit time at the warehouse (\$/unit – time unit)
h_i = The unit holding cost per unit time at retailer i (\$/unit – time unit)
K_w = The warehouse's major ordering cost per a replenishment cycle (\$/time)
K_r = The retailers' major ordering cost per a dispatch cycle (\$/time)
κ_i = The minor ordering cost at retailer i (\$)
$TC(c_i, S_i, S_0)$ = The total system-wide cost per unit time (\$/time unit)
HC_i = The total holding cost at location i over the time T units (\$)
MJ_r = The total major ordering cost at retailer echelon over the time T units (\$)

MN_r = The total minor ordering cost at retailer echelon over the time T units ($)

OC_w = The total major ordering cost at warehouse echelon over the time T units ($)

INV_i = The accumulated inventory over time period at location i (unit – time unit)

ND_r = The total number of dispatch cycle over the time T units (times)

NR_w = The total number of replenishment cycle over the time T units (times)

$\delta_{(i,j)}$ = An indicator which equals 1 when retailer i is included in the dispatch cycle j and equals 0 otherwise

Objective function:

Minimize $$TC(c_i, S_i, S_0) = \frac{\left(\sum_{i=0}^{n} HC_i + MJ_r + MN_r + MJ_w\right)}{T} \quad (3.1)$$

where $$HC_i = h_i \times INV_i \quad (3.2)$$

$$MJ_r = K_r \times ND_r \quad (3.3)$$

$$MN_r = \sum_{j=1}^{ND_r} \sum_{i=1}^{n} \delta_{(i,j)} \kappa_i \quad (3.4)$$

$$MJ_w = K_w \times NR_w \quad (3.5)$$

The objective function of the problem is to minimize the total system-wide cost per unit time. Since s_i and s_0 can be given by the zero-lead time assumption, the total system-wide cost per unit time can be a function of only three decision variables: c_i, S_i, S_0. This is able to simpler manipulate the problem. However, the problem remains the complications, such as demand uncertainty, variation of retailers' order quantity, and order-time synchronization at all locations.

3.2 Research Methodology

Dealing with the complication of the problem, the optimal solution cannot be simply derived from an analytical approach. Hence, we initially study the can-order policy on OWNR by using computer simulation. Computer simulation is an efficient approach representing the inventory process even in the complicated system [14, 88]. The preliminary study leads us to develop a heuristic approach. In addition, from the simulation we can determine the best-known solution used to measure the proposed heuristic approach's performance.

3.2.1 Computer simulation

The computer algorithm representing the inventory process is illustrated in Fig. III-2. The inputs for simulating the system can be divided into three groups as follows:

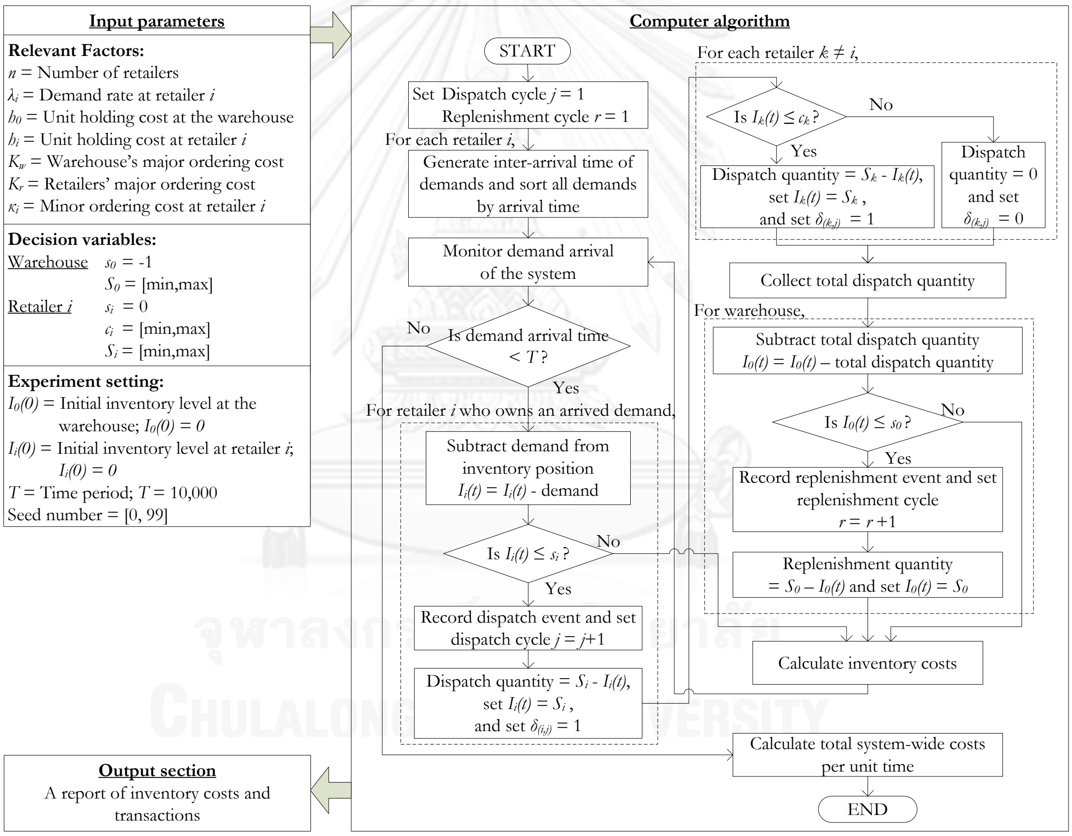


Figure III-2 The computer algorithm for simulation of Phase I

1) Decision variables (c_i, S_i, S_0): Each variable is inputted as a range of minimum and maximum values. A combination of (c_i, S_i, S_0) is called "solution". A solution provides a value of the total system-wide cost and its transaction (e.g. number of dispatch cycles, number of replenishment cycles).

2) Relevant factors (i.e. cost parameters, demand rates, and number of retailers): We set a combination of relevant factors to “scenario”. A scenario contains different solutions. The best solution providing the minimum total system-wide cost is selected for each scenario.

3) Experiment setting: Let $I_i(t)$ denote the inventory level of location i at time t. At the beginning of running period, all locations’ initial inventory levels start at zero, $I_i(0)$ = 0. According to the pilot testing, the stability of the system occurred after the first 8,000 running periods. So, we chose 10,000 running periods to provide the steady state for the system. The difference of computational time between the 10,000-running period and some other running periods in the range (8000, 10000) is too small and the 10,000-running period is a sufficient number to assure of the stability of the system. Additionally, various seed numbers are tested to verify the solutions since different seed numbers generate different inter-arrival time sets.

Finally, we obtain a report of the inventory costs and its transaction. In consequence, we can find the minimum total system-wide cost for each range of decision variables inputted under a given scenario.

3.2.2 The best solution finding

The best solution finding is composed of two steps: Input parameters and output validation. The following sub-sections explain each step in sequence.

3.2.2.1 Input parameters

First of all, we randomly select a seed number between [0, 99] to use for first replication (i.e. a replication comes from a seed number). Decision variables are inputted as a range of minimum and maximum values. The range is dynamic depending on our setting. In the experiment, we set the width of range are 5 units for c_i and S_i and 20 units for S_0. Since over 5 units of c_i and S_i creates multiplied combinations spending more running time. Whereas S_0 range is larger because S_0 linearly creates combinations. The first range can be set from the initial point of S_0 and S_i calculated by $S_0 = \sqrt{2K_w \sum_{i \in N} \lambda_i \Big/ h_0}$ and $S_i = \sqrt{2K_r \lambda_i / h_i}$ due to the zero-lead time assumption and the concept of economic order quantity. For example, initial S_0 = 45 and initial S_i = 14, the first ranges are identified as $S_0 \in$ [41, 60], $S_i \in$ [11, 15], and $c_i \in$ [10, 14].

The next step is the process of moving the ranges until the solution seems to be worse continuously. The S_0 range is moved upward and downward by fixing the range at all retailers. For example, the next range varied $S_0 \in$ [21, 40], and fixed $S_i \in$ [11, 15] and $c_i \in$ [10, 14]. Therefore, minimum and maximum values for inputting can be changed for each round of the simulation. Later, we determine the c_i and S_i ranges at the retailer i by keeping the same range of S_0 and the c_j and S_j ranges at the retailer $j \neq i$. S_0, c_i and S_i ranges are changed repeatedly. We select the best solution providing the minimized total system-wide cost for the first replication. After that, the validation process showed in the next part is utilized to get the typical best solution.

3.2.2.2 Output validation

The typical best solution is a representative of the best solutions from various replications. We define the typical best solution as “the best-known solution” to generally use in later sections. Since abundant combinations are run in the first replication, in this process we can reduce unnecessary ranges by starting at the best solution’s range from the first replication. By this process, we can find the best solution for other replications faster. If there is an error from the first replication, cross-checking is occurred.

In the pilot testing (10 scenarios), we tested on ten random seed numbers to determine the best solution for each seed number. We found that the best-known solution appeared since the first three random seed numbers were conducted. However, we chose to test on five random seed numbers instead to confirm the experimental results. Instead of a number of the experiments, we could save the computational time on five random seed numbers for determining each seed number’s best solution.

Consequently, we test another four replications on different seed numbers (after the first replication has been done previously). The first two seed numbers are randomized, whereas the last two seed numbers are fixed. We use this method to study two dimensions of the best solutions.

- For the first dimension, we aim at studying mean and variation of all best solutions' the total system-wide cost under the same scenario. The result of this study should provide indifferent total system-wide costs of the group if the best solutions are in the steady state. This process is to confirm that our experiments are conducted in the appropriate condition.
- For the second dimension, we fix seed numbers at 1 and 2 to study characteristics of the inventory policy parameters on different scenarios. For example, we vary the holding cost ratio (h_0 / h_i) from 0.1 to 1 and then we monitor trend of the inventory policy parameters on each scenario under the same seed number.

Most replications provide the same best solution; however, some different solutions can be appeared. Then, for each best solution we determine the average total system-wide cost by additional 10 random seed numbers. The best-known solution is provided by the best solution with the minimum of average total system-wide cost.

We handle all experiments by using methodology of the best solution finding as mentioned above. To gain more efficiency, all experiments are simultaneously run on 8 computers (Intel® Core™ i7-2600 CPU@ 3.4GHz. RAM 8 GB 64-bit Operating System). Simulation programming uses visual C# (2010). By the aforementioned methodology coupled with the efficient computers and programming, we are able to conduct various experiments.

3.2.3 Performance measurement

We use two measurements in the dissertation. The first one is a cost-saving measurement. We use it for evaluating the performance of the can-order policy as comparing to an independent (s, S) policy (called SI case in the dissertation). SI case meets stochastic demand and independent replenishment where each retailer is dispatched individually, so the major ordering cost of each retailer occurs without sharing. We determine the best solution of SI case by utilizing computer simulation. According to zero lead time, all retailers' reorder points (s_i)

are equal to zero. Meanwhile, warehouse's reorder point (s_0) is less than zero owning to retailers' batch-order size (s_0 = -1). Zero inventory position at warehouse can be occurred until the next order is replenished from the outside supplier. Thus, we can determine only the base-stock levels (S_i) for all retailers, and the base-stock level for the warehouse (S_0). From simulation, SI case can use the data from the can-order policy by c_i = -1.

Cost saving can be calculated by comparing the can-order policy with the SI case using the following equation:

$$Cost\,Saving\,(C.S.) = \frac{(TC^{(SI)} - TC^{(CAN)}) \times 100}{TC^{(SI)}} \quad (3.6)$$

where $TC^{(SI)}$ and $TC^{(CAN)}$ are the average total system-wide cost per unit time of SI case and the can-order policy, respectively.

Since this paper's objective is to propose a heuristic approach for setting the appropriate can-order policy, the best-known solution is utilized to compare with the heuristic's best solution. Heuristic's performance is measured in terms of the cost gap calculated from the following equation.

$$Cost\,Gap\,(C.G.) = \frac{(TC^{(HRT)} - TC^{(BS)}) \times 100}{TC^{(BS)}} \quad (3.7)$$

where $TC^{(HRT)}$ and $TC^{(BS)}$ are the average total system-wide cost per unit time of the heuristic approach and the average total system-wide cost per unit time of the best-known solution, respectively.

3.3 Preliminary Analysis

In the preliminary study, our experiments were conducted to study the relationship between relevant factors on 253 scenarios as showed in Table III-1 and Table III-2.

Table III-1: Numerical input for preliminary experiment under identical retailers

The asterisk (*) in the table means that parameter is varied.

Scenario No.	Fixed Parameters								Varied Parameters
	K_w	K_r	κ_i	h_0	h_i	h_0/h_i	λ_i	n	
1) Relationship between h_0 and h_i (80 scenarios)									
1-50	100	50	0	*	*	*	20	2	$h_i \in$ {10, 25, 50, 100, 250}; $h_0/h_i \in$ {0.1, 0.2, ...,1}
51-80	100	50	0	*	*	*	20	2	$h_i \in$ {0.1, 0.5, 1, 2.5, 5}; $h_0/h_i \in$ {0.1, 0.3, 0.5, 0.7, 0.9, 1}
2) Relationship between h_0, h_i and K_r (20 scenarios)									
81-92	100	*	0	*	25	*	20	2	$K_r \in$ {10, 90}; $h_0/h_i \in$ {0.1, 0.3, 0.5, 0.7, 0.9, 1}
93-100	100	*	0	*	10	*	20	2	$K_r \in$ {10, 90}; $h_0/h_i \in$ {0.2, 0.4, 0.6, 0.8}
3) Relationship between h_0, h_i and K_w (20 scenarios)									
101-112	*	50	0	*	25	*	20	2	$K_w \in$ {75, 200}; $h_0/h_i \in$ {0.1, 0.3, 0.5, 0.7, 0.9, 1}
113-120	*	50	0	*	10	*	20	2	$K_w \in$ {125, 250}; $h_0/h_i \in$ {0.2, 0.4, 0.6, 0.8}
4) Relationship between h_0, h_i, and K_w / K_r (14 scenarios)									
121-134	*	50	0	*	*	0.5	20	2	$K_w / K_r \in$ {1.5, 3, 4, 5, 10, 100, 1500}; $h_i \in$ {1, 25}
5) Relationship between h_0, h_i, and λ_i (10 scenarios)									
135-142	100	50	0	*	25	0.5	*	2	$\lambda_i \in$ {0.5, 1, 3, 5, 10, 40, 100, 500}
143-144	100	50	0	*	10	0.2	*	2	$\lambda_i \in$ {0.5, 10}
6) Relationship between h_0, h_i, λ_i and n (10 scenarios)									
145-148	100	50	0	*	25	0.5	20	*	$n \in$ {4, 8, 12, 20}
149-154	100	50	0	*	10	0.2	*	*	$\lambda_i \in$ {0.5, 10}; $n \in$ {4, 8, 12}
7) The effect of κ_i (54 scenarios)									
155-208	100	50	*	*	10	*	*	*	$\kappa_i \in$ {5, 10, 25}; $\lambda_i \in$ {0.5, 20}; $h_0/h_i \in$ {0.2, 0.4, 0.6}; $n \in$ {2, 4, 8}

Table III-2: Numerical input for preliminary experiment under non-identical retailers on two-retailer scenarios and three-retailer scenarios

All scenarios set identical cost components by K_w = 100, K_r = 50, κ_i = 0, h_0 = 2, and h_i = 10. Demand rate ratio be abbreviated to "DRR" in the table.

	Demand Rate					Demand Rate					Demand Rate			
	λ_1	λ_2	λ_3	DRR		λ_1	λ_2	λ_3	DRR		λ_1	λ_2	λ_3	DRR
1	20	20	-		16	20	1	-		31	20	0.67	0.67	30
2	10	10	-	1	17	10	0.5	-	20	32	20	0.5	0.5	40
3	40	40	-		18	40	2	-		33	20	20	10	2
4	20	10	-		19	20	0.67	-		34	20	20	5	4
5	10	5	-	2	20	10	0.33	-	30	35	20	20	2.5	8
6	40	20	-		21	40	1.33	-		36	20	20	2	10
7	20	5	-		22	20	0.5	-		37	20	20	1	20
8	10	2.5	-	4	23	10	0.25	-	40	38	20	20	0.67	30
9	40	10	-		24	40	1	-		39	20	20	0.5	40
10	20	2.5	-		25	20	20	20	1	40	20	10	5	2, 4
11	10	1.25	-	8	26	20	10	10	2	41	20	10	0.5	2, 20, 40
12	40	5	-		27	20	5	5	4	42	40	20	10	2, 4
13	20	2	-		28	20	2.5	2.5	8	43	40	20	1	2, 20, 40
14	10	1	-	10	29	20	2	2	10	44	20	2	0.5	4, 10, 40
15	40	4	-		30	20	1	1	20	45	40	4	1	4, 10, 40

We primarily analyze the experiments on identical retailers to study the effect of the relevant factors on the can-order policy. Specifically, from the existing literatures, the ratio of the major ordering cost and the minor ordering cost is one of the most significant factors for the can-order policy's performance, since such ratio affects the can-order level c_i to create a combination of retailers in an order. Therefore, we considered the experiments on identical retailers in case of zero minor ordering cost and non-zero minor ordering cost. To extend the experiment on non-identical retailers, we aimed at studying the can-order policy on the retailers' different demand rates because in reality we frequently encounter such situation. In addition, non-identical demands can create the different discount opportunities from the shared ordering cost. So, it is interesting to investigate and this inquiry has not been studied in the existing literatures.

Relating to the experimental design, we did not test on full combination of all factors, since each experiment was focused on different parameters. For example of scenario 1 – 50, we varied $h_i \in$ {10, 25, 50, 100, 250} and $h_0/h_i \in$ {0.1, 0.2, ..., 1} and fixed the other parameters to study the relationship between h_0 and h_i. In multi-echelon inventory problem, the relationship between h_0 and h_i directly affects stocking decisions of the warehouse and of the retailers. After first 50 scenarios, we also tested on the lower $h_i \in$ {0.1, 0.5, 1, 2.5, 5} but h_0/h_i can be reduced to smaller set $h_0/h_i \in$ {0.1, 0.3, 0.5, 0.7, 0.9, 1} as we found similar characteristics on the solutions. Similarly, other parameters were varied corresponding with the purpose of any experiment.

In reality, the ratio of h_0/h_i always appeared in the existing literatures is likely not over 1 since value of product increases from the warehouse echelon to retailer echelon according to, for instance, additional operations cost charged into product price, retail store rental price. If $h_0/h_i > 1$ means that all inventories should be hold at retailer echelon to reduce an expensive holding cost at warehouse echelon, except the case that there is any constraint for the warehouse's supplies. Similarly, the ratio of K_w/K_r is always equal or more than 1 because, in fact, warehouse's ordering cost deals with the external firms so that administrative and transportation costs are always more than internal management costs.

According to a study of non-identical demand rates for two-retailer scenarios and three-retailer scenarios as showed in Table III-2, we defined the demand rate ratio as the proportion of different retailers' demand rates. Demand rate ratio is used to analyze the effect of non-identical demand rates on the can-order policy.

For two-retailer scenarios, demand rate ratio can be simply identified as a proportion of the first retailer's fixed demand rate to the second retailer's varied demand rate (e.g. $\lambda_1 = 20, \lambda_2 = 10$ then the demand rate ratio is equal to 2).

For three-retailer scenarios, demand rate ratio is formed into three patterns:

- A proportion of a retailer's fixed demand rate to the other retailers' identically varied demand rates (e.g. fixed $\lambda_1 = 20$ and varied $\lambda_2 = 10, \lambda_3 = 10$; the demand rate ratio is equal to 2)
- A proportion of a retailer's varied demand rate to the other retailers' identically fixed demand rates (e.g. fixed $\lambda_1 = 20, \lambda_2 = 20$ varied $\lambda_3 = 10$; the demand rate ratio is equal to 2)

- A proportion of one retailer's varied demand rate to another retailer's varied demand rate (e.g. $\lambda_1 = 40, \lambda_2 = 4, \lambda_3 = 1$; three demand rate ratios for this case – λ_1 / λ_2 = 10, λ_1 / λ_3 = 40, and λ_2 / λ_3 = 4)

For each scenario, we found that the best solutions from various seed numbers had the same means proved by one way ANOVA (single factor) with 95% confidence interval[4]. In addition, trend of the inventory policy parameters seems not different even if random seed numbers were utilized. For example, a scenario at K_w = 100, K_r = 50, κ_i = 0, h_0 = 1, h_i = 10, λ_i = 20, n = 2, we found two best solutions from five replications. The best solutions (S_0, c_i, S_i) are (67,11,12) for replication 1, 2, 4, 5 and (67,10,12) for replication 3. Then, we determined ten total system-wide costs of two best solutions as demonstrated in Table III-3. We used these data for ANOVA. The ANOVA result is depicted in Table III-4. From Table III-4, F-critical value is less than F value, so we accept null hypothesis that all best solutions have indifferent means.

Table III-3: Ten total system-wide costs of two best solutions

	Replication No. (with random seed numbers)									
Best Solution	1	2	3	4	5	6	7	8	9	10
(67,11,12)	329.80	330.35	330.26	330.46	330.24	330.16	330.52	329.26	330.62	329.39
(67,10,12)	330.47	329.37	330.51	330.06	330.32	329.19	330.34	328.76	331.20	331.18

[4] For each best solution, we used ten values of total system-wide costs from ten seed numbers. Such costs of all best solutions are analyzed by ANOVA. Function "DATA ANALYSIS" from Microsoft Excel 2010 was utilized for ANOVA testing.

Table III-4: An example result of ANOVA testing

SUMMARY

Groups	*Count*	*Sum*	*Average*	*Variance*
Solution 1: (67,11,12)	10	3301.066	330.1066	0.219774
Solution 2: (67,10,12)	10	3301.407	330.1407	0.657857

ANOVA

Source of Variation	*SS*	*df*	*MS*	*F*	*P-value*	*F-critical*
Between Groups	0.005836	1	0.005836	0.0133	0.909465	4.413873
Within Groups	7.898677	18	0.438815			
Total	7.904514	19				

The significant findings are classified into three groups: 1) the effect of the can-order policy, 2) comparative analysis with an independent policy, and 3) inventory policy characteristics. Thus,

3.3.1 The effect of the can-order policy

From the general concept of the can-order policy, the major and minor ordering cost and the holding cost are traded off. The can-order level affects reduced major ordering costs, varied minor ordering costs, and increased holding cost from special replenishment. Hence, it is important to find a balance among all inventory costs.

For the experiment of identical retailers with zero minor ordering cost, it shows that when c_i increases, the retailers' holding cost increases, while the retailers' total major ordering cost decreases. Then, the total inventory cost at retailers also decreases when c_i increases. For warehouse echelon, the value of c_i affects its dispatch quantity and frequency, and therefore affects the warehouse's inventory costs. At the low level of S_0, increasing c_i can reduce the warehouse's inventory costs since higher c_i generates lower dispatch frequency. This also causes lower replenishment frequency at the warehouse. However, there is no obvious pattern reflecting relationships between all decision variables at the high level of S_0.

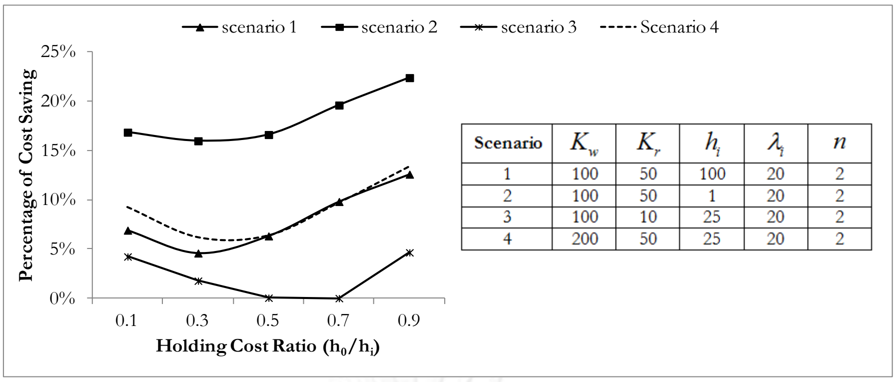
Considering the case of identical retailers with minor ordering cost, it directly affects the can-order level. If the minor ordering cost κ_i is large enough when comparing with the major ordering cost K_r, the retailers' total minor ordering cost is observed to be a convex function of the can-order level c_i. The increase of c_i reduces the retailers' total minor ordering cost until a value of c_i, that cost is then increased when c_i is large as too many retailers are included in an order. The c_i value affects number of retailers jointly replenished in the order, so it influences the retailers' total ordering cost per order and the retailers' total holding cost per dispatch cycle.

With regard to the case of non-identical retailers, different retailer's demand rates were tested. The minor ordering cost is neglectable to study only impact of non-identical demand rates. When retailer's demand rates are significantly different, the retailers with higher demand rates attempt to reduce c_i in order to have less residual stock, while the retailers with lower demand rates attempt to increase c_i in order to have more joint replenishment opportunity. Hence, all the can-order levels have to be traded off between the cost of residual stock and the cost of joint replenishment.

As these results, the can-order policy has an effect on the inventory costs at both echelons. It is related to retailers' residual stock, dispatch quantity and frequency at retailers, as well as replenishment quantity and frequency at the warehouse. All are necessary to be traded off to determine the best solution.

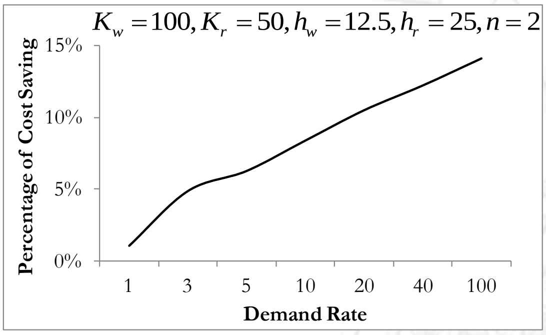
3.3.2 Comparative analysis

To study the can-order policy's performance and identify for which situation this policy is suitable, we compare the can-order policy with an independent (s, S) policy (called SI case). It has already been mentioned in section 3.2.3 in detail. From simulation, SI case can use the data from the can-order policy by c_i = -1. We vary a wide range of relevant factors according to various tested values showed in Table III-1 and Table III-2. Cost-saving measurement can use Equation (3.6). Figure III-3, Fig.III-4, and Fig.III-5 show experimental results in cases of identical retailers and non-identical retailers, in sequence.

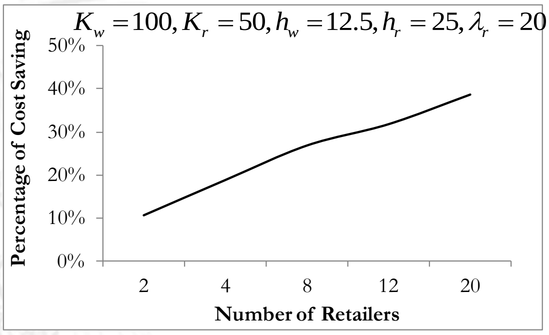


Scenario	K_w	K_r	h_i	λ_i	n
1	100	50	100	20	2
2	100	50	1	20	2
3	100	10	25	20	2
4	200	50	25	20	2

(a) Cost parameters



(b) Demand rates



(c) Number of retailers

Figure III-3 The cost saving of the can-order policy: Identical retailers with zero minor ordering cost

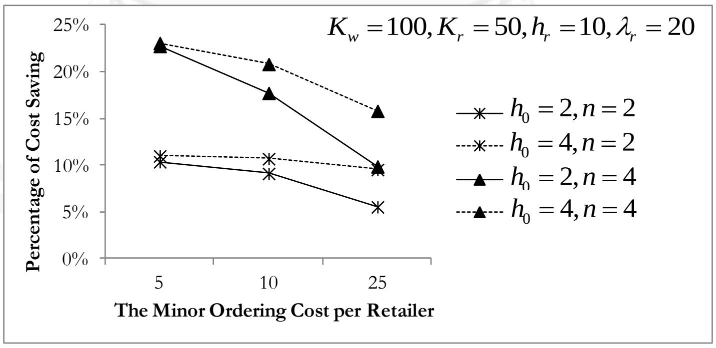
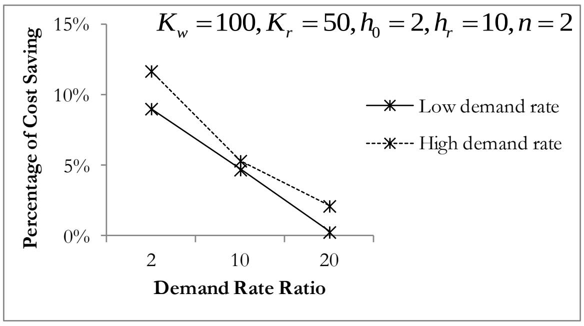
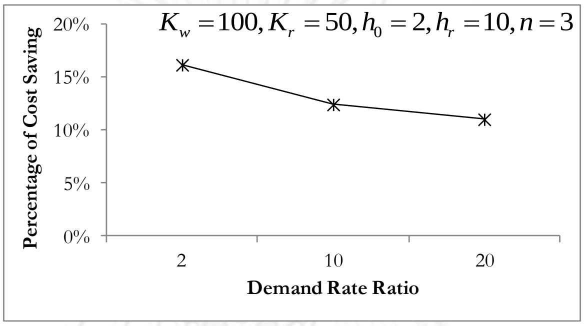


Figure III-4 The cost saving of the can-order policy: Identical retailers with non-zero minor ordering cost



(a) Two retailers



(b) Three retailers

Figure III-5 The cost saving of the can-order policy: Non-identical retailers

According to the experiments, we found that the can-order policy could reduce the total system-wide cost from the SI case for all scenarios. The amount of cost saving is depends on scenarios. The best (or useful) scenarios are identified that can save the greatest amount of the total system-wide cost. In the opposite way, the can-order policy does not outperform SI case when the total system-wide costs obtained by the can-order policy and by SI case are not different. The independent ordering decision by SI case should be a satisfactory policy for ease of control parameter determination.

The best scenarios for the can-order policy are addressed as follows:

- **Identical retailers**: The best scenario is when large K_r / h_i ratio, large K_r / κ_i ratio, high demand rate, and high number of retailers. High h_0 / h_i ratio is likely to gain more cost saving but it has to be high enough for trading off with other relevant factors.

Large K_r / h_i ratio and large K_r / κ_i ratio creates a large shared major ordering cost among retailers. High number of retailers also increases the opportunity to share the major ordering cost. High demand rate allows high level of

S_i so that the opportunity of joint replenishment can increase (i.e. longer dispatch cycle time makes the other retailers have a more change to include in an order), However, in the opposite way the can-order policy seems to be useless when small K_r / h_i ratio and low demand rate because both factors reduce the opportunity to share the major ordering cost. Both factors make very low level of S_i from which short dispatch cycle time happens. Total system-wide costs obtained by the can-order policy and by the (S_i, S_0) policy are not different.

- Non-identical retailers: The can-order policy is useful when low demand rate ratio. From low demand rate ratio, each retailer can create its own normal replenishment[5] nearly be about the same cycle time. So, it has more opportunity to share the major ordering cost with small residual stock[6]. On the other hand, high demand rate ratio might influence the can-order policy to be useless since a huge difference of normal replenishment cycle times reduces the sharing opportunity. We found that a retailer with higher demand rate reduced the can-order level near to the must-order level in order to reduce its order frequency together with another retailer with lower demand rate. The retailer with high demand rate has to tradeoff between the reduced ordering cost and the increased residual stock.

Like identical retailers, high demand rate and high number of retailers increase the opportunity to share the major ordering cost. Each factor allows high level of S_i so that the opportunity of joint replenishment can increase.

The can-order policy builds up cost saving at over 30%, compared to SI case. So, the application of the can-order policy into OWNR is considerably valuable.

[5] When the inventory position of retailer i drops to or below its must-order level s_i, an order is triggered to create normal replenishment.

[6] Residual stock is a stock left above the must-order level at the order-triggered point.

3.3.3 Inventory policy characteristics

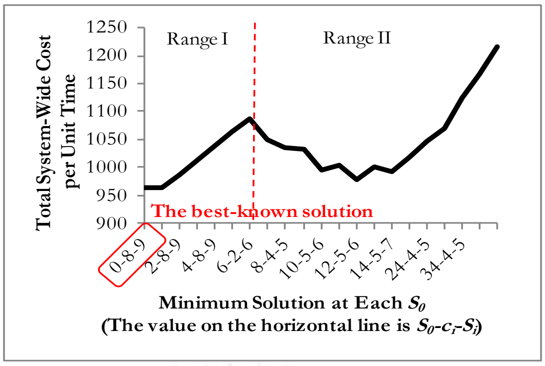
This section is separated into three sub-sections to elaborate the characteristic of each decision variable c_i, S_i, S_0 for the considered system.

3.3.3.1 The order-up-to level at the warehouse

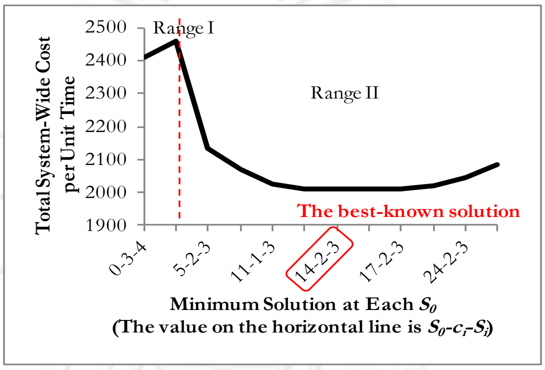
For a given S_0, we can find the solution of (c_i, S_i) providing the minimum average total system-wide cost as illustrated in Fig.III-6. There are two local minimum solutions located into two ranges: Range I – the solution occurs at S_0 = 0 and Range II – it occurs at S_0 > 0. For the Range I, S_0 starts from zero and then increases to reach the last value before the cost line turns to a convex function. For the Range II, it is defined after that last value to positive infinity. The best-known solution (global minimum solution) definitely occurs in either Range I or Range II.

For Range I, none of holding stock at the warehouse provides the lowest total system-wide cost since the increasing S_0 creates the excessive stock. Whenever retailer echelon triggers an order all excessive stock is consumed and the warehouse's must-order level is always reached. The warehouse is replenished every dispatch cycle; therefore, it is not necessary to keep stock waiting for the next dispatch cycle. For Range II, a trade-off between the increasing holding costs and the reduced ordering costs when increasing S_0 is occurred as found in the economic order quantity.

We can set S_0 = 0 when high h_0 / h_i ratio, since more stock creates more inventory cost (i.e. the increased holding cost is larger than the reduced ordering cost). However, there is a possibility that the best-known solution can move from Range I to Range II when relevant factor is changed, such as smaller h_0 / h_i ratio, higher K_w, or higher number of retailers since smaller h_0 / h_i ratio, higher K_w, or higher number of retailers affect the warehouse to hold inventories to reduce the frequency of replenishment.



(a) The best-known solution occurred in Range I

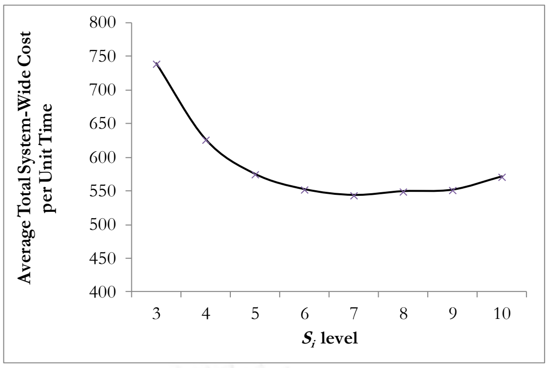


(b) The best-known solution occurred in Range II

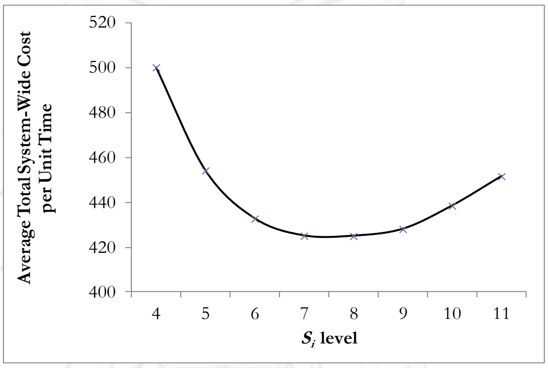
Figure III-6 Two ranges of the best-known solution

3.3.3.2 The order-up-to level at the retailers

When we fix inventory policy at the warehouse, the average total system-wide cost at retailer i is a convex (unimodal) function of S_i as showed in Fig. III-7. Figure III-7(a) and Fig.III-7(b) illustrate different scenarios but provide the same pattern. The convex function occurs from a trade-off between the increasing holding costs and the reduced ordering costs when increasing S_i, then the economic order quantity is determined.



(a) Scenario at K_w = 100, K_r = 50, κ_i = 0, h_0 = 2.5, h_i = 25, λ_i = 20, n = 2, S_0 = 48



(b) Scenario at K_w = 100, K_r = 50, κ_i = 25, h_0 = 2, h_i = 10, λ_i = 5, n = 4, S_0 = 27

Figure III-7 Convex function of S_i on given S_0

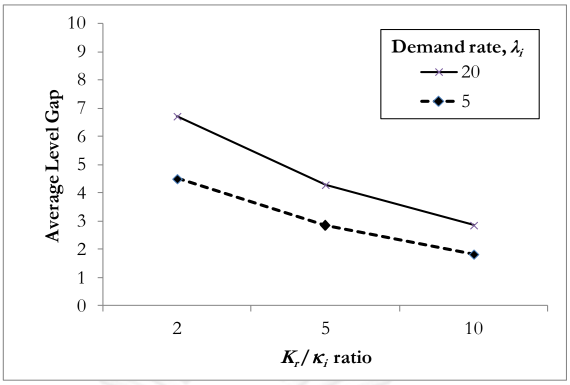
3.3.3.3 The can-order level at the retailers

From the existing literatures, the ratio of the major ordering cost and the minor ordering cost is one of the most significant factors for the can-order policy's performance, since such ratio affects the can-order level c_i to create a combination of retailers in an order. Therefore, we considered the experiments on identical retailers in case of zero minor ordering cost and non-zero minor ordering cost.

