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AN OVERBOOKING MODEL FOR AIR CARGO INDUSTRY

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A Thesis Submitted in Partial Fulfillment of the Requirements
for the Degree of Master of Engineering Program in Industrial Engineering
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การเปิดให้สำรองเกินเป็นหนึ่งในวิธีที่สำคัญของการบริหารรายได้ การเปิดให้สำรองเกินนี้เป็นการขายสินค้า หรือบริการที่มากกว่าที่มี เนื่องจากว่ามีโอกาสที่ลูกค้าบางรายจะยกเลิก ในปัญหาการเปิดให้สำรองเกินของการขนส่งสินค้าอากาศยานนี้มีความซับซ้อนมากกว่าการเปิดให้สำรองเกินของการขนส่งทั่วไปเนื่องจากลักษณะปัญหาที่เป็นแบบสองมิติ ซึ่งได้แก่ น้ำหนัก และปริมาตรของสินค้า

งานวิจัยนี้นำเสนอตัวแบบทางคณิตศาสตร์ของการขนส่งสินค้าอากาศยานที่เป็นแบบสองมิติเพื่อที่จะหาระดับการเปิดให้สำรองเกินที่เหมาะสมที่สุด โดยวัตถุประสงค์ของตัวแบบทางคณิตศาสตร์นี้คือการทำให้ค่าใช้จ่ายรวมมีค่าน้อยที่สุด ความต้องการในการจองสินค้า และอัตราการมาของสินค้าเป็นตัวแปรสุ่มที่ทราบการกระจายของทั้งสองตัวแปร งานวิจัยนี้นำเสนอวิธีการคิดค่าใช้จ่ายซึ่งไม่เคยมีงานวิจัยใดศึกษามาก่อน งานวิจัยนี้ได้แบ่งงานออกเป็นสี่ส่วน ส่วนแรกคือการสร้างตัวแบบทางคณิตศาสตร์ตัวเต็ม ส่วนที่สองคือการทดสอบการคำนวณของตัวแบบเพื่อที่จะสังเกตผลกระทบของปัจจัยที่สำคัญกับระดับการเปิดให้สำรองเกินที่เหมาะสมที่สุด ส่วนที่สามคือการลดความซับซ้อนของตัวแบบโดยมีสองวิธีที่นำเสนอคือ 1) วิธีการวิเคราะห์การถดถอยที่มีปฏิสัมพันธ์ระหว่างตัวแปร และ 2) การทำนายอย่างง่าย และส่วนสุดท้ายของงานวิจัยนี้คือการเปรียบเทียบผลที่ได้จากตัวแบบทั้งสาม และบ่งชี้สถานการณ์ที่เหมาะสมในการใช้งานแต่ละตัวแบบ

ผลของงานวิจัยนี้พบว่าระดับการเปิดให้สำรองเกินที่เหมาะสมที่ได้จากวิธีการวิเคราะห์การถดถอยนี้ใกล้เคียงกับค่าระดับการเปิดให้สำรองเกินที่เหมาะสมที่สุดมาก โดยมีค่า R-sq(adj) สูงถึง 98.3% ถึงแม้การทำนายอย่างง่ายจะใช้ตัวแปรน้อยที่สุด และให้คำตอบได้ง่ายที่สุด แต่ว่าการใช้วิธีนี้ในการทำนายระดับการเปิดให้สำรองเกินที่เหมาะสมที่สุดนี้ไม่สามารถให้คำตอบที่เหมาะสมได้เสมอไป ยกตัวอย่างเช่น การทำนายอย่างง่ายจะทำนายค่าระดับการเปิดสำรองเกินใกล้เคียงกับระดับการเปิดสำรองเกินที่เหมาะสมที่สุดเมื่อสัดส่วนระหว่างค่าใช้จ่ายส่วนเกิน และค่าเสียโอกาสมีค่าต่ำเนื่องจากว่าการทำนายอย่างง่ายนี้ไม่คำนึงถึงปัจจัยนี้

ภาควิชา วิศวกรรมอุตสาหการ ลายมือชื่อนิสิต ....

สาขาวิชา วิศวกรรมอุตสาหการ ลายมือชื่อ อ.ที่ปรึกษาหลัก ....

ปีการศึกษา 2557

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KEYWORDS: OVERBOOKING / AIR CARGO / REVENUE MANAGEMENT / SPOILAGE COST / OFFLOADING COST / OPTIMAL OVERBOOKING LEVEL

ANUPONG WANNAKRAIROT: AN OVERBOOKING MODEL FOR AIR CARGO INDUSTRY. ADVISOR: ASST. PROF. NARAGAIN PHUMCHUSRI, Ph.D., 137 pp.

Overbooking is one of the most important techniques in revenue management which sells goods or services in excess of the available capacity because there is a possibility that some customers might cancel or not show up. In air cargo overbooking problem, overbooking model is more complex as compared to others because of the two-dimensional characteristic, i.e., weight and volume of the booking requests.

This study presents two-dimensional air cargo overbooking models to find the optimal overbooking level with an objective of minimizing the total cost. Booking request and show-up rate are random variables with known distributions. This research proposes a new way of modeling the total cost that has not yet been presented before. There are four main parts in this research. First, the full mathematical model is formulated. Second, computational experiments are used to observe the impacts of important factors on the optimal overbooking level. Third, the other two models are proposed to reduce the complexity of the full model: 1) a regression model with interactions and 2) a naive method model. Last, the results of these three models are compared to identify which situation is suitable for each.

The results showed that the optimal overbooking level obtained from the simplified regression model is close to the optimal overbooking level obtained from the full model (R-sq(adj) = 98.3%). The naive method is the simplest method presented in this study; however, it does not always give the appropriate optimal overbooking level. For example, the naive method predicts the optimal overbooking level close to the real optimal overbooking level when the ratio between the offloading and spoilage costs are low because it does not consider this factor.

Department:	Industrial Engineering	Student's Signature	....
Field of Study:	Industrial Engineering	Advisor's Signature	....
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CHAPTER I
INTRODUCTION

1.1 Revenue Management

Revenue management (RM) is about analyzing the situation to predict the behavior of the customers, which is uncertain, and provide the right amount of products or services for the customers in order to maximize the profit. The main objective of revenue management is to sell the right products or services to the right customers at the right time with the right price in the right condition, in which the customers are satisfied. The importance of applying revenue management is understanding the behavior of customers and customers' perception about the product prices and values, defining prices, market positioning, and making the product available for every customer group. [1]

Business owners inevitably has to decide challenging and important decisions of what to sell, whom to be sold, when to sell, and what prices to sell. Revenue management uses strategies that analyze data to decide the answers to these decisions. The general procedure of revenue management consists of data collecting strategy, and operations research, in which the mathematical models and expressions are used in order to find the optimal decisions; furthermore, as stated before, understanding the customers' behavior, and cooperating with sales. A person or an organization that utilizes revenue management in order to increase their profit has to be analytical, rational, and detailed; moreover, they have to be able to think strategically. [2]

In the past, before revenue management was created, British Airlines tried adjusting fare of some of the seats. They controlled some of the seats for early bookings in order to encourage and provoke the customers' eagerness to book those seats that would have been empty [3]. Furthermore, American Airlines adopted a practice which focused on maximizing profit by analyzing and controlling the seats. After American Airlines had adopted this practice, the company's profit were increased more than 40% in the next year [1].

Revenue management has been utilized mainly on perishable products, for example, airlines' passenger seats, and hotel rooms. To explain further, in the case of airlines' passenger seats, the capacity of this problem situation is the passenger seats. The number of passenger seats is fixed. Every seat has its own price and can generate revenue for the airlines if the plane hasn't taken off yet. Once the airplane takes off, the left over capacity or seats can no longer generate revenue for the airlines. Moreover, the fuel consumption, the number of staffs, and other costs are still the same. In other words, the airlines' passenger seats that are empty or not sold are perished. Therefore, airlines should have done something in order to increase the opportunity to sell those seats and make more profit instead of wasting those left over seats by doing nothing. For example, airlines can give a discount to customers who book the seats very early, or alternatively, they can also give a discount to those who buy the tickets in the last minute before taking off. By doing that, the airlines can increase their profit. Even though the profit gained may be low, as compared to full price of the seats, it is better than flying with empty seats, gaining no profit. In the same way, hotel rooms are also classified as perishable products because if the hotel rooms cannot be sold in the right period, in which length-of-stay is also considered, the hotels lose their profit; therefore, hotel industry can also utilized revenue management techniques in order to increase their profit, e.g., appropriate pricing for different types and sizes hotel rooms.

1.2 Air Transportation

There are two types of air transportation, i.e., passenger and air cargo transportations. In this thesis, only air cargo transportation is considered. The trend for air cargo transportation weight is shown in Figure I-1. The data is obtained from the U.S. Department of Transportation [4].

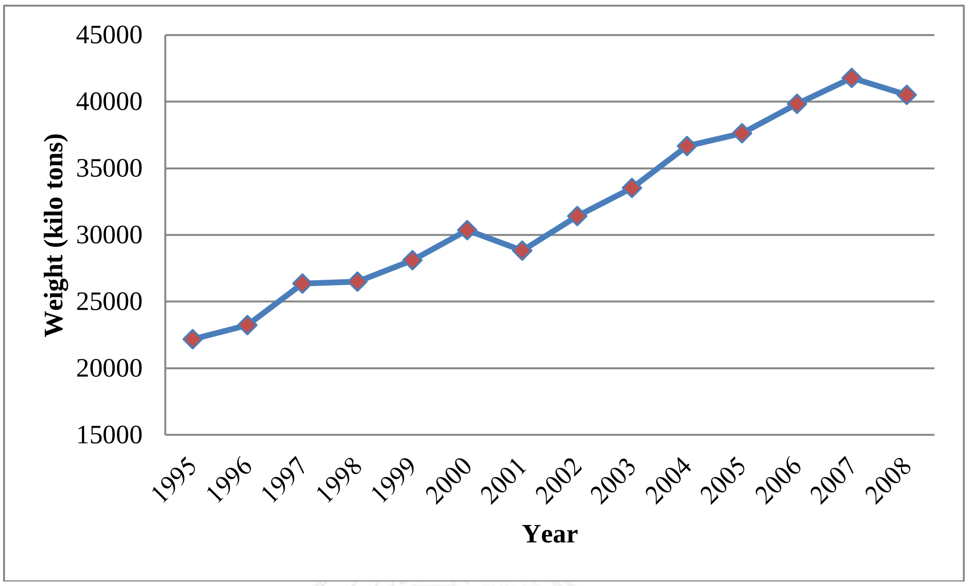

Figure I-1 Worldwide air cargo transportation weight: 1995-2008

As can be observed in Figure I-1, The trend is upward and there is a high possibility that it continues to increase in the future. Moreover, with convenient technology, fast and reliable internet connection nowadays, buying and selling products online is more convenient and reliable over time. This causes the increase of the demand for air cargo transportation.

Air cargo capacities can be managed by revenue management since they can be considered as perishable products. As mentioned before, there are many techniques within revenue management strategy; thus, there are also many techniques that an airline can utilize. Overbooking is one of the revenue management techniques that has also been utilized in many industries including the air cargo transportation. In this research, overbooking technique is applied to the air cargo transportation problem in order to increase the profit.

1.3 Overbooking

Overbooking is one of the most effective processes in a revenue management strategy, this includes the air cargo revenue management strategy. Overbooking is the process of selling and providing more products or services than the actual availability of that products or services because a company anticipates that there must be some customers who will not show up without notifying in advance, or some customers who will cancel their booking requests later.

When consider the air cargo overbooking problems, many airlines utilize this method of overbooking in order to increase their revenue by letting their customers book more volume and weight than the actual capacities that can be loaded onto the airplane. This main objective is to maximize the profit; or to minimize the opportunity lost cost caused by cancellations and no-shows, and offloading cost caused by too many show-up booking requests.

There are two costs that often have to be taken into account when speaking of overbooking, i.e., spoilage cost and offloading cost. Generally, spoilage cost occurs when there are some capacities spoiled due to the cancellations and no-shows, and the overestimated capacities. Spoilage cost is the opportunity lost cost as a result of the lack of ability to fully utilize the capacities. Spoilage cost can be considered as the revenue that airlines should have received if the customers had showed up and made the capacities fully utilized. On the other hand, offloading cost occurs when there are not enough capacities for serving the show-up customers. This results in finding alternative ways of providing the products or services to the show-up customers, in this case, for example, finding alternative airlines, organizations, or third-party companies to transport customers' cargos. This usually costs more than the original rates that airlines receive from their customers. Additionally, airlines may face more problematic situations and receive more penalty cost if they cannot find those alternative ways of serving their customers. These extra costs are the offloading cost for the airlines. In the same way of spoilage cost, offloading cost can be considered as the revenue of an alternative airline, organization, or a company. As far as this topic goes, it can be said that overbooking is a technique for an air cargo revenue management system which should not be ignored as it can help airlines maximize their revenue or minimize their total cost. If the overbooking level is set too low or it has been ignored and some of the booking requests made over this overbooking level have to be rejected, cancellations and no-shows may occur at the day of departure and cause the show-up booking requests to be lower than the actual capacity. Then, subsequently, this is the cause of the spoilage cost occurrence. Conversely, if the overbooking level is set too high and the demand of the booking requests is also high, the booking requests are over accepted and cause the show-up booking requests to be higher than the actual capacity. Then, subsequently, this is the cause of the offloading cost occurrence.

Overbooking has been a very popular technique in revenue management strategy because it can help companies increase their profit without having to invest much, if any, or with no costs. As a result, many airlines have adopted overbooking to passenger and air cargo transportation extensively. However, there is less research on air cargo overbooking as compared to passenger overbooking. Kasilingam [5] explained the difference between the complexities of passenger and air cargo transportation model. Also, Kasilingam [5] presented an overbooking model under discrete stochastic capacity. Kasilingam [6], whose model was continued from Kasilingam [5], expanded the model and presented an air cargo overbooking model under continuous stochastic capacity. Although Kasilingam [5] clearly stated that air cargo overbooking model was more complicated, especially with the multidimensional characteristic, the models presented by Kasilingam [5, 6] were still one-dimensional. Moreover, the models presented by Kasilingam [5, 6] did not consider the demand for booking requests; infinite number of booking requests assumption must be made which was contrary to real life situations. In reality, the number of booking requests is random and not always approaching infinity. Popescu [7] adopted the existing overbooking model to the newly estimated weight show-up rate and compared to the normal show-up rate. Gui, Gong, and Cheng [8] formulated two-dimensional air cargo overbooking model with an objective of maximizing profit. Luo, akanyıldırım, and Kasilingam [9] formulated two two-dimensional air cargo overbooking models, and compared between with the two models, i.e., Rectangle Acceptance Region Boundary (RAB) model and Curve Acceptance Region Boundary (CAB) model.

According to the aforementioned statement above, there is little research on air cargo overbooking model. Also, airline databases do not keep the record some of the necessary data for the model, e.g., the rejected booking requests [9], as the airline only keeps the record of the actual show-up booking requests. Therefore, formulating the overbooking model which requires all the data of the booking requests is not practical. Nevertheless, formulating the overbooking model with the censored booking requests data is too complicated. Consequently, most research on air cargo overbooking model still consider all of the booking request data. Furthermore, although most research on air cargo overbooking model describe the differences in complexities between air cargo and passenger, especially the difference in the dimensions, most research still formulate one-dimensional air cargo overbooking model. It should be noted that there are two main objective functions in regard to formulating a mathematical function for an overbooking model, i.e., minimizing the total cost, and maximizing the profit. Minimizing the total cost implement the idea of newsvendor model, and mainly concerns only the two costs, i.e., spoilage cost and offloading cost, and minimizing them. Maximizing the profit mainly concerns the actual revenue and actual costs, and try to find the optimal point in order to achieve the maximum the overall profit. To this extent, most research also adopt cost formulation concept from the newsvendor model whose spoilage cost occurs when there is too much capacity compared to the demand. Thus, the research in this thesis adopts the basic idea of the newsvendor costs in which two costs are considered, i.e., spoilage cost and offloading cost. The objective of this air cargo overbooking model is to minimize the total cost in order to find the optimal overbooking level. However, unlike previous studies, this thesis considers combination of the two costs in a different way.

1.4 Thesis Objective

This thesis has an objective of developing a mathematical model for finding the optimal air cargo overbooking level limit that can minimize the total cost..

1.5 Thesis Scope

1. This thesis is mainly concerned with the overbooking of air cargo transportation problem and not the passenger transportation problem.

2. This thesis has an assumption of knowing all the variables and parameters that are important to calculating the mathematical model.

3. Costs considered in this thesis are divided into two main costs. They are:

3.1. Spoilage cost: the opportunity lost cost which can be considered as the revenue that an airline should have received if the booking requests had not been rejected.

3.2. Offloading cost: the cost which an airline has to pay to alternative organizations in order to deliver promises of sending the customers' cargos when there are too many show-up booking requests.

4. This research involves three main processes. They are:

4.1. Developing a mathematical model for a two-dimensional air cargo overbooking problem.

4.2. Examining the effects of model parameters (such as the ratio between offloading and spoilage cost, capacities, and booking request level) on the optimal overbooking level via computational experiments.

4.3. Performing analysis to identify simplified methods for the two-dimensional air cargo overbooking model which can still deliver appropriate results, without having to run the full model for the optimal solutions.

5. This research considers the density of the booking requests one value at a time as it still makes sense for the behavior of the problem, and it simplifies the mathematical model.

6. Show-up rate is considered as a discrete random variable. The value of the show-up rate is in closed, bounded interval [0, 1].

1.6 Thesis Contribution

1. A mathematical model for the total cost of an air cargo transportation overbooking problem.

2. Help decide the optimal overbooking level limit for airlines whose characteristics and behavior match the assumptions of the overbooking model in this research.

3. This research can be a basis for future research extension as there is little research on this topic, especially on two-dimensional air cargo overbooking model. Therefore, this research is expected to add perspective and idea to two-dimensional air cargo overbooking topic.

4. The results in this research can also be applied to other industries whose characteristics are similar to air cargo transportation.

1.7 Thesis Methodology

To achieve insight information in this research and understand the two-dimensional air cargo overbooking the most, the process in this research is split into three phases as shown in Figure I-2.

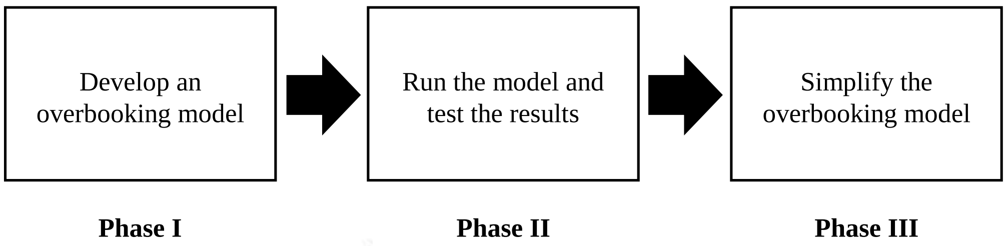

Figure I-2 Three phases for research methodology

The first phase (Phase I) is about developing a mathematical model for finding the optimal overbooking level. This mathematical model adopts the basic idea of newsvendor model and extends it further. The model has an objective function of minimizing the total cost, in which two costs are formulated, i.e., spoilage cost and offloading cost. The developed overbooking model is two-dimensional and has more than one random variables. Furthermore, spoilage cost in this model in the literature has never been formulated this way before. Therefore, this phase is the most important, difficult, and complex. There are two main processes in this phase. First, the two-dimensional overbooking model is formulated to match the parameters and the characteristics of the air cargo problem. The model developed in this phase is an unconstrained optimization. Second, in order to obtain the solution, which is the optimal overbooking level, to the first step, the method of finding the optimal overbooking level must be developed. In some overbooking models, this can be done by differentiating; but due to the complexity of this model, this cannot be achieved by the same method. Thus, writing a code file to be run through Matlab program has to be done in order to find the optimal overbooking level.

The second phase (Phase II) is running and testing all the random variables and parameters on the results. The effects of each random variables and parameters are observed in this phase. This phase gives the basic idea of how the optimal overbooking level is affected by the random variables and parameters. This phase involves running the program through different factors and graphing, mostly, the optimal overbooking level versus those factors. In addition, this phase also involves performing design of experiments (DOE) and analysis of variance (ANOVA) in order to know which factors affect the results significantly.

Lastly, the third phase (Phase III) is simplifying the two-dimensional overbooking model to make it easier and more practical to be applied in real life. However, simplifying the two-dimensional overbooking model by reducing the variables or alternating the form cannot be done easily as the original model is very complex and there are many random variables and parameters. Therefore, prediction methods have been utilized instead in order to give appropriate overbooking levels without having to run the full model. There are two methods of prediction presented in this research. First, naïve method, this method uses common sense to predict the optimal overbooking level and requires the least parameters. Second, after the design of experiments and analysis of variance has been accomplished, regression analysis is used to predict the optimal overbooking level according to only the significantly affected factors. Although, there are many types of regression available, this research adopts stepwise multiple regression with interactions method.

As the basic procedure of the thesis process are concerned, these three phases are carried out by the process flow as shown in Figure I-3.

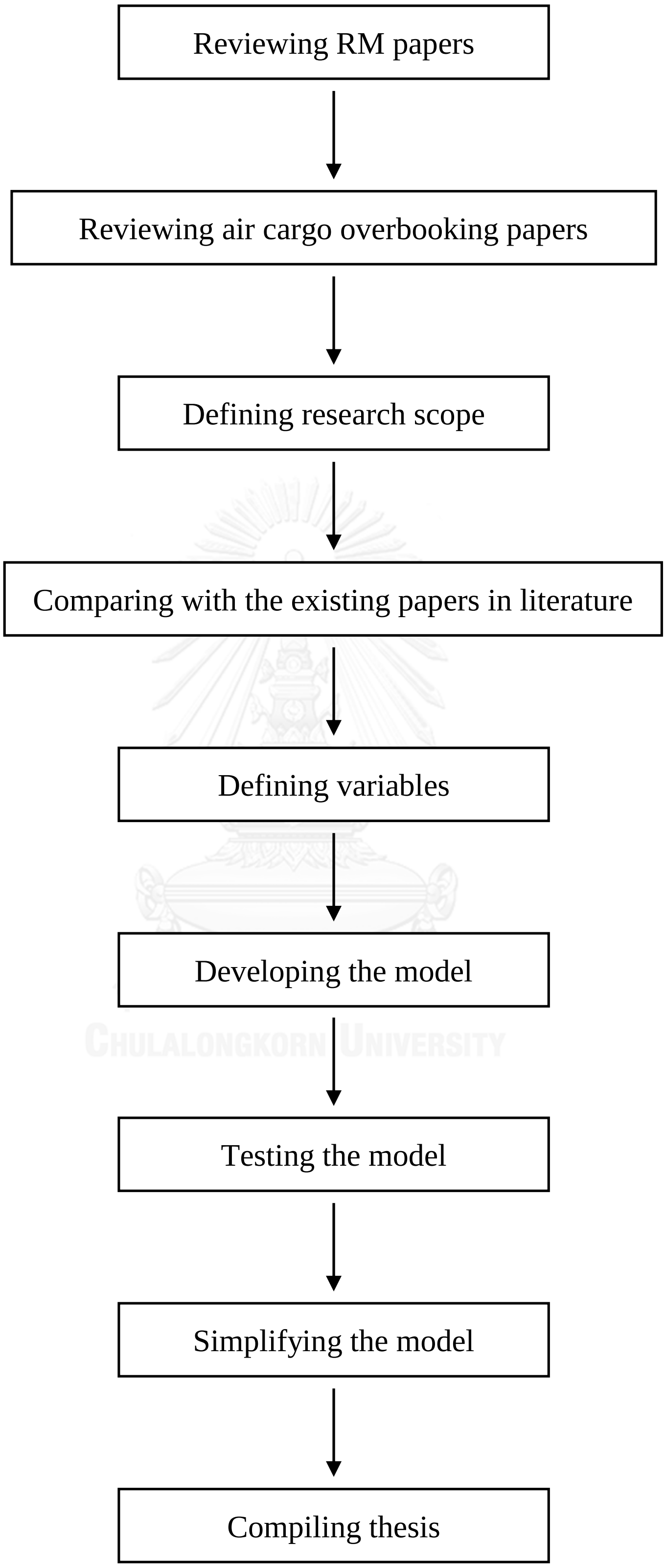

Figure I-3 Theses process flow

1) Reviewing RM papers

This is the first step of understanding the overall picture, and, consequently, the overbooking. This step gives information on what is being studied and the scope that is being covered. This thesis is a part in revenue management; thus knowing the overall image and what position this research stands is necessary.

2) Reviewing air cargo overbooking papers

In order to know and understand what has been done and what has not been covered before, reviewing others' papers on this topic is required. Air cargo overbooking problem related papers have to be studied in this step. Not only to know that what others have done, but also to be educated more on this topic. After knowing and understanding others' papers, research scope can then be defined.

3) Defining research scope

Defining the scope gives information about what is done and covered in this thesis and what is not. Without defining appropriate scope of the research, the results can be difficult to get and can be misleading to readers.

4) Analyzing and comparing with the existing papers in literature

This procedure has to be done with the intention to understand in detail about existing research on this topic. What is covered and excluded in each existing paper, advantages and disadvantages of each overbooking model, and how their models are formulated must be known in order to analyze and compare them properly. If this step were ignored, it would be impossible to thoroughly understand overall picture of this topic; and, subsequently, it would be impossible to know the gaps of others' research and what to be completed in this thesis.

5) Defining variables and parameters

In order to successfully formulate the two-dimensional air cargo overbooking model, a decision variable, important random variables and parameters must be defined properly. Failing to do so may result in an unexplainable, unusable, and useless overbooking model, if it can still be formulated.

6) Developing the model

After defining all the variables and parameters, the two-dimensional air cargo overbooking model can then be developed. This procedure is very crucial in this research. Operations research knowledge is applied along with the understanding in modeling costs for the two-dimensional air cargo overbooking model obtained from both the studied papers and original ideas. The overall two-dimensional air cargo overbooking model comprises more than one equation because the model is too complicated so that it cannot be combined into one equation. Each equation represents spoilage cost and offloading cost in a specific condition. Each condition depends on the combination of the variables and parameters. Therefore, as said before, due to the complexity of this model, obtaining the optimal overbooking level is not easy, and it has to be coded and run through Matlab program.

7) Testing the model

There are three processes after the two-dimensional air cargo overbooking model has been developed. The first thing that has to be done is testing the model in order to know whether or not it is working and correct. If the model is not working or correct, it will have to be corrected. After knowing that the model is correct and can give the optimal overbooking level precisely, the second thing is testing the effects of the variables and parameters. By alternating the variables and parameters one by one, we can observe the effect of each variables and parameters on the optimal overbooking level, in what direction they influence the optimal overbooking level, and try to understand the reason behind it. Lastly, design of experiments and analysis of variance are used in order to know which variables and parameters have impact on the results significantly.

8) Simplifying the model

In order to put the two-dimensional air cargo overbooking model into practice more easily, the model has to be simplified. Naïve method and regression method are presented in this thesis in order to predict the value of the optimal overbooking level. Common sense is used along with the knowledge of the two-dimensional characteristic in the naïve method. It requires less variables and parameters. On the other hand, regression method requires more variables and parameters in order to predict the optimal overbooking level. Necessary main effects and interactions obtained from the analysis of variance are considered in this regression model. Finally, stepwise method is used in order to obtain the appropriate final regression model with interactions.

9) Writing thesis

Last step is bringing all the works done and writing them into the thesis. Background knowledge, understandings, insights, results, and conclusions are gathered, organized, and written in this thesis.

CHAPTER II
LITERATURE REVIEW

This chapter explains background knowledge related to revenue management and goes through previous research in the area of air cargo overbooking problem. This thesis considers air cargo overbooking problem, which is a part of revenue management, air transportation, and overbooking technique. First, overall revenue management is reviewed. Second, passenger and air cargo transportations are explained in the air transportation section and are compared the difference between them. Last, the overbooking technique which is used in the air cargo transportation industry is reviewed. Others' research on this very specific topic are explained in detail in this section.

2.1 Revenue Management

Revenue management has become very popular nowadays. This section gives a brief review of basic knowledge of revenue management.

2.1.1 Main Categories for Revenue Management

Revenue management can be categorized and divided into four main categories. There are:

1) **Pricing**

This category deals with defining prices, pricing strategy, and improving pricing tactics. The main objective of pricing strategy is to match the product price to the product value. Product price can be adjusted according to situations. In some situations, product price has to be high in order to match the product value position, whereas, in some situations, product price may be reduced in order to compete with competitors. Therefore, appropriate pricing strategy can help companies increase their profit [10]. Pricing strategies must be dynamic in order to adapt and follow the market, demand, customers' behavior, and customers' perception.

2) Inventory Control

Revenue management considers inventory controlling and allocating to optimize the available space or capacity. Before inventory control becomes a problem, a company successfully increased its demand, which may be caused by discounting the product price. The lower the prices of the products, the higher the demand and the market share. This is great and the company will continuously increase its profit as long as the product selling price is higher than its cost. However, the more the demand, the harder the inventory control can be managed. There are sunk costs in keeping inventory for each product. Matching each product to its demand and allocating the right amount of capacity for each product are essential in inventory controlling. Also, there might be some cases where demand for the product is high, but there is a possibility that the cancellations may occur, e.g., passenger seats or hotel rooms, overbooking strategy may be applied in order to maximize the profit. Overbooking increases sales volume when cancellations may occur [3].

3) Marketing

There are many marketing strategy including advertisement, price promotion, etc. Advertisement concerns mainly with customers' perception about brand, quality, and value. Also, good advertisement makes customers accustom to the brand and would like to buy the product. Price promotion temporarily reduces the product price in some periods in order to increase sales volume. Revenue management concerns how customers react to the price promotion available in order to balance between an increase in the sales volume and a decrease in the selling price which causes the decrease in profitability. An effective price promotion increases revenue and profit when there is hesitation in customers' mind about whether or not to buy the product. In addition, in cases of products or services are in a form of long-term contracts between the seller or service provider and the customer, price promotions draw attention the customer who will commit to the contracts and generate profit for the company in a long period of time. Revenue management technique is proved to be useful in establishing appropriate promotions in order to maximizing the profit [11].

4) Distribution and Channels

There are many channels in which revenue management strategy will help companies increase their profit via those channels. Different channels serve different customers with different buying powers and sensitivities. To give an example, customer who shops online is more sensitive to price more than customer who shops at a local store. Also, the same products sold in many very different places often vary in prices. In addition, different channels have different costs. Product distribution through channels is another decision making problem that should not be overlooked in revenue management strategy. When businesses or companies face with a problem of distributing the product through multiple channels, revenue management can be a great technique to help calculate the amount to be distributed through each channel with appropriate promotions and discounts in order to distribute the products as much as they can, and gain more profit [11].

2.1.2 Basic Revenue Management Techniques

Revenue management strategy comprises of many techniques. There are some basic process in revenue management that can be divided into different categories.

1) **Data Collection**

As stated before, revenue management strategy is a data-driven and analytical process. This means that revenue management strategy greatly requires data in order to process and analyze afterwards. In other words, this can be said that data is the most important element in revenue management system [1]. Without appropriate data, a revenue management system cannot process and improve in other aspects in order to generate the revenue growth. Good data contains important, accurate, and precise information. This important information is useful in making many decisions and putting into practice. On the other hand, if this information is not collected, the data collected is not necessary, or the data collected is not accurate and precise, a company may not be able to correctly analyze the data and cannot put decisions into practice, or if they are put into practice, it may cause unpredictable results. Revenue management system needs to collect historical data as much as it can. These data contains information about customers, prices, promotions, demand, revenue, inventory, and other important influential factors. Furthermore, good data should contain these necessary information of the competitors'.

2) **Division and Segmentation**

Dividing customers into segments plays a vital role in revenue management strategy. One of the revenue management techniques involves defining price for each customer group in order to maximize the revenue. Succeeding in segmentation depends on the appropriateness of how the customers are divided into groups or market segments. The ability to divide customers with similar price sensitivities into groups is the key in this process.

3) Forecasting

Revenue management system needs various important data in advance. This can be achieved through forecasting. Relevant data such as demand for each customer segment, demand for the next month, etc. Estimating these data methods alone cannot succeed in forecasting process. It depends on the collected data and how these data is analyzed. The effectiveness of the revenue management is affected by the performance in forecasting the data. Forecasting process is absolutely essential in revenue management. It takes time to build up, maintain, put into practice, and improve. There are more than one forecast methods and types. For example, demand forecast, it forecasts the demand and the product sold or being booked in the near future, whereas another form of forecast, price-based forecast, tries to obtain such demand in a function of price or promotion. Developing these forecast techniques is a foundation of success in revenue management. Optimization can be applied after knowing necessary data obtained by forecasting.

4) Optimization

After obtaining the forecast data, which inform a company the expected behavior that the customers are tending to do in the near future, optimization is utilized in order to know what and how a company should response to that behavior of customers. Optimization is often considered as the most important role in a revenue management system. Optimization is about evaluating and calculating many decisions on how the products are sold. Such decisions include the amount of products, amount of advance booking requests, their prices, whom to sell, etc. [1]. Optimization performance is directly influenced by the collected data and the performance of the forecasts. There are two main elements in the optimization process in order to accomplish finding the answers to these decisions and generate the highest possible profit. First, select what factors should be calculated in objective function such as prices, sales, or costs; and decide what objective function to optimize such as minimizing the total costs or maximizing the profit. Second, a company has to decide which optimization method to employ. For example, linear programming is an optimization technique which is widely utilized in order to obtain the optimal decisions from a set of variables with linear relationships. There are plenty of optimization techniques and tools can be useful in increasing the companies' profit by deciding the optimal decisions on product prices, inventory levels, products to be sold, customers to sell, and other necessary decisions.