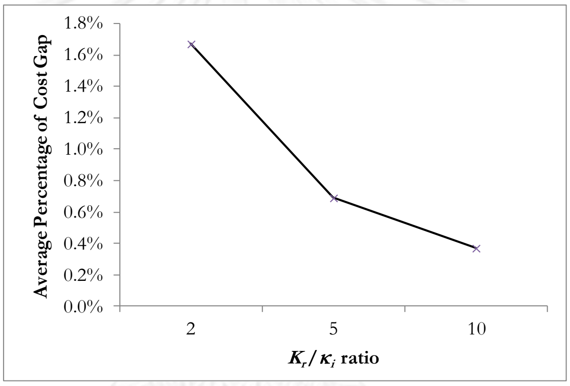
Considering the case of zero minor ordering cost (154 scenarios), a result demonstrates that 87.66% of all scenarios (135 scenarios) the value c_i^* = S_i^* - 1, where c_i^* and S_i^* denote the optimal can-order level and the optimal order-up-to level of retailer i. This result is consistent with the study of van Eijs [48] showed that when K_r / κ_i ratio is approaching infinity, then c_i^* = S_i^* - 1 for all items. It implies that all items are jointly replenished as soon as an item triggers an order. Other items are not ordered if there has been no demand after the preceding order. This concept's purpose is to most reduce the ordering cost from jointly replenishing all items in the order. For other 19 scenarios occurring the best solution at $c_i^* \neq S_i^*$ - 1, the result indicates that $TC^*_{(S_i-1)}$ is greater than TC^* 0.01% on average with a standard deviation 0.02% where TC^* is the optimal average total system-wide cost and $TC^*_{(S_i-1)}$ is the minimum average total system-wide cost of the solution at $c_i = S_i$ - 1.

In case of non-zero minor ordering cost (54 scenarios), smaller K_r / κ_i ratio influences father difference between c_i^* and S_i^* as showed in Fig.III-8(a). Since such difference can reduce the number of involved retailers in the order and dispatch quantity, but increase dispatch frequency. In multi-item single location problem, van Eijs [48] ruled that if K_r / κ_i ratio is less than 5, the can-order policy might not happen to be $c_i^* = S_i^*$ - 1. Additionally, high demand rate affects higher level gap between c_i^* and S_i^*. Comparing $TC^*_{(S_i-1)}$ and TC^*, the result indicates that $TC^*_{(S_i-1)}$ is greater than TC^* by 0.91% on average with a standard deviation of 1.85%. Smaller K_r / κ_i ratio increases cost gap as showed in Fig.III-8(b). Setting c_i near S_i increases the total ordering cost because of too many retailers included in an order.

As the results, we can simplify mathematical model by using the can-order level $c_i = S_i$ - 1 since small average cost gap between $TC^*_{(S_i-1)}$ and TC^* is occurred. Additionally, a convex function of S_i enable us to develop heuristic approach at ease with one-dimensional search.



(a) Average level gap between c_i^* and S_i^*



(b) Average percentage of cost gap between $TC^*_{(S_i-1)}$ and TC^*

Figure III-8 The effect of ratio on the can-order level at the retailers

However, computer simulation seems not to be an appropriate method for determining the best-known solution if the problem includes the large-size problem (e.g. number of retailers, high demand rates) and/or non-identical retailers (e.g. non-identical demand rates, non-identical cost components) because of a huge search space inputted in the simulation. Therefore, heuristic approach is interesting to systematically reduce the search space for determining the appropriate inventory policy parameters. As found in many literatures relating to the can-order policy, they used heuristics to accomplish their studies.

3.4 Heuristic I – Modified Deterministic Joint Replenishment (DJ)

Our purpose of developing heuristic approach is to provide an appropriate inventory policy (c_i, S_i, S_0). The total system-wide cost of mathematical model is able to be approximated as long as the acceptable solution is provided. This can reduce the complexity of our model. According to zero lead time and stationary demand, we consider an existing deterministic model to determine the best solution due to the same cost structure composed of the holding costs and ordering costs at both echelons. Schwarz [124] developed an analytical model for OWNR under deterministic demand for determining the optimal inventory policy, and then it has become a classical model referred in a lot of literatures.

In the pilot testing, we consider the case of identical retailers with zero minor ordering cost as we can simply modify Schwarz [124]'s model by jointly fulfilling all retailers' inventories in one order. This modification is consistent with the concept of van Eijs [48] and preliminary analysis (Section 3.3.3.2) mentioned previously. When K_r / κ_i ratio is approaching infinity, then c_i^* = S_i^*- 1 for all items. It implies that all items are jointly replenished as soon as an item triggers an order. This concept's purpose is to most reduce the ordering cost from jointly replenishing all items in the order.

3.4.1 Mathematical model and analytical approach

Since Schwarz [124]'s model is based on batch-ordering policies, we use terms of batch size to determine the minimum total system-wide cost and convert S_i, S_0 consistently to the batch-ordering policies. The deterministic model is developed under the property that the delivery to warehouse occurs only when the warehouse and at least one retailer have zero inventory. Relating to identical retailers with zero minor ordering cost, the cost model can be formulated for a given (S_i, S_0) policy by using the following equations.

$$TC(m_r, Q_r) = \left\{ \frac{K_w \lambda_r}{m_r Q_r} + \frac{K_r \lambda_r}{Q_r} + \frac{n h_0 m_r Q_r}{2} + \frac{n(h_r - h_0) Q_r + n h_r}{2} \right\} \tag{3.8}$$

$TC(m_r, Q_r)$ = The total system-wide cost per unit time respecting the batch-sizing model ($/unit time)

n = Number of retailers in the system

Q_r = A lot size of each identical retailer replenished by the warehouse (units)

m_r = Number of dispatch from the warehouse echelon to retailer echelon (times)

λ_r = Demand rate of an identical retailer (units/time unit)

h_0 = The unit holding cost per unit time at the warehouse ($/unit – time unit)

h_r = The unit holding cost per unit time at an identical retailer ($/unit – time unit)

K_w = The warehouse's major ordering cost per a replenishment cycle ($/time)

K_r = The retailers' major ordering cost per a dispatch cycle ($/time)

According to Equation (3.8), the first term is the warehouse's ordering cost per unit time and the second term is the retailers' ordering cost per unit time. The third term is the warehouse's holding cost per unit time considering the stock for all n identical retailers. The last term is the total retailers' holding cost accumulated on all n identical retailers.

The optimal solution can be determined by using the first order differential Equation (3.8) with respect to Q_r and m_r. It follows that,

$$Q_r^* = \sqrt{\frac{\left(\dfrac{2\lambda_r (K_r + K_w)}{nm_r}\right)}{(m_r - 1)h_0 + h_r}} \tag{3.9}$$

$$m_r^* = \sqrt{\frac{K_w (h_r - h_0)}{K_r h_0}} \tag{3.10}$$

where Q_r^* is the optimal lot size of each identical retailer replenished by the warehouse and m_r^* is the optimal number of dispatch from the warehouse echelon to retailer echelon. From Equation (3.10), the value of m_r^* and Q_r^* can be non-integer value. Therefore, we find the integer value of m_r^* by rounding down and rounding up them identified as m_r^- and m_r^+ respectively. Similarly, we find the integer value of Q_r^* by rounding down and rounding up them identified as Q_r^- and Q_r^+ respectively. Comparing $TC(m_r^-, Q_r^-)$, $TC(m_r^-, Q_r^+)$, $TC(m_r^+, Q_r^-)$, and $TC(m_r^+, Q_r^+)$, we select the solution providing the minimum total system-wide cost and assign new integer m_r^* and Q_r^*. We found that $\frac{nh_r}{2}$ of the last term could be ignorable since it does not affect to the optimal solution.

Consequently, S_i and S_0 can be determined by $S_i = Q_r^*$ for all identical retailers and $S_0 = (n-1)m_r^* Q_r^*$ for the warehouse. For any retailer, $S_i = Q_r^*$ occurs when unit Poisson demand. The replenishment quantity at warehouse is equal to $m_r^* Q_r^*$. Since in our system, warehouse inventory level can drop below zero ($I_0(t) = -Q_r^*$) when issuing a dispatch order (identified as pre-replenishing point) and then instant replenishment fulfills the warehouse's inventory level up to S_0 (identified as post-replenishing point). Thus, the order-up-to level at warehouse has to be subtracted Q_r^* from the replenishment quantity.

Lastly, we can determine the solution for the can-order policy from the modified Schwarz [124]'s model. Let DJ denote the case of determining the solution by using the modified Schwarz's model. Let $CAN^{(DJ)}$ denote the can-order policy which sets $S_0^{(CAN)} = S_0^{(DJ)}$, $S_r^{(CAN)} = S_r^{(DJ)}$, and $c_r^{(CAN)} = S_r^{(DJ)} - 1$ according to the preliminary analysis.

3.4.2 Pilot testing

We explore the cost gap when the modified Schwarz's solution is used in the can-order policy. The goal is to identify cost gap when $CAN^{(DJ)}$ is utilized. The cost gap can be calculated by using Equation (3.7). We tested on 20 scenarios following Table III-5. In consequence, the testing result can be summarized as showed in Fig.III-9.

Table III-5: Numerical input for pilot testing of the DJ heuristic

Scenario No.	Relevant Parameters						
	K_w	K_r	h_0	h_i	h_0/h_i	λ_i	n
1	100	50	20	100	0.2	20	2
2	100	50	40	100	0.4	20	2
3	100	10	20	100	0.2	20	2
4	100	90	20	100	0.2	20	2
5	75	50	20	100	0.2	20	2
6	500	50	20	100	0.2	20	2
7	100	50	20	100	0.2	20	4
8	100	50	20	100	0.2	20	8
9	100	50	2	10	0.2	20	2
10	100	50	4	10	0.4	20	2
11	100	10	2	10	0.2	20	2
12	100	90	4	10	0.2	20	2
13	125	50	2	10	0.2	20	2
14	250	50	2	10	0.2	20	2
15	100	50	2	10	0.2	20	4
16	100	50	2	10	0.2	20	8
17	100	50	2	10	0.2	20	12
18	100	50	2	10	0.2	10	4
19	100	50	2	10	0.2	10	8
20	100	50	2	10	0.2	10	12

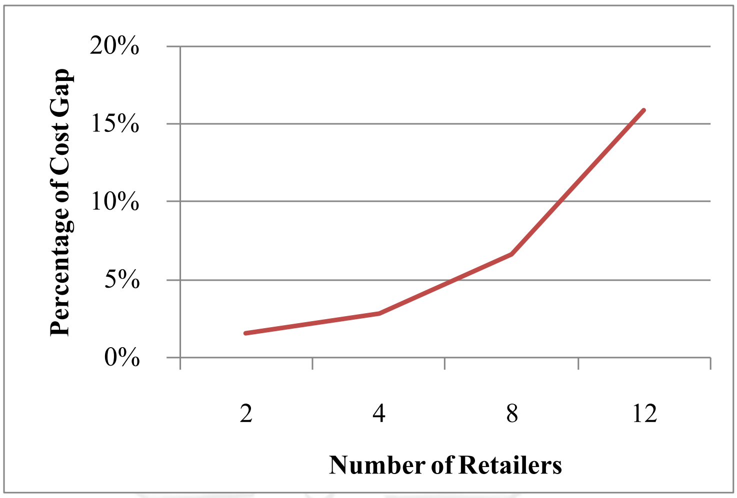


Figure III-9 Heuristic's performance on pilot testing

We found that this heuristic approach was not appropriate for the problem at higher number of retailers. Trend of cost gap is an exponential increase. The reason is that DJ case considers all retailers without uncertainty of demand, so zero residual stock occurs at every triggered order including all retailers, thus $S_0^{(BS)} > S_0^{(DJ)}$ and $S_r^{(BS)} > S_r^{(DJ)}$. Higher number of retailers increases the difference of holding amount between the best solution and DJ's solution due to residual stock. Similarly, higher number of retailers also increases the difference of retailers' major ordering cost per order-retailer between the best solution and DJ's solution. Meanwhile, other parameters cannot be clearly summarized because there is no obvious trend of cost gap.

This simple policy is useful in the case of identical retailers with low number of retailers (i.e. from the pilot test number of retailers should not be over 4 retailers). According to its limitation, we attempt to develop another heuristic approach to obtain better quality solution as demonstrated in the next solution.

3.5 Heuristic II – Approximate Mathematical Model based on EOQ (EOQ-Z)

We propose a new heuristic approach to determine an appropriate inventory policy. The approximate mathematical model is able to be employed as long as the acceptable solution is obtained. From the preliminary analysis, various interesting issues can be interpreted into the mathematical model and heuristic algorithm.

3.5.1 Mathematical model

Our purpose of developing heuristic approach is to provide an appropriate inventory policy (c_i, S_i, S_0). The total system-wide cost of mathematical model is able to be approximated as long as the acceptable solution is provided. This can reduce the complexity of our model. Hence, relating to the preliminary analysis our mathematical model utilizes the can-order level at $c_i = S_i - 1$. Then, there exists only two decision variables (S_i, S_0) concerned in the mathematical model. van Eijs [48] developed exact equations by using $c_i = S_i - 1$ for non-identical items on single location. His model used the exact probability of the special replenishment, unlike other models assuming Poisson distributions. It performed very well when K_r / κ_i ratio is more than 5. Hence, we adapt his work into our consideration.

Based on van Eijs [48], we can calculate the inventory cost at the retailer echelon close to the exact value. However, determination of inventory cost at warehouse is another difficult part. The warehouse's inventory level is consumed by an uncertain lot-sizing order from retailer echelon. From preliminary testing, we determine the expected dispatch quantity at retailer echelon by using the exact model of van Eijs [48]. We found that the expected dispatch quantity per dispatch cycle was always equal to the cumulative demand from all retailers. Thus, we simplify this part by assuming that the warehouse's inventory level is consumed continuously following the total Poisson demand cumulated from all retailers, $\lambda_0 = \sum_{i \in N} \lambda_i$. By this assumption, warehouse echelon and retailer echelon are independent to find the minimum inventory costs at each echelon. Even though the assumption provides the approximate warehouse's inventory cost higher than the warehouse's actual inventory cost, we compensate the approximate value by utilizing the minimum inventory cost at retailer echelon.

The cost model can be formulated for a given (S_i, S_0) policy. It follows that,

$$TC(S_i, S_0) = \frac{K_r + \sum_{i \in N} \left\{ \left(1 - \Phi(S_i)\right) \times \kappa_i \right) + E[H_i] \right\}}{E[DT]} + \frac{K_w + E[H_0]}{E[RT]} \tag{3.11}$$

$TC(S_i, S_0)$ = The long-run average total system-wide cost per unit time ($/unit time)

i = Index of the location i; the warehouse i = 0 and the retailer $i \in N$

S_0 = The order-up-to level at the warehouse (units)

S_i = The order-up-to level at retailer i (units)

λ_i = Demand rate of retailer i (units/time unit)

h_0 = The unit holding cost per unit time at the warehouse ($/unit – time unit)

h_i = The unit holding cost per unit time at retailer i ($/unit – time unit)

K_w = The warehouse's major ordering cost per a replenishment cycle ($/time)

K_r = The retailers' major ordering cost per a dispatch cycle ($/time)

κ_i = The minor ordering cost at retailer i ($)

$\Phi(S_i)$ = The probability that no demand arrives for retailer i during a dispatch cycle

$E[H_i]$ = The expected holding cost of retailer i during a dispatch cycle ($)

$E[H_0]$ = The expected holding cost of the warehouse during a replenishment cycle ($)

$E[DT]$ = The expected length of a dispatch cycle (unit time)

$E[RT]$ = The expected length of a replenishment cycle (unit time)

According to Equation (3.11), we consider the probability that at least one demand arrives for retailer i during a dispatch cycle to be consistent with the value $c_i = S_i - 1$. Such probability affects the occurrence of the minor ordering cost.

Retailer Echelon

The model is developed according to the independent Poisson process of demands for individual retailers, so inter-arrival times of demands are exponentially distributed. Suppose a dispatch cycle starts at time 0. We define the following variables according to stochastic process:

DT_i = Time until retailer i triggers an order to the warehouse

DT = Time until any retailer triggers an order to the warehouse; $DT = \min(DT_i)$

$f_i(t)$ = Probability density function of DT_i

$F_i(t)$ = Distribution function of DT_i

$f(t)$ = Probability density function of DT

$F(t)$ = Distribution function of DT

Retailer i will trigger an order if the total demand for retailer i from time 0 equals S_i. Thus, according to exponential distribution of inter-arrival times of demands, DT_i follows Erlang distribution with parameters λ_i and S_i. The value of $f_i(t)$ and $F_i(t)$ are determined by the general formula of Erlang distribution [125]. Then, the probability density function and distribution function of DT can be calculated by

$$f(t) = \sum_{i \in N} f_i(t) \prod_{j \neq i} \left(1 - F_i(t)\right) \tag{3.12}$$

$$F(t) = 1 - \prod_{i \in N} \left(1 - F_i(t)\right) \tag{3.13}$$

Thus, the expected length of a dispatch cycle is

$$E[DT] = \int_{t=0}^{\infty} tf(t)dt = \int_{t=0}^{\infty} \left(1 - F(t)\right)dt = \int_{t=0}^{\infty} \prod_{i \in N} \left(1 - F_i(t)\right)dt \tag{3.14}$$

The expected holding cost of retailer i during a dispatch cycle is associated with the retailer's inventory on hand at the beginning and at the end of the dispatch cycle. At the beginning of the cycle, setting $c_i = S_i - 1$ makes all retailers' inventory on hand equal S_i. At the end of the cycle, the inventory on hand depends on the residual stock level, which is a stock above the must-order level when an order is triggered. Thus, we define $\Phi_i(x)$ as the probability that at time DT the residual stock of retailer i equals x. There are two cases for determining $\Phi_i(x)$. The first case is when the residual stock level of retailer i is equal to zero; only retailer i triggers an order. The second case is when the residual stock level of retailer i is positive. So, an order is triggered by retailer $j \neq i$. Thus, the value of $\Phi_i(x)$ can be calculated by the following expressions:

$$\Phi_i(x)=\begin{cases}\int_{t=0}^{\infty} f_i(t)\prod_{j\neq i}\left(1-F_j(t)\right)dt & \text{if} \quad x=0,\\ \int_{t=0}^{\infty} Pois(\lambda_i t, S_i-x)f^{(-i)}(t)dt & \text{if} \quad 0<x\leq S_i\end{cases} \tag{3.15}$$

$$Pois(a,b)=\frac{a^b e^{-a}}{b!} \tag{3.16}$$

$$f^{(-i)}(t)=\sum_{j\neq i} f_j(t)\prod_{k\neq j,i}\left(1-F_k(t)\right) \tag{3.17}$$

where $Pois(a,b)$ is the probability density function of Poisson demand with parameter (a,b), and $f^{(-i)}(t)$ is the probability density function that at time t any retailer $j\neq i$ triggers an order. Thus, $\Phi(S_i)$ illustrated in Equation (3.11) can be calculated by using Equation (3.15) as well.

The expected holding cost of retailer i during a dispatch cycle is then given by

$$E[H_i]=\sum_{x=0}^{S_i}\left\{\Phi(x)\int_{t=0}^{\infty}\frac{h_i(S_i+x)t}{2}f(t)dt\right\} \tag{3.18}$$

According to Equation (3.14) and (3.18), we transform the expression to determine the expected holding cost of retailer i per unit time instead. Thus,

$$\frac{E[H_i]}{E[DT]}=\sum_{x=0}^{S_i}\left\{\Phi(x)\frac{h_i(S_i+x)}{2}\right\} \tag{3.19}$$

<u>Warehouse Echelon</u>

To simplify this part, we assume that the warehouse's inventory level is consumed continuously by all retailers' Poisson demands with rate λ_0. Inter-arrival times of demands are exponentially distributed, and then the distribution of time until warehouse triggers an order to an outside supplier is Erlang, similar to the retailer echelon. Let RT denote time until warehouse triggers an order to an outside supplier. The warehouse will trigger an order if the total demand from time 0 equals

S_0, so the distribution of RT is Erlang with parameters λ_0 and S_0. The expected length of a replenishment cycle is mean of Erlang distribution. Thus, $E[RT] = S_0/\lambda_0$.

In case of holding inventory at the warehouse, the expected holding cost of the warehouse during a replenishment cycle is estimated following the continuous demands from the retailer echelon. Then, we can determine the expected holding cost of the warehouse per unit time by $\frac{E[H_0]}{E[RT]} = \frac{h_0 S_0}{2}$. According to the expression at the warehouse, we can find the optimal order-up-to level at the warehouse S_0^* from the derivative of the cost function with respect to S_0. We found that S_0^* could be easily calculated from EOQ formula. Then, $S_0^* = \sqrt{2K_w\lambda_0/h_0}$

Consequently, we can figure out the long-run average total system-wide cost per unit time for a given (S_i, S_0) policy. Then, the next section will demonstrate the algorithm of heuristic approach to determine the appropriate decision variables by using the cost model.

3.5.2 Heuristic algorithm

With regard to the preliminary analysis and the mathematical model, the following analyses demonstrate our concept for developing heuristic approach.

1) According to two local minimum solutions located into two ranges, we can identify the value of S_0 to S_0 = 0 for Range I and $S_0 = \sqrt{2K_w\lambda_0/h_0}$ for Range II.

2) To develop initial solution at retailer echelon by assuming c_i = S_i - 1, we can use deterministic model to find economical joint ordering time when every retailer is replenished in an order.

3) Fixing inventory policy at retailer $j \neq i$ and at the warehouse, the total inventory cost at retailer i is a convex function of S_i. We can find the local minimum $TC(S_i, S_0)$ at the given $S_{j\neq i}$ and S_0. Therefore, Decomposition technique and iterative procedure can be applied to break multiple locations into single location and to recurrently find the minimum solution as far as the best solution has been found. Both techniques have been extensively used in JRP [34, 46, 51-56, 71, 73].

4) Since the total inventory cost at retailer i is a unimodal function under one-dimensional unconstrained problem. We apply the concept of line search called “golden section search” into discrete function [126]. Golden section search is a simple and efficient method for finding extremum of a unimodal function [48, 57, 58, 122]. Golden section search is suitable for the case of non-derivative function, like our model, by successively narrowing the range of search space until the desired accuracy in minimum value of the objective function is achieved. A golden ratio, which is a constant reduction factor for the size of the interval, is utilized to maintain the successive range of dynamic triples of points (i.e. upper point, middle point, and lower point). Advantageously, each successive range we only want to perform one new function evaluation. From this technique, we can determine the optimal S_i^* for the given $S_{j\neq i}$ and S_0 and save computational time.

Hence, the heuristic approach is outlined in the following algorithm illustrated in Fig.III-10.

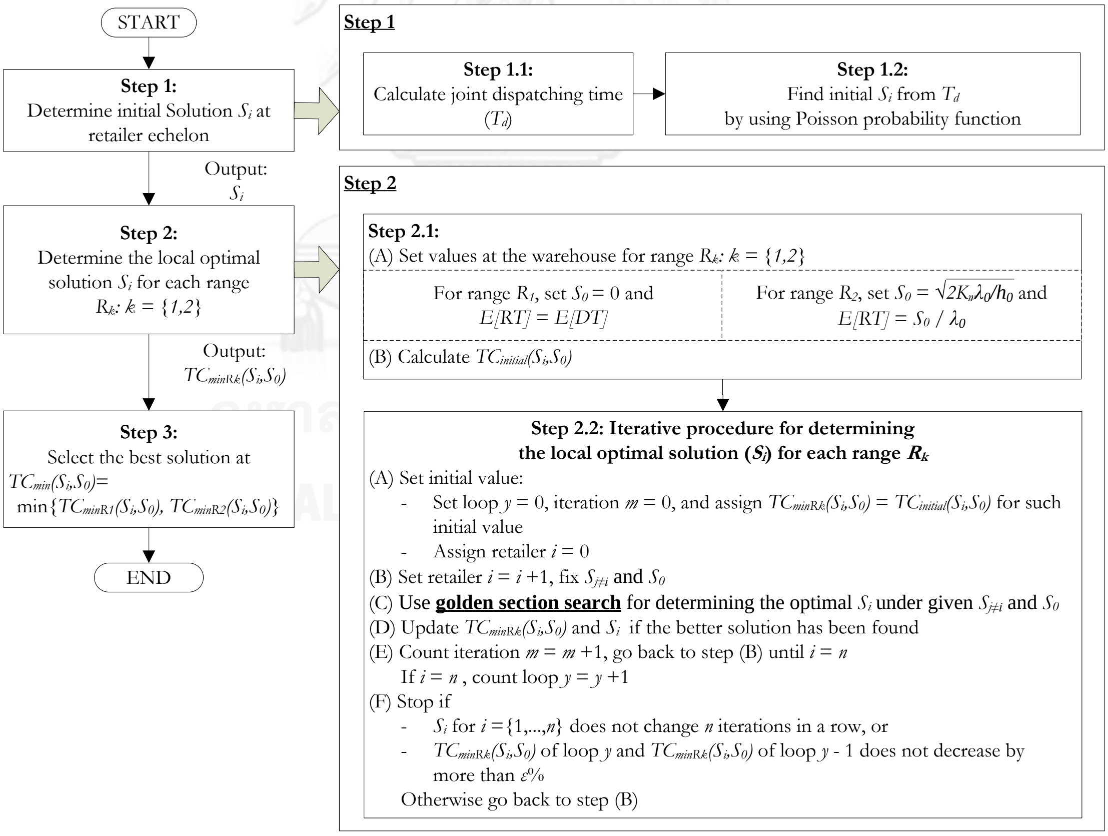


Figure III-10 The algorithm of the heuristic approach – EOQ-Z

In step 1 – determination of the initial solution S_i, we calculate joint dispatching time (T_d) by deterministic model according to the following expression:

$$T_d = \sqrt{\frac{2(K_w + \sum_{i\in N}\kappa_i)}{\sum_{i\in N}\lambda_i h_i}} \tag{3.20}$$

Then, the initial S_i for retailer i is determined by adapting Love [46]'s method. It is selecting S_i which provides the minimum gap between two probabilities: 1) the probability that demand for retailer i during time T_d is less than or equal to such S_i and 2) the probability that an order is triggered by any retailer (i.e. including normal replenishment and special replenishment). Thus,

$$S_i = \begin{cases} u & if \left\{ Pois(\lambda_i T_d, u+1) - \left(\frac{n}{n+1}\right) \right\} \geq \left\{ \left(\frac{n}{n+1}\right) - Pois(\lambda_i T_d, u) \right\} \\ u+1 & Otherwise \end{cases} \tag{3.21}$$

The initial S_i from Equation (3.18) is closer to the optimal solution than S_i obtained from $S_i = \lambda_i T_d$.

Step 2 is the most important procedure for the heuristic in order to determine the optimal S_i for each range of range R_1 and R_2 (note that for range R_1, the local optimal solution occurs at S_0 = 0 and $E[RT] = E[DT]$, and for range R_2, it occurs at $S_0 = \sqrt{2K_w\lambda_0/h_0}$ and $E[RT] = S_0/\lambda_0$). We use S_0 and initial S_i from step 1 to calculate the initial long-run average total system-wide cost per unit time, $TC_{initial}(S_i, S_0)$. The next step (2.2) is an iterative procedure containing step (A) to (F). For each iteration, the golden section search is carried out for retailer i: vary S_i and fix other retailers $S_{j\neq i}$ given from the previous iteration. $TC(S_i, S_0)$ is an objective function for the golden section search. The iterative process terminates as soon as every S_i does not change n iterations in a row, or the minimum long-run average total system-wide cost per unit time, $TC_{\min Rk}(S_i, S_0)$, from the current loop does not decrease from the previous loop by more than $\varepsilon\%$ (i.e. when all retailers have been run, one loop is counted). From step 2, we obtain the local minimum cost $TC_{\min Rk}(S_i, S_0)$ for $k \in \{1, 2\}$.

Lastly, the comparison of $TC_{\min Rk}(S_i, S_0)$ for $k \in \{1, 2\}$ is carried out in step 3. The minimum long-run average total system-wide cost per unit time is equal to $\min\{TC_{\min R1}(S_i, S_0), TC_{\min R2}(S_i, S_0)\}$.

To summarize, our heuristic approach (called EOQ-Z) is developed by using approximate mathematical model with heuristic algorithm to determine the appropriate inventory policy parameters. The mathematical model is formulated based on two compositions. The first one is the exact model for retailer echelon with relaxing synchronization of the dispatch cycle time between echelons. The other one is the EOQ model for warehouse echelon. We can interpret preliminary analysis into the heuristic algorithm consisting of decomposition technique, iterative procedure, and one-dimensional search called the golden section search. To measure heuristic's performance, we continue to the next section which various experiments are conducted and the findings are analyzed.

3.6 Experimental Results

In this section, we experimented on the EOQ-Z heuristic on various scenarios following Table III-1 and Table III-2. The experiments on identical retailers were analyzed, specifically in case of zero minor ordering cost and non-zero minor ordering cost. Since both cases affects the can-order policy at given $c_i = S_i - 1$ on different results as showed in the preliminary analysis. In addition, the experiment on non-identical retailers was also conducted to measure the heuristic's performance on the dissimilar situation. Moreover, we compared computational time between computer simulation and our EOQ-Z heuristic on the cases of identical and non-identical retailers.

3.6.1 Identical retailers with zero minor ordering cost

According to three relevant factors (i.e. cost parameters, demand rates, and number of retailers), they were designed to examine the heuristic's performance under 154 scenarios (showed in Table III-1). Table III-6 shows some numerical examples relating to the best-known solution of the system and the best solution from the heuristic approach. We found that the performance of heuristic approach depended on all relevant factors. It provided an average cost gap at 1.05%

with standard deviation 1.11% over various scenarios. Our approach performed well when high number of retailers, high K_w / K_r ratio, and high h_0 / h_i ratio.

Table III-6: Numerical examples for comparison of the best-known solution and the heuristic's best solution under identical retailers with zero minor ordering cost

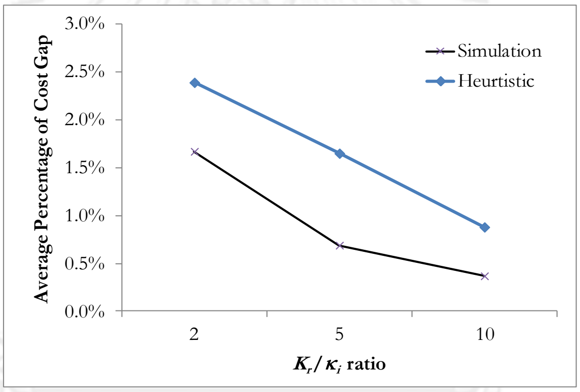
Instance	Relevant factors						Best-known Solution		Heuristic Approach	
	K_w	K_r	h_0	h_i	λ_i	n	S_0, c_i, S_i	$TC^{(BS)}$	S_0, c_i, S_i	$C.G.$
1	100	50	20	100	20	2	13,3,4	1,280.75	20,3,4	0.52%
2	100	50	40	100	20	2	0,5,6	1,420.94	0,5,6	0.00%
3	100	50	2	10	20	2	45,9,12	359.73	63,10,11	2.03%
4	100	50	4	10	20	2	25,12,13	392.37	0,18,19	1.88%
5	100	10	2	10	20	2	58,4,5	244.97	63,4,5	0.05%
6	100	90	2	10	20	2	31,15,16	424.21	63,14,15	4.34%
7	125	50	2	10	20	2	49,12,13	376.43	71,10,11	2.50%
8	250	50	2	10	20	2	86,11,12	436.63	100,10,11	0.98%
9	100	50	2	10	10	4	42,6,7	427.16	63,5,6	0.36%
10	100	50	2	10	10	8	78,4,5	697.16	89,4,5	0.11%
11	100	50	2	10	10	12	93,4,5	932.60	110,4,5	0.15%
12	100	50	2	10	20	4	79,8,9	576.83	89,8,9	1.03%
13	100	50	2	10	20	8	100,6,7	925.98	126,6,7	0.29%
14	100	50	2	10	20	12	142,5,6	1,230.39	155,5,6	0.17%

Let $S_i^{(BS)}$ and $S_0^{(BS)}$ denote the best-known order-up-to level at retailer i and at the warehouse determined from the computer simulation. Let $S_i^{(HRT)}$ and $S_0^{(HRT)}$ denote the best order-up-to level at retailer i and at the warehouse and they were calculated by the heuristic approach. Theoretically, when the number of retailers increases, it also increases the joint replenishment opportunity from special replenishment, which makes S_i decrease. Thus, higher number of retailers reduces $S_i^{(BS)}$ to be closer to $S_i^{(HRT)}$ and also increases $S_0^{(BS)}$ to be closer to $S_0^{(HRT)}$. Therefore, the cost gap can reduce. For higher K_w / K_r ratio, S_i and S_0 are affected in a similar pattern.

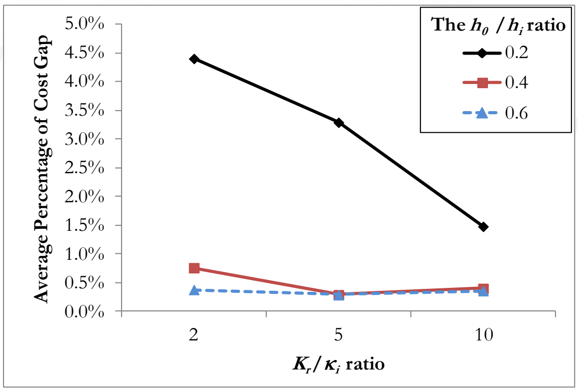
Regarding h_0 / h_i ratio, higher ratio influences the warehouse's stock equal to zero. Consequently, the inventory cost at retailer echelon becomes the main part of the system. Our mathematical model provided cost expression at retailer echelon near the exact value and heuristic approach could determine the minimum solution at retailer echelon. Then, the heuristic approach provided the (near) best-known solution.

3.6.2 Identical retailers with non-zero minor ordering cost

Although the can-order level is not necessary to be equal to S_i - 1 when there is a minor ordering cost, our heuristic approach can be applied into this problem in some situations. To identify such situation, we tested on 54 scenarios (showed in Table III-1) by mainly varying the minor ordering cost κ_i. The value of K_r / κ_i ratio are identified following van Eijs [48]'s work. The experimental results are depicted in Fig.III-11.



(a) Heuristic's performance and simulation's performance when fixing $c_i = S_i$ - 1



(b) The effect of h_0 / h_i ratio

Figure III-11 The effect of K_r / κ_i ratio on the can-order level at the retailers

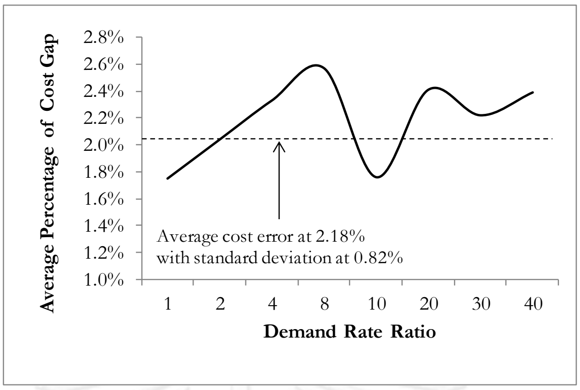
We found that the heuristic approach provided an average cost gap at 1.64% with standard deviation at 2.03% over various scenarios. Heuristic's performance is associated with two reasons. Firstly, our heuristic assumes $c_i = S_i$ - 1. As showed in Fig.III-11(a). Relating to computer simulation, we compared the best-known solution with the best solution fixing c_i = S_i - 1. Average percentage of cost gap provides in simulation's line. Smaller K_r / κ_i ratio provided larger cost gap in simulation's line, consequently our heuristic also performed in the same way. Secondly, the inventory cost at the warehouse is approximate. Cost gap of the heuristic's line is also added from the simulation's line.

Considering h_0 / h_i ratio, higher ratio (h_0 / h_i is 0.4 and 0.6) influences the warehouse's stock equal to zero. Then, the heuristic approach provided the (near) best-known solution. On the other hand, higher cost gap at the lower h_0 / h_i ratio comes from an approximate inventory cost at the warehouse, especially when small demand rate and high number of retailers by the reason that our heuristic obtains S_0 = 0 whereas the best-known solution is S_0 > 0. The difference of solution creates larger cost gap.

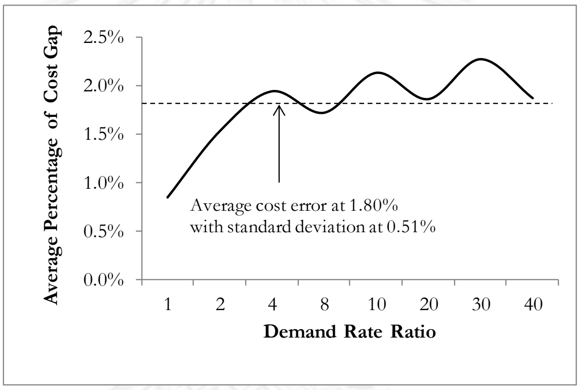
3.6.3 Non-identical retailers

To extend the experiment on non-identical retailers, we aim at studying the can-order policy on the retailers' different demand rates because in reality we frequently encounter such situation. In addition, non-identical demands can create the different discount opportunities from the shared ordering cost. Hence, it is interesting to investigate and this inquiry has not been studied in the existing literatures. We tested on two-retailer scenarios and three-retailer scenarios (showed in Table III-2). Figure III-12 depicts the cost gap from our heuristic approach, as compared to the best-known solutions.

The heuristic approach provided an average cost gap at 2.18% with standard deviation 0.82% for two-retailer scenarios, and an average cost gap at 1.80% with standard deviation 0.51% for three-retailer scenarios. At small demand rate ratio the heuristic approach performed well because order cycle of each retailer was quite not different. So, the retailers' ordering cost can be more shared with the balancing holding costs. However, at the higher demand rate ratio heuristic's performance does not depend on the different of demand rates (i.e. there is no trend of the cost gap following demand rate ratio).



(a)



(b)

Figure III-12 Heuristic's performance under non-identical retailers

3.6.4 Computational times

For the experiments on identical retailers as shown in Table III-1, computational time of our EOQ-Z heuristic was 2.37 seconds on average with a standard deviation at 0.78 seconds. EOQ-Z heuristic's computational times were much faster than computer simulation's computational times. Minimum time saving was 297 times and maximum time saving was 8722 times where time saving can be calculated by the following equation

$$Time\,Saving\,(T.S.) = \frac{CPU^{(SIM)} - CPU^{(HRT)}}{CPU^{(HRT)}} \tag{3.22}$$

$CPU^{(SIM)}$ and $CPU^{(HRT)}$ are the computational time of computer simulation and our heuristic, respectively.