5) Dynamic Evaluating

A company must continuously evaluate and follow up their performance, prices, products, and processes with the aim of unceasing maximum profit growth. With inconsistent, unpredictable, dynamic market, a revenue management system has to re-evaluate the factors in order to catch up with the dynamic behavior of the market. Therefore, an effective revenue management system must be re-evaluated, adjusted, and improved consistently as the market changes and evolves [1].

2.1.3 Adopting Industries

With the abilities of the revenue management that help a company achieve a massive revenue growth stated above, many companies that has sought ways to increase their profit and expand their own companies have a great desire for revenue management system. Therefore, there have been many industries adopting the principle of the revenue management strategy and apply such revenue management system to their own products and services in order to increase their revenue growth.

1) Bank Industry

Banks have many products to offer to wide variety groups of customers. They have been adopting customer segmentation, pricing policy in order to calculate the most appropriate prices, products, and groups of customers. This basically involves processing and analyzing massive amount of data, and calculating the highest interest rates without losing customer or the rates that a customer agrees to pay [12].

2) Media and Communication Industry

Price promotion based companies such as media companies and telecom companies draw customers by offering discounted prices and, often, long term commitment plans. More often than not, these price plans will increase profit by rising their promotion prices later. Businesses in this type of industry usually encounter dynamically changing in demand of different customer groups, e.g., business group, personal customer, or VIP customer. Understanding the behavior of the market is a difficult task, but revenue management can be a great help to these companies: forecasting the demand, and offering the optimal promotions to the right groups of customers in order to continuously gain the profit in a long term [13].

3) Retailers and Distributors

Many retailers and distributors encounter complicated problems which involve managing a large amount of stock keeping units (SKUs) of various products. Each product has a different life cycle. Retailers and distributors need to consider channels to be distributed, inventory management, and other important factors. Applying a revenue management system has been proved to be useful to these retailers and distributors in analyzing the promotions and making contract agreements in order to minimize the cost and maximize the profit [14].

4) Medication Industry

For example, hospitals, face a problem of variation and instability of the demand. The demand for medication products and services depends on what day in a week, e.g., weekdays or weekends; moreover, time of the day is also taken into account. More patients may have more time in weekends, which results in the increase in demand in the weekend period. On the other hand, in weekdays, the majority of people have to work; therefore, the demand in the middle of weekday period is lower. This principle is also applied to time of the day. This causes the employees to be free in some period of time, whereas, in another period of time, the employees are overworking. As the demand fluctuates between weekdays and weekends, and also time of the day, a hospital may try to properly adjust products and service levels according to the behavior of the demand. Revenue management techniques will help a hospital prepare its products and services adequately for serving its patients and satisfy them [15].

5) Hotel Industry

Hotels have to face the problem of the stochastic behavior of the customers in each day. The demand in each day consists of walk-in customers and booked customers. In the case of the customers who have booked before, there is a possibility that some customers may not cancel the bookings and some might not even show up without informing in advance. This may cause spoilage costs for the hotel. Moreover, hotels have to take pricing policy into account to cover all of the costs, e.g., maintenance costs, overhead costs, electricity cost, and employees' salary. Also, other than covering all of the costs, pricing strategy has to be appropriate with the service quality and the satisfaction of the customers. Pricing too high often cause the drop in customers' willingness to pay and, consequently, losing customers. Different customers have different levels of satisfaction to the service quality, and willingness to pay to the same particular service. This variation is the cause of the fluctuation of the customers to the change of the price [16]. In addition, hotels have to be concerned with the length of stay, customer segmentation which is providing different prices and quality room types for accommodating the needs of the different groups of customers. Hotels have to calculate appropriate promotions in order to stimulate the demand. Hotel room and service are one of many products and services that revenue management can help manage in order to increase the profit for hotels. One of popular revenue management techniques for hotel industry is overbooking [16], a technique that is useful for reserving large amount of demand when cancellations may be presented, which fits the customers' behavior of the hotels.

6) Other Industries

In addition to the industries indicated above, there are many more industries out there that utilize the practice of revenue management. Other industries such as car renting company, gas storage and transmission, electricity industry, and casinos [17]. The power and ability of increasing companies' profit of revenue management makes it widely recognize and employed.

2.2 Air Transportation

As aforementioned statement described above, revenue management is a technique used to deal with the stochastic behavior of customers in order to improve organizations' profit. Also, revenue management is used with perishable products and services, this includes air transportation industries.

2.2.1 Passenger Transportation

Airlines have to face a wide range of customers such as individual tourists, tour groups, business travelers, or vacationers. Segmentation between types of customers is crucial as there are differences between these customers. For example, customers travelling for business reason tend to be price-insensitive compared to vacationers; and require more flexibility of the schedule, bookings, and cancellations [17]. In addition to the customer segmentation, airlines involve pricing for different customers, itinerary control, itinerary pricing, and more.

As mentioned before, many industries have adopted revenue management strategy in order to maximize their revenue, this includes passenger airlines. Passenger airlines who desire the revenue growth can utilize one or many revenue management techniques, including overbooking technique. Overbooking technique can be used with passenger seats in order to sell more passenger seats and gain more revenue.

2.2.2 Air Cargo Transportation

Same as hotel rooms or passenger seats, air cargo capacities are considered as perishable products; thus, revenue management can be applied. There have been air cargo transportation for more than ten years. Air cargo industries have grown continuously for the past ten years due to the growth of economy. Air cargo transportation becomes very important for cargos that need short transportation time and high reliability. Figure I-1shows the trend for the total air cargo transportation worldwide. As seen in Figure I-1, the air cargo transportation weight in 2008 has almost doubled compared to that in 1995. Even though the movement of the graph may rise and fall at some points, it is ascending and there is a high possibility that it continues to grow steadily in the future. Moreover, with convenient technology, fast and reliable internet connection nowadays, buying and selling products online is more convenient and reliable over time. This causes the increase of the demand for air cargo transportation.

As said, air cargo transportation companies can utilize the power of revenue management in order to increase their profit; thus, overbooking technique can be applied to any air cargo transportation companies who wish to increase their revenue without having to invest much.

2.2.3 Passenger vs. Air Cargo Transportation

Although air cargo transportation is similar to passenger transportation, air cargo revenue management cannot be replaced by passenger revenue management [5] or revenue management used in ordinary industries, e.g., hotel, hospital. This is because most of their products and services are one-dimensional; meaning that it only concerns one factor, e.g., passenger airlines concern only the available seats of the airplane; size and weight of the passengers do not matter. In addition, the capacity, which is one-dimensional, is known and discrete. On the other hand, air cargo is at least two-dimensional; meaning that it concerns more than one factor, i.e., air cargo revenue management has to take weight and volume of the cargos into account. Besides, weight and volume capacities of the airplane may be unknown since the available weight capacity is the weight left over from the total weight capacity after subtracting weight of the passengers, passengers' luggage, fuel, etc., and the volume capacity is the volume left over from the total volume capacity after subtracting volume of the passengers' luggage, etc. Therefore, these considerations add more complexities to air cargo revenue management. The difference between passenger transportation and air cargo transportation can be summarized in Table II-1.

Table II-1: Comparison of the air transportation type summary

Air Transportation Type	
Passenger	**Air Cargo**
One-dimensional	Two-dimensional
Discrete number of seats	Continuous capacities
Different sizes don't matter	Different sizes matter
No stacking loss	There is stacking loss
Known capacity	Unknown capacities (in some cases)

2.3 Air Cargo Overbooking

Although the overbooking technique has been widely recognized and utilized, there are few articles and research in this area. This section explains and compares others' research related to air cargo overbooking problem.

Kasilingam [5] described the difference between passenger yield management (PYM) and air cargo revenue management (CRM). Reference [5] stated that there were four important differences between PYM and CRM, i.e., uncertain capacity, three-dimensional capacity, itinerary control, and allotments. These differences cause CRM to be more complex than PYM. Kasilingam [5] also proposed an overbooking mathematical model under discrete stochastic capacity. The model presented in this paper was one-dimensional despite the differences in the complexities. In addition, the number of booking requests and its probability density function were not considered as a part of the model. This means this model needs another assumption, i.e., the number of booking requests is high enough, unlimited, or approaching infinity. This assumption is not reasonable for all models and situations in reality.

Kasilingam [6] expanded the air cargo overbooking model from Kasilingam [5] by alternating the capacity from being discrete to continuous random variable. Although Kasilingam [6] expanded the air cargo overbooking model from Kasilingam [5], and have considered one of the random variables differently, the model and examples presented by Kasilingam [6] still needs the same assumption as that of the model presented by Kasilingam [5], i.e., the number of booking requests is high enough, unlimited, or approaching infinity. Also, the model was still one-dimensional which was contrary to the characteristics of the air cargo overbooking problem in real life situation.

Popescu [7] adopted the existing overbooking model to the newly estimated weight show-up rate and compared to the normal show-up rate. Also, the air cargo overbooking model used in Popescu [7] was run separately for weight and volume. That being said, the overbooking model in Popescu [7] was still equivalent to and can be considered as two separate one-dimensional models. In addition, similar to Kasilingam [5] and Kasilingam [6], the booking requests were not considered and assumed to be in excess of the available capacity at all times; that is, the booking requests were assumed to be approaching infinity. As mentioned before, such assumption is not valid for some overbooking models and situations.

Gui, Gong, and Cheng [8] formulated two-dimensional air cargo overbooking model with an objective of maximizing profit. Actual revenue and actual occurring cost, which is offloading cost, are considered in Gui, Gong, and Cheng [8]'s overbooking model. One advantage of the profit maximizing method is it requires only the revenue and offloading cost, and not spoilage cost. Spoilage cost is not the actual cost that occurs, instead, it is only an opportunity lost cost. Modeling with the actual cost that occurs is more straightforward.

Luo, akanyıldırım, and Kasilingam [9] presented a one-dimensional air cargo overbooking model and two two-dimensional air cargo overbooking models. Luo, akanyıldırım, and Kasilingam [9] also compared performance of the models with each other. All models presented in Luo, akanyıldırım, and Kasilingam [9] have the same objective of minimizing the total cost.

As mentioned above, there are few articles and research in air cargo overbooking problem, and despite the differences in characteristics of CRM and PYM, most of them still formulate one-dimensional air cargo overbooking model. The multidimensional characteristic has been surprisingly neglected by most research. Furthermore, although some of the research may formulate two-dimensional air cargo overbooking model, it is important to address that there are assumptions and cost modeling concept that may be not reasonable in some situations and can be improved. Therefore, the air cargo overbooking model presented in this paper is formulated two-dimensionally, which adopts the basic idea of the newsvendor costs in which two costs are considered, i.e., spoilage cost and offloading cost. The objective of this air cargo overbooking model is to minimize the total cost in order to find the optimal overbooking level. However, this research improves some of the assumptions, and models the two costs in a different way that has not yet been considered before.

CHAPTER III
OVERBOOKING MODEL

This chapter is associated to the first phase of thesis methodology. It is very crucial to give knowledge of how the overbooking model is formulated. The overbooking model is used throughout in this thesis, which is done in phase I and is important to understand. In order to formulate the two-dimensional air cargo overbooking model, there are subjects that need to be considered and completed. The subjects and details are explained below.

3.1 Problem Description

This section gives a basic knowledge of a two-dimensional overbooking problem. First, two important parameters need to be known, i.e., volume and weight capacities of the airplane. When the booking period starts, volume and weight of the booking requests rise up. Then, one of them will max out at one of the capacities if no overbooking level limit is applied. This is because the density of the booking requests is not equal to the ratio between weight and volume capacities. The same principle applies to the case that if there is an overbooking level limit, one of the booking request volume and weight will max out at the overbooking level limit as the booking requests that are over the overbooking level are rejected. The show-up rate of the booking requests is applied afterwards. If the booking request show-up rate is high, it means that the cancellations and no-shows are low; and the show-up booking requests will turn out to be high. On the other hand, if the booking request show-up rate is low, it means that the cancellations and no-shows are high; and the show-up booking requests will turn out to be low. The overall booking request level is multiplied by the booking request show-up rate in order to obtain the show-up booking request level. If the show-up booking requests are lower than the capacities when the booking requests have already been rejected, the left over available capacities are the spoiled capacities. If the show-up booking requests are higher than the capacities, the booking requests that cannot be loaded on the airplane are the offloaded booking requests. The show-up booking request level when there is an overbooking level and there is no overbooking level are illustrated in Figure III-1 and Figure III-2 below.

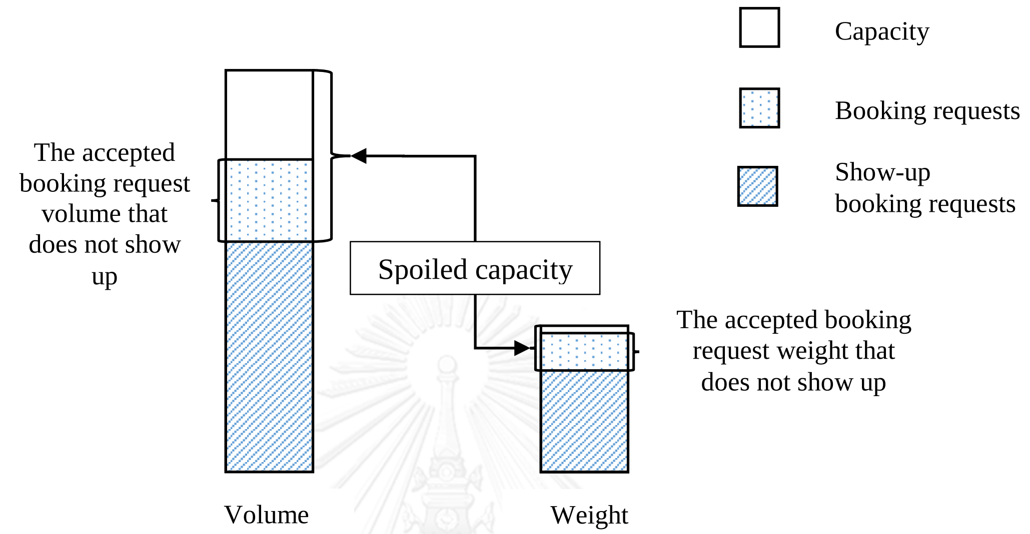

Figure III-1 The booking requests when no overbooking level is applied

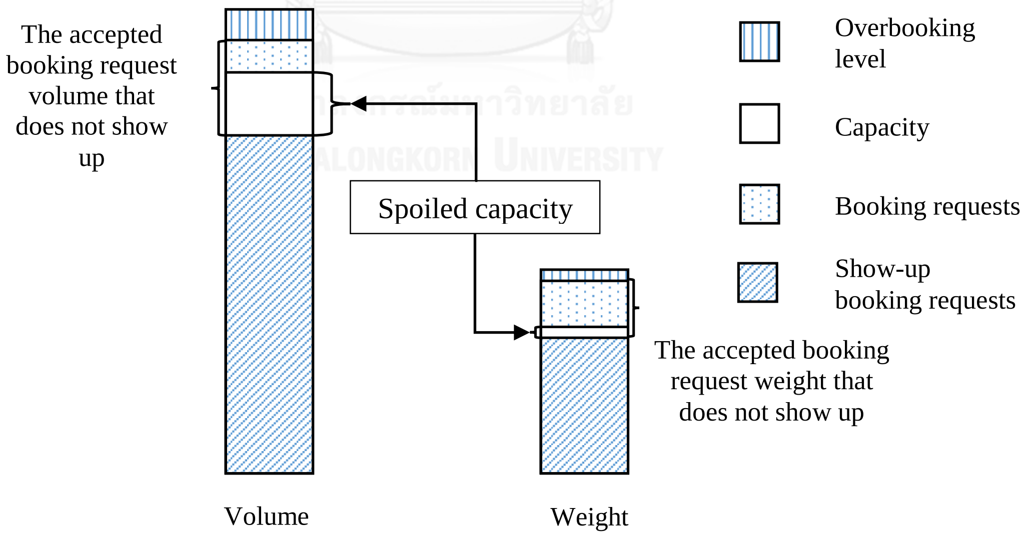

Figure III-2 The booking requests when overbooking level is applied

It can be seen that when no overbooking level is applied, the show-up booking requests are lower as the maximum booking requests that can be made is restricted by one of the capacities. On the other hand, if there is an overbooking level, the maximum booking requests that can be made is increased and is limited by the overbooking level, which is higher than the former capacities. This makes the show-up booking request level be higher than that if the overbooking level is not applied. Consequently, the possibility that the spoiled capacities happen is reduced by just applying an overbooking level. Spoilage cost occurs from the spoiled capacities; therefore, reducing the possibility that the spoiled capacities happen results in the decreased spoilage cost. However, nothing comes for free in life. Even though the possibility that the spoiled capacities happen can be reduced, the possibility that the offloaded booking requests occur can also be increased by an overbooking level. By defining an overbooking level, the booking request volume and weight can exceed the available capacities; and there is a chance that the show-up booking request volume and weight cannot be loaded on the airplane at the departure date.

3.2 Defining Variables and Parameters

There are four types of variables and parameters. First, they are parameters and constants. Second, they are random variables. Third, they are decision variables. Lastly, they are other variables which can be obtained by calculating from the preceding variables. Those variables and parameters are listed below.

3.2.1 Parameters and Constants

C : The total capacity that an airplane can carry

C_v : Volume capacity of an airplane

C_w : Weight capacity of an airplane

d : Standard density, the density that is used to convert the volume unit into weight unit

θ : Density of the booking requests, the ratio between the booking request weight and volume.

a : Spoilage cost per one unit of chargeable weight

b : Offloading cost per one unit of chargeable weight

3.2.2 Random Variables

p : Show-up rate with a probability mass function of $g_p(p_i)$

B : Booking request level with a probability density function of $f(B)$

3.2.3 Decision Variables

Q : Tested overbooking level

Q_v : Volume of an overbooking level

Q_w : Weight of an overbooking level

$Q*$: Optimal overbooking level

Q_v^* : Volume of the optimal overbooking level

Q_w^* : Weight of the optimal overbooking level

3.2.4 Other Input Variables

B_v : Volume of the booking requests

B_w : Weight of the booking requests

B_{sv} : Volume of the show-up booking requests

B_{sw} : Weight of the show-up booking requests

3.2.5 Cost Functions

CS : Spoilage cost

CO : Offloading cost

TC : Total cost

TC^* : The minimum total cost

The important variable and parameter notation declared above can be summarized in Table III-1 through Table III-5 below:

Table III-1: Summary of parameter notations

Category	Notation	Meaning
Capacity	C, C_v, C_w	Total, volume, weight capacity
Density	d, θ	Standard, booking request density
Cost per unit	a, b	Spoilage, offloading cost per unit

Table III-2: Summary of main input random variables and their probability function notations

Category	Notation	Meaning
Random variable	p	Show-up rate
	B	Booking request level

Probability function	$g_p(p_i)$	Probability mass function of p
	$f(B)$	Probability density function of B

Table III-3: Summary of other input variables notations

Category	Notation	Meaning	Derived from
Booking request	B_v, B_w	Volume, weight of the booking request	B, θ
	B_{sv}, B_{sw}	Show-up volume, weight of the booking request	B, p, θ

Table III-4: Summary of cost functions

Category	Notation	Meaning	Derived from
Cost	CS, CO, TC	Spoilage, offloading, total cost	All of the above variables with a Q
	TC^*	The minimum total cost	All of the above variables with a Q^*

Table III-5: Summary of decision variable notations

Category	Notation	Meaning
Tested variable	Q	An overbooking level
Decision variable	Q^*	The optimal overbooking level
Volume and weight overbooking level	Q_v, Q_w	Volume, weight of an overbooking level
	Q_v, Q_w	Volume, weight of the optimal overbooking level

3.3 Assumptions

For the developed two-dimensional air cargo overbooking model to be valid, there are required assumptions that are needed to be taken to account.

3.3.1 Divisibility

Volume and weight of the booking requests can be divided into small pieces. This assumption makes stacking loss, which is the capacity loss due to the shape differences of solid packages, is nearly zero. In other words, the capacities of the airplane can be used 100% efficiency. The capacities can be fully utilized.

3.3.2 Cost Calculation

Both spoilage cost and offloading cost are calculated in the same way of calculating the revenue. In addition, they are mainly calculated from chargeable weight.

3.3.3 Same Density for Booking Request

This assumption assumes that all booking requests have the same density. This includes show-up booking requests, the booking requests that can be loaded on the airplane, and the offloaded booking requests. This may seem unrealistic; but if the overall booking requests are considered, the average density for all booking requests is valid to be applied. All the booking requests are now use the average density to represent the overall density of the booking requests. Therefore, it can be assumed that all the booking requests have the same density.

3.3.4 Same Show-up Rate for Volume and Weight

Show-up rate of the booking requests is used with both volume and weight of the booking requests. This, again, may seem unrealistic. However, each booking request consists of both volume and weight; thus, considering the overall booking requests, if the weight of the booking requests does not show up, the volume of the booking requests must not show up too. Therefore, volume and weight can be assumed to use the same show-up rate.

3.3.5 Same Show-up Rate for Booking Request

In the same way as the assumption above, the same show-up rate is used with all the booking requests. This includes the approved booking requests and the rejected booking requests. The same principle as the above assumptions is applied. Show-up rate is the probability that the booking requests show up; therefore, considering that the overall booking requests have a show-up rate, the expected probability that the approved and rejected booking requests show up would be the same. Thus this assumption is perfectly valid.

3.3.6 Other Costs Negligibility

Costs that are being considered in this air cargo overbooking model are only spoilage cost and offloading cost. Other costs such as maintenance cost, or other processing costs are neglected. It can be implied that implementing only overbooking technique costs nothing; thus, there is no reason to include other costs in the model as these costs normally occur in the process with or without the overbooking technique. For that reason, other costs such those examples given above can be omitted in this model.

3.4 Show-up Rate

In this research, the show-up rate of the booking requests is a discrete random variable with the value between 0 and 1. This simplifies the process of formulating the two-dimensional air cargo overbooking model as there are already too many variables, parameters, and conditions. Using show-up rate as a continuous random variable complicates the model and the model cannot be integrated or give results, whereas alternating this random variable to be discrete makes the problem a lot easier. The model can then be developed, integrated, and can give answers. Not only defining show-up rate in this way simplifies the model, but it is also still realistic and practical. The show-up rate interval of 0 and 1 can be divided into multiple values. There is no limit for the numbers of the show-up rate values. It can be as many as two-thousand values or it can be as few as only one value. The more the numbers of the show-up rate values, the closer it is compared to a continuous show-up rate. Furthermore, the probability mass function, which is used with a discrete random variable, is easier to define compared to the probability density function, which is used with a continuous random variable. The probability mass function of the show-up rate is defined by the variable $g_p(p_i)$ which gives the probability of each given show-up rate value p_i. Moreover, this probability mass function can be defined as an equation or a specific probability for each show-up rate value p_i, i.e., the empirical probability. The probability mass function can be expressed as a mathematical expression as follow:

$$g_p(p_i) = \Pr(p = p_i) \qquad ;\ i = 1, 2, 3, \ldots, n$$

where n is the total number of show-up rate values.

Also, the summation of all possibilities of the show-up rate probabilities must equal to 1. Thus, we have:

$$\sum_{i=1}^{n} g_p(p_i) = \sum_{i=1}^{n} \Pr(p = p_i) = 1$$

Considering the show-up rate of the booking requests in this way, the expected total cost with uncertain show-up rates can be obtained by the normal method of calculating the conditional expected value. This is done by calculating the total cost for each show-up rate p_i, multiplying each total cost with the corresponding show-up rate, and summation of those terms. The equation of this method can be summarized as:

$$E\left[TC\right]=\sum_{i=1}^{n}\left[E\left[TC\mid p=p_i\right]\Pr\left(p=p_i\right)\right] \tag{3.1}$$

The show-up rate is also involved in finding the show-up booking request level. Not only the show-up rate of the booking requests is involved, but the booking request level and the overbooking level are also included. The show-up booking request level depends on the value of the booking request level and the overbooking level. If the booking request level is lower than the overbooking level, the show-up booking request level will be calculated from the booking request level. On the other hand, if the overbooking level is lower than the booking request level, the show-up booking request level will be calculated from the overbooking level because there is rejection and causes the final accepted booking requests to be equal to the overbooking level. From the explanations above, without considering the density, the show-up booking request level can be written in terms of the booking request level, the overbooking level, and the show-up rate as follow:

$$B_{sv}=\min\left(B_v,Q_v\right)p \tag{3.2}$$

$$B_{sw}=\min\left(B_w,Q_w\right)p \tag{3.3}$$

The $\min()$ function returns the minimum value between the booking request level and the overbooking level for both volume and weight.

3.5 Spoilage and Offloading Costs

With the purpose of formulating the two-dimensional air cargo overbooking model, it is essential that we have to know what are to be calculated in order to obtain the optimal overbooking level. We agree to use costs in order to calculate and decide the optimal overbooking level, and present the cost model in this research. Spoilage cost and offloading cost comprise the costs that are being considered in the model presented in this thesis. The two costs are outlined and explained in more detail below in this section.

To explain the general idea of how the two costs are calculated in the model, the meanings, and when they occur; the two-dimensional characteristic is not yet considered in this section. Without considering the two-dimensional characteristic of the actual model can help with understanding the general idea better. The two-dimensional characteristic of the model will be included in the later section in the thesis.

3.5.1 Spoilage Cost

Spoilage cost is mainly presented when there is a lack of the cargos compared to the capacities which is actually capable of carrying more cargos at the moment. If, at the moment, there are more cargos to be loaded in that unoccupied capacities, it will generate more revenue to the airline. That revenue lost is the spoilage cost. In other words, spoilage cost can be thought of an opportunity lost cost or the missing revenue for the airline. Nevertheless, in this research, the spoilage cost is considered in a different way of other research. In this research, the spoilage cost depends on the booking request level, show-up rate, and overbooking level. This can be broken up into three cases as described below:

1) The Booking Request Level is Lower than The Available Capacity

When the booking request level is lower than the available capacity of an airplane, airline does not have to reject any booking requests. Even if the airline had set the overbooking level to be higher, there would have been no difference at all as the booking requests are still the same, below the capacity level. As there is no rejection, an opportunity lost cost does not occur. Then, there is no spoilage cost in this case.

2) The Booking Request Level is Higher than The Available Capacity, but Lower than The Overbooking Level

In this second case, even though the booking request level is higher than the available capacity of an airplane, the booking request level is still lower than the overbooking level. Same explanation as case 1, when the booking request level is lower than the overbooking level, the airline does not have to reject any of the booking requests. Even if the airline had set the overbooking level to be higher, there would have been no difference at all as the booking requests are still the same, below the overbooking level. Therefore, same as case 1, there is no spoilage cost in this case.

As seen in the first two cases, the spoilage cost occurrence idea is different from other research in this field. Other research only consider the show-up booking request level compared to the available capacity of the airplane; if the show-up booking request level is lower than the available capacity of the airplane, then there is always spoilage cost presented no matter if the airline rejects some of the booking requests or not.

3) The Booking Request Level is Higher than The Overbooking Level

When the booking request level is higher than the overbooking level, airline has to reject some of the booking requests. When there is rejection, there is some circumstances that there is spoilage cost presented. There are three situations that can happen at the date of departure.

In the next three main situations, the show-up rate of the booking requests is one of the keys in defining what situation to be in. Not only the show-up rate of the booking requests, overall booking requests and the overbooking level also play a vital role in classifying the situation. However, the situations are described in terms of the show-up rate of the booking requests as it is more logical and easier to understand.

1. The show-up booking request level is equal to the capacity

This is the situation that, in reality, is not likely to occur. Anyway, taking this into consideration as there are always some possibilities of the situation occurrence. When the show-up booking request level is equal to the capacity, there are no rooms left for other booking requests, even if there are more, because the capacity is fully utilized. When this occurs, spoilage cost is not presented.

2. The show-up booking request level is more than the capacity

In this situation, because of the high show-up rate, i.e., the cancellations and no-shows are low, so the show-up booking request level ends up with a high amount. When the show-up booking request level is more than the capacity, all spaces of the available capacity are occupied. The left over number of booking requests cannot be loaded on the airplane. As a result of the fully utilized capacity, the spoilage does not occur.

3. The show-up booking request level is less than the capacity

In this situation, the show-up rate of the booking requests is low, i.e., there is a high amount of the cancellations and no-shows; thus, this causes the show-up booking requests to be lower than the available capacity of the airplane. When this occurs, there are spaces left in the capacity of the airplane, and they are spoiled. This is the situation where the airline loses its revenue. It goes without saying that if the airline set the overbooking level to be higher, the airline receives more revenue and gains more profit. The airline can gain more profit mainly because there are some booking requests, which the airline have rejected, that actually can be loaded on the airplane due to the cancellations and no-shows of the accepted booking requests. In conclusion, in this situation, spoilage cost is presented.

In this research, spoilage cost calculation needs comparing between the left over available capacity and the expected show-up rejected booking request level; and taking which one that is lower into account. As said before, the spoilage cost is the revenue that an airline should have received. Therefore, spoilage cost is presented when the airline unknowingly rejects some of the booking requests that can still be loaded on the airplane. If the rejected booking requests expectedly show up less than the left over available capacity at the departure date, spoilage cost is calculated from the expected show-up rejected booking request level. This is because if the airline had not rejected those booking requests, the airline should have received the revenue from all those rejected booking requests as there are available spaces in the capacity left for those booking requests. On the other hand, if the rejected booking requests expectedly show up more than the left over available capacity at the departure date, spoilage cost is, then, calculated from the left over spaces available in the capacity. This is because the airplane can only carry just the left over spaces available capacity no matter how many more the rejected booking requests would show up. All the above explanation can be summarized into two subcases in mathematical expressions for the spoilage cost as follow:

3.1. The expected show-up rejected booking request level is less than the left over available capacity:

$$CS = a(B-Q)p \tag{3.4}$$

where the $(B-Q)$ term is the rejected booking request level as the booking request level is more than the overbooking level. Thus, multiplying the show-up rate to this term makes the $(B-Q)p$ term to be the expected show-up rejected booking request level. The $(B-Q)p$ term has to be multiplied with a, which is the spoilage cost per unit, and that makes the total term to be spoilage cost for all the rejected booking requests.

3.2. The expected show-up rejected booking request level is more than the left over available capacity:

$$CS = a\left(C - Qp\right) \tag{3.5}$$

where the Qp term is the show-up booking request level after the booking requests have been approved or rejected. As the booking request level is higher than the overbooking level, the show-up booking request level is calculated from the overbooking level. Thus, the term $\left(C - Qp\right)$ is the left over available capacity. The $\left(C - Qp\right)$ term has to be multiplied with a, which is the spoilage cost per unit, and that makes the total term to be spoilage cost for all the rejected booking requests.

In this research, spoilage cost is considered in a very different way from spoilage cost in others' research. It depends on many situations in which many variables and parameters are compared. Furthermore, spoilage cost, in this research, occurs when there is a revenue that an airline should have received. This is more complex and more realistic than just plainly considering the spoilage cost is always presented when the show-up booking requests are less than the capacity. The above mathematical expressions for spoilage cost in two subcases can be merged into one mathematical expression using $\min()$ function as follow:

$$CS = a\min\left(\left(B - Q\right)p, C - Qp\right) \tag{3.6}$$

The $\min()$ function returns the minimum value inside it. In this mathematical function, the values of $\left(B - Q\right)p$ and $C - Qp$ are compared.

3.5.2 Offloading Cost

Offloading cost occurs when the show-up booking request level is more than the available capacity of an airplane. The booking requests that cannot be loaded on the airplane are the offloaded booking requests. The airline needs to deliver the promises made with its customers that the cargos are sent according to the date. The airline will have to pay a third-party deliverer in order to transport its offloaded booking requests. The cost which is caused by the offloaded booking requests is the offloading cost. In the same way as the spoilage cost, the cases are divided into three cases. The cases are described below:

1) The Booking Request Level is Lower than The Available Capacity

In this case, as there are no offloaded booking requests, there is no offloading cost. The show-up rate of the booking requests is not taken into account in this case because the booking request level is already lower than the available capacity.

2) The Booking Request Level is Higher than The Available Capacity, but Lower than The Overbooking Level

There may be offloading cost in this case, it depends on the show-up rate of the booking requests. Unlike previous case, the show-up rate of the booking requests is taken into account. As mentioned before, although there are other factors that affect the show-up booking request level, describing the show-up booking requests in terms of the show-up rate is easier to understand. Thus, the situations are divided into three situations according to the show-up rate of the booking requests.

1. The show-up booking request level is equal to the capacity.

As said before, this situation is unlikely to happen in reality. Although the booking request level is higher than the available capacity, the show-up booking request level is equal to the capacity at the departure date due to the cancellations and no-shows. There are no offloaded booking requests in this situation; therefore, there is no offloading cost.

2. The show-up booking request level is less than the capacity

There is a high amount of the cancellations and no-shows in this situation; that is, the show-up rate of the booking requests is low. This causes the show-up booking request level to be lower than the available capacity. When this occurs, there are no offloaded booking requests as the show-up booking request level is lower than the capacity. In the same way as the situation above, when there are no offloaded booking requests presented, there is no offloading cost.

3. The show-up booking request level is more than the capacity

Due to the high show-up rate, i.e., the cancellations and no-shows are low, the show-up booking request level is high. This causes the show-up booking requests to exceed the available capacity. There are some show-up booking requests that cannot be loaded on the airplane, i.e., offloaded booking requests. As there are offloaded booking requests, there is always offloading cost. The offloading cost in this case can be written in a mathematical form as shown below:

$$CO = b\left(Bp - C\right)$$

where Bp is the show-up booking requests after the acceptance and rejection of the booking requests have already occurred. The overall booking request level is being used in this case as the booking request level is lower than the overbooking level, and there is no rejection. Therefore, $\left(Bp - C\right)$ is the show-up booking requests that exceed the available capacity, and cannot be loaded on the airplane. In other words, $(Bp-C)$ is the offloaded booking requests. The $(Bp-C)$ term has to be multiplied with b, which is the offloading cost per unit, and that makes the total term to be offloading cost for all the offloaded booking requests.