We found how much the EOQ-Z heuristic could save computational time from computer simulation depended on various relevant factors. High value of S_0 and S_i affected a huge computational time of computer simulation, since we had to search on larger search space. Larger search space was associated with relevant factors at high K_w and K_r, low κ_i, low h_0 and h_i, high λ_i, and high n. Consequently, a lot of policy combinations were examined. Some examples of computational times between EOQ-Z heuristic and computer simulation are shown in Table III-7. It demonstrates trend of relevant factors affecting computational times.

Table III-7: Numerical examples for comparison of computational time between the EOQ-Z heuristic and computer simulation under identical retailers

All examples fixed K_w = 100, K_r = 50, h_0 / h_i = 0.2

Relevant Factors				Computational Times (seconds)		Time Saving (times)
λ_i	n	h_i	κ_i	EOQ-Z	Simulation	
0.5	2	10	-	1.58	473	297
10	2	10	-	2.04	1,702	833
20	2	10	-	2.89	4,131	1,429
0.5	4	10	-	3.05	946	322
10	4	10	-	2.92	2,269	757
20	4	10	-	2.99	6,262	1,742
0.5	8	10	-	3.22	2,837	881
10	8	10	-	3.18	5,949	1,871
20	8	10	-	3.17	7,502	2,364
20	2	100	-	2.26	836	836
20	4	100	-	3.02	1,249	1,249
20	8	100	-	3.18	1,783	1,783
20	8	10	5	3.24	27,358	8,446
20	8	10	10	3.35	27,580	8,722
20	8	10	25	3.79	27,593	7,276

Relating to the experiments on non-identical retailers as shown in Table III-2, average computational times of our EOQ-Z heuristic were 4.10 seconds (standard deviation at 2.87 seconds) for two-retailer scenarios and 37.84 seconds (standard deviation at 33.47 seconds) for three-retailer scenarios. Non-identical demand rates had a significant effect on computer simulation's computational time since huge combinations of inventory policy parameters were created. For our experiments, we spent more than 40 hours to determine the best solution of any scenario from computer simulation. As the results, the EOQ-Z heuristic's computational times were much faster than computer simulation's computational times.

3.7 Discussion

The EOQ-Z heuristic is based on preliminary analysis as shown in section 3.3. The important characteristics of inventory policy parameters are 1) two ranges of the warehouse's order-up-to level and 2) the fixed retailer's can-order level at the retailer's order-up-to level minus one. Such two characteristics can be intuitively described as the following contents.

The warehouse's order-up-to level S_0 is relative to the retailers' order-up-to level S_i . If $S_0 \leq S_i$, the warehouse's inventory is replenished every time when any retailer's triggers an order, because dispatch quantity is always larger than the warehouse's inventory level. So, the minimum total system-wide cost of this condition occurs at $S_0 = 0$. Meanwhile, if $S_0 > S_i$, it means that the warehouse holds stock for dispatching to the retailers more than one order. Trading off between the holding costs and the ordering costs has to be considered to decide how many order cycle the warehouse should serve retailer echelon. Then, there is a solution (or more than one solution) which $S_0 > S_i > 0$ providing the minimum total system-wide cost of this condition. According to these conditions, we can generally divide the system into two cases: case I – Cross-docking system, and case II – Stocking system at the warehouse. Therefore, our search algorithm can also be divided into two ranges (as described in section 3.5.2) to determine the minimum total system-wide cost of each case. Finally, we are able to decide for a given situation that the warehouse should apply either case I or case II, and then how the appropriate inventory policy parameters should be set.

Relating to the fixed retailer's can-order level at $c_i = S_i - 1$, the retailers' major ordering cost can be most shared if all retailers are included in an order to minimize the total system-wide cost [48]. The fixed retailer's can-order level at $c_i = S_i - 1$ can create the maximum opportunity of joint replenishment for all retailers. Unless all retailers are replenished, the total ordering cost will increase from the increased total ordering cost or/and the increased total holding cost. We call this concept as "All joint concept" in this dissertation. Then, the holding cost is traded off with the shared ordering cost in order to balance order frequency and holding stock. In the case of zero minor ordering cost, only the fixed ordering cost occurs once an order is triggered. The most sharing such cost among retailers according to the all joint concept is preferable. We also applied the all joint concept in the case of non-zero minor ordering cost. If the ratio of the major ordering cost to the minor ordering cost is not too small, this all joint concept can be utilized as van Eijs [48] recommended. Since the minor ordering cost has less effect on the total system-wide cost as comparing to the major ordering cost. If the minor ordering cost has major effect, not every retailer should be included in an order, since less number of retailers in an order might reduce the total system-wide cost from the reduced total ordering cost.

From preliminary analysis (section 3.3), our results were absolutely consistent with the all joint concept especially in the case of zero minor ordering cost. Whereas we tested the case of non-zero minor ordering cost by varying the ratio of the major ordering cost to the minor ordering cost from 2 to 10. The results were still consistent with the all joint concept. However, it seems that if such ratio is less than 2 the cost gap might be more than 1.67% on average. This means that the minor ordering cost has more effect on the system-wide cost. The fixed can-order level on the EOQ-Z heuristic has a very small effect on the cost gap in case of the zero minor ordering cost. The cost gap in this case majorly came from the fixed order-up-to level S_0 of Range II. Since we applied EOQ concept to determine the best value of S_0. However, we found that the cost gap from EOQ was not much (0.96% on average with standard deviation at 1.08%). Even though the fixed can-order level on the EOQ-Z heuristic influences the cost gap in case of non-zero minor ordering cost, we found that the cost gap was only 2.85% on average. It implies that EOQ application builds up the cost gap at 1.18% on average. If the ratio of the major ordering cost to the minor ordering cost is less than 2, the cost gap might be more than 2.85%. Neither the fixed can-order level nor the fixed order-up-to level at EOQ value might be applicable.

In addition, the fixed can-order level on the EOQ-Z heuristic has a significant effect on the cost gap in case of non-identical demand rates among retailers. From the best-known solutions, the retailers with higher demand rates attempt to reduce c_i under high S_i in order to have less residual stock, while the retailers with lower demand rates attempt to increase c_i in order to have more joint replenishment opportunity. Hence, all the can-order levels have to be traded off between the cost of residual stock and the cost of joint replenishment. It seems that the EOQ-Z heuristic is not consistent with this phenomenon. In our experiments, we varied demand rate ratios from 2 to 40. We found small cost gap (2.01% on average with standard deviation at 0.71%) under two-retailer scenarios and three-retailer scenarios. Interestingly, the EOQ-Z heuristic attempts to reduce S_i for less residual stock instead of reducing c_i as the best-known solutions. According to this mechanism, the EOQ-Z heuristic can provide the quality solutions.

Significant finding is an integration of the classical EOQ and the can-order policy for two-echelon inventory system. We simplified the EOQ concept to determine the warehouse's order-up-to level S_0. It relaxed dispatch quantity and frequency synchronized with retailer echelon, but utilized total demand rate which is a summation of all retailers' demand rates. From the experimental results, we found that $S_0^{(HRT)}$ was always higher than $S_0^{(BS)}$ where $S_0^{(HRT)}$ denote the best order-up-to level at the warehouse calculated by the heuristic approach and $S_0^{(BS)}$ denote the best-known order-up-to level at the warehouse determined from the computer simulation. It seems that holding cost at the warehouse obtained from the heuristic approach is higher than the best-known solution. However, the mechanism of trading off between warehouse echelon and retailer echelon occurs to rebalance with EOQ. Additionally, the EOQ concept has more performance for higher number of retailers. Theoretically, when the number of retailers increases, it also increases the joint replenishment opportunity from special replenishment, which makes S_i decrease. Thus, higher number of retailers reduces $S_i^{(BS)}$ to be closer to $S_i^{(HRT)}$ and also increases $S_0^{(BS)}$ to be closer to $S_0^{(HRT)}$. So, the cost gap is less. Even though we study the complicated system, the simple concept of EOQ remains useful and applicable for the case of zero lead time.

Another crucial finding is a characteristic of the retailer's order-up-to level S_i. Trading off between the holding costs and the ordering costs makes the total system-wide cost perform as a convex function relative to the value of S_i. So, we can determine the value of S_i providing the minimum total system-wide cost on one-dimensional search. Since our cost formulation is non-derivative function, we utilized a search algorithm called "Golden section search" by adapting for integer variable. This search algorithm performs better than other search algorithms, such as Fibonacci search and Half-interval search.[7] Previously, most researches used exhaustive search to determine the best solution, therefore large search space and long computational time occurred. Using one-dimensional search can reduce search space and computational time, especially when high number of non-identical retailers.

Decomposition technique and iterative procedure are the most common approach for the can-order policy determination. Decomposition technique helps breaking the complicated system (multiple retailers) into smaller part (single retailer). Determination of the can-order policy parameters seems easier than consideration of the whole parts together. However, this technique should be utilized with iterative procedure to consolidate all single retailers consistently. The solution can move to the better one and do until the best solution has been found for the whole system. From both techniques integrated with one-dimensional search, we can determine the best solution easier and faster than other approaches, especially computer simulation.

As the experimental results in various scenarios, the EOQ-Z heuristic provided the best solutions at a small average cost gap comparing to the best-known solution. Moreover, the heuristic approach's computational time can be saved from the reduced search space as comparing to the computer simulation's computational time. The EOQ-Z heuristic is a satisfactory approach to use for the can-order policy setting under OWNR with zero lead time. Note that the zero lead time assumption

[7] Golden section search is simpler than Fibonacci search whereas their computational time is not different in case of integer variable. Since Fibonacci search needs to know number of searching loops before starting search algorithm. Meanwhile number of searching loops between the golden section search and Fibonacci search is not different for integer variable. For half-interval search, the golden section search is faster than half-interval search because less number of variables has to be calculated.

can be interpreted and applied in the situation when the ratio of lead time to order cycle duration is very small.

3.8 Conclusion

This phase considered the basic model for the can-order policy on OWNR. The system assumed zero lead time to reduce decision variables remaining c_i, S_i, S_0. Single item consideration raised an interaction among retailers without joint ordering decision at the warehouse echelon. We studied the insight of the can-order policy on OWNR with three relevant factors, i.e. cost components, demand rates, and number of retailers. Then, we used the aforementioned insight to develop two heuristic approaches for determining the appropriate inventory policy setting.

The can-order policy had an effect on all inventory costs. It was connected with retailers' residual stock, dispatch quantity and frequency at retailers, as well as replenishment quantity and frequency at the warehouse. All are necessary to be traded off to determine the best solution. The can-order policy could save the total system-wide cost from an independent (s, S) policy at over 30%. Interestingly, application of this policy into such system is considerably valuable.

From preliminary analysis by computer simulation, the average total system-wide cost was a unimodal function of the retailer's order-up-to level S_i, when given $S_{j \neq i}$ and S_0 were fixed. Decomposition technique was applied to break multiple retailers into single retailer, as well as iterative procedure was utilized to successively find the best solution. Since our mathematical model was a non-derivative function, we utilized the golden section search for finding minimum of a unimodal function. This could save our computational time to find the appropriate inventory policy setting. The heuristic approach under simplified mathematical model and fixed $c_i = S_i - 1$ performed very well, especially in case of high K_r / κ_i ratio. Overall, the experiments tested on the wide range of data provided the cost gap of heuristic approach less than 2% on average. With satisfactory computational time and small cost gap, the heuristic approach is well worth using for the can-order policy setting under OWNR.

CHAPTER IV

THE CAN-ORDER POLICY FOR SINGLE-ITEM TWO-ECHELON INVENTORY SYSTEM WITH NON-ZERO LEAD TIME

The 2nd phase (Phase II) of dissertation methodology is addressed in this chapter. It is an extension of the basic model describe in previous chapter. Mostly, general industries encounter non-zero lead time which is the duration from the moment an order is placed to the warehouse (outside supplier) until the moment the order is received by the retailers (warehouse). Moreover, lead time can lead the system to occur backorder units. Thus, service level constraint is utilized to serve end customers with an acceptable service level. Research remains taking single item into consideration to study an interaction among retailers without joint ordering decision at warehouse echelon. The objective of this phase is to study inventory policy characteristics with the conditional relevant factors, as well as to develop the heuristic approach consistent with such characteristics provided. More complexity of the model is contributed to the research.

4.1 Problem Description

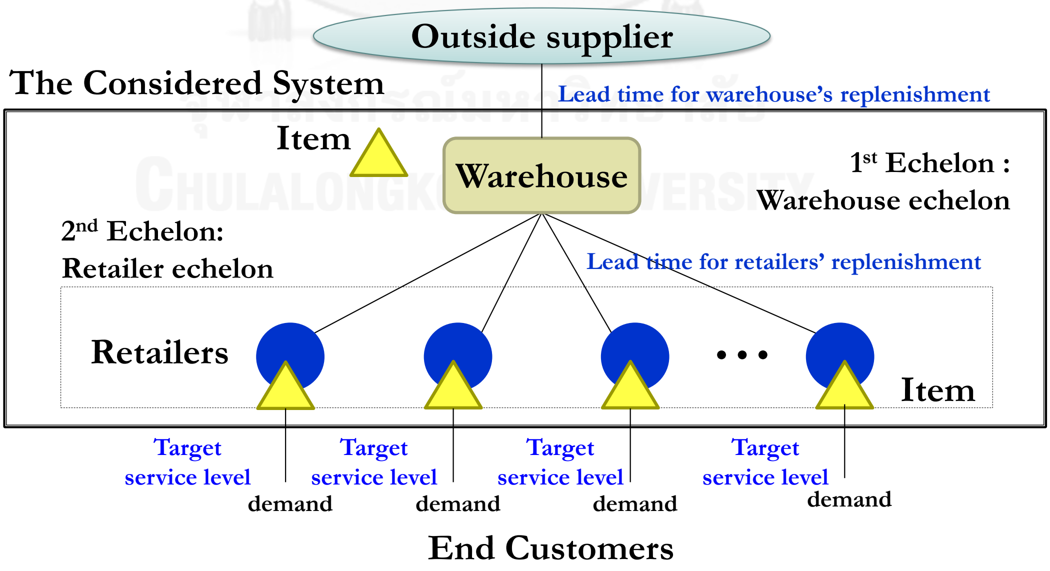


Figure IV-1 Single-item two-echelon inventory system with non-zero lead time

The system considers single commodity on a warehouse and multiple retailers. There are warehouse echelon and retailer echelon as described in Chapter III. Let n denote number of retailers and i denote the location i = {0, 1, 2, ..., n} where the warehouse is set by $i = 0$ and the retailer $i \in N$, N = {1, 2, ..., n}. Poisson demand is assumed to represent the end customer demands, denoted by λ_i which is a constant mean of customer demand at retailer i.

In the considered system, ordering process and replenishment process are more complicated as comparing to the basic model described in the Chapter III. First of all, there are four terms used throughout the dissertation [49].

(1) **Inventory on-hand** is the quantity of physical inventories at each location i.

(2) **Backorder** is the quantity that supplier[8] (predecessor) cannot fill a customer (successor)'s order, and then the customer is prepared to wait for some time.

(3) **Net inventory level** is the quantity representing the inventory status which is either available or reserved. Thus,

Net inventory level = Inventory on hand – Backorder

(4) **Inventory position** is the quantity includes the outstanding orders that have not yet arrived and backorders which units have been demanded but not yet delivered. Thus,

Inventory position = Inventory on hand + Outstanding order – Backorder

In Phase I – zero lead time, it is not necessary to identify various terms of inventory level because the zero-lead time condition allows the inventory position and net inventory level to be the same value. Backorder is not occurred due to instant replenishment. Similarly, outstanding order can be fulfilled immediately at once without waiting process. Unlike phase I, phase II need to differentiate these four terms since ordering process and replenishment process have more complexities. Each term is used in different purposes.

[8] The definition of supplier and customer in the context means about two levels of service: the first level – Warehouse and retailer, and the second level – Retailer and end customer.

The system employs the can-order (s_i, c_i, S_i) policy for ordering process. In the ordering process, we use the term of inventory position to determine the triggered point. At retailer echelon, the can-order (s_i, c_i, S_i) policy is applied into the system by coordinated ordering decision among retailers. When the inventory position of any retailer reaches its must-order level s_i, an order is triggered. Then, other retailers in the system can also be included by this order if their inventory position is at or below its can-order level c_i. All the involved retailers' inventories are fulfilled from the warehouse to their own order-up-to level S_i. Considering single commodity, the warehouse modifies the can-order policy to a traditional (s_0, S_0) policy by setting its can-order level equals its must-order level. The warehouse issues an order when its inventory position reaches its must-order level s_0. Then the outside supplier will replenish the warehouse's inventory to its order-up-to level S_0. For the system, we use the nested policy which the warehouse places an order to the outside supplier if and only if retailer echelon triggers an order to the warehouse [50, 88]. Note that we differentiate between order cycle at retailer echelon and order cycle at warehouse echelon by defining "dispatch cycle" and "replenishment cycle" for retailer echelon and warehouse echelon, respectively.

Whenever any retailer (warehouse) triggers an order to the warehouse (outside supplier), it needs to wait for some time that order arrives. The waiting time is called "lead time". In the problem, we assume constant lead time for each location (L_i). Relating to centralized control, the supplier can reduce and specify more certain lead time, so this assumption seems reasonable. However, the retailer's total lead time (TL_i, $i \in N$) can be longer than L_i depending on the warehouse's inventories. Meanwhile, the warehouse's total lead time (TL_0) is equal to L_0 due to ample stock of the outside supplier.

According to lead time, it can lead the system to occur backorder units which are the quantity that supplier[9] (predecessor) cannot fill a customer (successor)'s order, and then the customer is prepared to wait for some time. Thus, service level constraint is utilized to serve end customers with an acceptable service level. We measure such service level in term of "Fill Rate" widely used in industrial practice [22, 50]. Fill rate (FR) is a quantity-oriented performance measure describing the

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[9] The definition of supplier and customer in the context means about two levels of service: the first level – Warehouse and retailer, and the second level – Retailer and end customer.

proportion of total demand within a reference period delivered without delay from stock on hand. FR is measured only at retailer echelon since in a multi-echelon system the backorder at warehouse echelon has only a secondary effect on service. For this problem, retailer echelon must serve the end customer following a service constraint defined as target service level (TSL_{ij}, $i \in N$). By this constraint, the setting of the must-order levels at all locations is related.

On the assumption about no-splitting order, when the warehouse has insufficient inventory on-hand for dispatching all required quantities in an order to retailer echelon at once, the retailers have to wait for the next warehouse's order is arrived. It implies that the dispatching for that order is occurred if and only if there is sufficient inventory on-hand for all required quantities. Normally, the warehouse serve an order follows the First-In First-Out System (FIFO) except if there is an order issued to the warehouse and inventory on-hand is enough for this order we allow the warehouse to deliver it as special case to reduce the opportunity of stock-out at the retailers. This creates higher service level than FIFO. We illustrate the inventory process following this statement in Fig.IV-2. Let O_{Rk} represent a triggered order number k by retailer echelon, and A_{Rk} represent an arrived order number k fulfilled to retailer echelon. Similarly, let O_{Wk} represent a triggered order number k by warehouse echelon, and A_{Wk} represent an arrived order number k fulfilled to warehouse echelon. For retailer echelon, we use net inventory level to represent inventory on-hand if net inventory level is positive (≥ 0) and represent backorder if net inventory level is negative (< 0). On the other hand, for warehouse echelon net inventory level cannot be used for inventory on-hand since splitting lot is not allowed. The warehouse has to hold such inventory on-hand as soon as the next dispatch is occurred. Meanwhile backorders at the warehouse are accumulated from net inventory level as usual.

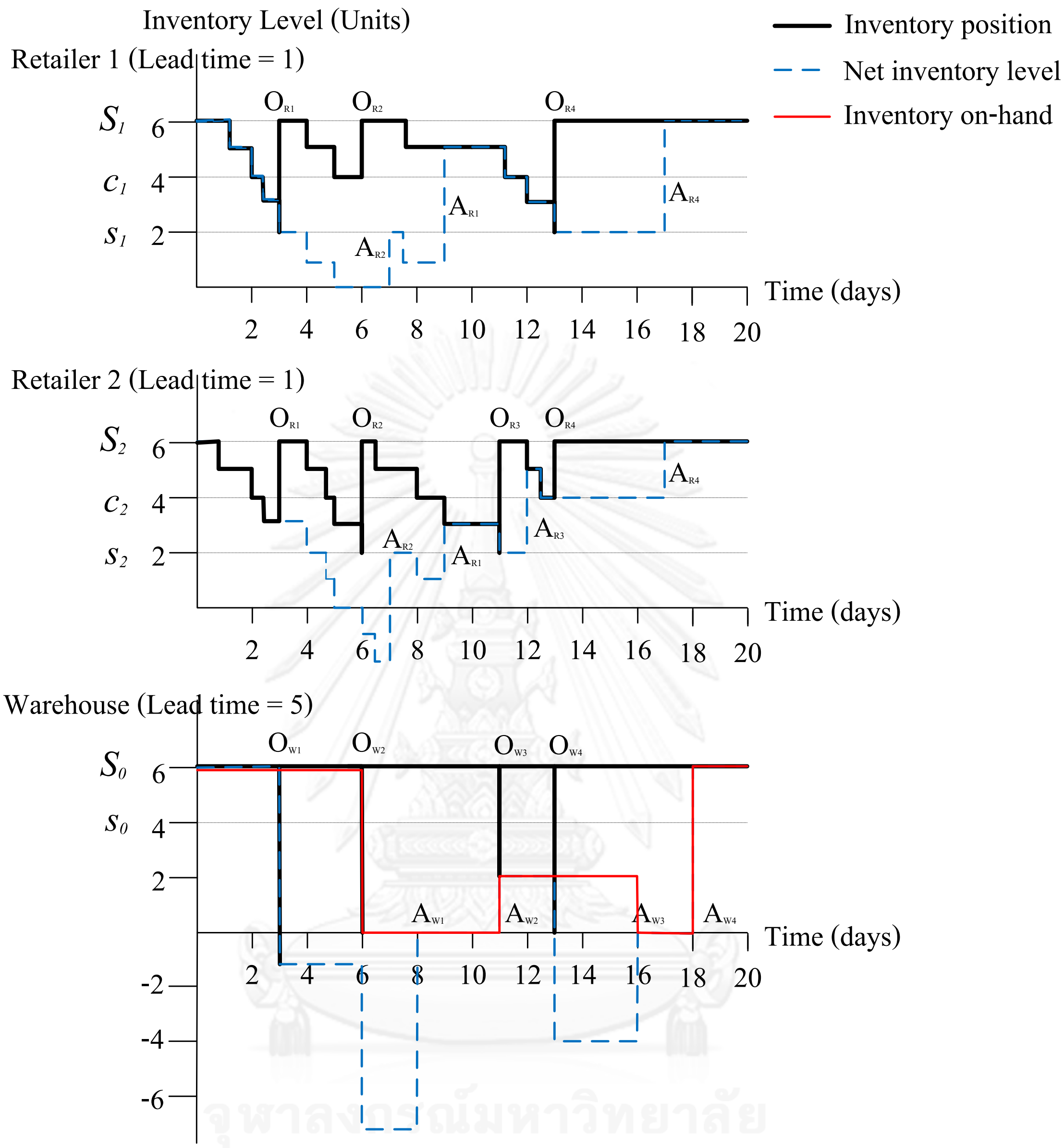

Figure IV-2 The inventory process of Phase II's problem

The system considers all inventory costs at both echelons. The inventory costs are composed of 1) The holding costs at the warehouse and all retailers, 2) The major ordering costs for warehouse echelon and retailer echelon, and 3) The minor ordering costs for retailer echelon. The holding cost occurs at each location having physical stock. The total holding cost over the time period at location i (HC_i) can be determined from the unit holding cost (h_i) and the accumulated inventory on-hand over the time period (INV_i). The major ordering cost, not depended on the number of retailers in the order, is the fixed cost occurring once an order is triggered[10]. The retailers can share the major ordering cost together for replenishing in one round trip. The total major ordering cost over the time period at retailer echelon (MJ_r) is the retailers' major ordering cost per an order (K_r) multiplied by the number of dispatch cycle (ND_r). Similarly, the total major ordering cost over the time period at warehouse echelon (MJ_w) is the multiplication of the warehouse' major ordering cost per an order (K_w) and the number of replenishment cycle (NR_w). The minor ordering cost depending on the number of involved retailers in that order is an additional cost of each retailer when replenishing their inventories, such as additional transportation cost relating to distance or other charges. The total minor ordering cost over the time period (MN_r) is accumulated from the involved retailers in each order multiplied by its minor ordering cost of retailer i (κ_i) over the time period. According to the system, target service level and retailers' lead time directly affect the must-order levels at the retailers. Hence, we have to consolidate all relevant costs to determine the appropriate inventory policy setting under the total system-wide cost minimization.

The notations and problem formulation are demonstrated as follows:

n = Number of retailers in the system
i = Index of the location i; the warehouse i = 0 and the retailer $i \in N$
T = The time period considered in the problem (time units)
s_0 = The must-order level at the warehouse (units)
S_0 = The order-up-to level at the warehouse (units)
s_i = The must-order level at retailer i (units)
c_i = The can-order level at retailer i (units)
S_i = The order-up-to level at retailer i (units)
λ_i = Demand rate of retailer i (units/time unit)
h_0 = The unit holding cost per unit time at the warehouse ($/unit – time unit)
h_i = The unit holding cost per unit time at retailer i ($/unit – time unit)
K_w = The warehouse's major ordering cost per a replenishment cycle ($/time)

[10] Even though the problem enable more than an order to be dispatched together in a trip, the major ordering cost is assumed that it occurs once an order is triggered, not once at the dispatching time. The problem relating to the shared major ordering cost at the dispatching time is in the area of shipment scheduling. It has been studied in one echelon holding stock.

K_r = The retailers' major ordering cost per a dispatch cycle (\$/time)

κ_i = The minor ordering cost at retailer i (\$)

L_0 = Lead time for the warehouse (time unit)

L_i = Lead time for the retailer i (time unit)

FR_i = Fill rate of the retailer i

TSL_i = Target service level of the retailer i

$TC(c_i, s_i, S_i, s_0, S_0)$ = The total system-wide cost per unit time (\$/time unit)

HC_i = The total holding cost at location i over the time T units (\$)

MJ_r = The total major ordering cost at retailer echelon over the time T units (\$)

MN_r = The total minor ordering cost at retailer echelon over the time T units (\$)

MJ_w = The total major ordering cost at warehouse echelon over the time T units (\$)

INV_i = The accumulated inventory on-hand over time period at location i (unit – time unit)

BO_i = The accumulated backorder unit over time period at location i (units)

ND_r = The total number of dispatch cycle over the time T units (times)

NR_w = The total number of replenishment cycle over the time T units (times)

$\delta_{(i,j)}$ = An indicator which equals 1 when retailer i is included in the dispatch cycle j and equals 0 otherwise

Objective function:

Minimize $$TC(c_i, s_i, S_i, s_0, S_0) = \frac{\left(\sum_{i=0}^{n} HC_i + MJ_r + MN_r + MJ_w\right)}{T} \quad (4.1)$$

where $$HC_i = h_i \times INV_i \quad (4.2)$$

$$MJ_r = K_r \times ND_r \quad (4.3)$$

$$MN_r = \sum_{j=1}^{ND_r} \sum_{i=1}^{n} \delta_{(i,j)} \kappa_i \quad (4.4)$$

$$MJ_w = K_w \times NR_w \quad (4.5)$$

Constraint $$FR_i = 1 - \frac{BO_i}{\lambda_i T} \quad (4.6)$$

$$FR_i \geq TSL_i \quad (4.7)$$

The objective function of the problem is to minimize the total system-wide cost per unit time. The total system-wide cost per unit time can be a function of five decision variables: c_i, s_i, S_i, s_0, S_0. This problem has more complicated than the problem in Phase I by the reason that it is a constraint problem with demand uncertainty, variation of retailers' order quantity, and order-time synchronization at all locations.

4.2 Research Methodology

According to the complication of the problem, we primarily study the can-order policy on OWNR by using computer simulation as utilized in Phase I. Computer simulation can represent the inventory process by inputting relevant parameters. The preliminary study leads us to develop a heuristic approach. We also determine the best-known solution used to measure the proposed heuristic approach's performance from the simulation.

4.2.1 Computer simulation

The computer algorithm representing the inventory process is illustrated in Fig.IV-3. The inputs for simulating the system can be divided into three groups: decision variables, relevant factors, and experiment setting. We use the same experiment setting as described in Chapter III (Section 3.2.1), and then only two groups are explained as follows:

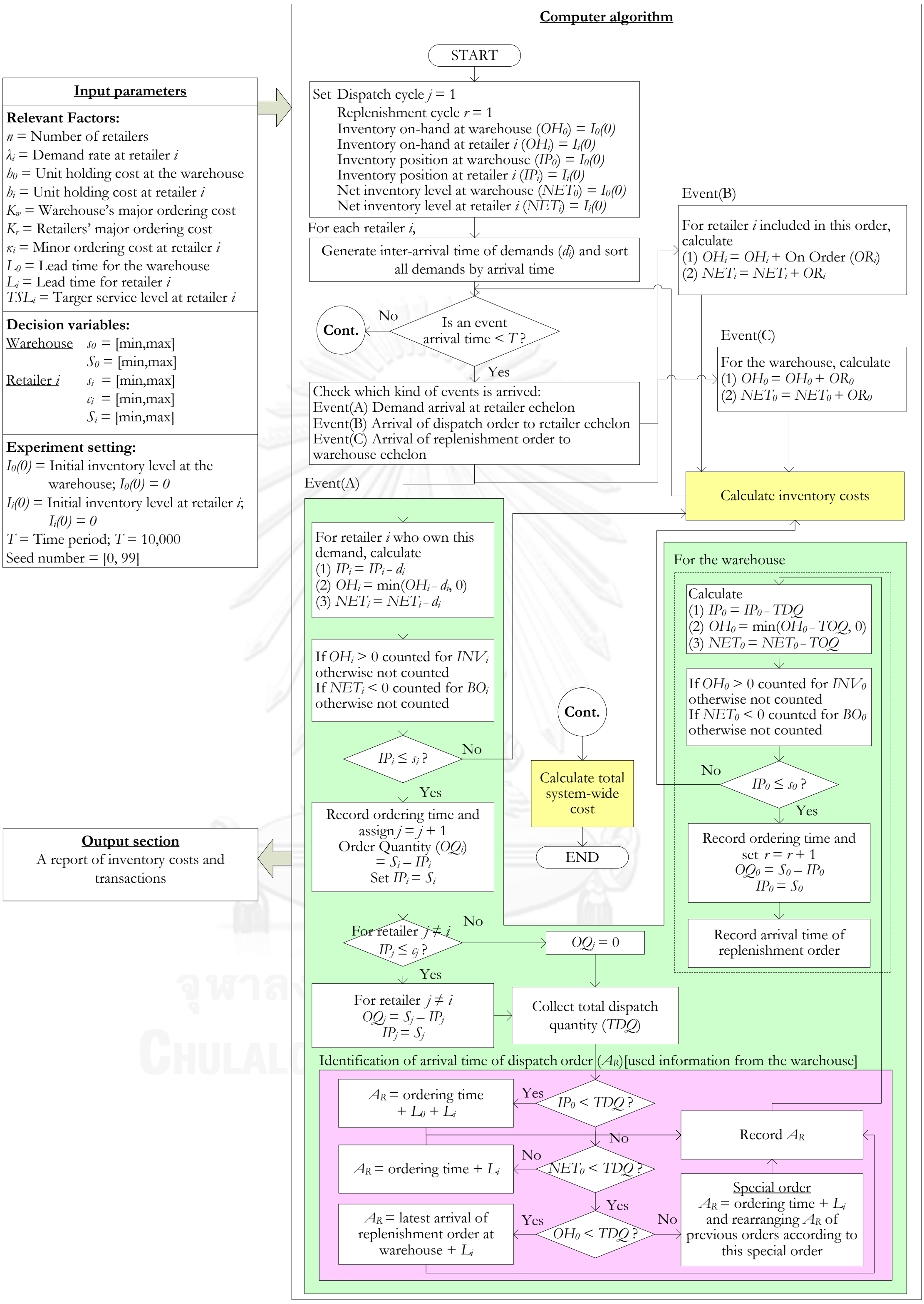

Figure IV-3 The computer algorithm for simulation of Phase II

1) Decision variables (c_i, s_i, S_i, s_0, S_0): Each variable is inputted as a range of minimum and maximum values. A combination of (c_i, s_i, S_i, s_0, S_0) is a solution providing a value of the total system-wide cost and its transaction. The transaction includes number of dispatch cycles, number of replenishment cycles, fill rate at each location, number of replenishment event for various situations[11].

2) Relevant factors: We consider five factors. Three factors are cost parameters, demand rates, and number of retailers already experimented in Phase I, as well as additional two factors are lead time and target service level. We set a combination of relevant factors to a scenario containing different solutions. The best solution providing the minimum total system-wide cost is selected for each scenario.

For the output section, we obtain a report of the inventory costs and its transaction. In consequence, we can find the minimum total system-wide cost for each range of decision variables inputted under a given scenario.

4.2.2 The best solution finding

This process has already been introduced in Chapter III (Section 3.2.2). So, we additionally elaborate some important features which are different from previous content. The best solution finding is composed of two steps: Input parameters and output validation. In this section we focus on the input parameters since there are additional two variables from Phase I. It makes the finding process more complicated. Meanwhile the output validation can follow in Chapter III (Section 3.2.2.2). It is a general procedure used for validation process throughout the dissertation.

With regard to the step of input parameters, we use a replication method for running simulation, so we randomly select a seed number between [0, 99] for first replication. Decision variables are inputted as a range of minimum and maximum values. In the experiment, we set the width of range are 5 units for

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[11] Various situations are identified as follows:

1) Situation that warehouse has sufficient stock to dispatch all lot and has not reached the reorder point,

2) Situation that warehouse has sufficient stock to dispatch all lot and also reached the reorder point,

3) Situation that warehouse has insufficient stock to dispatch all lot,

4) Situation that warehouse has no sufficient stock to dispatch all lot but enough for some retailers included in the order.

s_i, c_i, S_i, s_0 and 20 units for S_0. Since over 5 units of s_i, c_i, S_i, s_0 creates multiplied combinations spending more running time. Whereas S_0 range is larger because S_0 linearly creates combinations. The first range can be set from the initial point of s_i, S_i, s_0, S_0 calculated by

$$s_i^{initial} = \min\left\{ s_i : 1 - \frac{\sum_{y=s_i+1}^{\infty} (y - s_i) Pois(\lambda_i L_i, y)}{\sqrt{2\lambda_i (K_r + \sum_{i \in N} \kappa_i) / h_i}} \geq TSL_i \right\} \tag{4.8}$$

$$s_0^{initial} = \min\left\{ s_0 : 1 - \frac{\sum_{y=s_0+1}^{\infty} (y - s_0) Pois(\lambda_0 L_0, y)}{\sqrt{2\lambda_0 K_w / h_0}} \geq \max(TSL_i) \right\} \tag{4.9}$$

where $Pois(a,b)$ is the probability density function of Poisson demand with parameter (a,b) (using Equation (3.16) to calculate this term).

Then, S_i, S_0 can be determined by $S_0 = s_0 + \sqrt{2K_w \sum_{i \in N} \lambda_i / h_0}$ and $S_i = s_i + \sqrt{2K_r \lambda_i / h_i}$ due to the concept of economic order quantity.

The next step is the process of moving the ranges until the solution seems to be worse continuously. The s_0, S_0 ranges are moved upward and downward by fixing the range at all retailers. Later, we determine the s_i, c_i, S_0 ranges at the retailer i by keeping the same range of s_0, S_0 and the s_j, c_j, S_j ranges at the retailer $j \neq i$. All ranges are changed repeatedly. We select the best solution providing the minimized total system-wide cost for the first replication. Then, the validation process is utilized to get the best-known solution.

4.3 Preliminary Analysis

In the preliminary study, our experiments were conducted to study the relationship between relevant factors on 87 scenarios as showed in Table IV-1.

Table IV-1: Numerical input for preliminary experiment under identical retailers

The asterisk (*) in the table means that parameter is varied.

Scenario No.	Fixed Parameters								Varied Parameters
	K_w	K_r	κ_i	h_0	h_i	λ_i	n	L_i	
1-45	100	50	0	*	10	10	2	0.2	$h_0/h_i \in \{0.3, 0.5, 0.7\}$ $L_0/L_i \in \{0.25, 0.5, 1, 2, 4\}$ $TSL_i \in \{0.90, 0.95, 0.99\}$
46-75	100	50	0	*	10	10	2	1	$h_0/h_i \in \{0.3, 0.5\}$ $L_0/L_i \in \{0.25, 0.5, 1, 2, 4\}$ $TSL_i \in \{0.90, 0.95, 0.99\}$
76-87	100	50	25	3	10	10	2	*	$L_i \in \{2, 0.1\}$ $L_0/L_i \in \{0.5, 4\}$ $TSL_i \in \{0.90, 0.95, 0.99\}$

We primarily analyze the experiments on identical retailers to study the effect of the relevant factors on the can-order policy, since the case of non-identical retailers is very difficult to determine the best-known solution. Relating to the experimental design, the tested problem is generated according to Phase I's results which already studied insight of some relevant factors. From the Phase I, we recognize that the h_0/h_i ratio is one of the most important factors because it affects a decision whether holding stock at the warehouse should be occurred or not. Certainly, stock held at the warehouse bears upon retailer's lead time and service level. Therefore, the experiment is designed by mainly considering the following factors: h_0/h_i ratio, L_i, L_0/L_i ratio, and TSL_i.

According to the minor ordering cost, Phase I indicated that the small ratio of the major ordering cost and the minor ordering cost has an important effect on the coordinated ordering decision. Thus, we extend this finding into the experiment with low h_0/h_i ratio to study a change of decision variables (i.e. at high h_0/h_i ratio decision variables s_0, S_0 are fixed at -1 and 0, respectively, so we cannot clearly investigate the policy setting).

The significant findings are demonstrated as follows:

4.3.1 The effect of the can-order policy

From the general concept of the can-order policy, the major and minor ordering cost and the holding cost are traded off. The can-order level c_i affects reduced major ordering costs, varied minor ordering costs, and increased holding cost from special replenishment. However, c_i must consider target service level since cost saving can be reduced as much as the constraint is unmet. Hence, it is important to find a balance among all inventory costs with a service constraint.

The experiment shows that c_i can help the system sharing the ordering cost among retailers. The increase of c_i reduces the total system-wide cost until a value of c_i, that cost is then increased when c_i is large. The main factors affecting such results are

1) Target service level (TSL_i)

The decrease of c_i creates a possibility of reducing the retailer's fill rate (FR_i) since the average remnant inventory level decreases. The average remnant inventory level is the stock left when normal replenishment occurs as showed in Fig.IV-4 (referring to Fig.IV-2). It implies that the average reorder level occurs at the average remnant inventory level [46]. Therefore, the decrease of the average remnant inventory level increases the opportunity of stock-out influencing to reduce FR_i. Figure IV-5 also illustrates this statement. According to the effect on total system-wide cost, the best-known solution decides to reduce c_i to obtain smallest difference between FR_i and TSL_i ($FR_i \geq TSL_i$) providing lower total system-wide cost.

2) Minor ordering cost (κ_i):

If κ_i is large enough when comparing with the major ordering cost K_r, the increase of c_i reduces the retailers' total minor ordering cost until a value of c_i, that cost is then increased when c_i is large, by the reason that too many retailers are included in an order. The c_i value affects number of retailers jointly replenished in the order, so it influences all relevant inventory costs in the system.

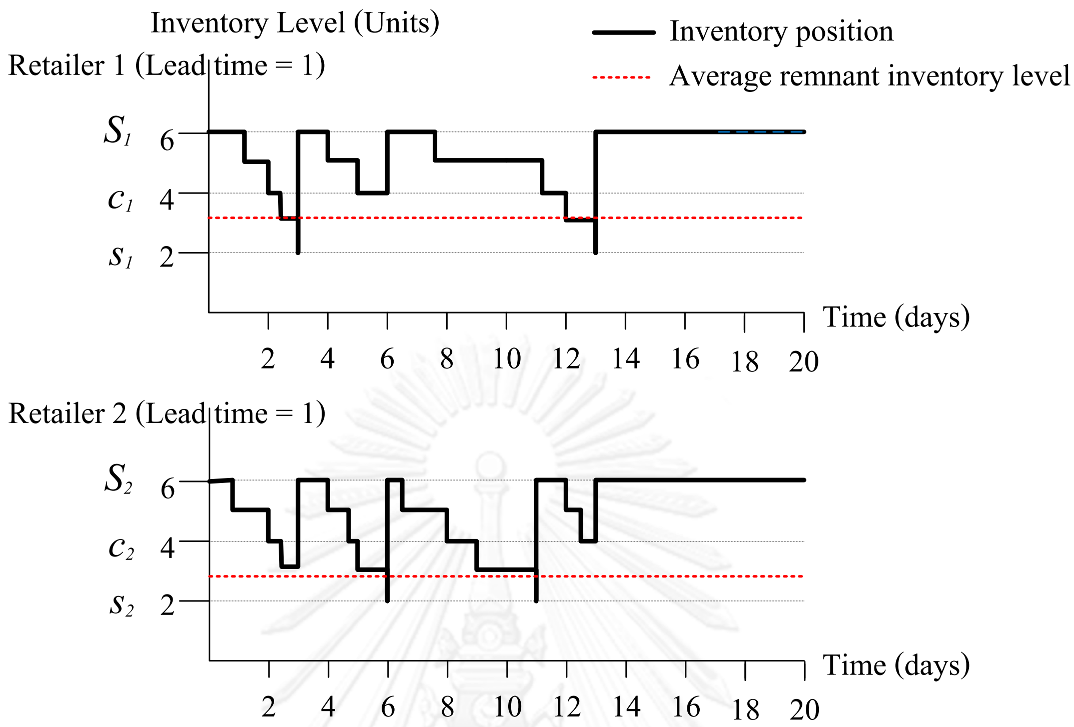

Figure IV-4 The effect of the can-order policy on target service level

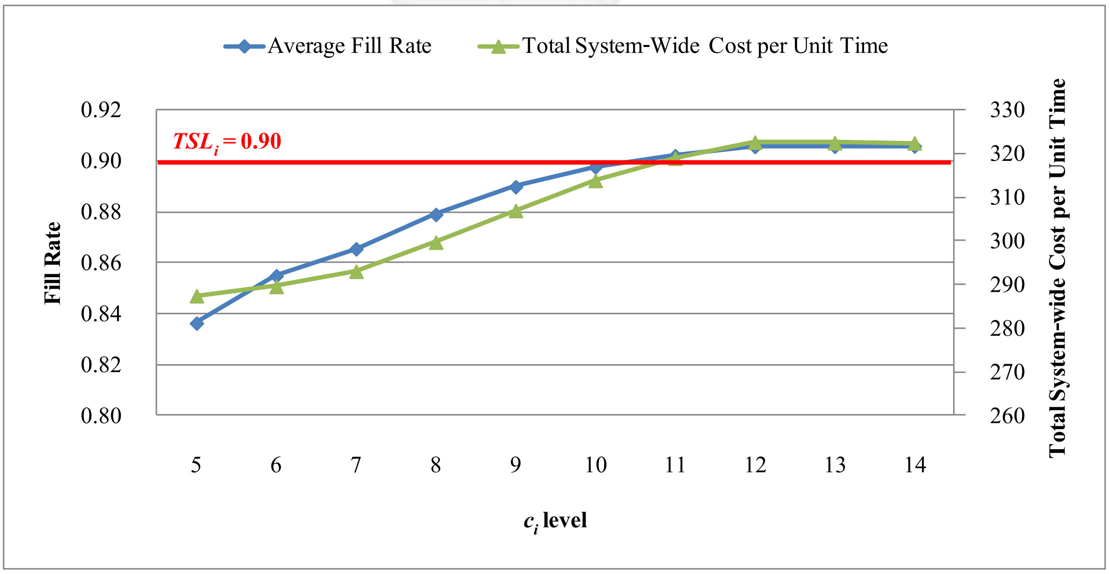

Figure IV-5 The effect of the can-order policy on target service level

As these results, the can-order policy has an effect on the inventory costs at both echelons. It is related to the average remnant inventory level directly associated with retailers' residual stock, as well as dispatch quantity and frequency at retailers. In addition, inventory process at retailer echelon also affect to the warehouse echelon in terms of replenishment quantity and frequency. All are necessary to be traded off with concerning a service constraint to determine the best solution. An interesting issue is that the average reorder level occurs at the average remnant inventory level, therefore $s_i^{(CAN)}$ can be lower than $s_i^{(SI)}$, where $s_i^{(CAN)}$ and $s_i^{(SI)}$ are the must-order level at retailer i of the can-order policy and the SI[12] case, respectively.

4.3.2 The best-known solutions

4.3.2.1 The inventory policies at the warehouse

For a given S_0, we can find the best solution of (s_i, c_i, S_i, s_0) providing the minimum average total system-wide cost as illustrated in Fig.IV-6. There are at least two local minimum solutions located into two ranges: Range I – one solution occurs at S_0 = 0 and Range II – at least one solution occurs at S_0 > 0. For the Range I, S_0 starts from zero and then increases to reach the last value before the cost line turns to resemble a convex function. For the Range II, it is defined after that last value to positive infinity. The best-known solution (global minimum solution) definitely occurs in either Range I or Range II. As a result, this phase provides two ranges as Phase I.

Relevant factors have an effect on determination of the best-known solution occurring in either range. For Range I, zero stock at the warehouse provides the lowest total system-wide cost since the increasing S_0 creates the excessive stock which should not be kept to wait for the next dispatch cycle. Since the warehouse's must-order level is always reached whenever retailer echelon triggers an order. For Range II, a trade-off between the increasing holding costs and the reduced ordering costs when increasing S_0 is occurred as found in the economic

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[12] SI case called in the dissertation is an independent (s, S) policy. SI case meets stochastic demand and independent replenishment where each retailer is dispatched individually, so the major ordering cost of each retailer occurs without sharing.

order quantity. However, this trade-off is restricted by a service constraint which provides the cost line in Fig.IV-6 not to be smooth as the curve found in Phase I.

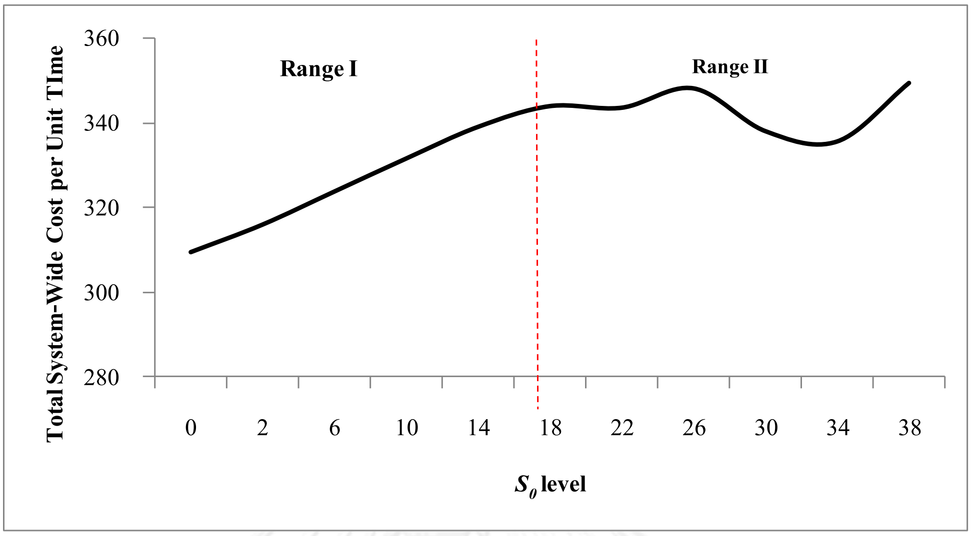


Figure IV-6 Two ranges of the best-known solution

At high h_0 / h_i ratio, we can set S_0 = 0 since increasing stock creates more total system-wide cost (i.e. the increased holding costs is larger than the reduced ordering costs). However, there is a possibility that the best-known solution can move from Range I to Range II when relevant factor is changed, such as smaller h_0 / h_i ratio, higher L_0, and higher TSL_i. At higher L_0, and higher TSL_i, they force the warehouse to hold more stocks to prevent the opportunity of stock-out.

Considering the value of s_0, it can be located in range $[0, s_0^{\max}]$ where $s_0^{\max}$ is the maximum value of the must-order level at the warehouse to serve end customers' demand. Therefore, L_i is included to allow the warehouse having sufficient stock for end customers. $s_0^{\max}$ can be determined by Equation (4.10). Note that $\lambda_0 = \sum_{i=1}^{n} \lambda_i$ with Poisson distribution.

$$s_0^{\max} = \min\left\{ s_0 : 1 - \frac{\sum_{y=s_0+1}^{\infty} (y - s_0) Pois(\lambda_0 TL_0, y)}{\sqrt{2\lambda_0 K_w / h_0}} \geq \max(TSL_i) \right\} \tag{4.10}$$

$$TL_0 = L_0 + \max(L_i) \tag{4.11}$$

4.3.2.2 The inventory policies at the retailers

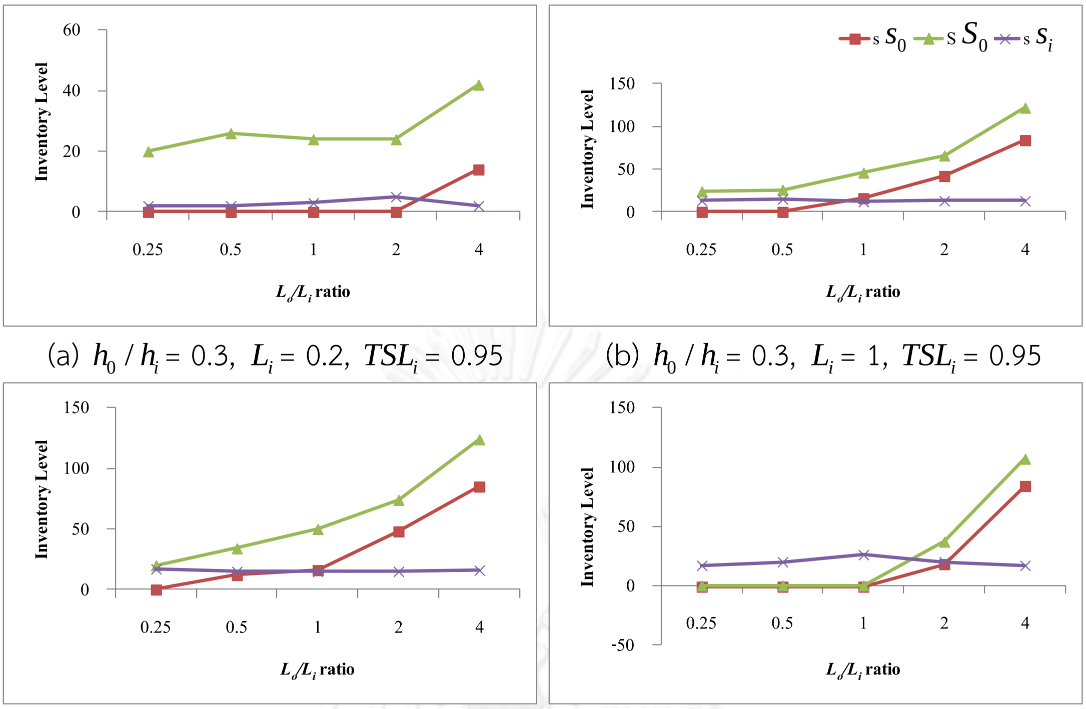
From the experiment (87 scenarios), a result demonstrates that 21.84% of all scenarios (19 scenarios) the value c_i^* = S_i^*- 1, where c_i^* and S_i^* denote the optimal can-order level and the optimal order-up-to level of retailer i. This result is different from Phase I due largely to target service level and the ratio between the major ordering cost and the minor ordering cost as mentioned in Section 4.3.1. However, the result indicates that $TC^*_{(S_i-1)}$ is greater than TC^* 0.15% on average with a standard deviation 0.34% where TC^* is the optimal average total system-wide cost and $TC^*_{(S_i-1)}$ is the minimum average total system-wide cost of the solution at c_i = S_i - 1. Therefore, the setting of c_i = S_i - 1 is still interesting for Phase II. The value of s_i is strongly related to L_i and TSL_i, as well as decision variables at the warehouse s_0, S_0. We deal with the value of s_i in more detail in the Section 4.3.3. Additionally, there is no obvious pattern for S_i because it depends on other decision variables. However, we can find some relationship between decision variables which will be explained in the Section 4.3.4.

4.3.3 Relationship between relevant factors

In this section, we consider the relationship between h_0/h_i ratio, L_i, L_0/L_i ratio, and TSL_i. The result demonstrates that these factors mainly affect the value of s_i, s_0, S_0 as depicted in Fig.IV-7.

Comparing Fig.IV-7(a) and Fig.IV-7(b), high value of L_i forces s_0 to increase even in low L_0/L_i ratio. Meanwhile, s_i increases following higher L_i to maintain service fill rate as targeted but it has to balance with increasing s_0 as well. At low value of L_i, the value of s_0 is close to zero and the warehouse keep a stock with the order-up-to at $S_0 > 0$. This stock compensates the fill rate from $s_0 = 0$.

In case of Fig.IV-7(b) and Fig.IV-7(c), TSL_i affects not only at the retailer echelon but also at the warehouse echelon. Higher TSL_i is able to increase the value of s_0 because it makes less holding costs than an increase of only s_0 due to the h_0 / h_i ratio.



(a) h_0 / h_i = 0.3, L_i = 0.2, TSL_i = 0.95 (b) h_0 / h_i = 0.3, L_i = 1, TSL_i = 0.95

(c) h_0 / h_i = 0.3, L_i = 1, TSL_i = 0.99 (d) h_0 / h_i = 0.5, L_i = 1, TSL_i = 0.95

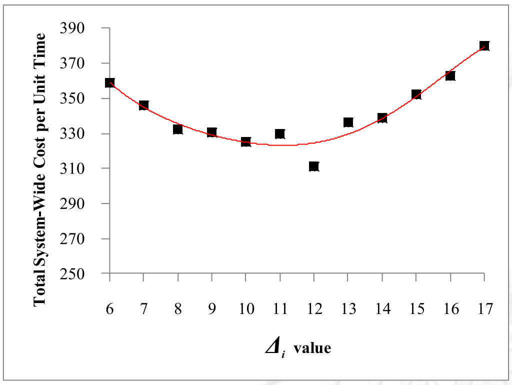
Figure IV-7 Relationship between relevant factors

Considering h_0 / h_i ratio in Fig.IV-7(c) and Fig.IV-7(d), the h_0 / h_i ratio largely influences the decision at the warehouse. When higher h_0 / h_i ratio, the warehouse should not keep stock in order to obtain the minimum total system-wide cost (i.e. S_0 = 0 and s_0 = -1). Thus, increasing s_i is executed to maintain service fill rate as targeted. The best-known solution can be moved from Range I to Range II when lower h_0 / h_i ratio (as stated in the Section 4.3.2.1).

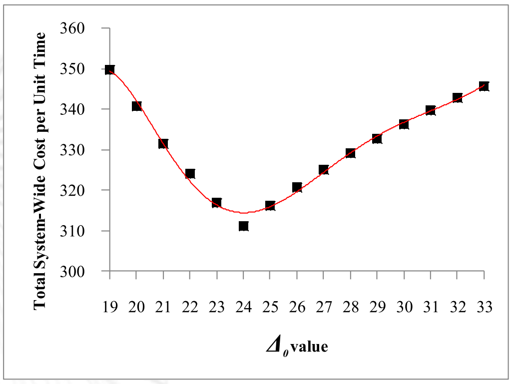
4.3.4 Relationship between decision variables

All decision variables are associated with each other, so it is hard to analyze their relationship obviously. In deterministic model, the major ordering cost and the holding cost are traded off to obtain economical order quantity as a classic EOQ. Thus, we consider $\Delta_k = S_k - s_k$ to represent location k including the warehouse and the retailers; for the warehouse $\Delta_0 = S_0 - s_0$ and for the retailers

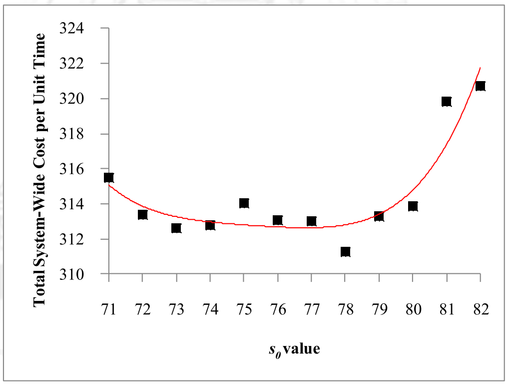
$\Delta_i = S_i - s_i$, $i \in N$. We analyze each decision variable by either fixing other variables or varying some needed variables. The results show various graphs as depicted in Fig.IV-8. Each point in the graph is the best solution on a given value on the horizontal axis. We can draw up a curve across most points in each graph. According to these curves, it is interesting to be a guideline for developing the heuristic approach.



(a) Varied Δ_i and fixed s_0, S_0



(b) Varied Δ_0 and fixed s_i, S_i, s_0



(c) Varied s_0 on which the best solution of (s_i, S_i, S_0) is provided

Figure IV-8 Relationship between decision variables

Due to the laborious process to determine the best-known solutions from computer simulation, an application of heuristic approach is more interesting to systematically reduce the search space for determining the appropriate inventory policy parameters. A lot of existing literatures on the can-order policy used heuristics to accomplish their studies. Section 4.4 and 4.5 will proposed our heuristic approaches with their performances and limitations.

4.4 Heuristic III – Joint Replenishment Model for Single Item and Non-Zero Lead Time

The retailers' major ordering cost can be most shared if all retailers are included in an order to minimize total system-wide cost as we discussed in section 3.7. Preliminary analysis also provided the results as this rationale. Therefore, to develop heuristic approach for the can-order policy we assume that all retailers are replenished together in an order to minimize the major ordering cost per retailers. Then, we can fix the retailer's can-order level at $c_i = S_i - 1$ to create the maximum opportunity of joint replenishment for all retailers. By the fixed retailer's can-order level, we can focus on determining other decision variables (s_i, S_i, s_0, S_0).

4.4.1 Approximate mathematical model with non-zero lead time (MMNZ)

From Phase I, we obtained two heuristics abbreviated to DJ and EOQ-Z (expressed in Section 3.4 and Section 3.5, respectively). EOQ-Z provides more efficient approach and preferable results than DJ does, so we extend EOQ-Z approach into non-zero lead time system.

4.4.1.1 Mathematical model

Our purpose of developing heuristic approach is to provide an appropriate inventory policy (c_i, S_i, S_0). The total system-wide cost of mathematical model is able to be approximated as long as the acceptable solution is provided. Hence, relating to the preliminary analysis our mathematical model utilizes the can-order level at c_i = S_i - 1. This fixed value of c_i can simplify the can-order policy into the regenerative process [22, 48, 127]. Each dispatch cycle is independently generated at the same starting point, which is the order-up-to level S_i for all retailers. In consequence, the cost model can be formulated for a given (s_i, S_i, s_0, S_0) policy.

We simplify this part by assuming the warehouse's inventory level is consumed continuously following total Poisson demand cumulated from all retailers, $\lambda_0 = \sum_{i \in N} \lambda_i$. Some equations in EOQ-Z (from the 1st phase) can be utilized. The cost model can be used Equation (3.10) for a given (s_i, S_i, s_0, S_0) policy, and also added the service constraint to the model. Note that Equation (4.7) can be reused. Thus,

$$TC(s_i, S_i, s_0, S_0) = \frac{K_r + \sum_{i \in N} \left\{ \left(1 - \Phi(S_i)\right) \times \kappa_i) + E[H_i] \right\}}{E[DT]} + \frac{K_w + E[H_0]}{E[RT]} \quad (4.12)$$

<u>Constraint</u>

$$FR_i = 1 - \frac{E[SH_i]}{E[Q_i]} \quad (4.13)$$

$$FR_i \geq TSL_i \quad (4.7)$$

$TC(s_i, S_i, s_0, S_0)$ = The long-run average total system-wide cost per unit time ($/unit time)

i = Index of the location i; the warehouse i = 0 and the retailer $i \in N$

s_0 = The must-order level at the warehouse (units)

s_i = The must-order level at retailer i (units)

S_0 = The order-up-to level at the warehouse (units)

S_i = The order-up-to level at retailer i (units)

λ_i = Demand rate of retailer i (units/time unit)

h_0 = The unit holding cost per unit time at the warehouse ($/unit – time unit)

h_i = The unit holding cost per unit time at retailer i ($/unit – time unit)

K_w = The warehouse's major ordering cost per a replenishment cycle ($/time)

K_r = The retailers' major ordering cost per a dispatch cycle ($/time)

κ_i = The minor ordering cost at retailer i ($)

L_0 = Lead time for the warehouse (time unit)

L_i = Lead time for the retailer i (time unit)

TSL_i = Target service level at the retailer i

$E[WT]$ = The expected waiting time at retailer echelon when the warehouse is unable to dispatch according to the committed lead time (time unit)

$E[L_i]$ = The expected total lead time for retailer i (time unit)

$\Phi(S_i)$ = The probability that no demand arrives for retailer i during a dispatch cycle

$E[H_i]$ = The expected holding cost of retailer i during a dispatch cycle ($)

$E[H_0]$ = The expected holding cost of the warehouse during a replenishment cycle ($)

$E[DT]$ = The expected length of a dispatch cycle (unit time)

$E[RT]$ = The expected length of a replenishment cycle (unit time)

FR_i = Long run fraction of demand satisfied from stock on-hand of the retailer i.

$E[SH_i]$ = The expected number of shortage per dispatch cycle of the retailer i (units/time unit)

$E[SH_0]$ = The expected number of shortage per replenishment cycle of the warehouse (units/time unit)

$E[Q_i]$ = The expected dispatch quantity per dispatch cycle of the retailer i (units/time unit)

Retailer Echelon

The model is developed according to the independent Poisson process of demands for individual retailers, so inter-arrival times of demands are exponentially distributed. Thus, time until retailer i triggers an order to the warehouse (DT_i) follows Erlang distribution with parameters λ_i and Δ_i. We can determine related probability function of DT_i and DT, where DT is time until any retailer triggers an order to the warehouse, by using Equation (3.12) and (3.13). Then, we are able to calculate the expected length of a dispatch cycle, $E[DT]$, by using Equation (3.14).

The expected holding cost of retailer i during a dispatch cycle is associated with the retailer's inventory on hand at the beginning and at the end of the dispatch cycle. At the beginning of the cycle, setting $c_i = S_i - 1$ makes all retailers' inventory on hand equal $S_i - y$ where y is total demands during lead time. At the end of the cycle, the inventory on hand depends on the residual stock. Thus, we define $\Phi_i(x)$ as the probability that at time DT the residual stock of retailer i equals x. There are two cases for determining $\Phi_i(x)$. The first case is when the residual stock level of retailer i is equal to zero; only retailer i triggers an order. The second case is when the residual stock level of retailer i is positive. So, an order is triggered by retailer $j \neq i$. From EOQ-Z, the value of $\Phi_i(x)$ can be calculated by the following expressions reused Equation (3.16) and (3.17) for $Pois(a,b)$ and $f^{(-i)}(t)$, respectively:

$$\Phi_i(x) = \begin{cases} \int_{t=0}^{\infty} f_i(t)\prod_{j\neq i}\left(1-F_j(t)\right)dt & \text{if } x = 0, \\ \int_{t=0}^{\infty} Pois(\lambda_i t, \Delta_i - x) f^{(-i)}(t)dt & \text{if } 0 < x \leq \Delta_i \end{cases} \tag{4.14}$$

Since any retailer's order has to wait until the warehouse's inventory on-hand is available. In case of sufficient stock on-hand at the warehouse, the retailer's replenishment just depends on its lead time L_i. On the other hand, if the warehouse has not enough stock on-hand, the retailer has to wait longer. The expected waiting time can be calculated by using Little's formula [50, 128]. However, in case of special order which does not follow FIFO as explained in Section 4.1 the waiting time might be shorter than FIFO. Therefore, we define the proportion of waiting time as comparing to FIFO case (p). This value is in a range of [0, 1]; it is defaulted at 1. We can determine the expected total lead time for retailer i by the following expressions.

$$E[WT]^{FIFO} = \frac{E[SH_0]}{\lambda_0} \tag{4.15}$$

$$E[WT] = pE[WT]^{FIFO} \tag{4.16}$$

$$E[L_i] = L_i + E[WT] \tag{4.17}$$

The expected holding cost of retailer i during a dispatch cycle is then given by

$$E[H_i] = \sum_{y=0}^{\infty}\left(Pois(\lambda_i E[L_i], y)\sum_{x=0}^{\Delta_i}\left\{ \Phi(x)\int_{t=0}^{\infty} H_i(S_i - y, s_i + x - y, t) f(t)dt \right\}\right) \tag{4.18}$$

$$H_i(z,q,t)=\begin{cases}\dfrac{h_i(z+q)t}{2} & if\ z>0,\ q\geq 0\\ \dfrac{h_i z^2 t}{2(z-q)} & if\ z>0,\ q<0\\ 0 & otherwise\end{cases} \tag{4.19}$$

where $H_i(z,q,t)$ is the expected total holding cost for retailer i during a dispatch cycle of t periods given that the inventory on hand equals z at the beginning and equals q at the end of the cycle.

According to Equation (3.14), (4.18) and (4.19), we can reduce the expression to determine the expected holding cost of retailer i per unit time by

$$\frac{E[H_i]}{E[DT]}=\sum_{y=0}^{\infty}\left(Pois(\lambda_i E[L_i],y)\sum_{x=0}^{\Delta_i}\left\{\Phi(x)H_i(S_i-y,s_i+x-y)\right\}\right) \tag{4.20}$$

$$H_i(z,q)=\begin{cases}\dfrac{h_i(z+q)}{2} & if\ z>0,\ q\geq 0\\ \dfrac{h_i z^2}{2(z-q)} & if\ z>0,\ q<0\\ 0 & otherwise\end{cases} \tag{4.21}$$

To determine long run fill rate at retailer i, the expected number of shortage per dispatch cycle of the retailer i, $E[SH_i]$, and the expected dispatch quantity per dispatch cycle of retailer i, $E[Q_i]$, are given by

$$E[SH_i]=\sum_{x=0}^{\Delta_i}\left\{\Phi(x)\sum_{y=s_i+x}^{\infty}(y-s_i-x)Pois(\lambda_i E[L_i],y)\right\} \tag{4.22}$$

$$E[Q_i]=\sum_{x=0}^{\Delta_i}\left\{\Phi(x)(\Delta_i-x)\right\} \tag{4.23}$$

Warehouse Echelon

We assume that the warehouse's inventory level is consumed continuously by all retailers' Poisson demands with rate λ_0. Inter-arrival times of demands are exponentially distributed, and then the distribution of time until warehouse triggers an order to an outside supplier is Erlang, similar to the retailer echelon. Let RT denote time until warehouse triggers an order to an outside supplier. The warehouse will trigger an order if the total demand from time 0 equals Δ_0, so the distribution of RT is Erlang with parameters λ_0 and Δ_0. The expected length of a replenishment cycle is mean of Erlang distribution. Thus, $E[RT] = \Delta_0 / \lambda_0$.

Similar to retailer echelon, we determine the expected holding cost of the warehouse per unit time by

$$\frac{E[H_i]}{E[RT]} = \sum_{y=0}^{\infty} Pois(\lambda_i L_0, y) H_0(S_0 - y, s_0 - y) \tag{4.24}$$

$$H_0(z,q) = \begin{cases} \dfrac{h_0(z+q)}{2} & if\ z > 0,\ q \geq 0 \\ \dfrac{h_0 z^2}{2(z-q)} & if\ z > 0,\ q < 0 \\ 0 & otherwise \end{cases} \tag{4.25}$$

The expected number of shortage per dispatch cycle of the warehouse is then given by

$$E[SH_0] = \sum_{y=s_i+1}^{\infty} (y - s_i) Pois(\lambda_i L_0, y) \tag{4.26}$$

Consequently, we can figure out the long-run average total system-wide cost per unit time for a given (s_i, S_i, s_0, S_0) policy. Later, the algorithm of heuristic approach is demonstrated to determine the appropriate decision variables by using the cost model mentioned above.

4.4.1.2 Heuristic algorithm

We use the concept of the EOQ-Z heuristic to develop heuristic algorithm for non-zero lead time consideration by the reason that the preliminary study provides the similar results. Therefore, the design of heuristic approach (named MMNZ for Phase II) is based on the following concept.

Table IV-2: Additional concept for developing the MMNZ heuristic as comparing to the EOQ-Z heuristic

Concept of the EOQ-Z heuristic (Phase I)	Concept of the MMNZ heuristic (Phase II)
1) The value of S_0 is identified to S_0 = 0 for Range I and $S_0 = \sqrt{2K_w\lambda_0/h_0}$ for Range II.	1) Regarding at least two local minimum solutions located into two ranges, procedure for determining the value of S_0 can be divided into such ranges. For Range I, the value of S_0 is set at 0 and then s_0 is also assigned to -1. For Range II, we apply a search algorithm to determine the value of S_0 which is more than 0.
2) To develop initial solution at retailer echelon, deterministic model is used to find economical joint ordering time when every retailer is replenished in an order.	2) We use the same concept as EOQ-Z to find out initial Δ_0 and Δ_i
3) Decomposition technique and iterative procedure are applied to break multiple locations into single location and to recurrently find the minimum solution as far as the best solution has been found.	3) We use the same concept as EOQ-Z to break multiple locations into single location and to recurrently determine the local minimum $TC(s_i, S_i, s_0, S_0)$ at the given $\Delta_{j \neq i}$ and Δ_0.
4) The concept of the golden section search is applied to determine the minimum value of S_i.	4) We apply the concept of the golden section search to determine the (near) minimum value of Δ_0, Δ_i, and s_0.

Hence, the heuristic approach is outlined in the following algorithm illustrated in Fig.IV-9.

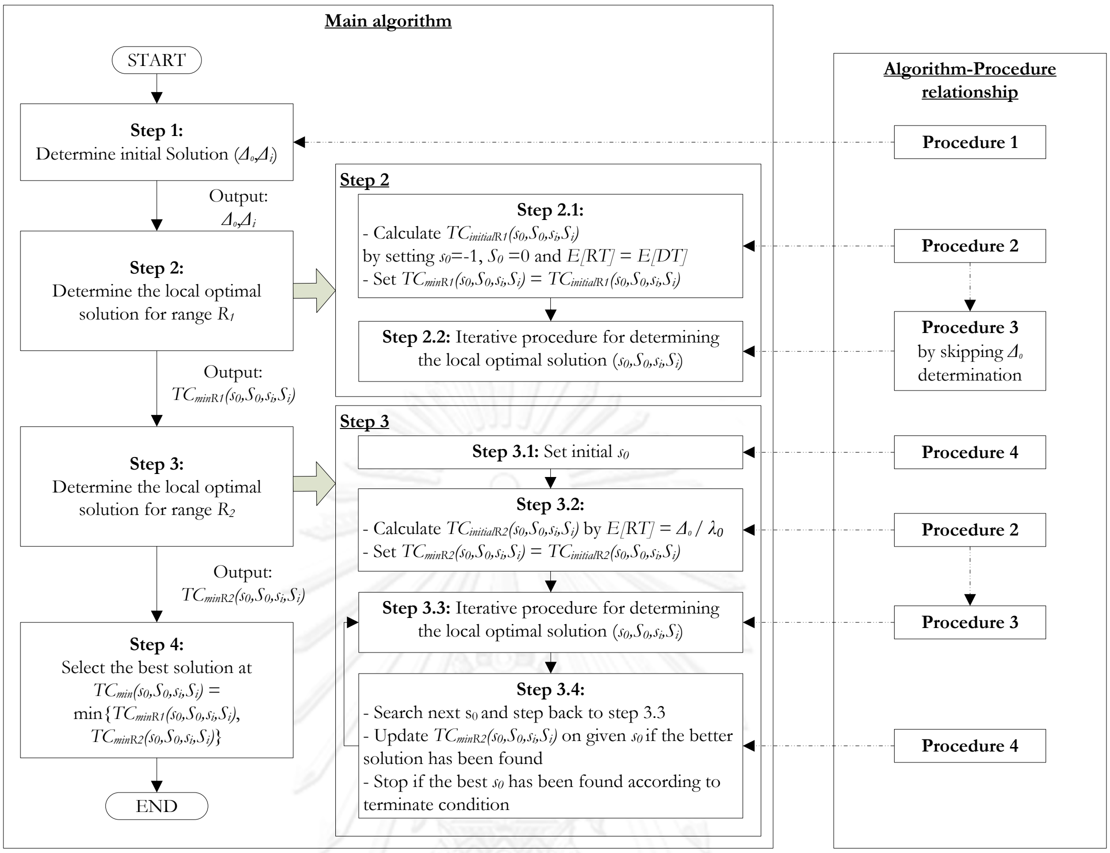


Figure IV-9 The algorithm of the heuristic approach – MMNZ

Procedure 1 – Determination of the initial solution Δ_0 and Δ_i

For the initial value of Δ_0, we simply determine by using EOQ formula, then $\Delta_0 = \sqrt{2K_w\lambda_0/h_0}$.

For the initial value of Δ_i, we calculate joint dispatching time (T_d) by deterministic model according to the following expression.

$$T_d = \sqrt{\frac{2(K_w + \sum_{i\in N}\kappa_i)}{\sum_{i\in N}\lambda_i h_i}} \qquad (4.27)$$

Later, the initial Δ_i for retailer i is determined by adapting Love [46]'s method. It is selecting Δ_i which provides the minimum gap between two probabilities: 1) the probability that demand for retailer i during time T_d is less than or equal to such Δ_i and 2) the probability that an order is triggered by any retailer (i.e. including normal replenishment and special replenishment). Thus,

$$\Delta_i = \begin{cases} u & if \left\{ Pois(\lambda_i T_d, u+1) - \left(\frac{n}{n+1}\right) \right\} \geq \left\{ \left(\frac{n}{n+1}\right) - Pois(\lambda_i T_d, u) \right\} \\ u+1 & Otherwise \end{cases} \quad (4.28)$$

The initial Δ_i from Equation (4.28) is closer to the optimal solution than Δ_i obtained from $\Delta_i = \lambda_i T_d$.

Procedure 2 – Determination of retailer's must-order level s_i

According to a service constraint, the value of s_i must be high enough to serve target service level (TSL_i). Whenever a change of Δ_0 or Δ_i is occurred, this procedure is needed to find out updated s_i. Therefore, the procedure is also used together with Procedure 3. It can be divided into 4 sub-procedures as follows:

<u>Procedure 2.1</u>: Determine the initial value of s_i for retailer i and repeat until all retailers has been done.

$$s_i^{initial} = \min \left\{ s_i : 1 - \frac{\sum_{y=s_i+1}^{\infty} (y - s_i) Pois(\lambda_i L_i, y)}{E[Q_i]} \geq TSL_i \right\} \quad (4.29)$$

Start at retailer i = 1 and follow operations below, then repeat until all retailers have been done.

<u>Procedure 2.2</u>: Calculate FR_i for each retailer by using Equation (4.13) and check the difference between FR_i and TSL_i with the tolerance $\hat{\varepsilon}$. If $|FR_i - TSL_i| \leq \hat{\varepsilon}$ go to Procedure 2.3, else go to Procedure 2.4. In case that $FR_i = TSL_i$, select the current s_i to the solution, and then terminate Procedure 2.

<u>Procedure 2.3</u>: Compare FR_i with TSL_i and use "Sequential Search" to find s_i.

- If $FR_i < TSL_i$, increase s_i by 1 until obtain $FR_i \geq TSL_i$. Select the current s_i to the solution, and then terminate Procedure 2.
- If $FR_i > TSL_i$, decrease s_i by 1 until obtain $FR_i < TSL_i$. Select the last s_i providing $FR_i \geq TSL_i$ to the solution, and then terminate Procedure 2.