3) The Booking Request Level is Higher than The Overbooking Level

There are some situations that offloading cost occurs in this case, it depends on the show-up rate of the booking requests. Same as the previous case, the show-up rate of the booking requests is taken into account. As mentioned before, although there are other factors that affect the show-up booking request level, describing the show-up booking requests in terms of the show-up rate is easier to understand. Thus, the situations are divided into three situations according to the show-up rate of the booking requests in order to clarify in which situations the offloading cost occurs.

1. The show-up booking request level is equal to the capacity.

As said before, this situation is unlikely to happen in reality. Although the booking request level is higher than the available capacity, the show-up booking request level is equal to the capacity at the departure date due to the cancellations and no-shows. There are no offloaded booking requests in this situation; therefore, there is no offloading cost.

2. The show-up booking request level is less than the capacity

There is a high amount of the cancellations and no-shows in this situation, i.e., the show-up rate of the booking requests is low. This causes the show-up booking request level to be lower than the available capacity. When this occurs, there are no offloaded booking requests as the show-up booking request level is lower than the capacity. In the same way as the situation above, when there are no offloaded booking requests presented, there is no offloading cost.

3. The show-up booking request level is more than the capacity

Due to the high show-up rate, i.e., the cancellations and no-shows are low, the show-up booking request level is high. This causes the show-up booking requests to exceed the available capacity. There are some show-up booking requests that cannot be loaded on the airplane, i.e., offloaded booking requests. As there are offloaded booking requests, there is always offloading cost. The offloading cost in this case can be written in a mathematical form as shown below:

$$CO = b(Qp - C)$$

where the Qp term is the show-up booking requests after the acceptance and rejection of the overall booking requests have occurred. The overbooking level, Q is being used in this case as the overall booking request level is higher than the overbooking level, and there are some booking requests that are rejected. Therefore, the accepted booking requests are represented by the overbooking level, Q. Thus, the $(Qp - C)$ term is the show-up booking requests that exceed the available capacity. In other words, in this case, $(Qp - C)$ is the offloaded booking requests. The $(Qp - C)$ term has to be multiplied with b, which is the offloading cost per unit, and that makes the total term to be offloading cost for all the offloaded booking requests.

In order to aggregate two mathematical expressions for the offloading cost shown above, once again, the $\min()$ function is used. The offloading cost can be summarized into one mathematical expression as shown below:

$$CO = b(\min(B, Q)\, p - C) \tag{3.7}$$

In the offloading cost function shown above, the $\min()$ function compares value between the overall booking request level, B, and the overbooking level, Q; and returns the minimum value between them.

3.5.3 Conclusion

This section gives a basic idea and summarizes all the cases elaborated above into two simple cases where spoilage cost and offloading cost occur.

1) Spoilage Cost

Spoilage cost occurs when there is rejection and the show-up booking request level is lower than the available capacity. It also depends on the left over available capacity and the expected show-up rejected booking request level. The occurrence of spoilage cost can be illustrated in a diagram in order to give visual understandings as shown in Figure III-3.

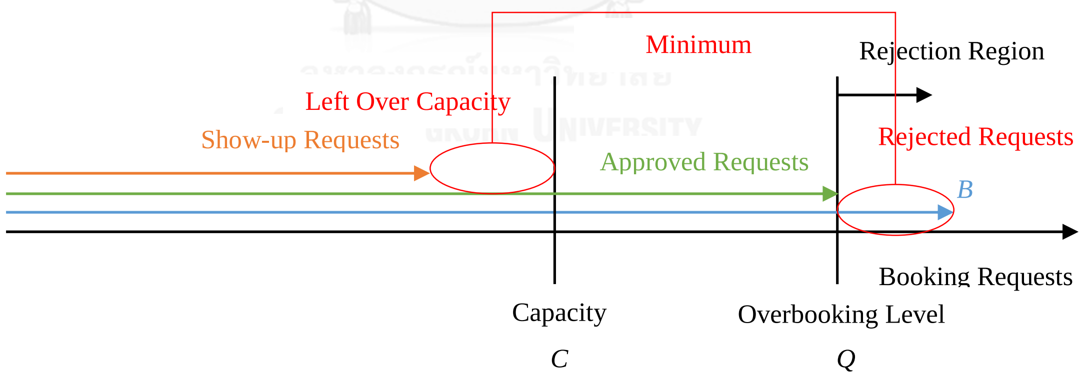

Figure III-3 Spoilage cost occurrence idea

The spoilage cost needs comparing between the left over available capacity and the expected show-up rejected booking requests to formulate the spoilage cost function. Without considering the two-dimensionality, the spoilage cost in this research can be explained in a mathematical expression summarized as:

$$CS = a\min\left(\left(B - Q\right)p, C - Qp\right)$$

where the $\min()$ function returns the minimum value between the expected show-up rejected booking requests and the left over available capacity.

2) Offloading Cost

Offloading cost occurs when there are higher level of show-up booking requests compared to the available capacity. The show-up booking request level depends on the booking request level, overbooking level, and show-up rate. The booking request level and overbooking level needs to be compared before multiplying by the show-up rate. To visualize the basic idea of occurrence of the offloading cost, it is illustrated as a chart as shown in Figure III-4 and Figure III-5.

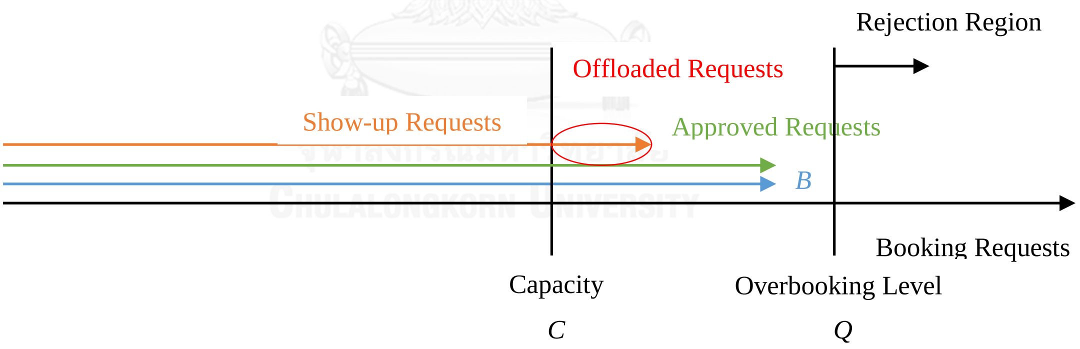

Figure III-4 Offloading cost occurrence when the booking request level is less than the overbooking level

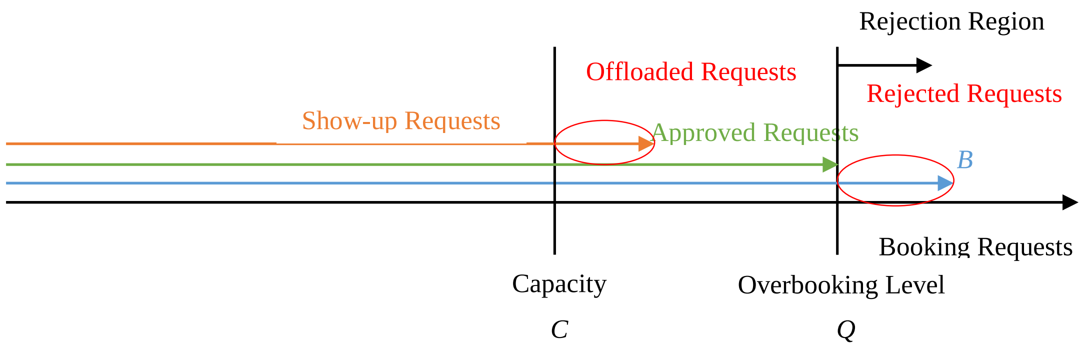

Figure III-5 Offloading cost occurrence when the booking request level is more than the overbooking level

The offloading cost calculation needs comparing between the booking request level and the overbooking level. In the cost function, it needs the minimum value between them. The offloading cost can be expressed in a mathematical form summarized as:

$$CO = b\left(\min\left(B, Q\right) p - C\right)$$

where the $\min()$ function returns the minimum value between the booking request level and the overbooking level.

3.6 Booking Request Density

In reality, the booking request level for air cargo is two-dimensional. This means that there are volume and weight of the booking requests. In practice, it is difficult to know the correlation between volume and weight of the booking requests. The joint probability density function of the booking request volume and weight are not simple to be obtained. Therefore, in this research, the booking request level, which represents the overall magnitude of volume and weight of the booking requests, is considered. Along with the booking request level, the density of the booking requests is used. The booking request density is used instead of the joint probability density function in order to define the correlation between the booking request volume and weight. In this way, the number of random variables and parameters can be vastly reduced. Moreover, defining variables and parameters is simpler. When the booking request density is changed, the correlation between volume and weight is also changed. The values and probabilities of both booking request volume and weight can be defined by just the booking request level and booking request density. The use of the booking request level and the density is shown in Figure III-6.

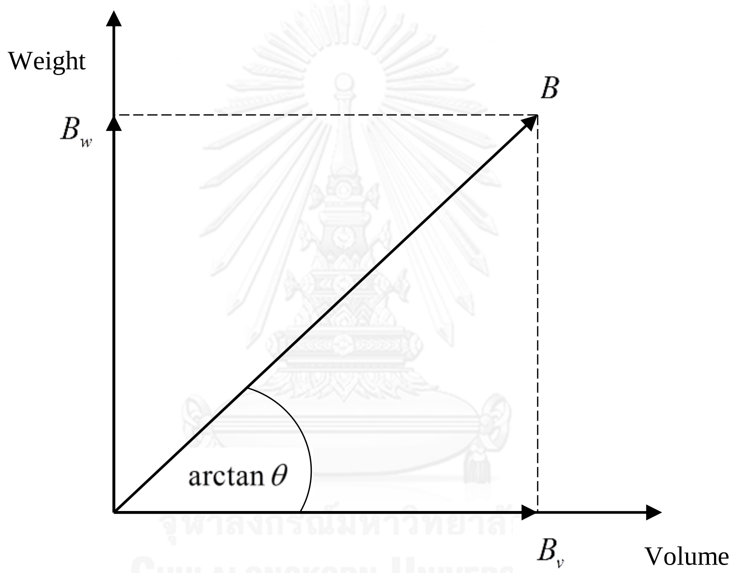

Figure III-6 The booking request level with the density in two dimensions

Without the booking request density, the probability density function of the booking request volume and weight will have to be separately defined. The independence status of the booking request volume and weight is unknown, so is the joint probability density function. Furthermore, if the magnitude of the overall booking requests has to be changed, it is simpler to change just one random variable than to define many random variables separately. There are relationships between variables and parameters related to the booking request density, and they can be presented as mathematical expressions.

$$\theta = \frac{B_w}{B_v} \tag{3.8}$$

$$B_v = \frac{B}{\sqrt{1+\theta^2}} \tag{3.9}$$

$$B_w = \frac{B\theta}{\sqrt{1+\theta^2}} \tag{3.10}$$

$$Q_v = \frac{Q}{\sqrt{1+\theta^2}} \tag{3.11}$$

$$Q_w = \frac{Q\theta}{\sqrt{1+\theta^2}} \tag{3.12}$$

In the same way, the show-up booking request volume and weight can also be presented as mathematical expressions in terms of the booking request density. However, the show-up booking request volume and weight have to be calculated from the accepted booking requests, which is in a function of the minimum value between the booking request level and the overbooking level. A comparison between the booking request level and the overbooking level has to be taken place before multiplying with the booking request density. Therefore, the show-up booking request volume and weight can be obtained as follow:

$$B_{sv} = \frac{\min(B,Q)\,p}{\sqrt{1+\theta^2}} \tag{3.13}$$

$$B_{sw} = \frac{\min(B,Q)\,p\theta}{\sqrt{1+\theta^2}} \tag{3.14}$$

3.7 Standard Density

For the two-dimensional characteristic of the air cargo transportation problem, the impact of both the booking request volume and weight is a very important issue to be realized. Spoilage cost and offloading cost calculations have to be considered which factor is more significant: volume, or weight of the booking request.

Spoilage cost can be considered as the revenue that an airline should have receive if the airline had not unintentionally rejected the booking requests that can be loaded on the airplane. Therefore, spoilage cost can be calculated in the same way as the revenue calculation.

In the same way as spoilage cost, offloading cost can be considered as the revenue of another airline which is hired in order to transport the offloaded booking requests.

Revenue of an airline has to be calculated from the chargeable weight. Chargeable weight considers the maximum value between the actual weight of the booking requests and the volumetric weight, which is transformed from the volume of the booking requests, in order for the airline to receive the most revenue. Calculation in this way involves the standard density in order to compare between volume and weight of the booking requests whether volume or weight generates the maximum revenue. Calculating the revenue for the two-dimensional characteristic of the air cargo transportation can be summarized as:

$$\text{Revenue} = a \max\left(Volume \times d, Weight\right) \tag{3.15}$$

where $Volume$ is the volume of the booking requests that can be loaded on the airplane; and $Weight$ is the weight of the booking requests that can be loaded on the airplane.

3.8 Cost Formulating

This section shows and explains how the two-dimensional air cargo overbooking model is formulated. Costs calculation has to consider the two-dimensional characteristics of the air cargo transportation problem. As stated before, volume and weight of the booking requests have to be compared in order to calculate the spoilage and offloading cost. This section clarifies more in depth about formulating the overbooking model which separately considers volume and weight of the booking requests. In the former sections, cost calculations including examples of other calculations are in a simple form to make it easier to understand. All of the cases and the two-dimensionality are joined together in this section with the use of the variables and parameters notations.

When comparing situations that can occur simultaneously with the two-dimensional characteristic of the air cargo transportation problem, this research divides cases based on the variables and parameters into two main cases and three subcases as shown below.

The spoilage cost and offloading cost notations are already defined in the previous section: *Defining Variables and Parameters*. The notations are used in this section; however, they are slightly modified in order to fit into many cases in this section.

Let $CS_{x.y}$ and $CO_{x.y}$ be the spoilage cost and offloading cost for main case x and subcase y; and *case* $x.y$ be the case for main case x and subcase y where $x \in \{1, 2\}$ and $y \in \{1, 2, 3\}$.

3.8.1 The Booking Request Level is Lower than The Overbooking Level $(B \leq Q)$

When the booking request level is lower than the overbooking level, an airline does not have to reject any of the booking requests. All the booking requests are accepted as shown in Figure III-7. The show-up booking request volume and weight depend on the booking request level, show-up rate, and booking request density.

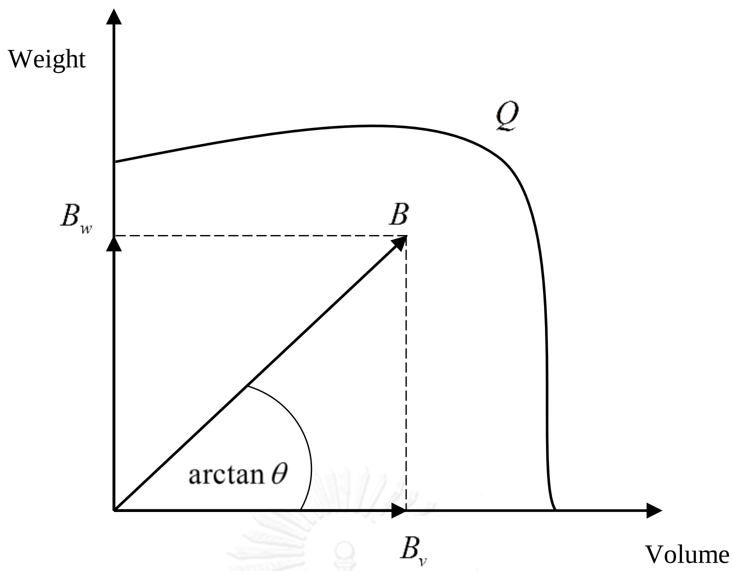

Figure III-7 The booking request level is lower than the overbooking level

1) The Show-up Volume and Weight are Less than The Volume and Weight Capacities ($B_{sv} \le C_v$ and $B_{sw} \le C_w$)

When this situation occurs, as there is no rejection, there is no spoilage cost. Also, the show-up booking request volume and weight are less than the volume and weight capacities, so there are no booking requests that cannot be loaded onto the airplane. In the same way as the spoilage cost, as there are no offloaded booking requests, there is no offloading cost. Therefore, the spoilage cost and offloading cost for main case 1 and subcase 1 can be summarized as:

$$CS_{1.1} = 0 \tag{3.16}$$

$$CO_{1.1} = 0 \tag{3.17}$$

2) The Booking Request Density is Less than The Ratio Between Weight and Volume Capacities, and The Show-up Volume is Higher than The Volume Capacity ($\theta \leq C_w / C_v$ and $B_{sv} > C_v$)

For the *case 1.2*, there is no rejection since the booking request level is lower than the overbooking level. Whenever the rejection is not presented, the spoilage cost does not occur. Thus, the spoilage cost for the *case 1.2* can be summarized as:

$$CS_{1.2} = 0 \tag{3.18}$$

Nonetheless, in this case, the volume of the booking requests is higher than the volume capacity of the airplane. There are absolutely some of the booking requests that cannot be loaded onto the airplane due to the fully utilized volume capacity. There are offloaded booking requests, so there is offloading cost. In this case, the volume of the booking requests reaches its capacity before weight. The weight of the booking requests may or may not reach its capacity. As a result, the volume capacity becomes the most important parameter in calculating the offloading cost. The offloading cost is mainly calculated in terms of the volume capacity. In order to accurately calculate the offloading cost, the offloaded volume and the offloaded weight due to the volume capacity needs to be compared. The volume that cannot be loaded on the airplane is straightforward because it is limited by the volume capacity. Unlike the offloaded volume, which is straightforward, some of the weight of the booking requests is also offloaded even though the actual show-up weight may or may not exceed the weight capacity. When the volume is offloaded, the corresponding weight to that volume is also offloaded. Calculating the corresponding offloaded weight to the offloaded volume can be tricky. The value that can generate the most offloading cost will, then, be selected to be in the function. Hence, there are two values that need to be considered in order to do a comparison between the offloaded volume and corresponding weight. With the aim of understanding more easily, the diagram for this subcase is presented in Figure III-8 below.