Procedure 2.4: Compare FR_i with TSL_i and use "Half-Interval Search" to reduce search space.

- Set initial boundary of s_i by the following conditions:
  - If $FR_i > TSL_i$, set $s_i^A = s_i$ and $s_i^B = 0$
  - If $FR_i < TSL_i$, increase $s_i = s_i + R$ where $R = round(\lambda_i L_i)$ to integer number until obtain $FR_i > TSL_i$. Assign the current s_i to s_i^A and the last s_i providing $FR_i \geq TSL_i$ to s_i^B
  - Then, the initial boundary of s_i (R') is equal to $\left|s_i^A - s_i^B\right|$
- Repeat the operations below and simultaneously evaluate with Procedure 2.2's condition.
  - If $FR_i > TSL_i$, assign $s_i = s_i^A - R'/2$, else $s_i = s_i^A + R'/2$
  - Set new value of s_i^A, s_i^B by assigning $s_i^B = s_i^A$ and $s_i^A = s_i$
  - Update new R' from new value of s_i^A, s_i^B
  - Range R' is reduced until Procedure 2.2's condition is met ($\left|FR_i - TSL_i\right| \leq \hat{\varepsilon}$), then go to Procedure 2.3.

Procedure 3 – Iterative procedure for finding the best combination of (Δ_0, Δ_i) on given s_0

This procedure applies Step 2.2 of EOQ-Z's heuristic algorithm which is an iterative procedure containing step (A) to (F) illustrated in Fig.III-10. Step 2.2 of EOQ-Z's heuristic algorithm is used for retailer echelon, but in Procedure 3 we extend to warehouse echelon as well.

We consider $\Delta_k = S_k - s_k$ to represent location k including the warehouse and the retailers; for the warehouse $\Delta_0 = S_0 - s_0$ and for the retailers $\Delta_i = S_i - s_i$, $i \in N$. We modify Step 2.2(A) by setting location k = -1 to cover both echelons, and Step2.2(B) set location $k = k$ + 1 by fixing other locations $\Delta_{j \neq k}$ given from the previous iteration. For each iteration, the golden section search is carried out for location k: vary Δ_k and fix $\Delta_{j\neq k}$. $TC(s_i, S_i, s_0, S_0)$ is an objective function for the golden section search. The iterative process terminates as soon as every Δ_k does not change $n+1$ iterations in a row, or the minimum long-run average total system-wide cost per unit time from the current loop does not decrease from the previous loop by more than $\varepsilon\%$. From Procedure 3, we get the local minimum long-run average total system-wide cost per unit time for either Range I or Range II on given s_0.

Procedure 4 – Determination of the warehouse's must-order level s_0

Procedure 4 includes 2 sub-procedures, the first is determination of the initial value of s_0 and the second is search algorithm for the best s_0. The initial s_0 is defined as the maximum value of s_0, so Equation (4.10) and (4.11) are utilized. Then, search space for s_0 is restricted within $[0, s_0^{\max}]$. Then, we apply gold section search to determine the best s_0.

In the last step of algorithm, the minimum long-run average total system-wide cost per unit time is equal to the minimum value of either ranges, $\min\{TC_{\min R1}(s_i, S_i, s_0, S_0), TC_{\min R2}(s_i, S_i, s_0, S_0)\}$.

To summarize, our heuristic approach (called MMNZ) is developed by using approximate mathematical model with heuristic algorithm to determine the appropriate inventory policy parameters. The mathematical model is extended from EOQ-Z integrating lead time consideration and service level. We can interpret preliminary analysis into the heuristic algorithm consisting of decomposition technique, iterative procedure, and one-dimensional search called the golden section search. To measure heuristic's performance, we carry out a pilot testing demonstrated in the next section.

4.4.1.3 Pilot testing

We explore the cost gap of the MMNZ heuristic and the best-known solution obtained from computer simulation by using Equation (3.7). We tested on 29 scenarios selected from Table IV-1 under considering zero minor ordering cost. Consequently, the testing result can be summarized as showed in Table IV-3 and Table IV-4.

Table IV-3: Pilot testing for comparison of the best-known solution and the MMNZ heuristic's best solution (Low lead

Fixed parameters $K_w = 100,\ K_r = 50,\ h_i = 10,\ \lambda_i = 10,\ n = 2,\ L_i = 0.2$

Instance	**Relevant Parameters**				**Best-known Solution (BS)**		**MMNZ heuristic**			
	h_0	L_0	L_0/L_i	TSL_i	$(s_i, c_i, S_i), (s_0, S_0)$	$TC^{(BS)}$	$(s_i, c_i, S_i), (s_0, S_0)$	$TC^{(HRT)}$	$FR^{(HRT)}$	$C.G.$
1	3	0.05	0.25	0.90	(1,10,11),(0,20)	253.25	(0,16,17),(-1,0)	251.66	0.899	- 0.63%
2	5	0.05	0.25	0.95	(2,13,16),(-1,0)	281.54	(2,14,15),(-1,0)	281.93	0.955	0.14%
3	7	0.05	0.25	0.99	(4,15,18),(-1,0)	321.02	(4,16,17),(-1,0)	321.33	0.991	0.10%
4	3	0.10	0.50	0.99	(4,16,17),(0,25)	322.14	(4,12,13),(2,39)	325.48	0.988	1.04%
5	5	0.10	0.50	0.90	(1,11,16),(-1,0)	254.65	(1,15,16),(-1,0)	254.73	0.905	0.03%
6	7	0.10	0.50	0.95	(2,16,19),(-1,0)	280.13	(2,18,19),(-1,0)	280.15	0.951	0.01%
7	3	0.20	1.00	0.99	(5,13,14),(2,33)	332.31	(4,11,12),(5,42)	332.27	0.986	-0.01%
8	7	0.20	1.00	0.90	(2,14,17),(-1,0)	255.11	(2,16,17),(-1,0)	255.11	0.900	0.00%
9	3	0.40	2.00	0.95	(5,16,17),(0,24)	275.12	(5,14,15),(3,40)	313.33	0.964	13.89%
10	5	0.40	2.00	0.99	(9,20,23),(-1,0)	350.89	(9,21,22),(-1,0)	351.19	0.991	0.09%
11	3	0.80	4.00	0.90	(1,9,12),(14,41)	265.86	(8,18,19),(3,42)	344.09	0.954	29.42%
12	5	0.80	4.00	0.95	(10,21,29),(-1,0)	309.65	(10,18,19),(3,34)	387.69	0.973	25.20%
13	5	0.80	4.00	0.99	(4,15,16),(10,23)	364.81	(8,16,170,(10,39)	397.45	0.975	8.95%
14	7	0.80	4.00	0.95	(10,21,29),(-1,0)	309.65	(10,27,28),(-1,0)	305.12	0.945	-1.46%

Table IV-4: Pilot testing for comparison of the best-known solution and the MMNZ heuristic's best solution (High lead

Fixed parameters $K_w = 100,\ K_r = 50,\ h_i = 10,\ \lambda_i = 10,\ n = 2,\ L_i = 1$

Instance	Relevant Parameters				Best-known Solution (BS)		MMNZ heuristic			
	h_0	L_0	L_0/L_i	TSL_i	$(s_i, c_i, S_i), (s_0, S_0)$	$TC^{(BS)}$	$(s_i, c_i, S_i), (s_0, S_0)$	$TC^{(HRT)}$	$FR^{(HRT)}$	$C.G.$
1	3	0.25	0.25	0.90	(11,22,24),(0,26)	279.73	(11,27,28),(-1,0)	272.97	0.898	-2.42%
2	5	0.25	0.25	0.99	(17,26,27),(0,20)	384.00	(17,31,32),(-1,0)	382.27	0.989	-0.45%
3	5	0.25	0.25	0.95	(13,26,31),(-1,0)	315.11	(13,29,30),(-1,0)	310.52	0.949	-1.45%
4	3	0.50	0.50	0.95	(15,24,28),(0,26)	324.23	(16,32,33),(-1,0)	320.42	0.940	-1.17%
5	5	0.50	0.50	0.90	(14,28,31),(-1,0)	282.73	(14,29,30),(-1,0)	278.61	0.897	-1.45%
6	5	0.50	0.50	0.99	(20,34,37),(-1,0)	398.79	(20,34,35),(-1,0)	392.17	0.989	-1.66%
7	3	1.00	1.00	0.90	(11,19,22),(12,42)	289.55	(18,29,30),(5,45)	391.43	0.960	35.18%
8	3	1.00	1.00	0.99	(15,27,28),(16,50)	397.30	(21,30,31),(10,48)	432.09	0.979	8.76%
9	5	1.00	1.00	0.95	(22,34,38),(-1,0)	335.79	(21,32,33),(3,36)	410.81	0.962	22.34%
10	3	2.00	2.00	0.95	(13,22,25),(42,66)	337.40	(24,35,36),(18,61)	438.68	0.969	30.02%
11	5	2.00	2.00	0.90	(30,41,49),(-1,0)	312.50	(25,38,39),(11,52)	481.38	0.982	54.04%
12	5	2.00	2.00	0.99	(20,33,39),(18,37)	452.16	(26,35,36),(22,56)	509.79	0.990	12.75%
13	3	4.00	4.00	0.90	(12,21,24),(82,106)	311.25	(19,30,31),(63,104)	383.24	0.940	23.13%
14	3	4.00	4.00	0.99	(16,24,25),(85,124)	424.51	(20,30,31),(72,110)	438.71	0.976	3.34%
15	5	4.00	4.00	0.95	(27,41,43),(42,61)	388.83	(28,39,40),(51,92)	465.05	0.951	19.60%

Let $FR^{(HRT)}$ is the average fill rate obtained from the best solution of heuristic approach. We found some limitations of the heuristic MMNZ as follows:

1) High cost gap (with maximum at 54.04%) was occurred when the best solution falls in Range II, i.e. $TC_{\min R2}(S_i, S_0) < TC_{\min R1}(S_i, S_0)$. It seemed that approximate mathematical model on the warehouse echelon was poor for the situations that have high L_i, high L_0 / L_i, and low h_0 / h_i. Cost gap was huge because

(1) The approximate mathematical model influenced the warehouse to hold more cycle stock and to reduce safety stock; therefore retailer's must-order level was higher than actual due to target service level or

(2) The approximate mathematical model provided lower cost than actual, so the best solution preferably fell into Range II instead of Range I. Range I used the (near) exact model which provided the total system-wide cost close to actual cost as van Eijs [48]'s formulation.

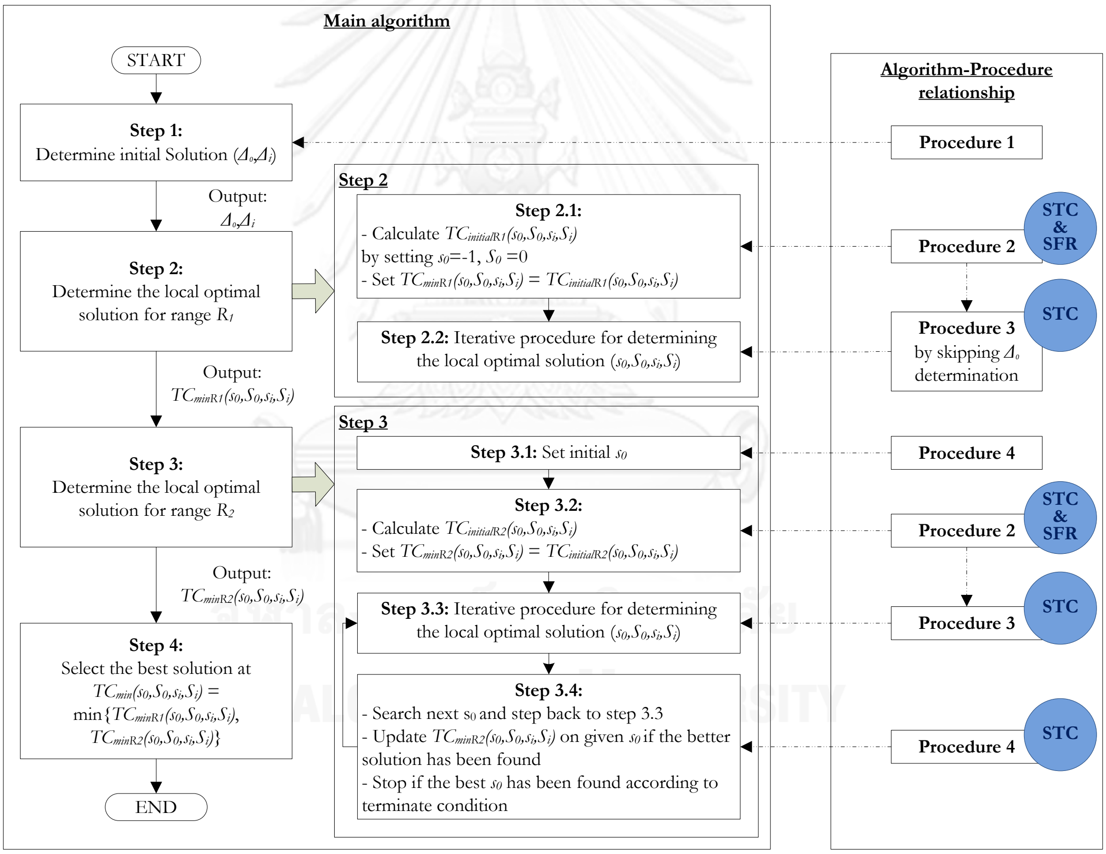
2) The best solution of heuristic approach provided the average total system-wide cost $TC^{(HRT)}$ lower than the best-known solution's cost $TC^{(BS)}$ due to the average fill rate.

The MMNZ heuristic seems to be useful if the cross-docking system performs better, but there is a possibility that the average fill rate is less than target service level. For pilot testing, average fill rate is less than target service level 0.29% on average with a standard deviation 0.32%. According to its limitation, we attempt to develop another heuristic approach to obtain better quality solution as demonstrated in the next solution.

4.4.2 Simulation cost model for single item and non-zero lead time (SIM/S/NZ)

We propose a new heuristic approach to determine an appropriate inventory policy. Since the approximate mathematical model is not suitable for the complicated system, we use the simulation cost model instead. This can reduce the cost error from an approximation. However, we apply heuristic algorithm from Section 4.4.1.2 into this approach called SIM/S/NZ.

The simulation cost model follows the algorithm illustrated in Fig.IV-3. Then, the total system-wide cost and fill rate from the simulation cost model are used in the heuristic algorithm instead of approximate mathematical model (Section 4.4.1.2). However, there are some equations of approximate mathematical model utilized to find out the initial values of s_i, s_0, Δ_i, and Δ_0. They have to be inputted in the simulation cost model to initiate the first solution for iterative procedure. The heuristic algorithm exists but we add the part of simulation cost model as depicted in Fig.IV-10.



STC = Simulated total system-wide cost per unit time obtained from the simulation cost model
SFR = Simulated fill rate obtained from the simulation cost model

Figure IV-10 The algorithm of the heuristic approach – SIM/S/NZ

Summarily, we conducted the research continuously to find out an appropriate solution approach. We first developed an approximate mathematical model and heuristic algorithm called "MMNZ" heuristic. We interpreted preliminary analysis into the heuristic algorithm consisting of decomposition technique, iterative procedure, and one-dimensional search called the golden section search. Its result provides huge cost gap in some situations because approximate mathematical model might not reflect actual process. Therefore, another approach was introduced by using simulation cost model instead of approximate mathematical model. It is called "SIM/S/NZ" heuristic. It could reduce cost error from approximation. To measure heuristic's performance, we continue to the next section which various experiments are carried out and the analysis of the results is demonstrated herein.

4.5 Experimental Results

The SIM/S/NZ heuristic was experimented on various scenarios following the Table IV-1 (87 scenarios). The experiments focused on identical retailers with and without minor ordering cost, since both cases affects the can-order policy at given c_i = S_i - 1 on different results as showed in the preliminary analysis. According to using simulated total system-wide cost and simulated fill rate, seed number is an important input to generate inter-arrival time of demand. Thus, we conducted the research by using the same method of output validation described in Section 3.2.2.2. We tested on five replications with different random seed numbers. Then, for each best solution we determined the average total system-wide cost by additional 10 random seed numbers. We define "the minimum solution" provided by the best solution with the minimum of average total system-wide cost. We use Equation (3.7) to measure heuristic's performance.

4.5.1 Identical retailers with zero minor ordering cost

According to 75 scenarios tested, Table IV-5 concludes the experimental result in four dimensions: the h_0 / h_i ratio, L_i, the L_0 / L_i ratio, and TSL_i. The SIM/S/NZ heuristic provides an average cost gap at 1.22% with standard deviation 1.52% over various scenarios. The obvious good performance of this heuristic was when high h_0 / h_i ratio providing cost gap only 0.81% on average. Moreover, at high TSL_i this heuristic performed well not depending on the h_0 / h_i ratio. However, for the situations having cost gap higher than 2% we found that the minimum solutions were s_0 = 0 and $S_0 \neq 0$ happening at low h_0 / h_i ratio together with low L_i or low L_0 / L_i ratio. However, the average cost gap of such situations was only 2.73% with standard deviation 2.03%.

Table IV-5: Cost gap between the best-known solution and the SIM/S/NZ heuristic's minimum solution under identical retailers with zero minor ordering cost

		The h_0 / h_i ratio	
		Low ($h_0 / h_i < 0.5$)	High ($h_0 / h_i \geq 0.5$)
L_i	Low (L_i = 0.2)	2.38% (2.74%)	0.70% (1.30%)
	High (L_i = 1)	1.24% (0.74%)	0.96% (1.49%)
L_0 / L_i	Low ($L_0 / L_i < 1$)	2.05% (1.32%)	1.44% (1.88%)
	High ($L_0 / L_i \geq 1$)	1.59% (1.92%)	0.33% (0.34%)
TSL_i	Low (TSL_i = 0.90)	1.46% (1.25%)	1.25% (1.79%)
	Middle (TSL_i = 0.95)	3.45% (2.31%)	1.09% (1.49%)
	High (TSL_i = 0.99)	1.07% (1.14%)	0.08% (0.16%)
	Average	1.72% (1.62%)	0.81% (1.31%)

The percentage values in the table is the average cost gap (standard deviation)

4.5.2 Identical retailers with non-zero minor ordering cost

In case of non-zero minor ordering cost, we tested on 12 scenarios (Scenario no. 76-87 in Table IV-1) to observe the minimum solution's trend and the cost gap as comparing to the best-known solution. Then, we illustrate the experimental result in Table IV-6.

The result showed that when considering the minor ordering cost the SIM/S/NZ heuristic provided an average cost gap at 0.93% with standard deviation 1.31% over various scenarios. An interesting issue was that the best-known solution moves from Range II to Range I when considering non-zero minor ordering cost, for example of Instance 1, the best-known solution $(s_i, c_i, S_i), (s_0, S_0)$ when zero minor ordering cost was (1,12,13),(0,24), whereas the best-known solution when non-zero minor ordering cost was (0,13,21),(-1,0). To explain this circumstance, when each retailer had the minor ordering cost charged into an order, the system attempted to rebalance new inventory policy by two mechanisms. The first mechanism was reducing number of dispatch cycle and holding more stock, and the second one was reducing its can-order level in order to reduce the opportunity of special replenishment. By this circumstance, cost gap seemed to be smaller as our heuristic could provide a little cost gap when the solution fell into Range I. Search algorithm on dimension of s_0 was not included for Range I, so the effect of search algorithm on multiple variables appears to diminish.

Table IV-6: Cost gap between the best-known solution and the SIM/S/NZ heuristic's minimum solution under identical retailers with non-zero minor ordering cost

Instance	Relevant Parameters			Best-known Solution		Heuristic Approach	
	L_i	L_0 / L_i	TSL_i	$(s_i, c_i, S_i), (s_0, S_0)$	$TC^{(BS)}$	$(s_i, c_i, S_i), (s_0, S_0)$	$C.G.$
1	0.2	0.5	0.90	(0,13,21),(-1,0)	290.49	(0,20,21),(-1,0)	0.11%
2	0.2	0.5	0.95	(2,12,19),(-1,0)	314.16	(2,18,19),(-1,0)	0.05%
3	0.2	0.5	0.99	(4,13,18),(0,28)	365.57	(4,24,25),(-1,0)	0.72%
4	0.2	4.0	0.90	(8,19,27),(-1,0)	301.89	(8,26,27),(-1,0)	0.15%
5	0.2	4.0	0.95	(10,21,29),(-1,0)	339.59	(10,28,29),(-1,0)	0.15%
6	0.2	4.0	0.99	(4,12,15),(20,42)	383.69	(4,15,16),(18,46)	1.83%
7	1	0.5	0.90	(13,26,34),(-1,0)	311.72	(14,30,31),(-1,0)	1.66%
8	1	0.5	0.95	(16,30,34),(-1,0)	356.99	(16,33,34),(-1,0)	0.02%
9	1	0.5	0.99	(20,30,37),(-1,0)	432.81	(20,36,37),(-1,0)	0.04%
10	1	4.0	0.90	(11,20,26),(78,114)	356.42	(10,25,26),(75,119)	0.63%
11	1	4.0	0.95	(12,22,28),(88,120)	401.48	(35,55,56),(40,41)	4.52%
12	1	4.0	0.99	(16,24,29),(87,118)	483.20	(15,25,26),(95,121)	1.23%

4.5.3 Computational times

For the experiments as shown in Table IV-1, computational time of our SIM/S/NZ heuristic was 811.90 seconds on average with a standard deviation at 521.78 seconds depending on lead times for the warehouse and the retailers. Longer lead time for the warehouse (retailers) increased the search range of the must-order level s_0 (s_i), so this also increased our heuristic's computational time. However, there was no obvious trend of computational times relative to target service level TSL_i ranged from 0.90 to 0.99. Most scenarios provided indifferent computational times among TSL_i values since numbers of iteration for each scenario were not different.

For our experiments, most scenarios spent more than 40 hours to determine the best solution of any scenario from computer simulation. As the results, the SIM/S/NZ heuristic's computational times were much faster than computer simulation's computational times. We found how much the SIM/S/NZ heuristic could save computational time from computer simulation majorly depended on lead times for the warehouse and for the retailers. Normally, longer lead times for the warehouse and for the retailers increase time saving. In some scenarios, although there were longer lead times for the warehouse and for the retailers, computational times of computer simulation were unchanged due to equal number of combinations. So, time saving of such scenarios could be reduced. However, time saving might increase again if number of combinations increases.

According to above result, it was only the case of identical retailers. Hence, for more complex case of non-identical retailers, we presume that the SIM/S/NZ heuristic's computational times are extremely much faster than computer simulation's computational times.

4.5.4 Comparative analysis

Dealing with the existing literatures, an interesting work being close to our problem is Özkaya [22]. Özkaya [22] proposed analytical models and heuristic approaches for four types of joint replenishment policy at the retailers, and utilized a traditional reorder point-based stock policy at the warehouse. At retailer echelon, zero minor ordering cost and target service level in terms of fill rate are also considered. Four types of joint replenishment policy are the (Q,S) policy, the (Q,S,T) policy, the $(Q,S|T)$ policy, and the $(s,S-1,S)$ policy. More details of all policies already explained in Chapter II. Özkaya [22] showed comparative results among these policies without comparing to the lower bound or the best-known solution. Therefore, in this section we attempt to compare his heuristic approach with the SIM/S/NZ heuristic.

Based on Özkaya [22]'s results, they can be separated into two groups: Group I – Cross-docking system and Group II – Holding stock at the warehouse. For

Group II, we cannot quantitatively compare our heuristic with Özkaya [22] since his system is different from ours. Özkaya [22]'s system applied First-In First-Out System (FIFO) for the warehouse replenishing to retailer echelon. Meanwhile, our system allows the warehouse to serve an order follows FIFO except if there is an order issued to the warehouse and inventory on-hand is enough for this order the warehouse can deliver it as special case to reduce the opportunity of stock-out at the retailers. This creates higher service level than FIFO. According to different systems, in case of holding stock at the warehouse Özkaya [22]'s cost and policy cannot compare with ours in detail. However, for the cross-docking system we can compare our heuristic with Özkaya [22] since no available stock at the warehouse allows FIFO for all orders. Therefore, our system acts as Özkaya [22]' system and they are comparable. Hence, the following content will demonstrate comparative analysis in case of the cross-docking system to illustrate our heuristic's performance over Özkaya [22].

For the cross-docking system, Group 1 of Özkaya [22] is consistent with Range I of our approach. Under this system, we can identify a simple lower bound determined by two steps. The first step is to find the order quantity Q_i for all retailers by assuming that they are replenished at the same order interval.

$$T_d = \sqrt{2K_w \Big/ \sum_{i \in N} \lambda_i h_i} \tag{4.30}$$

Thus, the order quantity for each retailer is $Q_i = \lambda_i T_d$. Determination of s_i can apply Equation (4.8) with considering total lead time for each retailer $TL_i = L_0 + L_i$.

$$s_i = \min\left\{ s_i : 1 - \frac{\sum_{y=s_i+1}^{\infty} (y - s_i) Pois(\lambda_i TL_i, y)}{Q_i} \geq TSL_i \right\} \tag{4.31}$$

Therefore, the total system-wide cost is given by

$$TC(s_i, Q_i) = \frac{\lambda_i (K_w + K_r)}{Q_i} + h_i \sum_{i \in N} \left(\frac{Q_i}{2} + s_i - \lambda_i TL_i \right) \tag{4.32}$$

The comparative results are depicted in Table IV-7 – Table IV-10. Different comparative analyses are illustrated by using the following equations.

For Table IV-7 and IV-9 we use the following equation to compare Özkaya [22]'s results with our heuristic approach.

$$Cost\,Gap\,(C.G.) = \frac{(TC^{(OZ)} - TC^{(HRT)}) \times 100}{TC^{(HRT)}} \qquad (4.33)$$

where $TC^{(HRT)}$ and $TC^{(OZ)}$ are the average total system-wide cost per unit time of the heuristic approach and the average total system-wide cost per unit time of Özkaya [22]'s policies, respectively.

For Table IV-8 and IV-10, we use Equation (4.32) to compare lower bound with proposed heuristics: SIM/S/NZ and Özkaya [22]'s the minimum result.

$$Cost\,Gap\,(C.G.) = \frac{(TC^{(hrt)} - TC^{(LB)}) \times 100}{TC^{(LB)}} \qquad (4.34)$$

where $TC^{(LB)}$ and $TC^{(hrt)}$ are the total system-wide cost per unit time of lower bound and the average total system-wide cost per unit time of heuristic approach, respectively.

Table IV-7: Comparison of heuristics with the warehouse employing cross-docking at $K_w = K_r, h_0 = h_i$, L_0 = 5, L_i = 1, and TSL_i = 0.95

Relevant Parameters				$TC^{(HRT)}$	Cost Gap (C.G.)			
λ_i	K_r	h_i	n	SIM/S/NZ	(Q,S,T)	$(Q,S\|T)$	(Q,S)	$(s,S-1,S)$
1	50	1	2	25.48	-40%	-40%	-38%	-37%
			4	42.43	-41%	-40%	-38%	-38%
			8	70.07	-33%	-32%	-31%	-31%
			16	123.00	-25%	-25%	-23%	-23%
1	50	1.2	2	29.30	-36%	-35%	-33%	-33%
			4	48.58	-34%	-33%	-32%	-32%
			8	80.22	-14%	-13%	-11%	-11%
			16	142.38	-23%	-23%	-21%	-20%
1	100	1	2	33.27	0%	1%	3%	3%
			4	51.73	25%	26%	28%	28%
			8	86.45	44%	44%	47%	46%
			16	145.56	64%	64%	66%	66%
1	100	1.2	2	36.95	21%	22%	25%	25%
			4	60.25	44%	46%	48%	48%
			8	98.90	90%	92%	95%	94%
			16	167.94	102%	102%	105%	105%
10	50	1	2	74.68	71%	73%	77%	79%
			4	120.07	81%	84%	87%	89%
			8	196.72	86%	87%	91%	92%
			16	339.01	120%	119%	124%	124%
10	50	1.2	2	83.64	88%	91%	95%	97%
			4	135.91	95%	98%	102%	103%
			8	225.15	151%	153%	157%	159%
			16	394.53	119%	118%	124%	126%
10	100	1	2	97.50	187%	189%	193%	195%
			4	151.92	256%	257%	262%	264%
			8	241.80	331%	332%	337%	338%
			16	398.01	396%	394%	400%	401%
10	100	1.2	2	109.93	246%	249%	254%	256%
			4	171.29	321%	324%	329%	330%
			8	274.54	495%	498%	505%	505%
			16	454.01	533%	532%	539%	540%

Table IV-8: Comparison of lower bound and heuristics with the warehouse employing cross-docking at $K_w = K_r, h_0 = h_i$, L_0 = 5, L_i = 1, and TSL_i = 0.95

Relevant Parameters				$TC^{(LB)}$	Cost Gap (C.G.)	
λ_i	K_r	h_i	n		SIM/S/NZ	Özkaya (2005)
1	50	1	2	24.00	6%	-37%
			4	36.28	17%	-30%
			8	64.00	9%	-27%
			16	104.57	18%	-12%
1	50	1.2	2	26.71	10%	-30%
			4	40.58	20%	-21%
			8	72.62	10%	-5%
			16	119.57	19%	-8%
1	100	1	2	30.28	10%	10%
			4	48.00	8%	35%
			8	72.57	19%	71%
			16	128.00	14%	86%
1	100	1.2	2	33.38	11%	34%
			4	53.42	13%	63%
			8	81.17	22%	132%
			16	145.24	16%	133%
10	50	1	2	71.25	5%	79%
			4	113.44	6%	92%
			8	190.49	3%	92%
			16	322.89	5%	130%
10	50	1.2	2	81.28	3%	93%
			4	126.78	7%	109%
			8	215.36	5%	162%
			16	368.76	7%	134%
10	100	1	2	93.44	4%	199%
			4	142.49	7%	279%
			8	226.89	7%	359%
			16	380.98	4%	416%
10	100	1.2	2	105.18	5%	262%
			4	162.56	5%	343%
			8	253.56	8%	544%
			16	430.73	5%	566%

Table IV-9: Comparison of heuristics with the warehouse employing cross-docking at $K_w = K_r, h_0 = h_i$, L_0 = 5, L_i = 1, and TSL_i = 0.99

Relevant Parameters				$TC^{(HRT)}$	Cost Gap (C.G.)			
λ_i	K_r	h_i	n	SIM/S/NZ	(Q,S,T)	$(Q,S\|T)$	(Q,S)	$(s,S-1,S)$
1	50	1	2	32.19	-29%	-28%	-26%	-28%
			4	53.61	-24%	-24%	-22%	-23%
			8	93.79	-17%	-17%	-15%	-17%
			16	165.52	-6%	-6%	-4%	-6%
1	50	1.2	2	36.33	-27%	-26%	-24%	-26%
			4	61.29	-17%	-16%	-14%	-16%
			8	108.75	-7%	-6%	-4%	-6%
			16	192.35	-17%	-17%	-15%	-17%
1	100	1	2	39.40	17%	18%	20%	19%
			4	65.99	43%	44%	46%	44%
			8	112.20	58%	59%	62%	60%
			16	196.14	81%	82%	84%	81%
1	100	1.2	2	46.14	32%	33%	36%	34%
			4	74.58	66%	68%	71%	69%
			8	127.70	117%	118%	122%	119%
			16	223.59	131%	132%	135%	132%
10	50	1	2	90.57	104%	106%	110%	108%
			4	150.79	120%	123%	127%	124%
			8	254.28	137%	139%	143%	140%
			16	453.68	184%	184%	189%	184%
10	50	1.2	2	102.01	133%	137%	142%	139%
			4	172.17	157%	160%	164%	161%
			8	295.79	230%	233%	237%	235%
			16	524.86	214%	216%	220%	214%
10	100	1	2	115.70	235%	238%	243%	240%
			4	186.14	324%	327%	332%	329%
			8	303.28	424%	426%	432%	427%
			16	511.91	509%	508%	514%	507%
10	100	1.2	2	129.43	303%	307%	313%	309%
			4	209.57	398%	402%	408%	404%
			8	346.08	518%	523%	530%	524%
			16	595.07	572%	571%	579%	571%

Table IV-10: Comparison of lower bound and heuristics with the warehouse employing cross-docking at $K_w = K_r, h_0 = h_i$, L_0 = 5, L_i = 1, and TSL_i = 0.99

Relevant Parameters				$TC^{(LB)}$	Cost Gap (C.G.)	
λ_i	K_r	h_i	n		SIM/S/NZ	Özkaya (2005)
1	50	1	2	28.00	15%	-19%
			4	48.28	11%	-16%
			8	80.00	17%	-3%
			16	136.57	21%	14%
1	50	1.2	2	31.51	15%	-16%
			4	54.98	11%	-8%
			8	91.82	18%	10%
			16	177.17	9%	-10%
1	100	1	2	36.28	9%	27%
			4	56.00	18%	68%
			8	96.57	16%	84%
			16	160.00	23%	122%
1	100	1.2	2	40.58	14%	50%
			4	63.02	18%	97%
			8	109.97	16%	152%
			16	183.64	22%	181%
10	50	1	2	85.25	6%	116%
			4	141.44	7%	135%
			8	238.49	7%	153%
			16	418.89	8%	207%
10	50	1.2	2	98.08	4%	143%
			4	160.38	7%	175%
			8	272.96	8%	257%
			16	483.96	8%	240%
10	100	1	2	109.44	6%	254%
			4	170.49	9%	363%
			8	282.89	7%	462%
			16	476.98	7%	552%
10	100	1.2	2	121.98	6%	328%
			4	196.16	7%	432%
			8	320.76	8%	567%
			16	545.93	9%	631%

As the experimental results, our heuristic extremely outperformed all policies proposed by Özkaya [22] when higher number of retailers and higher target service level together with the increase of major ordering costs, unit holding costs, and demand rates. However, the results seemed unusual because the cost gap was very huge. We inquired whether some errors arise on either our heuristic or Özkaya [22]'s. Hence, Table IV-8 and IV-10 were carried out on the cross-docking system and the results proved that the errors arise on Özkaya [22]'s work. Özkaya [22]'s result provided the total system-wide cost less than lower bound in the situation that our heuristic was beaten. According to the error of Özkaya [22]'s cost model along with insufficient information, it was not worth comparing our heuristic's performance with Özkaya [22]'s by quantitative analysis. Then, we will compare them by qualitative analysis instead in the next paragraph. However, specifically comparing our heuristic's result with lower bound, the SIM/S/NZ heuristic provided less cost gap when higher demand rate and/or higher major ordering costs. The reason is that such situations create more opportunity of special replenishment, so the SIM/S/NZ heuristic can share the major ordering costs as much as lower bound obtains.

We make a qualitative comparison between the SIM/S/NZ heuristic and Özkaya [22]'s approach by considering their search algorithms. Özkaya [22]'s approach uses a combination of the iterative and the exhaustive search procedures. His approach sets search ranges for each variable s_0, S_0, s_i, S_i. It seems that our heuristic is better than Özkaya [22]'s because

1) Our heuristic contains smaller search ranges than Özkaya [22]'s. For example, search range for s_0 – denote that s_0^{hrt} and s_0^{OZ} are the warehouse's order-up-to level for the SIM/S/NZ heuristic and for Özkaya [22]'s approach, respectively. Under a scenario at λ_i = 10, L_0 = 1, TSL_i = 0.90, and n = 2, search range for s_0^{hrt} has 39 values obtained by equation (4.10) and (4.11) whereas search range for s_0^{OZ} has 200 values obtained by $10\left(L_0 \sum_{i=1}^{n} \lambda_i\right)$.

2) Since Özkaya [22]'s approach uses the exhaustive search procedures. It means that all values over search ranges need to be calculated. Thus, a lot of combinations have to be considered. On the contrary, our search algorithm applies the golden section search to reduce number of search points. Therefore, our heuristic with smaller search ranges and the golden section search can reduce the computational time from Özkaya [22]'s approach. According to the exhaustive search

used by Özkaya [22], it should bring to the better solution which provides lower total system-wide cost than our heuristic. This statement is not always correct because of iterative procedure. There is a possibility that the solution is unable to the optimal solution.

According to this comparative analysis, the SIM/S/NZ heuristic appears to be better than the existing research. It can save the computational time by reducing number of search space. However, the best solutions obtained from the SIM/S/NZ heuristic and Özkaya [22] cannot be guaranteed as the optimal solutions.

4.6 Discussion

For this phase, the SIM/S/NZ Heuristic extends search algorithm from the EOQ-Z heuristic. Decomposition technique, iterative procedure, and golden section search are utilized. We added two procedures to determine the retailers' must-order levels s_i and the warehouse's must-order level s_0. Since all values of s_i interrelate with s_0, searching the best values of s_i and s_0 together is quite hard. We set a search range for s_0, and then varied s_0 until the best value of s_0 has been found. Under a fixed value of s_0, the best values of s_i are determined. This algorithm can simplify the complication from interrelationship between s_i and s_0.

In deterministic model, the major ordering cost and the holding cost are traded off to obtain economical order quantity as a classic EOQ. Thus, we considered $\Delta_k = S_k - s_k$ to represent an order quantity for location k including the warehouse and the retailers; the warehouse $\Delta_0 = S_0 - s_0$ and the retailers $\Delta_i = S_i - s_i$, $i \in N$. We found the characteristic of Δ_k by trading off between the holding cost and the ordering cost. The total system-wide cost performs as a curve containing the minimum point relative to the value of Δ_k as shown in Fig.IV-8. Interestingly, even though the curves are not unimodal continuous function because of discrete numbers and the must-order levels, the golden section search with iterative procedure can be applied to determine the appropriate value of Δ_k. The reason is that the cost difference between two connected points is small enough to lead the successive search ranges from the golden section search meet the minimum point. Similarly, we also used the golden section search for determining the best values of s_0. Based on the same reason of small cost difference between two connected points, the best values of s_0 can be reached. According to the experimental results, it is fascinating to apply the golden section search into our system in order to shorten the computational time with the appropriate inventory system-wide cost.

For determining the retailers' must-order level s_i, most previous researches used the exhaustive search. Certainly, it is not worth using this search for high value of s_i. Therefore, we combined a half-interval search to the exhaustive search (also called sequential search in the dissertation). We used the half-interval search to find an acceptable search range of s_i which provides the fill rate FR_i to be close to the target service level TSL_i (by %tolerance). Then, we applied the sequential search to determine the best value of s_i. According to this mixed approach, it can reduce the successive search range of s_i, thus it can reduce the computational time as well.

The SIM/S/NZ heuristic assumes the fixed retailer's can-order level at $c_i = S_i - 1$ as the rationale that the retailers' major ordering cost can be most shared if all retailers are included in an order to minimize the total system-wide cost. Then, the holding cost is traded off with the shared ordering cost in order to balance order frequency and holding stock. The fixed retailer's can-order level at $c_i = S_i - 1$ can create the maximum opportunity of joint replenishment for all retailers. From Phase I, this assumption performs well for non-zero lead time. According to our experiments on identical retailers as shown in Table V-1, we found that service fill rate FR_i affected number of retailers included in an order. Most of the best-known solutions occur at $c_i \neq S_i - 1$. The decrease of c_i creates a possibility of reducing FR_i since the average remnant inventory level decreases. The average remnant inventory level is the stock left when normal replenishment occurs. It implies that the average reorder level occurs at the average remnant inventory level [46]. Therefore, the decrease of the average remnant inventory level increases the opportunity of stock-out influencing to reduce FR_i. According to the effect on total system-wide cost, the best-known solution chooses to reduce c_i to obtain the smallest difference between FR_i and TSL_i ($FR_i \geq TSL_i$) providing lower total system-wide cost. However, the cost gap between TC^* and $TC^*_{(S_i-1)}$ is very small (0.15% on average) where TC^* is the optimal average total system-wide cost and $TC^*_{(S_i-1)}$ is the minimum average total system-wide cost of the solution at c_i = S_i - 1. The reason is that difference between FR^* and $FR^*_{(S_i-1)}$ is very small (0.05% on average) where FR^* is the average fill rate of the best-known solution and $FR^*_{(S_i-1)}$ is the average fill rate of the solution at c_i = S_i - 1. In conclusion, the fixed retailer's can-order level at $c_i = S_i - 1$ is applicable. As the experimental results in various scenarios, the SIM/S/NZ heuristic provided the best solutions at a small average cost gap comparing to the best-known solution.

4.7 Conclusion

Phase II studied an extension of the basic model from the first phase by considering non-zero lead time and target service level as found in general industry. Research remained taking single item into consideration to study an interaction among retailers without joint ordering decision at the warehouse echelon. The objective of this phase was to study the can-order policy characteristics with the conditional relevant factors, as well as to develop the heuristic approach consistent with such characteristics provided.

The objective function of the problem was to minimize the total system-wide cost per unit time. The total system-wide cost per unit time could be a function of five decision variables: c_i, s_i, S_i, s_0, S_0. This problem had more complicated than the problem in Phase I by the reason that was a constraint problem with a service constraint. We provided insight of the can-order policy through preliminary analysis. Like the first phase, the fixed can-order level $c_i = S_i - 1$ was applicable (i.e. all retailers were replenished together in an order). The relationship between decision variables could be analyzed. We found the curve patterns of the total system-wide cost relative to decision variables. It was interesting to apply one-dimensional search with them.

Consequently, we developed the SIM/S/NZ heuristic with an extension of the EOQ-Z heuristic's search algorithm. The SIM/S/NZ heuristic used decomposition technique, iterative procedure, and one-dimensional search called golden section search. We also added two procedures to determine the retailers' must-order levels and the warehouse's must-order level. In comparison with the best-known solution obtained from computer simulation, the SIM/S/NZ heuristic provided an average cost gap at 1.01% on average. The good performance of this heuristic was when high h_0 / h_i ratio and high TSL_i. Additionally, the SIM/S/NZ heuristic spent the computational time less than computer simulation more than 177 times on average. We also provided comparative analysis with Özkaya [22]. We used the golden section search with smaller search range, whereas Özkaya [22] used the exhaustive search. Hence, the SIM/S/NZ heuristic should be better than Özkaya [22]'s approach.

Advantageously, the SIM/S/NZ gave small cost gap as comparing to the best-known solution obtained from computer simulation. The SIM/S/NZ heuristic also provided less computational time than computer simulation and the existing approach. We proposed the systematic approach to reduce search space by synchronizing with the inventory policy characteristics. Hence, the SIM/S/NZ heuristic is interesting for the can-order policy setting under OWNR with single-item and non-zero lead time consideration.

CHAPTER V
THE CAN-ORDER POLICY FOR MULTI-ITEM TWO-ECHELON INVENTORY SYSTEM WITH NON-ZERO LEAD TIME

According to Phase I and Phase II, we obtained the single-item two-echelon inventory models and heuristic approaches to determine the appropriate inventory policy setting. Later, we extend the single-item model into the multi-item model in order to consider coordinated ordering decisions at both echelons. So, this chapter demonstrates Phase III's system which comprises multiple items on OWNR. The warehouse's items are jointly replenished. However, the structure of the ordering cost is different from previous chapters since we consider the ordering cost following location-item ij instead of only location i. To determine the inventory policy parameters for controlling multiple items, we propose three models of joint replenishment described in the section of problem description. Throughout this chapter, we present such three models comparatively. The aim of this chapter is to analyze the proposed models and identify the relationship of such models and the significant relevant factors.

5.1 Problem Description

The system considers multiple commodities on a warehouse and multiple retailers. Let index i represents location i where i = 0 for the warehouse and $i \in N$, N = {1, 2, ..., n} for the retailers. Considering multi-item inventory system, such system comprises an item set with m items. Let index j denote item j in the system, so that $j \in M$, M = {1, 2, ..., m}. Thus, the whole system is composed of multiple location-items indexed by ij representing item j at location i. Totally, the system has $(n+1)\times m$ location-items. The customer demands are identical Poisson distributed with rate λ_{ij}.

In general, the system employs the can-order (s_{ij}, c_{ij}, S_{ij}) policy for ordering process at both echelons. At retailer echelon, the can-order (s_{ij}, c_{ij}, S_{ij}) policy is applied into the system by coordinated ordering decision among retailer-items. When the inventory position of any retailer-item reaches its must-order level s_{ij}, an order is triggered. Then, other retailer-items in the system can also be included by this order if their inventory position is at or below its can-order level c_{ij}. All the involved retailer-items' inventories are fulfilled from the warehouse to their own order-up-to level S_{ij}. Similarly, the warehouse employs the can-order ($s_{(0,j)}, c_{(0,j)}, S_{(0,j)}$) policy. Coordinated ordering decision occurs when any warehouse-item triggers an order by the must-order level $s_{(0,j)}$ and when other warehouse-items' inventory position is at or below its can-order level $c_{(0,j)}$. All the involved warehouse-items' inventories are fulfilled from the outside supplier to its order-up-to level $S_{(0,j)}$. For this phase, we assume that for each echelon all location-items are replenished together in an order. Then, we can fix the can-order level at $c = S - 1$ to create the maximum opportunity of joint replenishment for all location-items. By the fixed can-order level, we can focus on determining other decision variables $(s_{ij}, S_{ij}, s_{(0,j)}, S_{(0,j)})$.

Whenever an order is triggered at an echelon, such echelon needs to wait for some time that order arrives called "lead time". In the problem, we assume constant lead time for each location-item (L_{ij}). For the retailer echelon, the total lead time (TL_{ij}, $i \in N$ $j \in M$) can be longer than L_{ij} depending on the warehouse's inventories. Meanwhile, the warehouse's total lead time ($TL_{(0,j)}$) is equal to $L_{(0,j)}$ due to ample stock of the outside supplier. Lead time enable the system to face backorder units. Then, service level constraint is utilized to serve end customers with an acceptable service level. We measure such service level in term of "Fill Rate" (FR) which is a quantity-oriented performance measure describing the proportion of total demand within a reference period delivered without delay from stock on hand. FR is measured only at retailer echelon since in a multi-echelon system the backorder at warehouse echelon has only a secondary effect on service. For this problem, retailer echelon must serve the end customer following a service constraint defined as target service level (TSL_{ij}, $i \in N$).

We assume the system with no-splitting order, when the warehouse has insufficient inventory on-hand for dispatching all required quantities in an order to retailer echelon at once, the retailers have to wait for the next warehouse's order is arrived. It implies that the dispatching for that order is occurred if and only if there is sufficient inventory on-hand for all required quantities. Normally, the warehouse serve an order follows the First-In First-Out System (FIFO) except if there is an order issued to the warehouse and inventory on-hand is enough for this order we allow the warehouse to deliver it as special case to reduce the opportunity of stock-out at the retailers. This creates higher service level than FIFO.

The system considers all inventory costs at both echelons. We concern different cost structure as comparing to Phase I and II, since the major and minor ordering costs are identified following the location-item ij. In general, the inventory costs are composed of 1) The holding costs at the warehouse and all retailers, 2) The major ordering costs for warehouse echelon and retailer echelon, and 3) The minor ordering costs for warehouse echelon and retailer echelon. The structure of cost component is illustrated as Fig.V-1.

As usual, the warehouse's cost structure comprises 1) The holding cost occurring for each item, 2) The major ordering cost which is a fixed cost occurring once any item triggers an order, and 3) the minor ordering cost which is an additional cost of item j when it is included in the order. So, multiple items at the warehouse enable the system to share its major ordering cost by coordinated ordering decision.

Unlike Phase I and II, the retailer echelon has to concern two types of the major ordering cost and the minor ordering cost which is charged at the retailer-item instead of the retailer as in the single-item model. To define each type of the major ordering cost, the major ordering cost Type I is the fixed ordering cost occurring once any retailer-item in the system triggers an order. So, the major ordering cost Type I is a typical of fixed ordering cost mentioned in Phase I and Phase II. All retailer-items can be shared the major ordering cost Type I together. Meanwhile, the major ordering cost Type II is an additional fixed ordering cost when retailer i is included by this order (e.g. transportation cost or additional charge when visiting retailer i). Even though it looks like the minor ordering cost in previous phases, but it can be shared among items of such retailer. Hence, we assume it as a type of the major ordering cost according to the main character of the fixed ordering cost. However, we can manage the major ordering cost Type II in different ways following the joint replenishment model proposed in the next section. The minor ordering cost at the retailers is an additional cost of retailer-item ij when it is included in the order. This additional cost is considered like other literatures relating to the multi-item single-location inventory system.

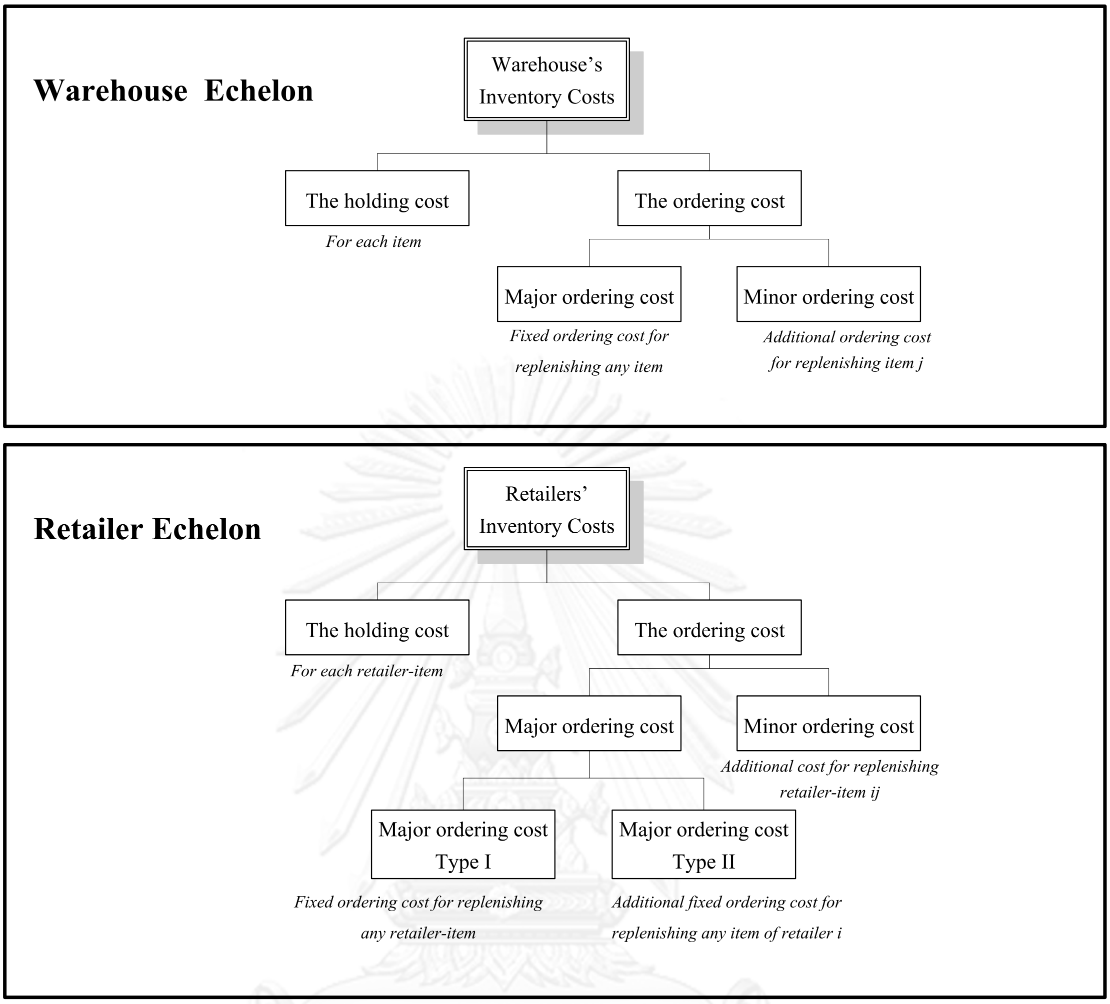


Figure V-1 Cost structure for Phase III

In consequence, the holding cost occurs at each location-item having physical stock. The total holding cost over the time period at location-item ij (HC_{ij}) can be determined from the unit holding cost (h_{ij}) and the accumulated inventory on-hand over the time period (INV_{ij}). The major ordering cost Type I and Type II can be generalized into a term of the major ordering cost (More detail of generalization will be explained in Section 5.1.4). The total major ordering cost over the time period at retailer echelon (MJ_r) is the retailers' major ordering cost per an order (K_r) multiplied by the number of dispatch cycle (ND_r). Similarly, the total major ordering cost over the time period at warehouse echelon (MJ_w) is the multiplication of the warehouse' major ordering cost per an order (K_w) and the number of replenishment cycle (NR_w). At the retailer echelon, the total minor ordering cost over the time period (MN_r) is accumulated from the involved retailer-items in each order multiplied by its minor ordering cost of retailer-item (κ_{ij}) over the time period. In the same way, at the warehouse the total minor ordering cost over the time period (MN_w) is collected from the involved warehouse-items in each order multiplied by its minor ordering cost of warehouse-item ($\kappa_{(0,j)}$) over the time period. Hence, we have to consolidate all relevant costs to determine the appropriate inventory policy setting under the total system-wide cost minimization.

The notations and problem formulation are demonstrated as follows:

n = Number of retailers in the system
m = Number of items in the system
i = Index of the location $\mathrm{i} = \{1,2,\ldots,\mathrm{n}\}$; the warehouse i = 0 and the retailer $i \in N$
j = Index of the item $j \in M$
T = The time period considered in the problem (time units)
s_{ij} = The must-order level at location-item ij (units)
c_{ij} = The can-order level at location-item ij (units); assign $c_{ij} = S_{ij} - 1$
S_{ij} = The order-up-to level at location-item ij (units)
λ_{ij} = Demand rate of retailer-item ij (units/time unit)
h_{ij} = The unit holding cost per unit time at location-item ij (\$/unit – time unit)
K_w = The warehouse's major ordering cost per a replenishment cycle (\$/time)
K_r = The retailers' major ordering cost per a dispatch cycle (\$/time)
κ_{ij} = The minor ordering cost at location-item ij (\$)
L_{ij} = Lead time for location-item ij (time unit)
FR_{ij} = Fill rate of retailer-item ij
TSL_i = Target service level of retailer-item ij
$TC(s_{ij}, c_{ij}, S_{ij})$ = The total system-wide cost per unit time (\$/time unit)
HC_{ij} = The total holding cost of location-item ij over the time T units (\$)
MJ_r = The total major ordering cost at retailer echelon over the time T units (\$)
MN_r = The total minor ordering cost at retailer echelon over the time T units (\$)
MJ_w = The total major ordering cost at warehouse echelon over the time T units (\$)

MN_w = The total minor ordering cost at warehouse echelon over the time T units ($)

INV_{ij} = The accumulated inventory on-hand over time period at location-item ij (unit – time unit)

BO_{ij} = The accumulated backorder unit over time period at location-item ij (units)

ND_r = The total number of dispatch cycle over the time T units (times)

NR_w = The total number of replenishment cycle over the time T units (times)

$\delta_{(ij,x)}$ = An indicator which equals 1 when retailer-item ij is included in the dispatch cycle x and equals 0 otherwise

$\delta_{(ij,y)}$ = An indicator which equals 1 when warehouse-item $(0,j)$ is included in the replenishment cycle y and equals 0 otherwise

Objective function:

Minimize $$TC(s_{ij},c_{ij},S_{ij}) = \frac{\sum_{j\in M}\sum_{i=0}^{n} HC_{ij} + \left(MJ_r + MN_r\right) + \left(MJ_w + MN_w\right)}{T} \quad (5.1)$$

where $$HC_{ij} = h_{ij} \times INV_{ij} \quad (5.2)$$

$$MJ_r = K_r \times ND_r \quad (5.3)$$

$$MN_r = \sum_{x=1}^{ND_r}\sum_{j\in M}\sum_{i=1}^{n} \delta_{(ij,x)}\kappa_{ij} \quad (5.4)$$

$$MJ_w = K_w \times NR_w \quad (5.5)$$

$$MN_r = \sum_{y=1}^{ND_w}\sum_{j\in M} \delta_{(0,j,y)}\kappa_{(0,j)} \quad (5.6)$$

<u>Constraint</u> $$FR_{ij} = 1 - \frac{BO_{ij}}{\lambda_{ij}T} \quad (5.7)$$

$$FR_{ij} \ge TSL_{ij} \quad (5.8)$$

The interesting issue is how to coordinate multiple items and multiple retailers in order to minimize the total system-wide cost, since the ordering cost structure has been changed and it is very significant to decide how to manage our considered system. For example questions according to the new ordering cost structure,

- Is it worth managing all retailer-items together due to a lot of decision variables concerned in the system?
- In case of large value of the major ordering cost Type II, do we prefer to manage each retailer individually and coordinate only multiple items?
- In case of large value of the minor ordering cost of retailer-item ij, do we prefer to manage each item individually and coordinate only multiple retailers?

These questions issued from the new ordering cost structure lead us to develop three joint replenishment models. We aim at analyzing the proposed models and identifying the relationship of such models and the significant relevant factors. Ultimately, we expect to clarify which joint replenishment model is preferable to any situation. Consequently, the proposed models are demonstrated as follows:

5.1.1 Model 1 – Joint replenishment with item-based model

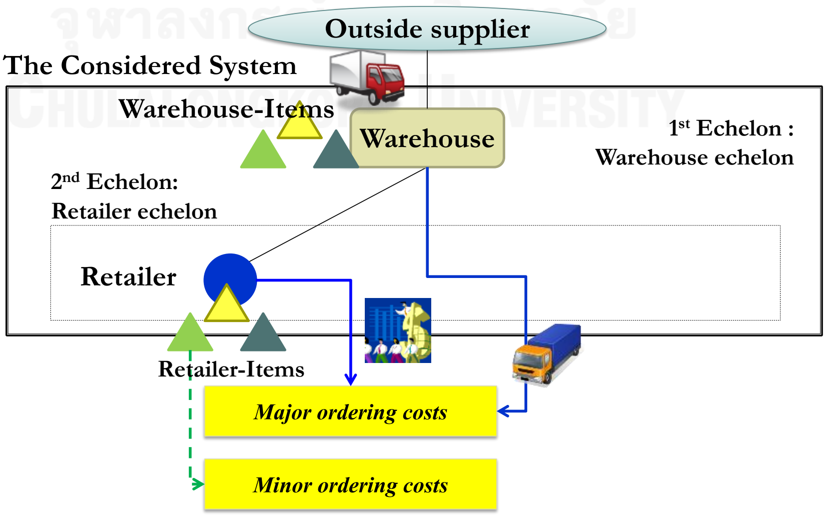


Figure V-2 Model 1 – Joint replenishment with item-based model

This model considers each retailer individually and coordinates only multiple items under such retailer. It appears to a serial system as one warehouse, one retailer, and multiple items. At retailer echelon, the retailers' major ordering cost can be shared among multiple items. In the same way, the warehouse's major ordering cost can be also shared among multiple items. According to this model, we can determine the inventory policy parameters by considering only individual retailer, and the warehouse's inventory is individually stocked for that retailer. Thus, sum of n serial systems is the total system-wide cost. The following expressions demonstrate the model formulation to determine the inventory policy setting. Equation (5.2) – (5.8) can be used for the Equation (5.9).

For retailer i ,

Objective function:

Minimize

$$TC_i(s_{ij}, c_{ij}, S_{ij}) = \frac{\sum_{j \in M} (HC_{(0,j)} + HC_{ij}) + (MJ_r + MN_r) + (MJ_w + MN_w)}{T} \quad (5.9)$$

The total system-wide cost is then given by

$$TC(s_{ij}, c_{ij}, S_{ij}) = \sum_{i \in N} TC_i(s_{ij}, c_{ij}, S_{ij}) \quad (5.10)$$

5.1.2 Model 2 – Joint replenishment with retailer-based model

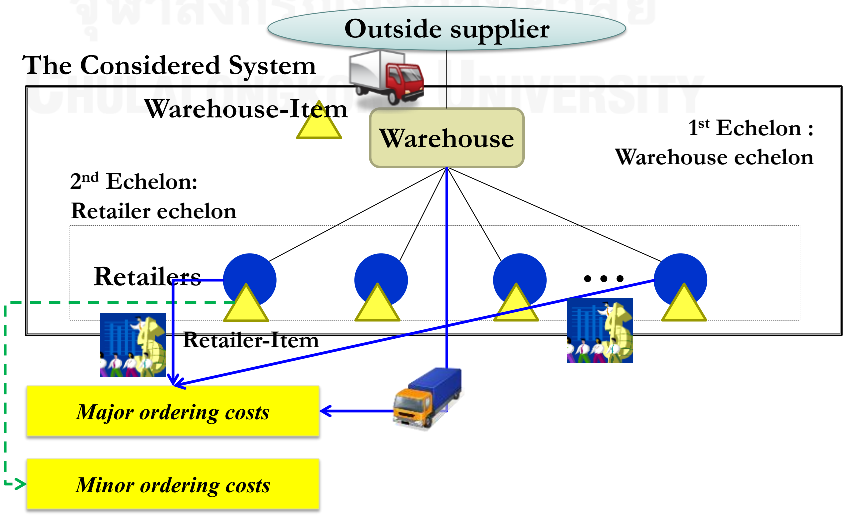


Figure V-3 Model 2 – Joint replenishment with retailer-based model

The model considers each item individually and coordinates only multiple retailers. It has the same structure as single-item two-echelon inventory system demonstrated in Phase II. Therefore, sum of m single-item systems is the total system-wide cost. The model is formulated to determine the inventory policy setting as follows. Equation (5.2) – (5.8) can be used for the Equation (5.11).

For item j,

Objective function:

$$\text{Minimize} \quad TC_j(s_{ij}, c_{ij}, S_{ij}) = \frac{\sum_{i=0}^{n} HC_{ij} + (MJ_r + MN_r) + (MJ_w + MN_w)}{T} \tag{5.11}$$

The total system-wide cost is then given by

$$TC(s_{ij}, c_{ij}, S_{ij}) = \sum_{j \in M} TC_j(s_{ij}, c_{ij}, S_{ij}) \tag{5.12}$$

5.1.3 Model 3 – Completely joint replenishment model

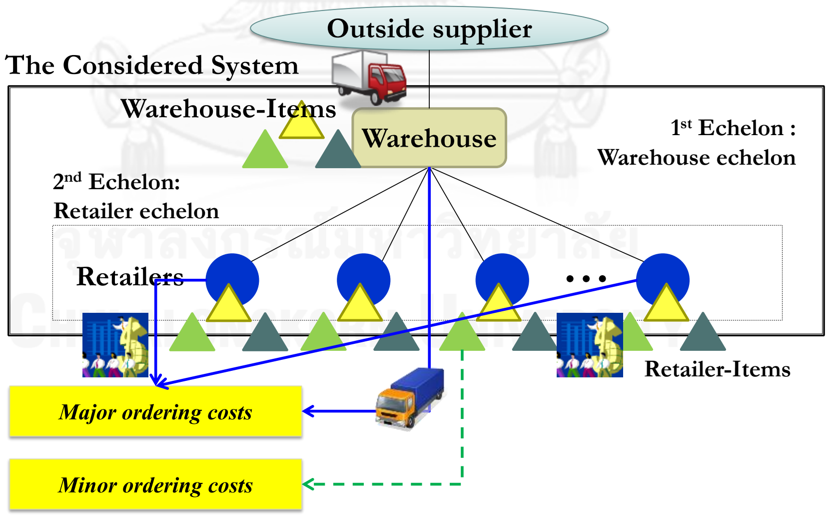


Figure V-4 Model 3 – Completely joint replenishment model

The model includes all location-items to determine the inventory policy setting. So, the retailers' major ordering cost can be shared among multiple retailer-items, as well as the warehouse' major ordering cost can also be shared among multiple warehouse-items. Thus, we can formulate the model to determine the inventory policy setting as Equation (5.13).

Objective function:

$$\text{Minimize} \quad TC(s_{ij}, c_{ij}, S_{ij}) = \frac{\sum_{j \in M} \sum_{i=0}^{n} HC_{ij} + \left(MJ_r + MN_r\right) + \left(MJ_w + MN_w\right)}{T} \tag{5.13}$$

5.1.4 Generalization of the major ordering cost at the retailers

Due to the new ordering cost structure, we need to make some assumptions to generalize the major ordering cost Type I and Type II for each joint replenishment model into a term of the major ordering cost, K_r. The generalization is used for integrating the major ordering cost Type I and Type II into our models. The following table summarizes the value of K_r for each model. Given that major order cost Type I and Type II are represented by K^{TypeI} and K_i^{TypeII}, respectively.

Table V-1: Generalization of the ordering cost structure

<table>
<tr><th>Model</th><th>K_r</th></tr>
<tr><td>1</td><td>$K^{TypeI} + K_i^{TypeII}$</td></tr>
<tr><td>2</td><td rowspan="2">$K^{TypeI} + \sum_{i \in N} K_i^{TypeII}$</td></tr>
<tr><td>3</td></tr>
</table>

For Model 1, it is not complicated to set K_r since only single retailer is considered in the optimization model. Thus, the major order cost Type I and Type II can be combined to set $K_r = K^{TypeI} + K_i^{TypeII}$.

Differently, Model 2 and Model 3 coordinate all retailers into the optimization model. Even though in the reality there is a possibility that not every retailer is included in an order, we assume that in ordering decision all retailers are considered to jointly replenish once an order is triggered. Therefore, the major order cost Type I and Type II can be transformed into $K_r = K^{TypeI} + \sum_{i \in N} K_i^{TypeII}$.

To summarize this section, three joint replenishment models are proposed for managing multiple items and multiple retailers in OWNR. All proposed models will be analyzed to identify the relationship of such models and the significant relevant factors. This enables us to clarify which joint replenishment model is preferable to any situation. Definitely, before obtaining any results we will address our research methodology utilized throughout this phase in the next section.

5.2 Research Methodology

Like previous chapters, we use computer simulation to represent the inventory process of the system containing multiple items and multiple retailers coordinated replenishment. Due to the extremely complicated system, it is herculean task to find out the best-known solution from computer simulation. However, we integrate the simulation cost model obtained from computer simulation into our heuristic approach to determine the appropriate inventory policy setting. Another methodology is determination of lower/upper bound for Model 1 and Model 2. Since such two models are decomposed into smaller parts (i.e. Model 1 – n serial systems implying there are n-warehouse for all retailers, and Model 2 – m single-item systems implying there are m-warehouse for all items), meanwhile our computer simulation is based on single warehouse. Therefore, we need to figure out lower bound and upper bound to represent a range of the total system-wide cost instead of any point value.

5.2.1 Computer simulation

The computer algorithm representing the inventory process is illustrated in Fig.V-5. The inputs for simulating the system can be divided into three groups: decision variables, relevant factors, and experiment setting. We use the same experiment setting as described in Chapter III (section 3.2.1), and then only two groups are explained as follows:

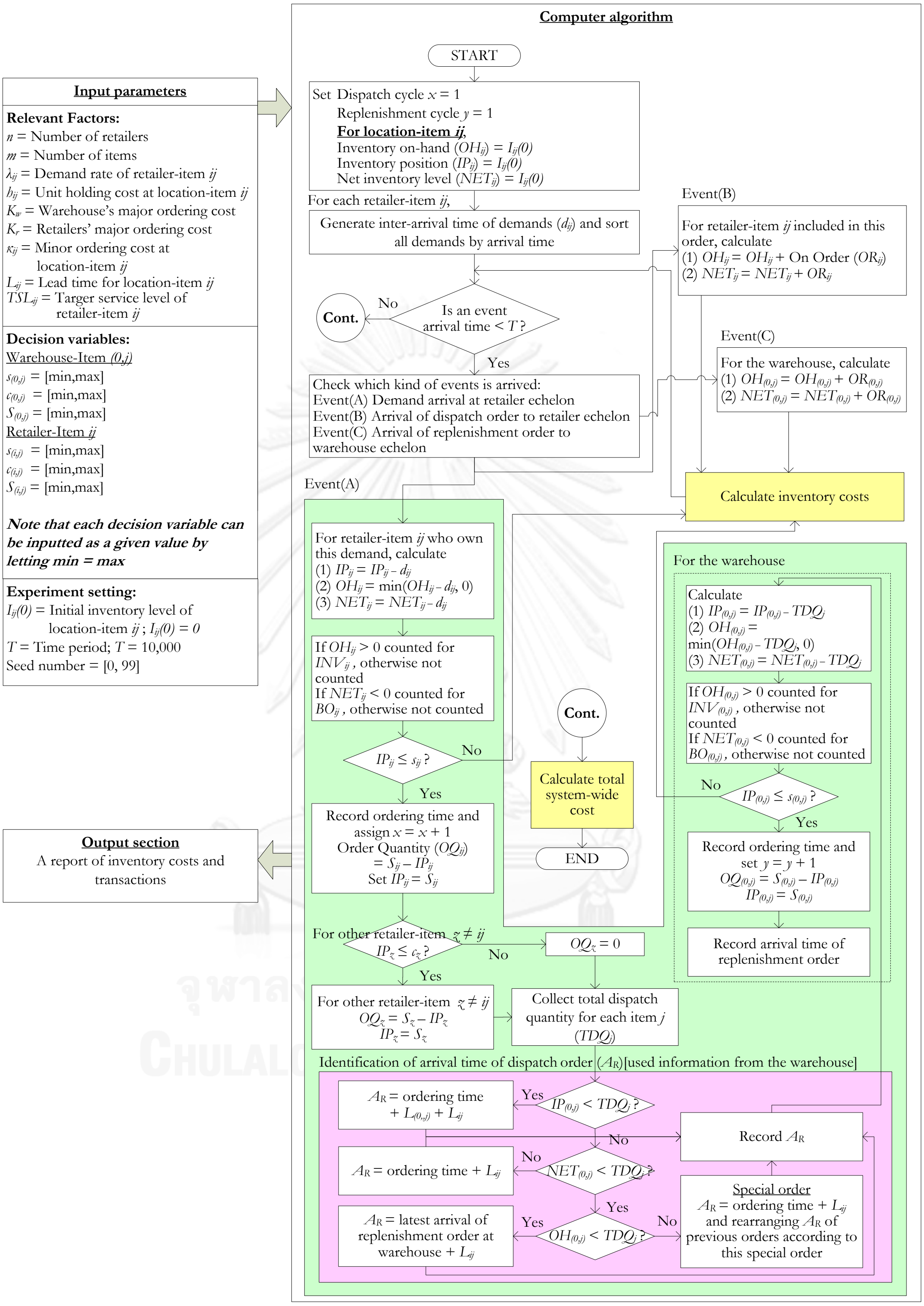


Figure V-5 The computer algorithm for simulation of Phase III

1) Decision variables (c_{ij}, s_{ij}, S_{ij}) where the warehouse i = 0 and the retailer $i \in N$: Each variable is inputted either as a range of minimum and maximum values or a point value by setting identical minimum and maximum values. A combination of (c_{ij}, s_{ij}, S_{ij}) is a solution providing a value of the total system-wide cost and its transaction. The transaction includes, for example, number of dispatch cycles, number of replenishment cycles, and fill rate at each location.

2) Relevant factors: we consider five basic factors as previous phase, i.e. cost parameters, demand rates, number of retailers, lead time and target service level. We set a combination of relevant factors to a scenario containing different solutions. Specifically, cost parameters are different from prior chapters as we mentioned the new ordering cost structure. However, in computer simulation we only input the major ordering cost K_r after generalizing the major ordering cost Type I and Type II.

For the output section, we obtain a report of the inventory costs and its transaction if we input decision variables as a range. On another hand, we get the total system-wide cost for a combination of (c_{ij}, s_{ij}, S_{ij}) which is a feasible solution under a given scenario. Then, we use the heuristic approach to determine the best solution among these feasible solutions.

5.2.2 Determination of lower/upper bound for Model 1 and Model 2

Model 1 and Model 2 are decomposed into n serial systems and m single-item systems respectively. It implies that Model 1 has n-warehouse for all retailers and Model 2 has m-warehouse for all items, but our computer simulation is based on only one warehouse. Then, we need to figure out lower bound and upper bound to represent a range of the total system-wide cost instead of any point value. This range is used to compare with the total system-wide cost obtained from Model 3 so as to measure each model's performance.

The warehouse's major ordering cost K_w is necessary to be transformed to a decomposed value of K_w. Let $\underline{K}_w$ denote the warehouse's major ordering cost for determining lower bound and $\bar{K}_w$ denote the warehouse's major ordering cost for determining upper bound. Then, we replace K_w with either $\underline{K}_w$ or $\bar{K}_w$ in Equation (5.5). The largest cost-saving of the warehouse's major ordering cost is when all items at the warehouse are jointly replenished. So, we identify such situation to a lower bound case. On the contrary, when each item is separately replenished bring the non-shared warehouse's major ordering cost to an upper bound case. The above concept assumes that the warehouse's minor ordering cost is small as comparing to the warehouse's major ordering cost, Table V-2 summarizes the calculation of lower and upper bounds for such two models.

Table V-2: The calculation of lower/upper bound for Model 1 and Model 2

Model	Lower Bound	Upper Bound
1	$\underline{K}_w = K_w / n$	$\bar{K}_w = K_w$
2	$\underline{K}_w = K_w / m$	$\bar{K}_w = K_w$

5.3 Heuristic IV – Joint Replenishment Model for Multiple Location-Items and Non-Zero Lead Time (SIM/M/NZ)

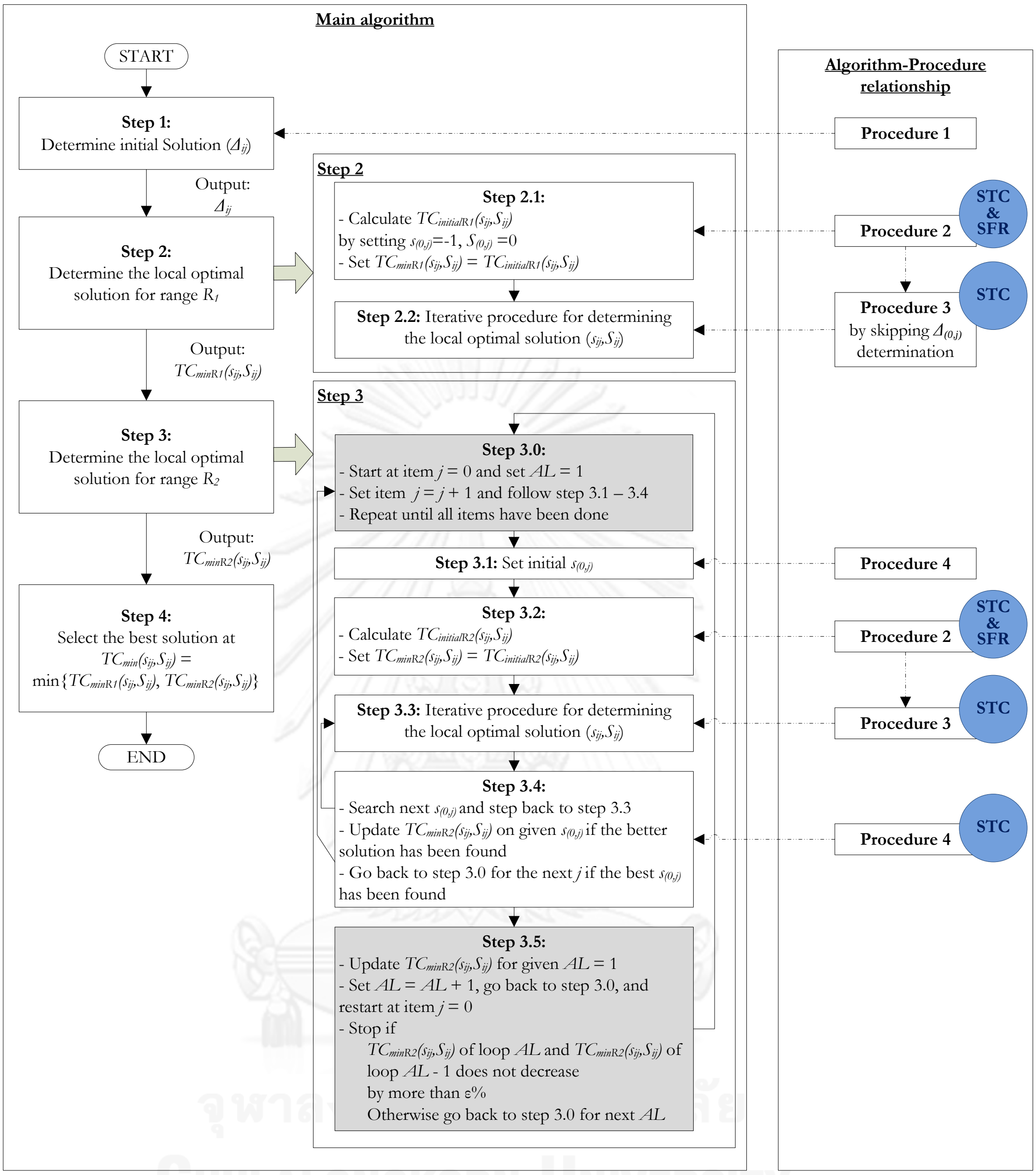
To develop this phase's heuristic called SIM/M/NZ, we use the same concept as the SIM/S/NZ heuristic mentioned in the Phase II for the single-item two-echelon inventory system. Decomposition technique, iterative procedure and one-dimensional search are employed into the SIM/M/NZ heuristic. The SIM/S/NZ heuristic is used as a part of the proposed SIM/M/NZ heuristic for determining decision variables of each item j. We utilize the simulation cost model to reduce the cost error from an approximation. The simulation cost model follows the algorithm illustrated in Fig.V-5. Then, the total system-wide cost and fill rate from the simulation cost model are used in the heuristic algorithm. However, there are some equations of approximate mathematical model utilized to find out the initial values of related decision variables. They have to be inputted in the simulation cost model to initiate the first solution for iterative procedure.