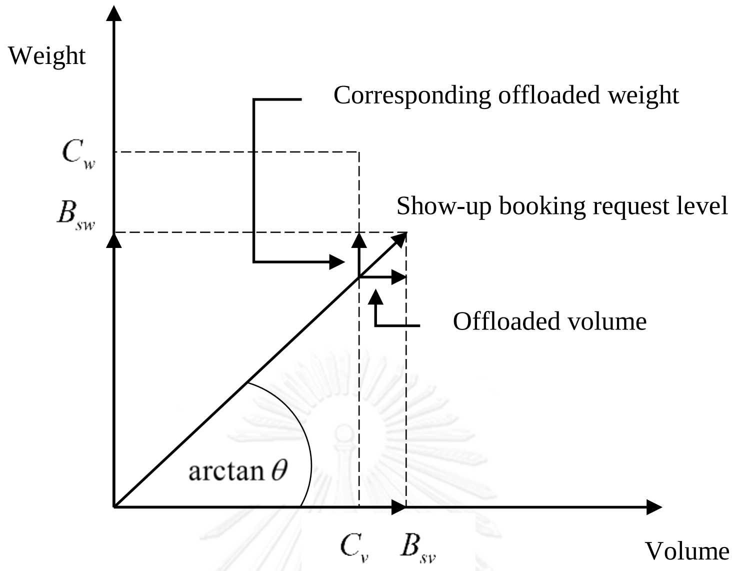

Figure III-8 Diagram summary for the *case 1.2* and *case 2.2*

1. Offloaded volume

The offloaded volume for this case is simple to calculate. It can be obtained by subtracting the volume capacity from the show-up volume of the booking requests. The offloaded volume can be written in a mathematical expression as follow:

$$Volume = B_{sv} - C_v$$

2. Corresponding offloaded weight

The corresponding offloaded weight to the offloaded volume can be obtained by subtracting the weight that can be loaded on the airplane from the actual show-up weight of the booking requests. By applying the rule of three in arithmetic, the corresponding offloaded weight can be easily obtained. The corresponding offloaded weight can be written in a mathematical expression as follow:

$$Weight = B_{sw} - \frac{B_{sw}C_v}{B_{sv}}$$

The latter term, $\frac{B_{sw}C_v}{B_{sv}}$, is the weight of the booking requests that can be loaded onto the airplane. By subtracting them out of the show-up booking request weight makes the whole term be the corresponding offloaded weight.

In order to find the maximum revenue generated by these two values, the offloaded volume needs to be multiplied by the standard density to transform it to be a chargeable weight and compared with the corresponding offloaded weight, without transforming anything to it as it is already in the weight unit. They are combined into one mathematical expression and compared by the $\max()$ function. Therefore, the offloading cost for *case 1.2* can be summarized as:

$$CO_{1.2} = b\max\left(\left(B_{sv} - C_v\right)d, B_{sw} - \frac{B_{sw}C_v}{B_{sv}} \right)$$

The $\max()$ function returns the maximum value between the chargeable weight transformed from the offloaded volume, and the corresponding offloaded weight.

By substituting the relationships between the show-up booking requests, show-up rate of the booking requests, and the density; the offloading cost can be summarized as:

$$CO_{1.2} = b\max\left(\left(\frac{Bp}{\sqrt{1+\theta^2}} - C_v \right)d, \frac{Bp\theta}{\sqrt{1+\theta^2}} - C_v\theta \right) \tag{3.19}$$

The Bp term is the show-up booking request level for this main case. The booking request level, B, is being used as it is lower than the overbooking level and there is no rejection. Therefore, the booking request level is equal to the accepted booking request level; and is used in order to find the show-up booking request level.

3) The Booking Request Density is More than The Ratio Between Weight and Volume Capacities, and The Show-up Weight is Higher than The Weight Capacity ($\theta > C_w / C_v$ and $B_{sw} > C_w$)

For the *case 1.3*, there is no rejection since the booking request level is lower than the overbooking level. Whenever the rejection is not presented, the spoilage cost does not occur. Thus, the spoilage cost for the *case 1.3* can be summarized as:

$$CS_{1.3} = 0 \tag{3.20}$$

Nevertheless, in this case, the weight of the booking requests is higher than the weight capacity of the airplane. There are absolutely some of the booking requests that cannot be loaded onto the airplane due to the fully utilized weight capacity. There are offloaded booking requests, so there is offloading cost. In this case, the weight of the booking requests reaches its capacity before volume. The volume of the booking requests may or may not reach its capacity. As a result, the weight capacity becomes the most important parameter in calculating the offloading cost in this case. The offloading cost is mainly calculated in terms of the weight capacity. In order to accurately calculate the offloading cost, the offloaded weight and the offloaded volume due to the weight capacity needs to be compared. The weight that cannot be loaded on the airplane is straightforward because it is limited by the weight capacity. Unlike the offloaded weight, which is straightforward, some of the volume of the booking requests is also offloaded even though the actual show-up volume may or may not exceed the volume capacity. When the weight is offloaded, the corresponding volume to that weight is also offloaded. Calculating the corresponding offloaded volume to the offloaded weight can be somewhat difficult. The value that can generate the most offloading cost will, then, be selected to be in the function. Hence, there are two values that need to be considered in order to do a comparison between the offloaded weight and corresponding volume. In order to understand more easily, the diagram for the *case 1.3* and *case 2.3* is shown in Figure III-9 below.

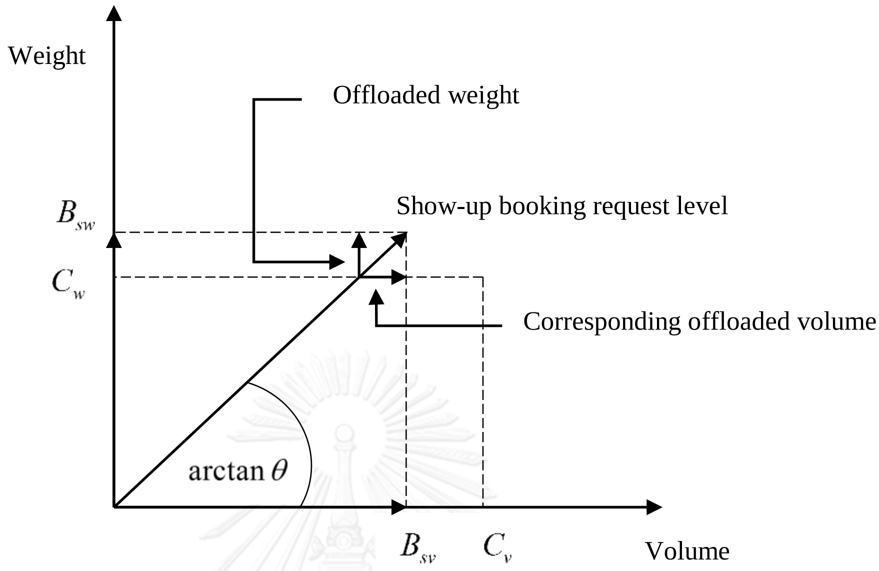

Figure III-9 Diagram summary for the *case 1.3* and *case 2.3*

1. Corresponding offloaded volume

The offloaded volume for this case is simple to calculate. It can be obtained by subtracting the volume capacity from the show-up volume of the booking requests. The offloaded volume can be written in a mathematical expression as follow:

The corresponding offloaded volume to the offloaded weight can be obtained by subtracting the volume that can be loaded onto the airplane from the actual show-up volume of the booking requests. By applying the rule of three in arithmetic, the corresponding offloaded volume can be easily obtained. The corresponding offloaded volume can be written in a mathematical expression as follow:

$$Volume = B_{sv} - \frac{B_{sv}C_w}{B_{sw}}$$

The latter term, $\frac{B_{sv}C_w}{B_{sw}}$, is the weight of the booking requests that can be loaded onto the airplane. By subtracting them out of the show-up booking request weight makes the whole term be the corresponding offloaded weight.

2. Offloaded weight

The offloaded weight for this case is simple to calculate. It can be obtained by subtracting the weight capacity from the show-up weight of the booking requests. The offloaded weight can be written in a mathematical expression as follow:

$$Weight = B_{sw} - C_w$$

In order to find the maximum revenue generated by these two values, the corresponding offloaded volume needs to be multiplied by the standard density to transform it to be a chargeable weight and compared with the offloaded weight, without transforming anything to it as it is already in the weight unit. They are combined into one mathematical expression and compared by the $\max()$ function. Therefore, the offloading cost for *case 1.3* can be summarized as:

$$CO_{1.3} = b\max\left(\left(B_{sv} - \frac{B_{sv}C_w}{B_{sw}}\right)d, B_{sw} - C_w\right)$$

The $\max()$ function returns the maximum value between the chargeable weight transformed from the corresponding offloaded volume, and the offloaded weight.

By substituting the relationships between the show-up booking requests, show-up rate of the booking requests, and the density; the offloading cost can be summarized as:

$$CO_{1.3} = b\max\left(\left(\frac{Bp}{\sqrt{1+\theta^2}} - \frac{C_w}{\theta}\right)d, \frac{Bp\theta}{\sqrt{1+\theta^2}} - C_w\right) \quad (3.21)$$

The Bp term is the show-up booking request level for this main case. The booking request level, B, is being used as it is lower than the overbooking level and there is no rejection. Therefore, the booking request level is equal to the accepted booking request level; and is used in order to find the show-up booking request level.

3.8.2 The Booking Request Level is Higher than The Overbooking Level $(B > Q)$

When the booking request level is higher than the overbooking level, an airline has to reject some of the booking requests that exceed the overbooking level. Not all the booking requests are accepted: the accepted booking request level is equal to the overbooking level. The show-up booking request volume and weight depend on the overbooking level, show-up rate of the booking requests, and booking request density

1) The Show-up Volume and Weight are Less than The Volume and Weight Capacities ($B_{sv} \le C_v$ and $B_{sw} \le C_w$)

As stated above, in this main case, the booking request level is higher than the overbooking level; therefore, the rejection is certainly presented. Also, in this subcase, the show-up volume and weight of the booking requests are less than the volume and weight capacities; when this main case and subcase occur simultaneously, there will be spoiled capacities. Whenever spoiled capacities are presented, the spoilage cost occurs. Spoiled capacities and spoilage cost occur due to the underestimated overbooking level. Spoilage cost can be calculated in the same way as the revenue calculation as stated in the previous section: *Standard Density*. Thus, the general form of the spoilage cost function can be presented as:

$$CS = a\max\left(Volume \times d, Weight\right) \tag{3.22}$$

However, the spoilage cost in this research is complicated. Without considering the two-dimensional characteristic of the air cargo overbooking problem, there are two situations that can occur: the expected show-up rejected booking request level is less than the left over available capacity, and the expected show-up rejected booking request level is more than the left over available capacity, as mentioned in the previous section: *Costs*. Therefore, spoilage cost calculation in this research considers volume and weight separately. The separately considered volume and weight each has their own two situations that can occur. This makes a total of four cases: two from volume and two from weight. Then, all the situations are merged into one mathematical expression in the last process. The volume and weight consideration and their cases are shown below:

1. Volume consideration

First, the volume dimension is considered. The cases can be divided into two subcases as follow:

1.1. The expected show-up rejected volume of the booking requests is less than the left over available volume capacity

In this subcase, all of the rejected booking request volume can be loaded onto the airplane. Thus, the $\boldsymbol{Volume}$ that is used to calculate the spoilage cost is the total booking request volume that the airline has rejected multiplied by the show-up rate of the booking requests. Therefore, the $\boldsymbol{Volume}$ for the spoilage cost calculation can be written in a mathematical expression as:

$$Volume = \left(B_v - Q_v\right) p$$

Where the $\left(B_v - Q_v\right)$ term is the booking request volume that has been rejected. As the booking request level is higher than the overbooking level, $\left(B_v - Q_v\right)$

is a positive value. When it is multiplied by the show-up rate of the booking requests, p, the value represents the volume that would have showed up if the volume had not been rejected before. By applying the relationships in the section *Booking Request Density* and substituting them in the above mathematical expression, the $\boldsymbol{Volume}$ can be modified as:

$$Volume = \frac{(B-Q)\,p}{\sqrt{1+\theta^2}}$$

Where the $(B-Q)$ term is the booking request level that has been rejected in the two-dimensional form. This term is, then, changed into a volume form by dividing the $\sqrt{1+\theta^2}$ term as a rule of Pythagoras's.

1.2. The expected show-up rejected volume of the booking requests is more than the left over available volume capacity

In this subcase, all of the rejected booking request volume cannot be loaded onto the airplane due to the lack of the left over available volume capacity. No matter how much the booking request volume is, it can be loaded on to the airplane just the left over available volume capacity. Thus, the $\boldsymbol{Volume}$ that is used to calculate the spoilage cost is the left over available volume capacity. This can be calculated by subtracting the show-up booking request volume from the volume capacity. The $\boldsymbol{Volume}$ can be put into a mathematical expression as follow:

$$Volume = C_v - B_{sv}$$

The show-up booking request volume for this case must be calculated from the overbooking level as it is lower than the booking request level. The accepted booking request level is limited by the overbooking level. By applying the relationships in the section *Booking Request Density*, and substituting them with the notations in the above mathematical expression, the $\boldsymbol{Volume}$ term can be modified as:

$$Volume = C_v - \frac{Qp}{\sqrt{1-\theta^2}}$$

As previously mentioned, the smallest value between these two subcases has to be identified and used in order to calculate the spoilage cost. The $\min()$ function is used, and the $\boldsymbol{Volume}$ in this two subcases can be merged into one mathematical expression as:

$$Volume = \min\left(\frac{(B-Q)\,p}{\sqrt{1+\theta^2}}, C_v - \frac{Qp}{\sqrt{1+\theta^2}}\right)$$

Where $\min()$ function returns the minimum value between the expected show-up rejected volume and the left over available volume capacity.

2. Weight consideration

Second, the weight dimension is considered. The cases can be divided into two subcases below:

2.1. The expected show-up rejected volume of the booking requests is less than the left over available volume capacity

In this subcase, all of the rejected booking request weight can be loaded onto the airplane. Thus, the $\boldsymbol{Weight}$ that is used to calculate the spoilage cost is the total booking request weight that the airline has rejected multiplied by the show-up rate of the booking requests. Therefore, the $\boldsymbol{Weight}$ for the spoilage cost calculation can be written in a mathematical expression as:

$$Weight = (B_w - Q_w)\,p$$

where the $(B_w - Q_w)$ term is the booking request weight that has been rejected. As the booking request level is higher than the overbooking level, $(B_w - Q_w)$

is a positive value. When it is multiplied by the show-up rate of the booking requests, p, the value represents the weight that would have showed up if the weight had not been rejected before. By applying the relationships in the section *Booking Request Density* and substituting them in the above mathematical expression, the $Weight$ can be modified as:

$$Weight = \frac{(B-Q)\,p\theta}{\sqrt{1+\theta^2}}$$

where the $(B-Q)$ term is the booking request level that has been rejected in the two-dimensional form. This term is, then, changed into a weight form by multiplying the booking request density, θ, and dividing the $\sqrt{1+\theta^2}$ term as a rule of Pythagoras's.

2.2. The expected show-up rejected volume of the booking requests is more than the left over available volume capacity

In this subcase, all of the rejected booking request weight cannot be loaded onto the airplane due to the lack of the left over available weight capacity. No matter how much the booking request weight is, it can be loaded on to the airplane just the left over available weight capacity. Thus, the $Weight$ that is used to calculate the spoilage cost is the left over available weight capacity. This can be calculated by subtracting the show-up booking request weight from the weight capacity. The $Weight$ can be put into a mathematical expression as follow:

$$Weight = C_w - B_{sw}$$

The show-up booking request weight for this case must be calculated from the overbooking level as it is lower than the booking request level. The accepted booking request level is limited by the overbooking level. By applying the relationships in the section *Booking Request Density*, and substituting them with the notations in the above mathematical expression, the $Weight$ term can be modified as:

$$Weight = C_w - \frac{Qp\theta}{\sqrt{1-\theta^2}}$$

As previously mentioned, the smallest value between these two subcases has to be identified and used in order to calculate the spoilage cost. The $\min()$ function is used, and the $Weight$ in this two subcases can be merged into one mathematical expression as:

$$Weight = \min\left(\frac{(B-Q)\,p\theta}{\sqrt{1+\theta^2}}, C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}}\right)$$

where $\min()$ function returns the minimum value between the expected show-up rejected weight and the left over available weight capacity.