With regard to the single-item model dividing two ranges for determining the value of S_0, in this phase we use the same two ranges applied for all items. It means that we consider all items controlled in either Range I or Range, for example, we determine the inventory policy setting for item 1 and item 2 in case of the cross-docking system (Range I) and also determine the setting for both items in case of which the warehouse is allowed to hold stock (Range II). We do not concern the combination of Range I and Range II, such as Range I for item 1 and Range II for item 2 and vice versa.

Additionally, we set the can-order level for all location-items equal to c_{ij} = S_{ij} - 1. Then, we can determine only $TC(s_{ij}, S_{ij})$ on given $c_{ij} = S_{ij}$ - 1. We use the same concept as the SIM/S/NZ heuristic to determine $\Delta_{ij} = S_{ij} - s_{ij}$ for location-item ij, and decompose multiple location-items into single location-item to recurrently determine the local minimum $TC(s_{ij}, S_{ij})$ at the given $\Delta_{k \neq ij}$. We also apply the concept of the golden section search to determine the (near) minimum value of Δ_{ij} and $s_{(0,j)}$. Hence, the SIM/M/NZ heuristic is outlined in the following algorithm illustrated in Fig.V-6.

The SIM/M/NZ heuristic still use the same structure as the single-item heuristic, but we need to add some steps for this multi-item heuristic: Step 3.0 and Step 3.5 showed in Fig.V-6. Step 3.0 defines item j concerned in the loop number AL, then the given item j is processed according to Step 3.1 – 3.4 as the single-item heuristic (already described in Section 4.4.2). Step 3.0 is repeated until all items have been done. Step 3.5 is a conditional step of termination. The iterative process terminates as soon as the minimum long-run average total system-wide cost per unit time of Range II, $TC_{\min R2}(s_{ij}, S_{ij})$, from the current loop AL does not decrease from the previous loop $AL-1$ by more than $\varepsilon\%$. Meanwhile, Procedure 1 – 4 are the same as the single-item heuristic on a given item j.

Model 1 applies the SIM/M/NZ heuristic for a given retailer i . The best solution is determined for such retailer, and then the heuristic is repeated until all retailers have been done. Similarly, Model 2 utilizes the SIM/M/NZ heuristic for a given item j. We find out the best solution is found for such item, and repeat the heuristic until all items have been done. Alternatively, Model 2 is able to employ the SIM/S/NZ heuristic from the single-item model. Model 3 uses the SIM/M/NZ heuristic for full combination of retailer-item ij. According to the aim of this chapter which is to analyze the proposed models and identify the relationship of such models and the significant relevant factors, we continue to the next section. Various experiments are carried out and the analysis of the results is demonstrated herein.



STC = Simulated total system-wide cost per unit time obtained from the simulation cost model
SFR = Simulated fill rate obtained from the simulation cost model

Figure V-6 The algorithm of the heuristic approach – SIM/M/NZ

5.4 Experimental Results

The SIM/S/NZ heuristic was experimented on various scenarios shown in Table V-3 (35 scenarios). The experiments focused on identical items and identical retailers, thus an identical inventory policy is employed to all identical retailer-item

ij as well as at the warehouse an identical inventory policy is used for all identical item j.

We tested each scenario on five replications with different random seed numbers. Then, for each best solution we determined the average total system-wide cost by additional 10 random seed numbers. We defined "the minimum solution" provided by the best solution with the minimum of average total system-wide cost. For Model 1 and Model 2, we first determined the lower bound. If the minimum of average total system-wide costs obtained from these lower bounds are less than the minimum of average total system-wide costs obtained from Model 3, the upper bound will be determined later. The experimental results are demonstrated in Table V-4 – Table V-6.

Table V-3: Test problems for the multi-item one-warehouse n-retailer inventory system with identical items and identical retailers

Fixed identical parameters for retailer echelon λ_{ij} = 1, TSL_{ij} = 0.95, h_{ij} = 10

Fixed identical parameters for warehouse echelon $h_{(0,j)}$ = 3

Fixed identical parameters for both echelons L_{ij} = 1

Scenario No.	Varied Parameters				
	K_w	K^{TypeI}	K_i^{TypeII} ($i \in N$)	k_{ij} ($i \in N$)	Combination of parameter n and m
1-5	100	50	5	0	
6-10	100	50	5	5	
11-15	100	50	5	25	
16-20	100	50	500	0	
21-25	100	50	500	25	
26-30	100	100	5	0	
31-35	500	50	5	0	

n	m
2	2
2	8
4	4
8	2
8	8

Table V-4: Comparison of joint replenishment models: The result of Scenario 1 – 15

Fixed parameters at $K_w = 100,\ K^{TypeI} = 50,\ K_i^{TypeII} = 5$

The inventory policy setting $(s_{ij}, c_{ij}, S_{ij}), (s_{(0,j)}, c_{(0,j)}, S_{(0,j)})$, except upper bound case $(s_{ij}, c_{ij}, S_{ij}), (s_{(0,j)}, S_{(0,j)})$

Scenario No.	Parameters			Model 1				Model 2				Model 3	
				Lower Bound		Upper Bound		Lower Bound		Upper Bound			
	k_{ij}	n	m	TC	Policy	TC	Policy	TC	Policy	TC	Policy	TC	Policy
1	0	2	2	223.13	(3,6,7),(-1,-1,0)	-	-	226.62	(3,6,7),(-1,-1,0)	-	-	217.76	(2,5,6),(0,6,7)
2	0	2	8	704.00	(2,6,7),(-1,-1,0)	747.33	(2,6,7),(-1,0)	784.64	(3,6,7),(-1,-1,0)	-	-	733.43	(1,4,5),(3,5,6)
3	0	4	4	756.12	(3,5,6),(-1,-1,0)	-	-	795.15	(2,4,5),(3,11,12)	-	-	713.27	(1,4,5),(5,13,14)
4	0	8	2	787.66	(3,6,7),(-1,-1,0)	-	-	734.33	(2,6,7),(-1,-1,0)	-	-	699.82	(1,4,5),(9,21,22)
5	0	8	8	2,731.71	(3,5,6),(-1,-1,0)	-	-	2,807.32	(2,6,7),(-1,-1,0)	-	-	2,247.19	(1,3,4),(9,18,19)
6	5	2	2	229.69	(3,6,7),(-1,-1,0)	260.73	(3,7,8), (-1,0)	233.18	(3,6,7),(-1,-1,0)	-	-	230.43	(2,4,5),(0,10,11)
7	5	2	8	736.50	(2,6,7),(-1,-1,0)	779.83	(2,6,7), (-1,0)	827.89	(3,6,7),(-1,-1,0)	-	-	765.39	(2,6,7),(-1,-1,0)
8	5	4	4	797.67	(3,5,6),(-1,-1,0)	-	-	836.71	(2,4,5),(3,11,12)	-	-	756.48	(1,4,5),(5,13,14)
9	5	8	2	813.91	(3,6,7),(-1,-1,0)	-	-	764.83	(2,6,7),(-1,-1,0)	-	-	743.03	(1,4,5),(9,21,22)
10	5	8	8	2,808.02	(2,6,7),(-1,-1,0)	-	-	2,929.35	(2,6,7),(-1,-1,0)	-	-	2,494.52	(1,3,4),(8,20,21)
11	25	2	2	254.61	(3,7,8),(-1,-1,0)	-	-	257.31	(3,7,8),(-1,-1,0)	-	-	250.11	(2,8,9),(-1,-1,0)
12	25	2	8	856.53	(2,6,7),(-1,-1,0)	899.86	(2,6,7), (-1,0)	932.90	(3,6,7),(-1,-1,0)	-	-	873.41	(1,7,8),(-1,-1,0)
13	25	4	4	944.58	(3,6,7),(-1,-1,0)	-	-	942.10	(2,8,9),(-1,-1,0)	-	-	876.57	(1,7,8),(-1,-1,0)
14	25	8	2	918.92	(3,6,7),(-1,-1,0)	-	-	886.86	(2,6,7),(-1,-1,0)	-	-	882.89	(1,7,8),(-1,-1,0)
15	25	8	8	3,296.14	(2,6,7),(-1,-1,0)	3,599.44	(2,6,7), (-1,0)	3,417.44	(2,6,7),(-1,-1,0)	3,720.77	(2,6,7), (-1,0)	3,428.98	(1,6,7),(-1,-1,0)

Table V-5: Comparison of joint replenishment models: The result of Scenario 16 – 25

Fixed parameters at $K_w = 100,\ K^{TypeI} = 50,\ K_i^{TypeII} = 500$

The inventory policy setting $(s_{ij}, c_{ij}, S_{ij}), (s_{(0,j)}, c_{(0,j)}, S_{(0,j)})$, except upper bound case $(s_{ij}, c_{ij}, S_{ij}), (s_{(0,j)}, S_{(0,j)})$

Scenario No.	Parameters			Model 1				Model 2				Model 3	
				Lower Bound		Upper Bound		Lower Bound		Upper Bound			
	k_{ij}	n	m	TC	Policy	TC	Policy	TC	Policy	TC	Policy	TC	Policy
16	0	2	2	387.67	(2,10,11),(-1,-1,0)	401.59	(2,10,11),(-1,0)	479.92	(1,14,15),(-1,-1,0)	-	-	394.75	(1,11,12),(-1,-1,0)
17	0	2	8	1,115.53	(2,7,8),(-1,-1,0)	-	-	1,893.91	(1,14,15),(-1,-1,0)	-	-	1,067.08	(1,7,8),(-1,-1,0)
18	0	4	4	1,280.91	(1,11,12),(-1,-1,0)	-	-	2,043.63	(1,12,13),(-1,-1,0)	-	-	1,273.42	(0,9,10),(-1,-1,0)
19	0	8	2	1,508.93	(2,10,11),(-1,-1,0)	1,606.37	(2,10,11),(-1,0)	2,077.43	(0,13,14),(-1,-1,0)	-	-	1,658.29	(0,10,11),(-1,-1,0)
20	0	8	8	4,361.98	(2,7,8),(-1,-1,0)	-	-	8,276.89	(0,13,14),(-1,-1,0)	-	-	4,311.41	(0,7,8),(-1,-1,0)
21	25	2	2	401.53	(2,10,11),(-1,-1,0)	411.19	(1,14,15),(-1,0)	488.50	(1,14,15),(-1,-1,0)	-	-	407.68	(1,11,12),(-1,-1,0)
22	25	2	8	1,167.73	(1,9,10),(-1,-1,0)	-	-	1,928.22	(1,14,15),(-1,-1,0)	-	-	1,157.20	(0,9,10),(-1,-1,0)
23	25	4	4	1,332.66	(1,11,12),(-1,-1,0)	1,371.54	(1,11,12),(-1,0)	2,090.24	(1,12,13),(-1,-1,0)	-	-	1,349.70	(0,9,10),(-1,-1,0)
24	25	8	2	1,564.37	(2,10,11),(-1,-1,0)	1,644.76	(1,14,15),(-1,0)	2,121.19	(0,13,14),(-1,-1,0)	-	-	1,725.30	(0,10,11),(-1,-1,0)
25	25	8	8	4,612.89	(1,9,10),(-1,-1,0)	4,748.27	(1,9,10),(-1,0)	8,451.90	(0,13,14),(-1,-1,0)	-	-	4,824.69	(0,7,8),(-1,-1,0)

Table V-6: Comparison of joint replenishment models: The result of Scenario 26 – 35

The inventory policy setting $(s_{ij}, c_{ij}, S_{ij}), (s_{(0,j)}, c_{(0,j)}, S_{(0,j)})$, except upper bound case $(s_{ij}, c_{ij}, S_{ij}), (s_{(0,j)}, S_{(0,j)})$

Scenario No.	Parameters		Model 1				Model 2				Model 3	
			Lower Bound		Upper Bound		Lower Bound		Upper Bound			
	n	m	TC	Policy	TC	Policy	TC	Policy	TC	Policy	TC	Policy
Fixed parameters at $K_w = 100,\ K^{TypeI} = 100,\ K_i^{TypeII} = 5,\ k_{ij} = 0$												
26	2	2	255.50	(3,7,8),(-1,-1,0)	-	-	258.21	(3,7,8),(-1,-1,0)	-	-	239.34	(2,8,9),(-1,-1,0)
27	2	8	747.33	(2,6,7),(-1,-1,0)	790.66	(2,6,7),(-1,0)	924.44	(3,6,7),(-1,-1,0)	-	-	756.85	(2,6,7),(-1,-1,0)
28	4	4	877.02	(3,6,7),(-1,-1,0)	-	-	877.02	(3,6,7),(-1,-1,0)	-	-	751.38	(5,13,14),(1,4,5)
29	8	2	927.46	(3,6,7),(-1,-1,0)	-	-	777.66	(2,6,7),(-1,-1,0)	-	-	737.93	(9,21,22),(1,4,5)
30	8	8	2,859.31	(2,6,7),(-1,-1,0)	-	-	2,980.64	(2,6,7),(-1,-1,0)	-	-	2,356.33	(9,18,19),(1,3,4)
Fixed parameters at $K_w = 500,\ K^{TypeI} = 50,\ K_i^{TypeII} = 5,\ k_{ij} = 0$												
31	2	2	298.76	(2,4,5),(0,9,10)	-	-	282.86	(2,4,5),(0,17,18)	329.43	(2,4,5),(0,23,24)	285.26	(2,4,5),(0,16,17)
32	2	8	877.32	(2,6,7),(-1,-1,0)	-	-	924.44	(3,6,7),(-1,-1,0)	-	-	861.95	(1,4,5),(1,12,13)
33	4	4	930.43	(2,8,9),(-1,-1,0)	-	-	885.50	(2,4,5),(0,19,20)	-	-	823.10	(1,4,5),(2,24,25)
34	8	2	927.46	(3,6,7),(-1,-1,0)	-	-	852.06	(1,4,5),(12,44,45)	-	-	806.32	(1,4,5),(4,46,47)
35	8	8	3,003.31	(2,6,7),(-1,-1,0)	3,003.31	(2,6,7),(-1,0)	2,980.64	(2,6,7),(-1,-1,0)	2,980.64	(2,6,7),(-1,-1,0)	2,484.58	(1,3,4),(6,30,31)

According to the scenario 1 – 15, the result is showed in Table V-6. We found that Model 3 outperformed other models in almost all scenarios except some scenarios that Model 1 seemed to be more interesting (i.e. Model 3 provides the minimum of average total system-wide cost in the bounds of Model 1). Model 1 is interesting in any scenario with low number of retailers and in the scenario of high minor ordering cost, high number of items, and high number of retailers. Due to low number of retailers, Model 3 appears to share the fixed ordering cost among retailers indifferent from Model 1. Furthermore, in case of high minor ordering cost, high number of items and high number of retailers the total ordering cost including the major and minor ordering cost seems not different. Even though Model 3 can save the fixed ordering cost, it can increase the minor ordering cost as well. So, all components have to be traded off.

We then analyzed the scenario 16 – 25 with higher the retailers' major ordering cost Type II, K_i^{TypeII}. Model 1 is preferable when high minor ordering cost. It can provide the lower average total system-wide costs than other models, since coordinated ordering decision among retailer-items of Model 3 produces larger total minor ordering cost MN_r. Additionally, we tested on the scenario 26 – 30 with higher retailers' major ordering cost Type I, K^{TypeI}, and on the scenario 31 – 35 with higher warehouse's major ordering cost K_w. The results showed that Model 3 outperforms the others. Mostly, the minimum solutions allow the warehouse to hold stock for all retailer-items.

An interesting issue is which scenario Model 2 is suitable for. We found that Model 2 could not outperform Model 3 in any scenarios. However, Model 2 was more interesting when low number of items and high number of retailers, because it provided lower minimum of average total system-wide cost than that of Model 1. Advantageously, Model 2 could be used in such scenarios if Model 3's computational time is too long.

According to our experiments, we found that Model 1 and Model 2 spent computational times less than Model 3, especially for the scenarios having high number of retailers and high number of items. For eight retailers and eight items, Model 3's computational time was found to be 3.77 hours on average. Then, for such scenarios Model 1's computational time was around 52 times faster than Model 3's, and Model 2's computational time was around 40 times faster than Model 3's. The reason is that Model 3 has a lot of interactions for joint replenishment (64 interactions among location-items). Meanwhile, Model 1 and Model 2 have a few interactions for joint replenishment (8 interactions among locations or items). Certainly, if number of retailers and number of items are more than 8, Model 1 and Model 2 are more interesting. As comparison between Model 1 and Model 2, Model 1's computational time is faster than Model 2's for high number of retailers. On the other hand, Model 1's computational time is slower than Model 2's for high number of items. It is not surprising because Model 2 is based on interaction among retailers and Model 1 is based on interaction among items. High number of retailers (items) increases more interactions, and then increases computational time. Since we tested only the case of identical retailers, we presume that for more complex case of non-identical location-items, Model 1 or Model 2 can be a good choice for managing the multi-item system under OWNR.

We can identify the relationship of such models and the significant relevant factors by considering the lowest total system-wide costs among three models. Figure V-7 illustrates the relationships of three models and the relevant factors to decide which model is suitable for given relevant factors. As comparing to the other models, although Model 3 provides the smallest minimum of average total system-wide cost, it spends a lot of computational time especially for high number of items and high number of retailers. Therefore, which model is suitable for any scenario should be measured not only by costs but also by computational times.

According to Fig.V-7, it can be explained as follows:

(1) Model 3 provided low total system-wide cost on any values of K_w. Meanwhile Model 1 should be interesting for the low value of K_w and Model 2 should be interesting for the high value of K_w. The reason is that Model 2 influences the warehouse to hold stock for all retailers, then the high value of K_w could be saved from the reduced replenishment frequency. Model 1 is suitable for the lower value of K_w because the best solutions often occur for the cross-docking system (i.e. high replenishment frequency).

(2) Then, for the high value of K_w, Model 2 and 3 are considered. We found that for low number of retailers and low number of items, Model 2 could be a good choice since it provided low total system-wide cost as Model 3. Small effect of sharing the ordering costs between items provides Model 2's performance be close to Model

3's. However, other situations for the high value of K_w, Model 3 still outperforms.

(3) We found that for the low value of K_w along with the low value of K^{TypeI}, Model 1's and model 3's performances were not difference because the ordering costs shared among location-items were small. However, we could not summarize to select either model from only the values of K_w and K^{TypeI}.

(4) Then, we analyzed the relationships between the values of K_i^{TypeII} and k_{ij}. Model 1 performs well for high value of K_i^{TypeII} along with high value of k_{ij}. The reasons are that Model 3 considers too many retailers in an order might increase the total ordering cost from the high value of K_i^{TypeII}, and the high value of k_{ij} also affects high total ordering cost from all retailer-items included in an order. However, for the low value of k_{ij}, Model 3 outperforms due to small effect of all retailer-items' total minor ordering costs.

(5) Interestingly, for the low value of K_i^{TypeII} and the high value of k_{ij}, Model 1 and 3 should be considered together since each model takes an effect of either K_i^{TypeII} or k_{ij}. We found that for such situation along with high number of items, Model 1 could be a good choice since it provided low total system-wide cost as Model 3. The reason is that Model 1 can more reduce the major ordering costs due to the high number of items. High effect of sharing the major ordering costs between items provides Model 1's performance be close to Model 3's.

In conclusion, considering all location-items together for the whole system is the best option as shown in Model 3. However, we provide other options for some situations in order to save computational times from high interrelationships between all location-items. Model 1 should be interesting for 1) the scenarios having low K_w, high k_{ij}, high K_i^{TypeII}, and 2) the scenarios having low K_w, high k_{ij}, low K_i^{TypeII}, and high number of items. Meanwhile, Model 2 should be interesting for the scenarios having high K_w, low number of retailers, and low number of items.

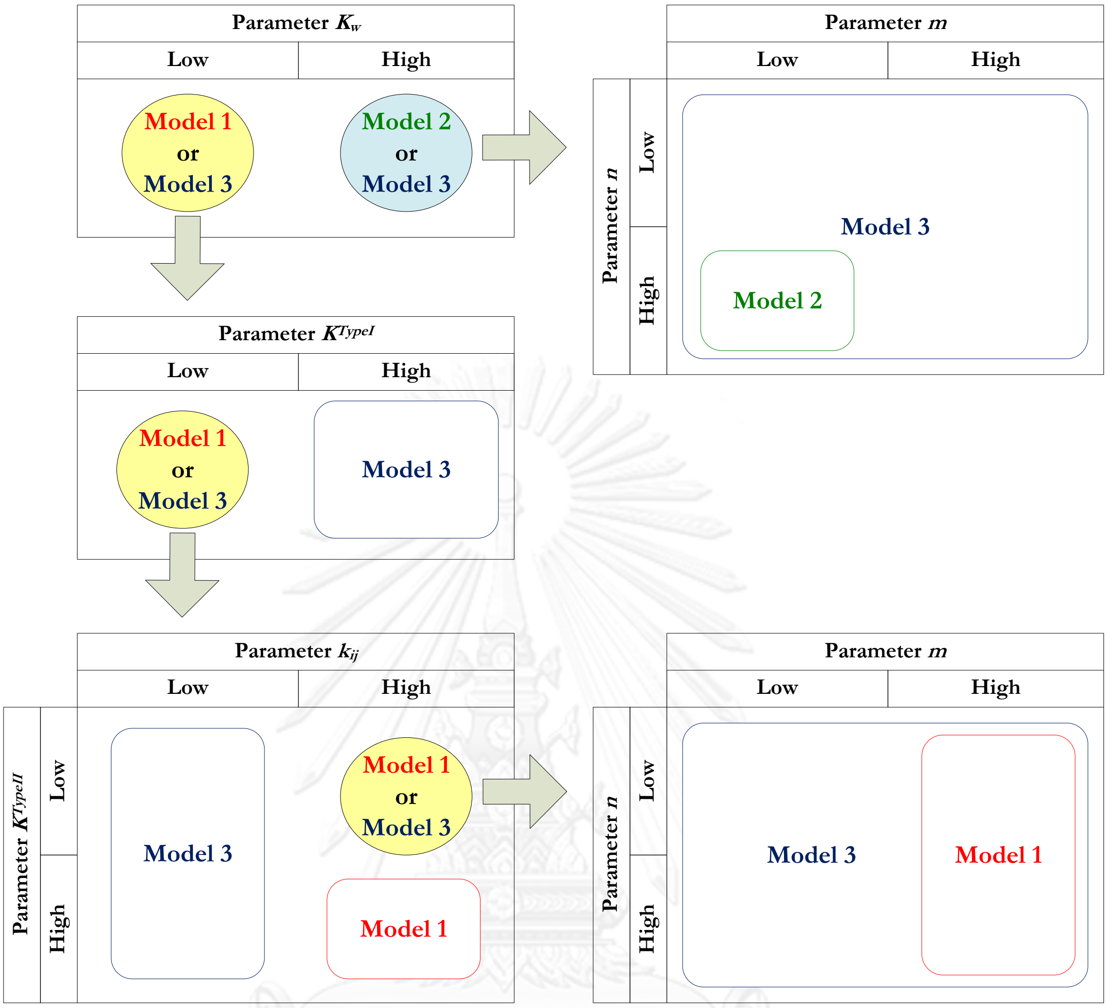


Figure V-7 Relationship of the proposed models and the significant relevant factors

5.5 Discussion

Under multi-item OWNR system, we are interested how to coordinate multiple items and multiple retailers in order to minimize the total system-wide cost. According to the ordering cost structure as shown in Fig.V-1, it is very significant to decide how to manage our system. Some questions have been raised, e.g.

- Is it worth managing all retailer-items together due to a lot of decision variables concerned in the system?
- If there are large values of the major ordering cost Type II, do we prefer to manage each retailer individually?

- If there are large values of the minor ordering cost of retailer-item ij, do we prefer to manage each item individually but coordinate all retailers for economic order quantities?

These questions lead us to develop three joint replenishment models: Model 1 – joint replenishment with item-based model, Model 2 – joint replenishment with retailer-based model, and Model 3 – completely joint replenishment model (more details of each model have already been mentioned in section 5.1). One hypothesis is that Model 3 should provide better performance than others since all location-items are coordinated to share all ordering costs. Disadvantageously, Model 3 contains a lot of interrelationship between decision variables, so it takes a lot of computational time to determine the best solution. Consequently, the above questions motivate us to study other models if they provide indifferent results as comparing to Model 3.

We extended the SIM/S/NZ heuristic from Phase II to the SIM/M/NZ heuristic. Dimension of multiple items was added into search algorithm. We still applied decomposition technique, iterative procedure, and golden section search, by the reason that inventory policy characteristics of each location-item have not been changed. For Model 2, the SIM/S/NZ heuristic can also be applied because Model 2 is based on the single-item multi-retailer model.

From the experimental results, it was not surprising that Model 3 provided the lowest total system-wide cost in many scenarios since all location-items were coordinated to share all ordering costs. However, huge number of interrelated decision variables is the weakness of Model 3. In consequence, Model 3 takes a lot of computational times, especially for the scenarios at high number of retailers and high number of items. For eight retailers and eight items, Model 3's computational time is 3.77 hours on average. Whereas for such scenarios Model 1's computational time is faster than Model 3's around 52 times, and Model 2's computational time is faster than Model 3's around 40 times. Certainly, if number of retailers and number of items are more than 8, Model 3 will spend even more computational times than other models due to multiplication of interrelated decision variables.

Focusing on Model 1, it should be suitable for a scenario under high major ordering cost Type II, since considering too many retailers in an order might increase the total ordering cost (instead of taking an advantage from sharing the fixed ordering costs). From the experimental results, Model 1 performs well for high major ordering cost Type II along with high retailer-item's minor ordering cost. Whereas Model 3 performs better if there are high major ordering cost Type II and low retailer-item's minor ordering cost. The reason is that high retailer-item's minor ordering cost affects high total ordering cost from all retailer-items included in an order. Hence, Model 1 decomposes all retailer-items into single retailer with multiple items to determine the best solutions of all items for such retailer. Interestingly, when there are high number of items along with low major ordering cost Type II, Model 1 should be replaceable Model 3 to save computational times with the maximum cost gap[13] at 5.19% (based on our experiments). The reason is that for Model 1 high number of items can more reduce the major ordering costs.

Even though Model 2 does not outperform other models, especially for Model 3, it still is interesting for the situation under high warehouse's major ordering cost, low number of items, and high number of retailers. Under such situation, Model 2 performs quite well because high number of retailers can more reduce the major ordering costs and there is more possibility that the warehouse is allowed to hold stock. Moreover, it has less effect of low number of items; therefore its performance is close to Model 3's. Since the warehouse has high value of Δ_0, the opportunity of joint replenishment is very high in spite of low number of items. Therefore, the upper bound should not be used to compare with Model 3 due to overestimate. Hence, the lower bound should be used instead. Under such situation, Model 2 can save computational times 10 with the cost gap at 5.67% (based on our experiments).

For more complex case, such as high number of retailers, high number of items, non-identical location-items, the decomposed models like Model 1 or Model 2 should be another good choice for managing the multi-item system under OWNR. However, Model 1 and Model 2 were developed based on the simulation model as shown in Fig.V-5, the warehouse cannot identify an exact number of retailers (or number of items) included in an order sent to the outside supplier. We have to estimate number of retailers per order (or number of items per order) to use for the models.

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[13] The largest cost-saving of the warehouse's major ordering cost is when all items at the warehouse are jointly replenished. So, such situation can be identified to a lower bound case. On the contrary, when each item is separately replenished bring the non-shared warehouse's major ordering cost to an upper bound case (already mentioned in section 5.2.2). By this concept, the maximum cost gap is a comparative measurement between an upper bound from Model 1 (or 2) and the lowest total system-wide cost from Model 3.

5.6 Conclusion

This chapter demonstrated Phase III's system comprising multiple items on OWNR. The warehouse's items were jointly replenished, and then the can-order level at the warehouse was employed. Unlike Phase I and Phase II, we considered the ordering cost following location-item ij instead of only location i. The ordering costs were restructured to be consistent with multiple items and multiple retailers.

We proposed three joint replenishment models to manage multiple items and multiple retailers: Model 1 was a joint replenishment with item-based model, Model 2 was as joint replenishment with retailer-based model, and Model 3 was a completely joint replenishment model for all retailer-items. Comparative analysis on three joint replenishment models was conducted.

Heuristic algorithm called SIM/M/NZ was developed to determine the inventory policy setting for location-item ij. We extended the SIM/S/NZ heuristic which was proposed in Phase II into this SIM/M/NZ heuristic. Decomposition technique, iterative procedure, and one-dimensional search were still applied by adding a dimension of multiple items. The experimental results showed that Model 3 provided the lowest total system-wide cost in many scenarios, but it spent much more computational time specifically high number of items and high number of retailers. By this result, a selection of joint replenishment model (three proposed models) employing to the multi-item inventory system should be based on the compromise between "total system-wide cost" and "computational time".

CHAPTER VI
CONCLUSION

This chapter summarizes the dissertation deliverables: analyses of the can-order policies as well as joint replenishment models and solution approaches. Moreover, the future research directions are also provided to fulfill the research gaps in the area of an integration of joint replenishment problem and multi-echelon inventory system.

6.1 Dissertation Deliverables

In this dissertation, we studied the can-order policies employed into two-echelon inventory system composing of one warehouse and multiple retailers with multiple commodities. Regarding a few of literatures studied on the shared ordering costs among retailers/items, it was interesting to apply joint replenishment policy into the one-warehouse n-retailer inventory system (OWNR) under continuous replenishment and stochastic demand. Then, the system including all inventory costs were taken into consideration in order to determine the inventory policy parameters for all stores in the system as the general inventory control process. Our objective was to develop the stochastic joint replenishment model and the solution approach for determining inventory policy parameters under such system so as to obtain the expected minimum total system-wide cost.

We conducted the research by using two methods: computer simulation and heuristic approach. Due to the system's complexity, computer simulation was an efficient approach representing the complicated inventory process. We used computer simulation to preliminarily study the can-order policy, and also to obtain the best-known solution providing the minimum of average total system-wide cost. We made an effort to determine the best-known solution due to a large search space; therefore, the heuristic approaches were proposed to solve this problem. Lot of literatures on the can-order policy used the heuristic approach to determine the appropriate inventory policy setting as it was an NP-hard problem.

To obtain insights of the can-order policy on OWNR, the dissertation methodology is divided into three phases: Phase I – The single-item model with zero lead time, Phase II – The single-item model with non-zero lead time, and Phase III – The multi-item model with non-zero lead time. We studied the can-order policy on each Phase, observed its characteristics, and analyzed what we found to develop the joint replenishment models and heuristic approaches following each phase.

Hence, we summarize the significant deliverables of our dissertation as follows:

6.1.1 Analyses of the can-order policies

We studied the can-order policy on six relevant factors which are the most important as found in many kinds of inventory problem. There were cost components, demand rates, lead times, target service levels (fill rates), number of retailers, and number of items. Specifically, cost components were classified into three components, i.e. unit holding cost per unit time, major ordering cost per order, and minor ordering cost per location-item. The experimental results showed that all relevant factors had an effect on the can-order policy, especially for the holding cost ratio which is a ratio of unit holding cost per unit time at the warehouse echelon to unit holding cost per unit time at the retailer echelon. We found that it highly affected the decision on the warehouse echelon whether or not the warehouse would employ the cross-docking system.

Rationally, the warehouse's order-up-to level $S_{(0,j)}$ is relative to the retailers' order-up-to level S_{ij}. If $S_{(0,j)} \leq S_{(i,j)}$, the warehouse's inventory is replenished every time when any retailer's triggers an order, because dispatch quantity is always larger than the warehouse's inventory level. So, the minimum total system-wide cost of this condition occurs at $S_{(0,j)} = 0$. Meanwhile, if $S_{(0,j)} > S_{ij}$, it means that the warehouse holds stock for dispatching to the retailers more than one order. Trading off between the holding costs and the ordering costs has to be considered to decide how many order cycle the warehouse should serve retailer echelon. Then, there is a solution (or more than one solution) which $S_0 > S_i > 0$ providing the minimum total system-wide cost of this condition. According to these conditions, we could generally divide the system into two cases: case I – Cross-docking system, and case II – Stocking system at the warehouse. The best-known solution definitely occurred in either case I or II.

The fixed can-order level at $c = S - 1$ can create the maximum opportunity of joint replenishment for all retailers (items). The major ordering cost can be most shared if all retailers (items) are included in an order to minimize the total system-wide cost [48]. Unless all retailers (items) are replenished, the total ordering cost will increase from the increased total ordering cost or/and the increased total holding cost. Then, the holding cost is traded off with the shared ordering cost in order to balance order frequency and holding stock. We found that the experimental results were consistent with this joint replenishment concept. The can-order level could be approximated to S_{ij} - 1 since $TC^*_{(S_{ij}-1)}$ was greater than TC^* not over 1% on average where TC^* was the optimal average total system-wide cost and $TC^*_{(S_{ij}-1)}$ was the minimum average total system-wide cost of the solution at c_{ij} = S_{ij} - 1. Even though the minor ordering costs and target service levels influenced the can-order level is not equal to S_{ij} - 1, we obtained a small cost gap between TC^* and $TC^*_{(S_{ij}-1)}$. Mainly, if the ratio of the major ordering cost to the minor ordering cost was not too small, this all joint concept could be utilized as van Eijs [48] recommended. Since the minor ordering cost had less effect on the total system-wide cost as comparing to the major ordering cost.

All decision variables were associated with each other. For example, the retailers' must-order level affected the warehouse's must-order level to hold sufficient stock for serving target service levels, each location's order-up-to level was relative to its must-order level. Therefore, it was hard to analyze their relationship obviously. Thus, we considered $\Delta_{ij} = S_{ij} - s_{ij}$ where index ij represents location-item ij. The value of Δ_{ij} was originated from an economical order quantity as a classic EOQ which the ordering cost and the holding cost were traded off. We analyzed each decision variable by either fixing other variables or varying some needed variables. The results showed that the total system-wide cost line turns to resemble a curve containing a minimum point. The knowledge from this study was very significant for solution approaches.

6.1.2 Joint replenishment models and solution approaches

We proposed various joint replenishment models and solution approaches to solve the inventory problems for each phase. To illustrate the overall of what we developed for each phase, Fig.VI-1 shows a summary of the dissertation with our aimed deliverables. The existing research and the best-known solutions obtained from computer simulation were compared with our heuristics for performance evaluation. The best-known solution can be determined by systematic approaches: input determination and output validation.

From Fig.V-1, each phase was related to indifferent number of decision variables reflecting indifferent dimensions. Since the fixed can-order level $c_{ij} = S_{ij} - 1$ was utilized, we could reduce number of decision variables. It was an easier approach to develop the can-order policy according to a regenerative process. The simple phase was Phase I. We developed two heuristic approaches, and each approach employed its joint replenishment model.

- Heuristic I called DJ was proposed by using the concept of a classical deterministic model of Schwarz [124]. The pilot testing on DJ showed this simple policy was useful in the case of identical retailers with low number of retailers.

- We attempted to develop heuristic II called EOQ-Z to obtain better quality solution than the DJ heuristic. We modified the model of van Eijs [48] which was developed following Erlang distribution. We approximated continuous arrival of demand at warehouse echelon, so it enabled us to use EOQ. Decomposition technique, iterative procedure, and one-dimensional search called golden section search were employed into the heuristic algorithm. Overall, the experiments provided the cost gap of heuristic approach less than 2% on average as compared to the best-known solution. With satisfactory computational time and small cost gap, heuristic II (EOQ-Z) is well worth using for the can-order policy setting under OWNR with zero lead time.