After the volume and weight has been separately considered with all the subcases, they can, then, be merged together. The $Volume$ term from the first consideration is in a volume unit, so it has to be multiplied by the standard density parameter, d, in order to change it to a chargeable weight and can be used to calculate the spoilage cost. For the second consideration, the $Weight$ term is already in a weight unit, and can be used immediately to calculate the spoilage cost. The spoilage cost calculation, still, needs to compare between these two chargeable weights: $Volume \times d$ and $Weight$. The maximum value between them is selected as it can generate the most revenue. This can be done using the $\max()$ function. Then, multiply the maximum value selected by a spoilage cost per unit, a. The explanation above uses the same principle and logic as the revenue calculation and the general form of the spoilage cost calculation; and the spoilage cost for *case 2.1* can be written in a mathematical expression as:

$$CS_{2.1} = a\max\left(Volume \times d, Weight\right)$$

By substituting the $Volume$ in the mathematical expression above with the $Volume$ calculated in the first consideration, and the $Weight$ in the mathematical expression above with the $Weight$ calculated in the second consideration; the spoilage cost for *case 2.1* can be summarized as:

$$CS_{2.1} = a\max\left\{ \begin{array}{l} d\min\left(\dfrac{(B-Q)p}{\sqrt{1+\theta^2}}, C_v - \dfrac{Qp}{\sqrt{1+\theta^2}} \right) \\ \min\left(\dfrac{(B-Q)p\theta}{\sqrt{1+\theta^2}}, C_w - \dfrac{Qp\theta}{\sqrt{1+\theta^2}} \right) \end{array} \right\} \tag{3.23}$$

2) The Booking Request Density is Less than The Ratio Between Weight and Volume Capacities, and The Show-up Volume is Higher than The Volume Capacity ($\theta \le C_w / C_v$ and $B_{sv} > C_v$)

For the *case 2.2*, there is no rejection since the booking request level is lower than the overbooking level. Whenever the rejection is not presented, the spoilage cost does not occur. Thus, the spoilage cost for the *case 2.2* can be summarized as:

$$CS_{2.2} = 0 \tag{3.24}$$

Nonetheless, in this case, the volume of the booking requests is higher than the volume capacity of the airplane. There are absolutely some of the booking requests that cannot be loaded onto the airplane due to the fully utilized volume capacity. There are offloaded booking requests, so there is offloading cost. In this case, the volume of the booking requests reaches its capacity before weight. The weight of the booking requests may or may not reach its capacity. As a result, the volume capacity becomes the most important parameter in calculating the offloading cost. The offloading cost is mainly calculated in terms of the volume capacity. In order to accurately calculate the offloading cost, the offloaded volume and the offloaded weight due to the volume capacity needs to be compared. The volume that cannot be loaded on the airplane is straightforward because it is limited by the volume capacity. Unlike the offloaded volume, which is straightforward, some of the weight of the booking requests is also offloaded even though the actual show-up weight may or may not exceed the weight capacity. When the volume is offloaded, the corresponding weight to that volume is also offloaded. Calculating the corresponding offloaded weight to the offloaded volume can be tricky. The value that can generate the most offloading cost will, then, be selected to be in the function. Hence, there are two values that need to be considered in order to do a comparison between the offloaded volume and corresponding weight. The same principle as the *case 1.2* is applied, and the diagram for this case is used together with that case and can be seen in Figure III-8.

1. Offloaded volume

The offloaded volume for this case is simple to calculate. It can be obtained by subtracting the volume capacity from the show-up volume of the booking requests. The offloaded volume can be written in a mathematical expression as follow:

$$Volume = B_{sv} - C_v$$

2. Corresponding offloaded weight

The corresponding offloaded weight to the offloaded volume can be obtained by subtracting the weight that can be loaded on the airplane from the actual show-up weight of the booking requests. By applying the rule of three in arithmetic, the corresponding offloaded weight can be easily obtained. The corresponding offloaded weight can be written in a mathematical expression as follow:

$$Weight = B_{sw} - \frac{B_{sw} C_v}{B_{sv}}$$

The latter term, $\frac{B_{sw}C_v}{B_{sv}}$, is the weight of the booking requests that can be loaded onto the airplane. By subtracting them out of the show-up booking request weight makes the whole term be the corresponding offloaded weight.

In order to find the maximum revenue generated by these two values, the offloaded volume needs to be multiplied by the standard density to transform it to be a chargeable weight and compared with the corresponding offloaded weight, without transforming anything to it as it is already in the weight unit. They are combined into one mathematical expression and compared by the $\max()$ function. Therefore, the offloading cost for *case 2.2* can be summarized as:

$$CO_{2.2} = b\max\left(\left(B_{sv} - C_v\right)d, B_{sw} - \frac{B_{sw}C_v}{B_{sv}} \right)$$

The $\max()$ function returns the maximum value between the chargeable weight transformed from the offloaded volume, and the corresponding offloaded weight.

By substituting the relationships between the show-up booking requests, show-up rate of the booking requests, and the density; the offloading cost can be summarized as:

$$CO_{2.2} = b\max\left(\left(\frac{Qp}{\sqrt{1+\theta^2}} - C_v \right)d, \frac{Qp\theta}{\sqrt{1+\theta^2}} - C_v\theta \right) \tag{3.25}$$

The Qp term is the show-up booking request level for this main case. The overbooking level, Q, is being used as it is lower than the booking request level and the rejection is presented. Therefore, the accepted booking request level is equal to the overbooking level; and the overbooking level is used in order to find the show-up booking request level.

3) The Booking Request Density is More than The Ratio Between Weight and Volume Capacities, and The Show-up Weight is Higher than The Weight Capacity ($\theta > C_w / C_v$ and $B_{sw} > C_w$)

For the *case 2.3*, there is no rejection since the booking request level is lower than the overbooking level. Whenever the rejection is not presented, the spoilage cost does not occur. Thus, the spoilage cost for the *case 2.3* can be summarized as:

$$CS_{2.3} = 0 \tag{3.26}$$

Nevertheless, in this subcase, the weight of the booking requests is higher than the weight capacity of the airplane. There are absolutely some of the booking requests that cannot be loaded onto the airplane due to the fully utilized weight capacity. There are offloaded booking requests, so there is offloading cost. In this case, the weight of the booking requests reaches its capacity before volume. The volume of the booking requests may or may not reach its capacity. As a result, the weight capacity becomes the most important parameter in calculating the offloading cost in this case. The offloading cost is mainly calculated in terms of the weight capacity. In order to accurately calculate the offloading cost, the offloaded weight and the offloaded volume due to the weight capacity needs to be compared. The weight that cannot be loaded on the airplane is straightforward because it is limited by the weight capacity. Unlike the offloaded weight, which is straightforward, some of the volume of the booking requests is also offloaded even though the actual show-up volume may or may not exceed the volume capacity. When the weight is offloaded, the corresponding volume to that weight is also offloaded. Calculating the corresponding offloaded volume to the offloaded weight can be somewhat difficult. The value that can generate the most offloading cost will, then, be selected to be in the function. Hence, there are two values that need to be considered in order to do a comparison between the offloaded weight and corresponding volume. The same principle as the *case 1.3* is applied, and the diagram for this case is used together with that case and can be seen in Figure III-9.

1. Corresponding offloaded volume

The offloaded volume for this case is simple to calculate. It can be obtained by subtracting the volume capacity from the show-up volume of the booking requests. The offloaded volume can be written in a mathematical expression as follow:

The corresponding offloaded volume to the offloaded weight can be obtained by subtracting the volume that can be loaded onto the airplane from the actual show-up volume of the booking requests. By applying the rule of three in arithmetic, the corresponding offloaded volume can be easily obtained. The corresponding offloaded volume can be written in a mathematical expression as follow:

$$Volume = B_{sv} - \frac{B_{sv}C_w}{B_{sw}}$$

The latter term, $\frac{B_{sv}C_w}{B_{sw}}$, is the weight of the booking requests that can be loaded onto the airplane. By subtracting them out of the show-up booking request weight makes the whole term be the corresponding offloaded weight.

2. Offloaded weight

The offloaded weight for this case is simple to calculate. It can be obtained by subtracting the weight capacity from the show-up weight of the booking requests. The offloaded weight can be written in a mathematical expression as follow:

$$Weight = B_{sw} - C_w$$

In order to find the maximum revenue generated by these two values, the corresponding offloaded volume needs to be multiplied by the standard density to transform it to be a chargeable weight and compared with the offloaded weight, without transforming anything to it as it is already in the weight unit. They are combined into one mathematical expression and compared by the $\max()$ function. Therefore, the offloading cost for *case 2.3* can be summarized as:

$$CO_{2.3} = b\max\left(\left(B_{sv} - \frac{B_{sv}C_w}{B_{sw}}\right)d, B_{sw} - C_w\right)$$

The $\max()$ function returns the maximum value between the chargeable weight transformed from the corresponding offloaded volume, and the offloaded weight.

By substituting the relationships between the show-up booking requests, show-up rate of the booking requests, and the density; the offloading cost can be summarized as:

$$CO_{2.3} = b\max\left(\left(\frac{Qp}{\sqrt{1+\theta^2}} - \frac{C_w}{\theta}\right)d, \frac{Qp\theta}{\sqrt{1+\theta^2}} - C_w\right) \tag{3.27}$$

The Qp term is the show-up booking request level for this main case. The overbooking level, Q, is being used as it is lower than the booking request level and the rejection is presented. Therefore, the accepted booking request level is equal to the overbooking level; and the overbooking level is used in order to find the show-up booking request level.

3.8.3 Conclusion

From all the explanation above in the section *Cost Formulating*, the spoilage cost and offloading cost calculations, and all the cases can be summarized in terms of variables and parameters. The spoilage cost and offloading cost for each case are summarized below.

1. The booking request level is lower than the overbooking level ($B \le Q$)

1.1. $B_{sv} \le C_v$ *and* $B_{sw} \le C_w$

There is no rejection and there is no offloaded booking requests; thus, the spoilage cost and offloading cost for the *case 1.1* can be summarized as:

$$CS_{1.1} = 0$$

$$CO_{1.1} = 0$$

1.2. $\theta \le \frac{C_w}{C_v}$ *and* $B_{sv} > C_v$

There is no rejection, but there are some offloaded booking requests due to the volume capacity; thus, the spoilage cost and offloading cost for the *case 1.2* can be summarized as:

$$CS_{1.2} = 0$$

$$CO_{1.2} = b\max\left(\left(\frac{Bp}{\sqrt{1+\theta^2}} - C_v\right)d, \frac{Bp\theta}{\sqrt{1+\theta^2}} - C_v\theta\right)$$

1.3. $\theta > \frac{C_w}{C_v}$ *and* $B_{sw} > C_w$

There is no rejection, but there are some offloaded booking requests due to the weight capacity; thus, the spoilage cost and offloading cost for the *case 1.3* can be summarized as:

$$CS_{1.3} = 0$$

$$CO_{1.3} = b\max\left(\left(\frac{Bp}{\sqrt{1+\theta^2}} - \frac{C_w}{\theta}\right)d, \frac{Bp\theta}{\sqrt{1+\theta^2}} - C_w\right)$$

2. The booking request level is higher than the overbooking level ($B > Q$)

2.1. $B_{sv} \le C_v$ *and* $B_{sw} \le C_w$

There is rejection and there are no offloaded booking requests; thus, the spoilage cost and offloading cost for the *case 2.1* can be summarized as:

$$CS_{2.1} = a\max\left\{\begin{array}{l} d\min\left(\frac{(B-Q)p}{\sqrt{1+\theta^2}}, C_v - \frac{Qp}{\sqrt{1+\theta^2}}\right) \\ \min\left(\frac{(B-Q)p\theta}{\sqrt{1+\theta^2}}, C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}}\right) \end{array}\right\}$$

$$CO_{2.1} = 0$$

2.2. $\theta \le \frac{C_w}{C_v}$ *and* $B_{sv} > C_v$

There is rejection, but there are some offloaded booking requests due to the volume capacity; thus, the spoilage cost and offloading cost for the *case 2.2* can be summarized as:

$$CS_{2.2} = 0$$

$$CO_{2.2} = b\max\left(\left(\frac{Qp}{\sqrt{1+\theta^2}} - C_v\right)d, \frac{Qp\theta}{\sqrt{1+\theta^2}} - C_v\theta\right)$$

2.3. $\theta > \frac{C_w}{C_v}$ *and* $B_{sw} > C_w$

There is rejection, but there are some offloaded booking requests due to the weight capacity; thus, the spoilage cost and offloading cost for the *case 2.3* can be summarized as:

$$C_{2.3} = 0$$

$$CO_{2.3} = b\max\left(\left(\frac{Qp}{\sqrt{1+\theta^2}} - \frac{C_w}{\theta}\right)d, \frac{Qp\theta}{\sqrt{1+\theta^2}} - C_w\right)$$

The spoilage cost and offloading cost in each case above can be briefly summarized in Table III-6.

Table III-6: Summary of the costs occurred in each case

Main Case	Subcase	Occurred Cost
$B \le Q$	$B_{sv} \le C_v$ and $B_{sw} \le C_w$	No costs
	$\theta \le \frac{C_w}{C_v}$ and $B_{sv} > C_v$	$CO_{1.2} = b\max\left(\left(\frac{Bp}{\sqrt{1+\theta^2}} - C_v\right)d, \frac{Bp\theta}{\sqrt{1+\theta^2}} - C_v\theta\right)$
	$\theta > \frac{C_w}{C_v}$ and $B_{sw} > C_w$	$CO_{1.3} = b\max\left(\left(\frac{Bp}{\sqrt{1+\theta^2}} - \frac{C_w}{\theta}\right)d, \frac{Bp\theta}{\sqrt{1+\theta^2}} - C_w\right)$
$B > Q$	$B_{sv} \le C_v$ and $B_{sw} \le C_w$	$CS_{2.1} = a\max\left\{ d\min\left(\frac{(B-Q)p}{\sqrt{1+\theta^2}}, C_v - \frac{Qp}{\sqrt{1+\theta^2}}\right), \min\left(\frac{(B-Q)p\theta}{\sqrt{1+\theta^2}}, C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}}\right)\right\}$
	$\theta \le \frac{C_w}{C_v}$ and $B_{sv} > C_v$	$CO_{2.2} = b\max\left(\left(\frac{Qp}{\sqrt{1+\theta^2}} - C_v\right)d, \frac{Qp\theta}{\sqrt{1+\theta^2}} - C_v\theta\right)$
	$\theta > \frac{C_w}{C_v}$ and $B_{sw} > C_w$	$CO_{2.3} = b\max\left(\left(\frac{Qp}{\sqrt{1+\theta^2}} - \frac{C_w}{\theta}\right)d, \frac{Qp\theta}{\sqrt{1+\theta^2}} - C_w\right)$

After spoilage costs and offloading costs for each case are specified, the total cost can be obtained by the summation of spoilage and offloaded costs from all 6 cases. Note that both CS and CO are a function of continuous random variable B (the booking request level), whose probability density function is given by $f(B)$. For any given show-up rate p_i, the conditional expected total cost can be calculated by taking the integral of the booking requests for all possibilities of the booking requests. Let $E[TC \mid p = p_i]$ be the expected total cost for any given show-up rate p_i, we have:

$$E[TC \mid p = p_i] = \int \sum_{x=1}^{2} \sum_{y=1}^{3} \left(CS_{x.y}(p_i) + CO_{x.y}(p_i) \right) f(B)\, d(B) \tag{3.28}$$

where $CS_{x.y}(p_i)$ and $CO_{x.y}(p_i)$ are the spoilage cost and offloading cost for main case x and subcase y, respectively, for any given show-up rate p_i.

In order to calculated the expected total cost, all possibilities of the show-up rate must be taken into account. Note that the show up rate p_i is considered as a discrete random variable with probability mass function of $g_p(p_i)$. By doing that, the expected total cost can be written as:

$$E[TC] = \sum_{i=1}^{n} \left(E[TC \mid p = p_i]\, g_p(p_i) \right) \tag{3.29}$$

where n is the total number of the show-up rate values and $i = 1, 2, 3, \ldots, n$.

3.9 Model Formulating

In order to obtain the optimum overbooking level from the cost function given in the previous section, the integral interval of the booking request level must be defined. In this section, the integral interval of the booking request level is calculated based on cases and in terms of important variables and parameters.

The case notations are adopted from the previous section: *Cost Formulating*; that is, *case* $x.y$, $CS_{x.y}$, and $CO_{x.y}$ are the case where main case x and subcase y occurs, the spoilage cost, and offloading cost, respectively, for the main case x and subcase y where $x \in \{1,2\}$ and $y \in \{1,2,3\}$. The procedure of determining the integral interval for the booking request level is outlined below.