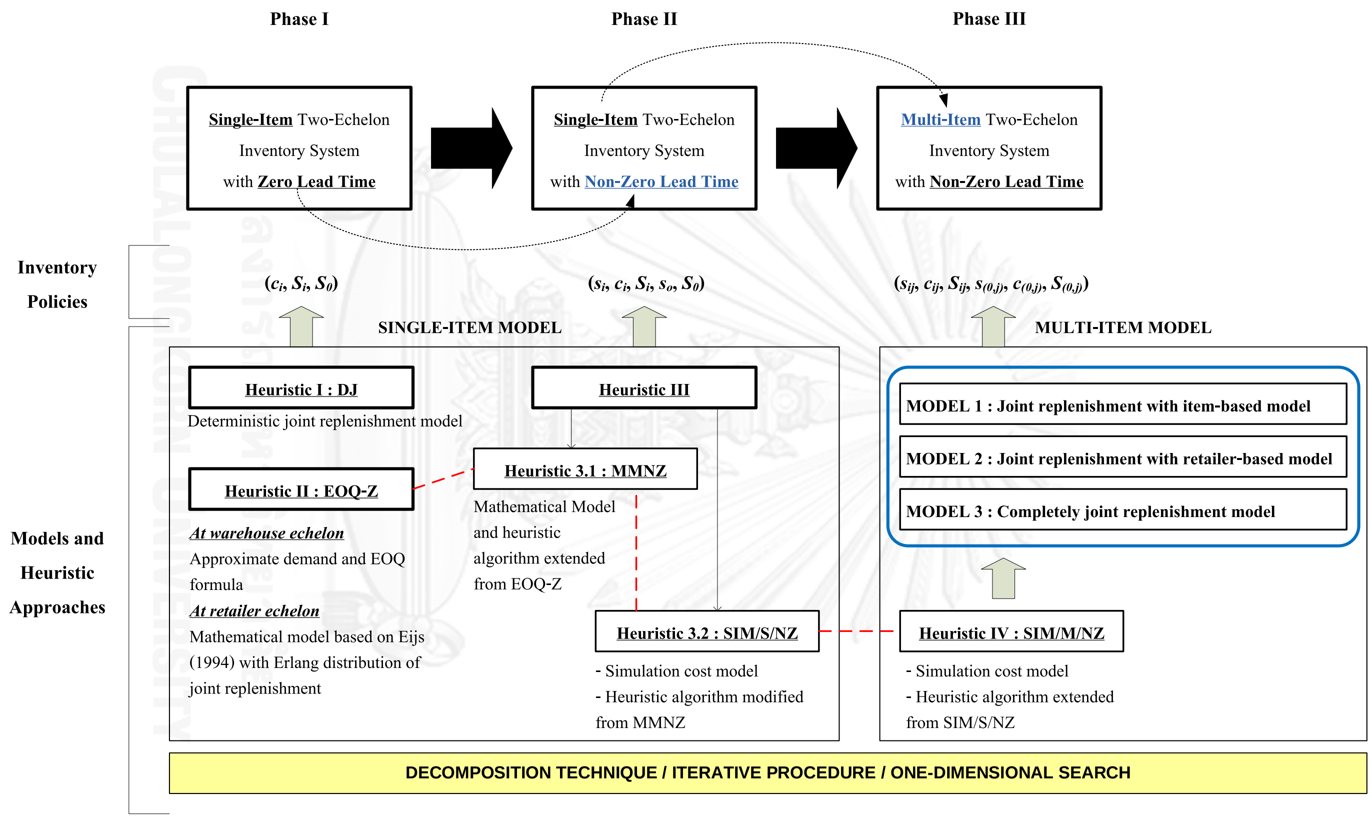

Figure VI-1 Summary of three phases of the dissertation with the deliverables

Significant finding was an integration of the classical EOQ and the can-order policy for two-echelon inventory system. We simplified the EOQ concept to determine the warehouse's order-up-to level S_0. It relaxed dispatch quantity and frequency synchronized with retailer echelon, but utilized total demand rate which is a summation of all retailers' demand rates. From the experimental results, the mechanism of trading off between warehouse echelon and retailer echelon occurred to rebalance with EOQ. The research showed that even though we studied the complicated system, the simple concept of EOQ remained useful and applicable for the case of zero lead time (i.e. the zero lead time assumption could be interpreted and applied in the situation when the ratio of lead time to order cycle duration was very small).

Later, Phase II was examined to determine the appropriate inventory policy setting by heuristic III. We classified heuristic into two sub-approaches: heuristic 3.1 and 3.2.

- Firstly, we proposed heuristic 3.1 named MMNZ. Approximate mathematical model extended the concept of the EOQ-Z heuristic from Phase I added lead time and target service level. We extended an application of decomposition technique, iterative procedure, and golden section search to heuristic algorithm. The MMNZ heuristic was useful if the cross-docking system was preferable, but it was quite poor if the warehouse was allowed to hold stock.

- Heuristic 3.2 called SIM/S/NZ was introduced. We used simulation cost model instead of approximate mathematical model, but yet the same heuristic algorithm. It could reduce the cost error from an approximation. The performance of the SIM/S/NZ heuristic was measured into two methods. The first method was a comparison with the best-known solution obtained from computer simulation. The SIM/S/NZ heuristic provided an average cost gap not over 2% on average. The second method was a comparison with Özkaya [22]. Qualitative analysis was provided that the SIM/S/NZ heuristic should be better than Özkaya [22]'s approach.

Advantageously, the SIM/S/NZ heuristic gave the best performance in terms of cost gap and their computational time could be saved from the reduced search space as comparing to the computer simulation's computational time. We provided systematically reduced search space. Hence, the SIM/S/NZ heuristic was interesting for the can-order policy setting under OWNR with single-item and non-zero lead time consideration.

For the last phase, we proposed three joint replenishment models to manage multiple items and multiple retailers: Model 1 was a joint replenishment with item-based model, Model 2 was as joint replenishment with retailer-based model, and Model 3 was a completely joint replenishment model for all retailer-items. We developed mathematical models with a generalization of the ordering cost structure, and extended the heuristic algorithm from SIM/S/NZ into the multi-item model. We called the SIM/M/NZ heuristic. Dimension of multiple items was added into search algorithm. We still applied decomposition technique, iterative procedure, and golden section search, by the reason that inventory policy characteristics of each location-item have not been changed. For Model 2, the SIM/S/NZ heuristic can also be applied because Model 2 is based on the single-item multi-retailer model.

From the experimental results, it is not surprising that Model 3 provided the lowest total system-wide cost in many scenarios since all location-items are coordinated to share all ordering costs. However, huge number of interrelated decision variables is the weakness of Model 3. In consequence, Model 3 takes a lot of computational times, especially for the scenarios at high number of retailers and high number of items. Certainly, in reality there are many retailers or items considered in the system, Model 3 will spend even more computational times than other models due to multiplication of interrelated decision variables. Hence, we provided insights of which situation is suitable for each joint replenishment model. Some situations, Model 3 could be replaced by Model 1 or Model 2 by making a decision based on "total system-wide cost" and "computational time".

The most significant deliverable of our dissertation was the proposed solution approaches for determining the appropriate inventory policy parameters. Each approach was consistent with the inventory policy characteristics obtained from preliminary analyses. For all phases, we used the same basis for developing the solution approaches: decomposition technique, iterative procedure, and golden section search. Decomposition technique and iterative procedure were the most common approach for the can-order policy determination. Decomposition technique helped breaking the complicated system (multiple location-items) into smaller part (single location-item). Determination of the can-order policy parameters seems easier than consideration of the whole parts together. However, this technique should be utilized with iterative procedure to consolidate all single location-items consistently. The solution could move to the better one until the best solution has been found for the whole system. From both techniques integrated with one-dimensional search, we can determine the best solution easier and faster than other approaches, especially computer simulation and the exhaustive search (e.g. Özkaya [22]).

From the first phase, trading off between the holding costs and the ordering costs makes the total system-wide cost performed as a convex function relative to the value of S_i. So, we could determine the value of S_i providing the minimum total system-wide cost on one-dimensional search. Since our cost formulation was non-derivative function, we utilized a search algorithm called “Golden section search” by adapting for integer variable. This search algorithm performed better than other search algorithms, such as Fibonacci search and Half-interval search. Later phases, we considered $\Delta_k = S_k - s_k$ to represent an order quantity for location k including the warehouse and the retailers; for the warehouse, $\Delta_0 = S_0 - s_0$ and for the retailers, $\Delta_i = S_i - s_i$, $i \in N$. We found the characteristic of Δ_k by trading off between the holding cost and the ordering cost. The total system-wide cost performed as a curve containing the minimum point relative to the value of Δ_k. Interestingly, even though the curves were not unimodal continuous function because of discrete numbers and the must-order levels, the golden section search with iterative procedure was applicable for determining the appropriate value of Δ_k. The reason was that the cost difference between two connected points was small enough to lead the successive search ranges from the golden section search meet the minimum point. Similarly, we also used the golden section search for determining the best values of s_0. Based on the same reason of small cost difference between two connected points, the best values of s_0 could be reached. According to the experimental results, it was fascinating to apply the golden section search into our system in order to shorten the computational time with the appropriate inventory system-wide cost.

Advantageously, the dissertation provided various joint replenishment models and heuristic approaches suitable for each part of the OWNR. We considered the warehouse and the retailers from the small part of OWNR as the single-item model to the multi-item model. Therefore, we believe that our contribution is not only limited to a specific area but also a good starting point for a research of more complex environment such as the joint replenishment policies in multi-echelon inventory systems.

6.1.3 Application of the can-order policy

This dissertation is generalized for any industry which matches the considered system. However, this section exemplifies a specific industry to show the real situation. Since the research problem originally surveyed in the healthcare industry, we would apply the can-order policy for such industry (or others which have the similar system) to express the employment and its limitation. The research can be applied into many parts of healthcare industry for pharmaceuticals and medical supplies management such as hospital's internal chain (central storeroom and multiple departments), hospital network (central warehouse and multiple hospitals), and drug store chain (central warehouse and multiple drug stores).

For the inventory policy setting, healthcare services have implemented both types of inventory reviews: continuous review and periodic review. Each type is considered depending on item types, demands, suppliers, replenishment and distribution operations, and resource constraints. Mostly, healthcare inventory management has commonly adopted "par level" policy which is special feature only in healthcare. There are two kinds of par levels. The minimum par level is equivalent to the reorder point and the maximum par level is equivalent to the order-up-to level (or base stock). Each kind of par levels can be used separately or together such as

- The (s,S) policy where s represents the reorder point or the minimum par level and S represents the based stock or the maximum par level.
- The (r,Q) policy where r represents the reorder point or the minimum par level and Q represents the fixed order quantity.
- The (R,S) policy where R represents the length of review period and S represents the based stock or the maximum par level.

Focusing on continuous review, the (s, S) policy and the (r, Q) policy are based on independent ordering decision. Instead of both policies, we can employ the can-order policy to coordinate multiple items and/or multiple locations (e.g. departments, patient care units, hospitals, drug stores). Since we consider the OWNR system, central warehouse can also employ the can-order policy to coordinate multiple items.

The can-order policy is not suitable for a large group of items (locations) since there are a lot of interactions between items (locations). Decomposing a large group into various small groups is preferable to reduce interactions. In addition, due to the fixed can-order level $c_{ij} = S_{ij} - 1$, all items (locations) in a group have to be replenished in the same order. Therefore, small groups are also useful to apply our heuristics. We suggest to group items (locations) which have minor variation of demand rates in order to synchronize the same order cycles.

For some systems, periodic review seems to be more popular than continuous review, because it is easier to set joint replenishment period. However, a lot of stock has to be hold to cover the review period. Therefore, the can-order policy is able to use for specific group of items, such as items with high service level, in order to reduce safety stock.

6.2 Future Research Directions

In this section, we recommend some possible research extensions. We categorize the interesting research into three groups as follows:

1) Heuristic approach

According to the golden section search which is actually used for unimodal function, we attempted to apply its concept to our problem even the function seems to be multimodal as depicted in Fig.IV-8. The experimental results provide a quality solution, so we chose to use only simple method to determine the appropriate inventory policy setting. However, to verify the can-order policy's performance without heuristic's error, global search methods with derivative-free optimization might be another option to conduct a research. A review of Rios and Sahinidis [123] is recommended to study the derivative-free optimization with comparison of software implementation. They provided the insights of the derivative-free optimization in both academic and practical aspects. Finally, comparative analysis between global search methods and our approach should also be carried out.

Even though fixing the can-order level at $S_{ij}-1$ provides a small cost gap as comparing to the best-known solution, it seems not suitable if the minor ordering cost is quite large when comparing to the major ordering cost. Thus, our heuristics can be extended to the search of the can-order level on given s_{ij} and S_{ij}. Recently, we found an interesting work of Nagasawa et al. [77]. They applied genetic algorithm to determine the can-order level on given s_{ij} and S_{ij}. This work is a starting point to extend our heuristic for determining the can-order level.

In this study, location-items are jointly ordered according to the predetermined inventory policy setting. Interaction among location-items is one of the most important effects to the system. If number of items and/or number of retailers are large, the system needs a lot of computational time to determine the appropriate inventory policy setting. So, it can reduce the efficiency and advantage of the multi-item multi-location inventory control. Clustering location-items into small groups is important to reduce the complexity of the joint ordering decision. Tsai et al. [76] proposed an association clustering algorithm applying to the can-order policy for multi-item single-location inventory system to evaluate the correlated demands among items. Clustering method was developed to group items with close demand in a hierarchal way. The results of the experiments showed that the proposed method outperformed several replenishment models. Therefore, the extension of clustering location-items would be an interesting issue to focus on. Moreover, from our experiment results the considered system seems to contain two sub-systems: the cross-docking system (with no stock at the warehouse) and the stocking system (allowing the warehouse to stock). It is possible that some location-items are stored at the warehouse and the others utilize the cross-docking system. Hence, clustering location-items can be applied for coordinated ordering decision and choose the proper system for each location-item.

2) System complexities

According to the growing trend of information technology, the warehouse can obtain the real time information about the status of the retailers and also be in charge of the allocation of goods to the retailers. Therefore, it is possible to split lot to allocate the units in an order to the involved retailers according to a predetermined allocation rule. Since a joint order has larger lot size than an independent order, the average waiting time from the warehouse to retailer echelon for all-lot replenishment is much longer. Consequently, an integration of allocation problem into joint replenishment model is able to reduce the average waiting time and it can be interesting to study in more details for a future research extension.

With regard to centralized control for OWNR, there are various options to practically increase service level. An outside supplier can directly deliver to the retailers with an additional cost in case of insufficient stock at the warehouse. Another option is that the warehouse replenishes the involved retailers with an emergency order immediately dispatched to the retailers ($< L_{ij}$), and an additional cost is charged. Such two options have to concern the additional costs charged to the system. The system needs to tradeoff between the holding costs and the additional costs under target service levels at the retailers.

In determining which the cross-docking system is preferable, it is interesting to include truck capacity constraint (i.e. limited dispatch quantity is needed) in order to synchronize with shipment scheduling problem.

3) <u>Other joint replenishment policies</u>

According to the can-order policy selected in our dissertation, we raise its advantages in practical and academic aspects as mainly demonstrated in Section 1.3.1. Moreover, we compared our approach to Özkaya [22] proposed four joint replenishment policies, and we evaluated that our heuristic approach has an advantage over Özkaya [22]'s approach. Later, to enhance our approach's performance and to identify which situation is suitable for each joint replenishment policy considered on OWNR, it is interesting to analyze our can-order policy on OWNR with other joint replenishment policies, especially periodic replenishment policies would be focused on both the single-item and multi-item models.

As various directions recommended, the integration of joint replenishment problem and multi-echelon inventory system is extended into more complex system. In addition, comparative analyses with other joint replenishment policies or other heuristic approach would be focused on. These are great opportunities to enhance the knowledge in the field of inventory problem.

REFERENCES

1. Silver, E.A., D. Pyke, and R. Peterson, *Inventory management and production planning and scheduling*. 3rd ed. 1998, New York: Wiley.
2. Kim, D., *An integrated supply chain management system: A case study in healthcare sector*, in *E-Commerce and Web Technologies*. 2005. p. 218-227.
3. Nilsson, A., *Essays on joint replenishment and multi-echelon inventory systems*, in *Department of Business Administration and Social Sciences*. 2006, Luleå University of Technology: Luleå, Sweden.
4. Kelle, P., et al., *Healthcare supply chain management*, in *Inventory management : Non-classical views* M.Y. Jaber, Editor. 2009, CRC Press: Baco Raton. p. 99-128.
5. Woosley, J.M., *Improving healthcare supply chains and decision making in the management of pharmaceuticals*, in *Information Systems and Decision Sciences*. 2009, Louisiana State University and Agricultural and Mechanical: Baton Rouge. p. 112.
6. Freudenheim, M., *All about/medical supplies; removing the warehouse from cost-conscious hospitals*, in *The New York Times*. 1991, The New York Times.
7. Jarrett, P.G., *Logistics in the health care industry.* International Journal of Physical Distribution & Logistics Management, 1998. **28**(9/10): p. 741-772.
8. Rivard-Royer, H., S. Landry, and M. Beaulieu, *Hybrid stockless: a case study lessons for health-care supply chain integration.* International Journal of Operations & Production Management, 2002. **22**(4): p. 412-424.
9. Nicholson, L., A.J. Vakharia, and S.S. Erenguc, *Outsourcing inventory management decisions in healthcare: models and application.* European Journal of Operational Research, 2004. **154**(2004): p. 271-290.
10. Moschuris, S.J. and M.N. Kondylis, *Outsourcing in public hospitals: a Greek perspective.* Journal of Health Organisation and Management 2006. **20**(1): p. 4-14.
11. Baboli, A., M.P. Neghab, and R. Haji, *Order quantity in two-level supply chain with transportation costs: Comparison of decentralized with centralized decision.* J Syst Sci Syst Eng, 2008. **17**(3): p. 353-366.
12. Williams, B.D. and T. Tokar, *A review of inventory management research in major logistics journals: themes and future Directions.* The International Journal of Logistics Management, 2008. **19**(2): p. 212-232.

13. Foxx, W.K., M.D. Bunn, and V. McCay, *Outsourcing services in the healthcare sector.* J Med Market, 2009. **9**(1): p. 41-55.
14. Gou, Q., et al., *A modified joint inventory policy for VMI systems.* International Journal of Information Technology & Decision Making (IJITDM), 2008. **07**(02): p. 225-240.
15. Schneider, H., D.B. Rnks, and P. Kelle, *Power approximations for a two-echelon inventory system using service levels.* Production and Operations Management, 1995. **4**(4): p. 381-400.
16. Axsäter, S., S.C. Graves, and A.G.d. Kok, *Supply chain operations: Serial and distribution inventory systems*, in *Handbooks in operations research and management science*. 2003, Elsevier. p. 525-559.
17. Wang, Q., T.-M. Choi, and T.C.E. Cheng, *Control policies for multi-echelon inventory systems with stochastic demand*, in *Supply chain coordination under uncertainty*. 2011, Springer Berlin Heidelberg. p. 83-108.
18. Graves, S.C., *On the deterministic demand multi-product single-machine lot scheduling problem.* Management Science, 1979. **25**(3): p. 276-280.
19. Özkaya, B.Y., Ü. Gürler, and E. Berk, *The stochastic joint replenishment problem: A new policy, analysis, and insights.* Naval Research Logistics (NRL), 2006. **53**(6): p. 525-546.
20. Khouja, M. and S. Goyal, *A review of the joint replenishment problem literature: 1989-2005.* European Journal of Operational Research, 2008. **186**(1): p. 1-16.
21. Cheung, K.L. and H.L. Lee, *The inventory benefit of shipment coordination and stock rebalancing in a supply chain.* Management Science, 2002. **48**(2): p. 300-306.
22. Özkaya, B.Y., *Stochastic joint replenishment problem: A new policy and analysis for single location and two echelon inventory systems*, in *Industrial Engineering*. 2005, Bilkent: Turkey. p. 199.
23. Cetinkaya, S. and C.-Y. Lee, *Stock replinishment and shipment scheduling for vendor-managed inventory systems.* Management Science, 2000. **46**(2): p. 217-232.
24. Gürbüz, M.Ç., K. Moinzadeh, and Y.-P. Zhou, *Coordinated Replenishment Strategies in Inventory/Distribution Systems.* Management Science, 2007. **53**(2): p. 293-307.

25. Axsäter, S. and W.-F. Zhang, *A joint replenishment policy for multi-echelon inventory control.* International Journal of Production Economics, 1999. **59**: p. 243-250.
26. Cohen, M., et al., *Optimizer: IBM's multi-echelon inventory system for managing service logistics.* Interfaces, 1990. **20**(1): p. 65-82.
27. Hopp, W., R. Zhang, and M. Spearman, *An easily implementable hierarchical heuristicfor a two-echelon spare parts distribution system.* IIE Transactions, 1999. **31**(10): p. 977-988.
28. Qu, W.W., J.H. Bookbinder, and P. Iyogun, *An integrated inventory-transportation system with modified periodic policy for multiple products.* European Journal of Operational Research, 1999. **115**(2): p. 254-269.
29. Sindhuchao, S., *An inventory-transportation system for multi-item joint replenishment with limited vehicle capacity*, in *Industrial Engineering*. 2004, Chulalongkorn University: Bangkok. p. 123.
30. Al-Rifai, M.H. and M.D. Rossetti, *An efficient heuristic optimization algorithm for a two-echelon (R, Q) inventory system.* International Journal of Production Economics, 2007. **109**(1-2): p. 195-213.
31. Topan, E., Z. Pelin BayIndIr, and T. Tan, *An exact solution procedure for multi-item two-echelon spare parts inventory control problem with batch ordering in the central warehouse.* Operations Research Letters, 2010. **38**(5): p. 454-461.
32. Topan, E., Z. Pelin BayIndIr, and T. Tan, *Heuristics for multi-item two-echelon spare parts inventory control problem with batch ordering in the central warehouse*. 2010, Research School for Operations Management and Logistics.
33. Zhou, W.-Q., L. Chen, and H.-M. Ge, *A multi-product multi-echelon inventory control model with joint replenishment strategy.* Applied Mathematical Modelling, 2013. **37**(4): p. 2039-2050.
34. Dellaert, N. and E. van de Poel, *Global inventory control in an academic hospital.* International Journal of Production Economics, 1996. **46-47**: p. 277-284.
35. Totrakool, P., *A heuristic search method for a vehicle routing problem in a medical supplies distribution system*, in *Industiral Engineering*. 2003, Chulalongkorn University: Bangkok. p. 139.
36. Rattanasin, N., *A heuristic search method for a hub location problem in a medical supplies distribution system*, in *Industiral Engineering*. 2003, Chulalongkorn University: Bangkok. p. 164.

37. Belson, D., *Storage, distribution, and dispensing of medical supplies*. 2005, CREATE, University of Southern California: Los Angeles.
38. Tongrod, D., *Medical supplies fulfillment system for central medical supplies management*, in *Industrial Engineering*. 2007, Chulalongkorn University: Bangkok. p. 208.
39. Rudeejaroensakul, D., *Design of medical supplies replenishing system*, in *Industrial Engineering*. 2008, Chulalongkorn University: Bangkok.
40. Arshinder, A. Kanda, and S.G. Deshmukh, *Supply chain coordination: Perspectives, empirical studies and research directions.* International Journal of Production Economics, 2008. **115**(2): p. 316-335.
41. Portes, A.N. and G.E. Vieira. *The impact of vendor managed inventory (VMI) on the bullwhip effect in supply chains*. in *Proceedings of Third International Conference on Production Research - Americas' Region 2006 ICPR-AM06).* 2006.
42. Balintfy, J.L., *On a basic class of multi-item inventory problems.* Management Science, 1964. **10**(2): p. 287-297.
43. Pantumsinchai, P., *A comparison of three joint ordering inventory policies.* Decision Sciences, 1992. **23**(1): p. 111-127.
44. Viswanathan, S., *Periodic review (s, S) policies for joint replenishment inventory systems.* Management Science, 1997. **43**(10): p. 1447-1454.
45. Nielsen, C. and C. Larsen, *An analytical study of the Q(s,S) policy applied to the joint replenishment problem.* European Journal of Operational Research, 2005. **163**(3): p. 721-732.
46. Love, S.F., *Coordinated replenishment of multiple items*, in *Inventory Control*. 1979, McGraw-Hill: Auckland. p. 111-141.
47. Atkins, D.R. and P.O. Iyogun, *Periodic versus 'Can-Order' policies for coordinated multi-item inventory systems.* Management Science, 1988. **34**(6): p. 791-796.
48. van Eijs, M.J.G., *On the determination of the control parameters of the optimal can-order policy.* Zeitschrift für Operations Research, 1994. **39**(3): p. 289-304.
49. Axsäter, S., *Inventory control*. 2007: Springer.
50. Diks, E.B., A.G. de Kok, and A.G. Lagodimos, *Multi-echelon systems: A service measure perspective.* European Journal of Operational Research, 1996. **95**(2): p. 241-263.

51. Silver, E.A., *A control system for coordinated inventory replenishment.* International Journal of Production Research, 1974. **12**(6): p. 647-671.
52. Silver, E.A., *Establishing reorder points in the (S,c,s) coordinated control system under compound Poisson demand.* International Journal of Production Research, 1981. **19**(6): p. 743-750.
53. Federgruen, A., H. Groenevelt, and H.C. Tijms, *Coordinated replenishments in a multi-item inventory system with compound Poisson demands.* Management Science, 1984. **30**(3): p. 344-357.
54. Zheng, Y.-S., *Optimal control policy for stochastic inventory systems with Markovian discount opportunities.* Operations Research, 1994. **42**(4): p. 721-738.
55. Schultz, H. and S.G. Johansen, *Can-order policies for coordinated inventory replenishment with Erlang distributed times between ordering.* European Journal of Operational Research, 1999. **113**(1): p. 30-41.
56. Melchiors, P., *Calculating can-order policies for the joint replenishment problem by the compensation approach.* European Journal of Operational Research, 2002. **141**(3): p. 587-595.
57. Evans, M., N.A.J. Hastings, and J.B. Peacock, *Statistical distributions*. 3rd ed. / Merran Evans, Nicholas Hastings, J. Brian Peacock ed. 2000: Wiley.
58. Press, W.H., et al., *Numerical recipes 3rd edition: The art of scientific computing*. 2007: Cambridge University Press. 1256.
59. Arkin, E., D. Joneja, and R. Roundy, *Computational complexity of uncapacitated multi-echelon production planning problems.* Operations Research Letters, 1989. **8**(2): p. 61-66.
60. van Eijs, M.J.G., R.M.J. Heuts, and J.P.C. Kleijnen, *Analysis and comparison of two strategies for multi-item inventory systems with joint replenishment costs.* European Journal of Operational Research, 1992. **59**(3): p. 405-412.
61. Kaspi, M. and M.J. Rosenblatt, *On the economic ordering quantity for jointly replenished items.* International Journal of Production Research, 1991. **29**(1): p. 107-114.
62. Goyal, S.K. and S.G. Deshmukh, *Discussion a note on 'The economic ordering quantity for jointly replenishing items'.* International Journal of Production Research, 1993. **31**(12): p. 2959-2961.
63. Hariga, M., *Two new heuristic procedures for the joint replenishment problem.* J Oper Res Soc, 1994. **45**(4): p. 463-471.

64. Ben-Daya, M. and M. Hariga, *Comparative study of heuristics for the joint replenishment problem.* Omega, 1995. **23**(3): p. 341-344.
65. Viswanathan, S., *A new optimal algorithm for the joint replenishment problem.* J Oper Res Soc, 1996. **47**(7): p. 936-944.
66. Khouja, M., Z. Michalewicz, and S.S. Satoskar, *A comparison between genetic algorithms and the RAND method for solving the joint replenishment problem.* Production Planning & Control, 2000. **11**(6): p. 556-564.
67. Lee, F.-C. and M.-J. Yao, *A global optimum search algorithm for the joint replenishment problem under power-of-two policy.* Computers & Operations Research, 2003. **30**(9): p. 1319-1333.
68. Olsen, A.L., *An evolutionary algorithm to solve the joint replenishment problem using direct grouping.* Computers & Industrial Engineering, 2005. **48**(2): p. 223-235.
69. Ignall, E., *Optimal continuous review policies for two product inventory systems with joint setup costs.* Management Science, 1969. **15**(5): p. 278-283.
70. Silver, E.A., *Some characteristics of a special joint-order inventory model.* Operations Research, 1965. **13**(2): p. 319-322.
71. Thompstonej, R.M. and E.A. Silver, *A coordinated inventory control system for compound Poisson demand and zero lead time.* International Journal of Production Research, 1975. **13**(6): p. 581-602.
72. Kayiş, E., T. Bilgiç, and D. Karabulut, *A note on the can-order policy for the two-item stochastic joint-replenishment problem.* IIE Transactions, 2007. **40**(1): p. 84-92.
73. Johansen, S.G. and P. Melchiors, *Can-order policy for the periodic-review joint replenishment problem.* Journal of the Operational Research Society, 2003. **54**(3): p. 283-290.
74. Duyn Schouten, F.A.V.D., M.J.G.V. Eijs, and R.M.J. Heuts, *Coordinated replenishment systems with discount opportunities.* International Journal of Production Research, 1994. **32**(12): p. 2879-2895.
75. Liu, L. and X.-M. Yuan, *Coordinated replenishments in inventory systems with correlated demands.* European Journal of Operational Research, 2000. **123**(3): p. 490-503.
76. Tsai, C.-Y., C.-Y. Tsai, and P.-W. Huang, *An association clustering algorithm for can-order policies in the joint replenishment problem.* International Journal of Production Economics, 2009. **117**(1): p. 30-41.

77. Nagasawa, K., et al. *Applying genetic algorithm for can-order policies in the joint replenishment problem*. in *14th Asia Pacific Industrial Engineering and Management Systems Conference 2013*. 2013. Cebu, Philippines.

78. Renberg, B. and R. Planche, *Un modèle pour la gestion simultanèe des n articles d'un stock.* Revue Francaise d'Informatique et de Recherche Opérationnelle, 1967. **6**(Serie Blue): p. 47-59.

79. Cachon, G., *Managing a retailer's shelf space, inventory, and transportation.* Manufacturing & Service Operations Management, 2001. **3**(3): p. 211-229.

80. Mustafa Tanrikulu, M., A. Şen, and O. Alp, *A joint replenishment policy with individual control and constant size orders.* International Journal of Production Research, 2009. **48**(14): p. 4253-4271.

81. Roushdy, B., et al., *Inventory control for a joint replenishment problem with stochastic demand.* World Academy of Science, Engineering and Technology, 2011. **52**: p. 812-186.

82. Axsäter, S., *Inventory control*. 2004: Kluwer Academic Publisher Group.

83. Blanc, H., *Advanced inventory management: models and algorithms*. 2005, Tilburg University. p. 56.

84. Fan, Y., *Strategic outsourcing: evidence from British companies.* Marketing Intelligence & Planning, 2000. **18**(4): p. 213-219.

85. Lunn, T., *Ways to reduce inventory.* Hosp Mater Manage Q., 2000. **21**(4): p. 1-7.

86. Li, L.X. and W.C. Benton, *Performance measurement criteria in health care organizations: Review and future research directions.* European Journal of Operational Research, 1996. **93**(3): p. 449-468.

87. Arshinder, K., et al., *A Review on supply chain coordination: Coordination mechanisms, managing uncertainty and research directions*, in *Supply Chain Coordination under Uncertainty*. 2011, Springer Berlin Heidelberg. p. 39-82.

88. Axsäter, S. and L. Juntti, *Comparison of echelon stock and installation stock policies for two-level inventory systems.* International Journal of Production Economics, 1996. **45**(1-3): p. 303-310.

89. Sherbrooke, C.C., *Metric: A multi-echelon technique for recoverable item control.* Operations Research, 1968. **16**(1): p. 122-141.

90. Federgruen, A. and P. Zipkin, *Allocation policies and cost approximations for multilocation inventory systems.* Naval Research Logistics Quarterly, 1984. **31**(1): p. 97-129.

91. Matta, K.F. and D. Sinha, *Policy and cost approximations of two-echelon distribution systems with a procurement cost at the higher echelon.* IIE Transactions, 1995. **27**(5): p. 646-656.
92. Chu, L.Y. and Z.-J.M. Shen, *A power-of-two ordering policy for one-warehouse multiretailer systems with stochastic demand.* Operations Research, 2010. **58**(2): p. 492-502.
93. Shang, K.H. and S.X. Zhou, *Optimizing replenishment intervals for two-echelon distribution systems with fixed order costs.* Working paper, 2010.
94. Shang, K.H. and S.X. Zhou, *Optimizing replenishment intervals for two-echelon distribution systems with stochastic demand.* Working paper, 2011.
95. Eppen, G. and L. Schrage, *Centralized ordering policies in a multi-warehouse system with lead times and random demand.* TIMS Studies in the Management Sciences, 1981. **16**: p. 51-67.
96. Rogers, D.F. and S. Tsubakitani, *Inventory positioning/partitioning for backorders optimization for a class of multi-echelon inventory problems.* Decision Sciences, 1991. **22**(3): p. 536-558.
97. Axsäter, S., *Optimization of order-up-to-s policies in two-echelon inventory systems with periodic review.* Naval Research Logistics (NRL), 1993. **40**(2): p. 245-253.
98. Diks, E.B. and A.G. de Kok, *Optimal control of a divergent multi-echelon inventory system.* European Journal of Operational Research, 1998. **111**(1): p. 75-97.
99. Axsäter, S., *Approximate optimization of a two-level distribution inventory system.* International Journal of Production Economics, 2003. **81**(0): p. 545-553.
100. Rao, U.S., *Properties of the periodic review (R, T) inventory control policy for stationary, stochastic demand.* Manufacturing & Service Operations Management, 2003. **5**(1): p. 37-53.
101. Li, Z., *Comparative research on three-echelon and two-echelon medicine inventory model with positive lead-time.* Journal of Southeast University, 2005. **Vol.21**(no.4).
102. Wang, L., et al., *An effective and efficient differential evolution algorithm for the integrated stochastic joint replenishment and delivery model.* Knowledge-Based Systems, 2012. **36**(0): p. 104-114.

103. Wang, Q., *A periodic-review inventory control policy for a two-level supply chain with multiple retailers and stochastic demand.* European Journal of Operational Research, 2013. **230**(1): p. 53-62.

104. Wang, Q. and S. Axsäter, *Fixed-interval joint-replenishment policies for distribution systems with multiple retailers and stochastic demand.* Naval Research Logistics (NRL), 2013. **60**(8): p. 637-651.

105. De Bodt, M.A. and S.C. Graves, *Continuous-review policies for a multi-echelon inventory problem with stochastic demand.* Management Science, 1985. **31**(10): p. 1286-1299.

106. Chen, F. and Y.-S. Zheng, *Evaluating echelon stock (R, nQ) policies in serial production/inventory systems with stochastic demand.* Management Science, 1994. **40**(10): p. 1262-1275.

107. Chen, F. and Y.-S. Zheng, *Near-optimal echelon-stock (R, nQ) policies in multistage serial systems.* Operations Research, 1998. **46**(4): p. 592-602.

108. Deuermeyer, B.L., *A model for the analysis of system service level in warehouse-retailer distribution systems : the identical retailer case*. 1981.

109. Svoronos, A. and P. Zipkin, *Estimating the performance of multi-level inventory systems.* Operations Research, 1988. **36**(1): p. 57-72.

110. Axsäter, S., *Exact and approximate evaluation of batch-ordering policies for two-level inventory systems.* Operations Research, 1993. **41**(4): p. 777-785.

111. Axsäter, S., *Simple solution procedures for a class of two-echelon inventory problems.* Operations Research, 1990. **38**(1): p. 64-69.

112. Axsäter, S., *Evaluation of installation stock based (R, Q)-policies for two-level inventory systems with Poisson demand.* Operations Research, 1998. **46**(3-Supplement-3): p. S135-S145.

113. Axsäter, S., *Simple evaluation of echelon stock (R,Q) policies for two-level inventory systems.* IIE Transactions, 1997. **29**(8): p. 661-669.

114. Axsäter, S., *Exact analysis of continuous review (R,Q) policies in two-echelon inventory systems with compound Poisson demand.* Operations Research, 2000. **48**(5): p. 686-696.

115. Schwarz, L.B., B.L. Deuermeyer, and R.D. Badinelli, *Fill-rate optimization in a one-warehouse n-identical retailer distribution system.* Management Science, 1985. **31**(4): p. 488-498.

116. Ahire, S.L. and C.P. Schmidt, *A model for a mixed continuous-periodic review one-warehouse, n-retailer inventory system.* European Journal of Operational Research, 1996. **92**(1): p. 69-82.

117. Chen, F. and Y.-S. Zheng, *One-warehouse multiretailer systems with centralized stock information.* Operations Research, 1997. **45**(2): p. 275-287.

118. Tee, Y.-S. and M.D. Rossetti. *Using simulation to evaluate a continuous review (R,Q) two-echelon inventory model*. in *The 6th Annual International Conference on Industrial Engineering - Theory, Applications, and Practice*. 2001. San Francisco, CA, USA.

119. Hill, R.M., M. Seifbarghy, and D.K. Smith, *A two-echelon inventory model with lost sales.* European Journal of Operational Research, 2007. **181**(2): p. 753-766.

120. Jha, J.K. and K. Shanker, *Two-echelon supply chain inventory model with controllable lead time and service level constraint.* Computers & Industrial Engineering, 2009. **57**(3): p. 1096-1104.

121. Moin, N.H. and S. Salhi, *Inventory routing problems: a logistical overview.* J Oper Res Soc, 2006. **58**(9): p. 1185-1194.

122. Antoniou, A. and W.-S. Lu, *One-dimensional optimization*, in *Practical optimization*. 2007, Springer US. p. 81-117.

123. Rios, L. and N. Sahinidis, *Derivative-free optimization: a review of algorithms and comparison of software implementations.* Journal of Global Optimization, 2013. **56**(3): p. 1247-1293.

124. Schwarz, L.B., *A simple continuous review deterministic one-warehouse n-retailer inventory problem.* Management Science, 1973. **19**(5): p. 555-566.

125. Forbes, C., et al., *Erlang distribution*, in *Statistical distributions*. 2010, John Wiley & Sons, Inc. p. 84-85.

126. Keane, A. and P. Nair, *Computational approaches for aerospace design: The pursuit of excellence*. 2005: Wiley.

127. Sigman, K. and R.W. Wolff, *A Review of regenerative processes.* SIAM Review, 1993. **35**(2): p. 269-288.

128. Little, J.D.C., *A proof for the queuing formula: L= λW.* Operations Research, 1961. **9**(3): p. 383-387.

VITA

Varaporn Pukcarnon was born on the 20th January 1981 in Bangkok, Thailand. She completed her undergraduate studies in academic year 2001 with the Bachelor’s Degree of Engineering the 2nd class honor at the Department of Industrial Engineering, Chulalongkorn University. Consecutively, she enrolled in the Master’s Degree program at the Department of Industrial Engineering, Chulalongkorn University. She was a research assistant under the research unit named “ROM” (Resources and Operations Management). She graduated the Master’s Degree in academic year 2003 with cumulative grade point average at 3.88. After that, she started working as a supply planner in the department of supply chain and logistics management, Siam Michelin Co.,Ltd. In 2005, she came back to the “ROM” and worked as a researcher to strengthen her research ability and academic experience. In 2008, she enrolled in the Doctor’s Degree program at the Department of Industrial Engineering, Chulalongkorn University. During the doctoral study, she was granted the Scholarship from the Graduate School, Chulalongkorn University to commemorate the 72nd anniversary of his Majesty King Bhumibala Aduladeja.