For the *case 1.1*, since there are no spoilage cost and offloading cost, this case can be omitted. Therefore, there are five cases left to be considered. First, $CO_{1.2}$ and $CO_{2.2}$ in the *case 1.2* and *case 2.2*, respectively, are used simultaneously in order to determine the integral interval because there are only offloading costs for each case, and the condition for their subcases, i.e., subcase 2, are the same. Although their main cases are different, the condition for each case is the basic interval of the booking request level and can be merged together, i.e., $B \le Q$ and $B > Q$. Second, $CO_{1.3}$ and $CO_{2.3}$ in the *case 1.3* and *case 2.3*, respectively, are used simultaneously in order to determine the integral interval, once more, because there are only offloading costs for each case, and the condition for their subcases, i.e., subcase 2, are the same. Again, although their main cases are different, the condition for each case is the basic interval of the booking request level and can be merged together, i.e., $B \le Q$ and $B > Q$. Last, $CS_{2.1}$ in the *case 2.1* is used in order to determine the integral interval. This is done in the last step because of the large and complex function.

3.9.1 Defining Integral Interval for Case 1.2 and Case 2.2

As stated before, there is a reason why these two cases are grouped together. The main case conditions of both cases are the base interval for the booking request level, i.e., $B < Q$ and $B > Q$. The subcase conditions of both cases are the same and are used in order to obtain the final conditions and integral interval.

1) *Case 1.2* (Main Case 1 and Subcase 2)

There are three conditions in this case, two of which is important to determine the integral interval of the booking request level for $CO_{1.2}$. The three conditions are:

$B \le Q$: important as it involves the booking request level

$\theta \le \frac{C_w}{C_v}$: not important as it is a condition based on parameters

$B_{sv} > C_v$: important as it involves the booking request level

The first two conditions are $B \le Q$ and $\theta \le \frac{C_w}{C_v}$, and they requires no solving. The last condition is $B_{sv} > C_v$ which requires a little bit of solving. First, the B_{sv} term must be transformed into the B term by applying relationships in section *Booking Request Density*. By doing that, the last condition becomes:

$$\frac{Bp}{\sqrt{1+\theta^2}} > C_v$$

By solving the inequation of the condition above, the interval of the booking request level can be obtained as shown below:

$$B > \frac{C_v\sqrt{1+\theta^2}}{p}$$

When merging this condition with the first condition: $B < Q$, the overall condition can be written as:

$$Q > B > \frac{C_v\sqrt{1+\theta^2}}{p}$$

Thus, the integral interval for the booking request level is from $\frac{C_v\sqrt{1+\theta^2}}{p}$ to Q, given that $Q > \frac{C_v\sqrt{1+\theta^2}}{p}$. Therefore, $Q > \frac{C_v\sqrt{1+\theta^2}}{p}$ is another condition for this case, and $\left[\frac{C_v\sqrt{1+\theta^2}}{p}, Q\right]$ is the integral interval. The offloading cost for the *case 1.2* can be written in form of integral as:

$$CO_{1.2} = \int_{\frac{C_v\sqrt{1+\theta^2}}{p}}^{Q} b\max\left(\left(\frac{Bp}{\sqrt{1+\theta^2}} - C_v\right)d, \frac{Bp\theta}{\sqrt{1+\theta^2}} - C_v\theta\right) f\left(B\right)dB \qquad (3.30)$$

given that $Q > \frac{C_v\sqrt{1+\theta^2}}{p}$.

The last step for this case is comparing the value inside of the $\max()$ function. $\left(\frac{Bp}{\sqrt{1+\theta^2}} - C_v\right)d$ and $\frac{Bp\theta}{\sqrt{1+\theta^2}} - C_v\theta$ are compared. By solving that, it is found that $\left(\frac{Bp}{\sqrt{1+\theta^2}} - C_v\right)d$ will be more than $\frac{Bp\theta}{\sqrt{1+\theta^2}} - C_v\theta$ if $\theta \le d$, and same goes the other way around.

To summarize for the *case 1.2*, $Q > \frac{C_v\sqrt{1+\theta^2}}{p}$ and $\theta \le \frac{C_w}{C_v}$ are the conditions for this case, and $\left[\frac{C_v\sqrt{1+\theta^2}}{p}, Q\right]$ is the integral interval of the booking request level. Last, the value of θ and d determine the value of $CO_{1.2}$ to be integrated. This can be shown briefly in Table III-7.

Table III-7: Cost function for *case 1.2* based on conditions

1st condition	$Q > \frac{C_v\sqrt{1+\theta^2}}{p}$ and $\theta \le \frac{C_w}{C_v}$	
2nd condition	$\theta \le d$	$\theta > d$
$CO_{1.2}$	$\int_{\frac{C_v\sqrt{1+\theta^2}}{p}}^{Q} \left(b\left(\frac{Bp}{\sqrt{1+\theta^2}} - C_v\right)d\right) f\left(B\right)dB$	$\int_{\frac{C_v\sqrt{1+\theta^2}}{p}}^{Q} b\left(\frac{Bp\theta}{\sqrt{1+\theta^2}} - C_v\theta\right) f\left(B\right)dB$

2) *Case 2.2* **(Main Case 2 and Subcase 2)**

There are three conditions in this case, two of which is important to determine the integral interval of the booking request level for $CO_{2.2}$. The three conditions are:

$B > Q$: important as it involves the booking request level

$\theta \leq \frac{C_w}{C_v}$: not important as it is a condition based on parameters

$B_{sv} > C_v$: important as it involves the booking request level

The first two conditions are $B > Q$ and $\theta \leq \frac{C_w}{C_v}$, and they requires no solving. The last condition is $B_{sv} > C_v$ which requires a little bit of solving. First, the B_{sv} term must be transformed into the Q term by applying relationships in section *Booking Request Density*. By doing that, the last condition becomes:

$$\frac{Qp}{\sqrt{1+\theta^2}} > C_v$$

By solving the inequation of the condition above, the interval of the booking request level can be obtained as shown below:

$$Q > \frac{C_v\sqrt{1+\theta^2}}{p}$$

When merging this condition with the first condition: $B < Q$, the overall condition can be written as:

$$B > Q > \frac{C_v\sqrt{1+\theta^2}}{p}$$

Thus, the integral interval for the booking request level is from Q to infinity, given that $Q > \frac{C_v\sqrt{1+\theta^2}}{p}$. Therefore, $Q > \frac{C_v\sqrt{1+\theta^2}}{p}$ is another condition for this case, and $[Q,\infty)$ is the integral interval. The offloading cost for the *case 2.2* can be written in form of integral as:

$$CO_{2.2} = \int_{Q}^{\infty} b \max\left(\left(\frac{Qp}{\sqrt{1+\theta^2}} - C_v\right)d, \frac{Qp\theta}{\sqrt{1+\theta^2}} - C_v\theta\right) f(B)\,dB \tag{3.31}$$

given that $Q > \frac{C_v\sqrt{1+\theta^2}}{p}$.

The last step for this case is comparing the value inside of the $\max()$ function. $\left(\frac{Qp}{\sqrt{1+\theta^2}} - C_v\right)d$ and $\frac{Qp\theta}{\sqrt{1+\theta^2}} - C_v\theta$ are compared. By solving that, it is found that $\left(\frac{Qp}{\sqrt{1+\theta^2}} - C_v\right)d$ will be more than $\frac{Qp\theta}{\sqrt{1+\theta^2}} - C_v\theta$ if $\theta \le d$, and same goes the other way around.

To summarize for the *case 2.2*, $Q > \frac{C_v\sqrt{1+\theta^2}}{p}$ and $\theta \le \frac{C_w}{C_v}$ are the conditions for this case, and $[Q,\infty)$ is the integral interval of the booking request level. Last, the value of θ and d determine the value of $CO_{2.2}$ to be integrated. This can be shown briefly in Table III-8.

Table III-8: Cost function for *case 2.2* based on conditions

1st condition	$Q > \frac{C_v\sqrt{1+\theta^2}}{p}$ and $\theta \le \frac{C_w}{C_v}$	
2nd condition	$\theta \le d$	$\theta > d$
$CO_{2.2}$	$\int_{Q}^{\infty} \left(b\left(\frac{Qp}{\sqrt{1+\theta^2}} - C_v\right)d\right) f(B)\,dB$	$\int_{Q}^{\infty} b\left(\frac{Qp\theta}{\sqrt{1+\theta^2}} - C_v\theta\right) f(B)\,dB$

After determining the integral interval for the subcase 2 for both main case 1 and 2, in the next section, the integral interval for the subcase 3 for both main case 1 and 2 are obtained.

3.9.2 Defining Integral Interval for Case 1.3 and Case 2.3

In the same way as the previous group. As stated before, there is a reason why these two cases are grouped together. The main case conditions of both cases are the base interval for the booking request level, i.e., $B < Q$ and $B > Q$. The subcase conditions of both cases are the same and are used in order to obtain the final conditions and integral interval.

1) *Case 1.3* (Main Case 1 and Subcase 3)

There are three conditions in this case, two of which is important to determine the integral interval of the booking request level for $CO_{1.3}$. The three conditions are:

$B \leq Q$: important as it involves the booking request level

$\theta > \frac{C_w}{C_v}$: not important as it is a condition based on parameters

$B_{sw} > C_w$: important as it involves the booking request level

The first two conditions are $B \leq Q$ and $\theta > \frac{C_w}{C_v}$, and they requires no solving. The last condition is $B_{sw} > C_w$ which requires a little bit of solving. First, the B_{sw} term must be transformed into the B term by applying relationships in section *Booking Request Density*. By doing that, the last condition becomes:

$$\frac{Bp\theta}{\sqrt{1+\theta^2}} > C_w$$

By solving the inequation of the condition above, the interval of the booking request level can be obtained as shown below:

$$B > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$$

When merging this condition with the first condition: $B < Q$, the overall condition can be written as:

$$Q > B > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$$

Thus, the integral interval for the booking request level is from $\frac{C_w\sqrt{1+\theta^2}}{p\theta}$ to Q, given that $Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$. Therefore, $Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$ is another condition for this case, and $\left[\frac{C_w\sqrt{1+\theta^2}}{p\theta}, Q\right]$ is the integral interval. The offloading cost for the *case 1.3* can be written in form of integral as:

$$CO_{1.3} = \int_{\frac{C_w\sqrt{1+\theta^2}}{p\theta}}^{Q} b\max\left(\left(\frac{Bp}{\sqrt{1+\theta^2}} - \frac{C_w}{\theta}\right)d, \frac{Bp\theta}{\sqrt{1+\theta^2}} - C_w\right) f(B)\,dB \tag{3.32}$$

given that $Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$.

The last step for this case is comparing the value inside of the $\max()$ function. $\left(\frac{Bp}{\sqrt{1+\theta^2}} - \frac{C_w}{\theta}\right)d$ and $\frac{Bp\theta}{\sqrt{1+\theta^2}} - C_w$ are compared. By solving that, it is found that $\left(\frac{Bp}{\sqrt{1+\theta^2}}-\frac{C_w}{\theta}\right)d$ will be more than $\frac{Bp\theta}{\sqrt{1+\theta^2}}-C_w$ if $\theta \le d$, and same goes the other way around.

To summarize for the *case 1.3*, $Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$ and $\theta > \frac{C_w}{C_v}$ are the conditions for this case, and $\left[\frac{C_w\sqrt{1+\theta^2}}{p\theta}, Q\right]$ is the integral interval of the booking request level. Last, the value of θ and d determine the value of $CO_{1.3}$ to be integrated. This can be shown briefly in Table III-9.

Table III-9: Cost function for *case 1.3* based on conditions

1st condition	$Q > \frac{C_v\sqrt{1+\theta^2}}{p}$ and $\theta \le \frac{C_w}{C_v}$	
2nd condition	$\theta \le d$	$\theta > d$
$CO_{1.3}$	$\int_{\frac{C_w\sqrt{1+\theta^2}}{p\theta}}^{Q}\left(b\left(\frac{Bp}{\sqrt{1+\theta^2}}-\frac{C_w}{\theta}\right)d\right)f(B)dB$	$\int_{\frac{C_w\sqrt{1+\theta^2}}{p\theta}}^{Q} b\left(\frac{Bp\theta}{\sqrt{1+\theta^2}}-C_w\right)f(B)dB$

2) *Case 2.3* (Main Case 2 and Subcase 3)

There are three conditions in this case, two of which is important to determine the integral interval of the booking request level for $CO_{2.3}$. The three conditions are:

$B > Q$: important as it involves the booking request level

$\theta > \frac{C_w}{C_v}$: not important as it is a condition based on parameters

$B_{sw} > C_w$: important as it involves the booking request level

The first two conditions are $B > Q$ and $\theta > \frac{C_w}{C_v}$, and they requires no solving. The last condition is $B_{sw} > C_w$ which requires a little bit of solving. First, the B_{sv} term must be transformed into the Q term by applying relationships in section *Booking Request Density*. By doing that, the last condition becomes:

$$\frac{Qp\theta}{\sqrt{1+\theta^2}} > C_w$$

By solving the inequation of the condition above, the interval of the booking request level can be obtained as shown below:

$$Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$$

When merging this condition with the first condition: $B > Q$, the overall condition can be written as:

$$B > Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$$

Thus, the integral interval for the booking request level is from Q to infinity, given that $Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$. Therefore, $Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$ is another condition for this case, and $[Q,\infty)$ is the integral interval. The offloading cost for the *case 2.3* can be written in form of integral as:

$$CO_{2.3} = \int_Q^\infty b\max\left(\left(\frac{Qp}{\sqrt{1+\theta^2}} - \frac{C_w}{\theta}\right)d, \frac{Qp\theta}{\sqrt{1+\theta^2}} - C_w\right) f(B)\,dB \tag{3.33}$$

given that $Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$.

The last step for this case is comparing the value inside of the $\max()$ function. $\left(\frac{Qp}{\sqrt{1+\theta^2}}-\frac{C_w}{\theta}\right)d$ and $\frac{Qp\theta}{\sqrt{1+\theta^2}}-C_w$ are compared. By solving that, it is found that $\left(\frac{Qp}{\sqrt{1+\theta^2}}-\frac{C_w}{\theta}\right)d$ will be more than $\frac{Qp\theta}{\sqrt{1+\theta^2}}-C_w$ if $\theta \le d$, and same goes the other way around.

To summarize for the *case 2.3*, $Q > \frac{C_w\sqrt{1+\theta^2}}{p\theta}$ and $\theta > \frac{C_w}{C_v}$ are the conditions for this case, and $[Q,\infty)$ is the integral interval of the booking request level. Last, the value of θ and d determine the value of $CO_{2.3}$ to be integrated. This can be shown briefly in Table III-10.

Table III-10: Cost function for *case 2.3* based on conditions

1st condition	$Q > \frac{C_v\sqrt{1+\theta^2}}{p}$ and $\theta \le \frac{C_w}{C_v}$	
2nd condition	$\theta \le d$	$\theta > d$
$CO_{2.3}$	$\int_Q^\infty \left(b\left(\frac{Qp}{\sqrt{1+\theta^2}}-\frac{C_w}{\theta}\right)d\right)f(B)dB$	$\int_Q^\infty b\left(\frac{Qp\theta}{\sqrt{1+\theta^2}}-C_w\right)f(B)dB$

3.9.3 Defining Integral Interval for Case 2.1

For *case 2.1*, the spoilage cost is presented. In this case, the booking request level is higher than the overbooking level, and both show-up volume and weight are less than the capacities. There are three conditions in this case, only one of which is involved with the booking request level. The three conditions are:

$B > Q$: important as it involves the booking request level

$B_{sv} \leq C_v$: not important as it is a condition based on the parameters

The show-up volume in this case can be calculated from the overbooking level and not the booking request level. Therefore, this condition is not derived from the booking request level, and only based on parameters.

$B_{sw} \leq C_w$: not important as it is a condition based on the parameters

The show-up weight in this case can be calculated from the overbooking level and not the booking request level. Therefore, this condition is not derived from the booking request level, and only based on parameters.

The $B > Q$ term is an integral interval for the booking request level and requires no transformation. The B_{sv} and B_{sw} term must be transformed into the Q term by applying relationships in section *Booking Request Density*. By doing that, the last two conditions become:

$$\frac{Qp}{\sqrt{1+\theta^2}} \leq C_v$$

$$\frac{Qp\theta}{\sqrt{1+\theta^2}} \leq C_w$$

By solving the inequations of the conditions above, the interval of the booking request level can be obtained as shown below:

$$Q \leq \frac{C_v\sqrt{1+\theta^2}}{p}$$

$$Q \leq \frac{C_w\sqrt{1+\theta^2}}{p\theta}$$

To merge these two conditions with the first condition: $B > Q$, the value of $\frac{C_v\sqrt{1+\theta^2}}{p}$ and $\frac{C_w\sqrt{1+\theta^2}}{p\theta}$ must be compared. By solving that, it is found that $\frac{C_v\sqrt{1+\theta^2}}{p}$ will be more than $\frac{C_w\sqrt{1+\theta^2}}{p\theta}$ if $\theta > \frac{C_w}{C_v}$, and same goes the other way around. This causes the integral interval to be different in each case. Nevertheless, in both case, the integral interval can be divided into three parts. The integral interval for this case can be illustrated in Figure III-10.

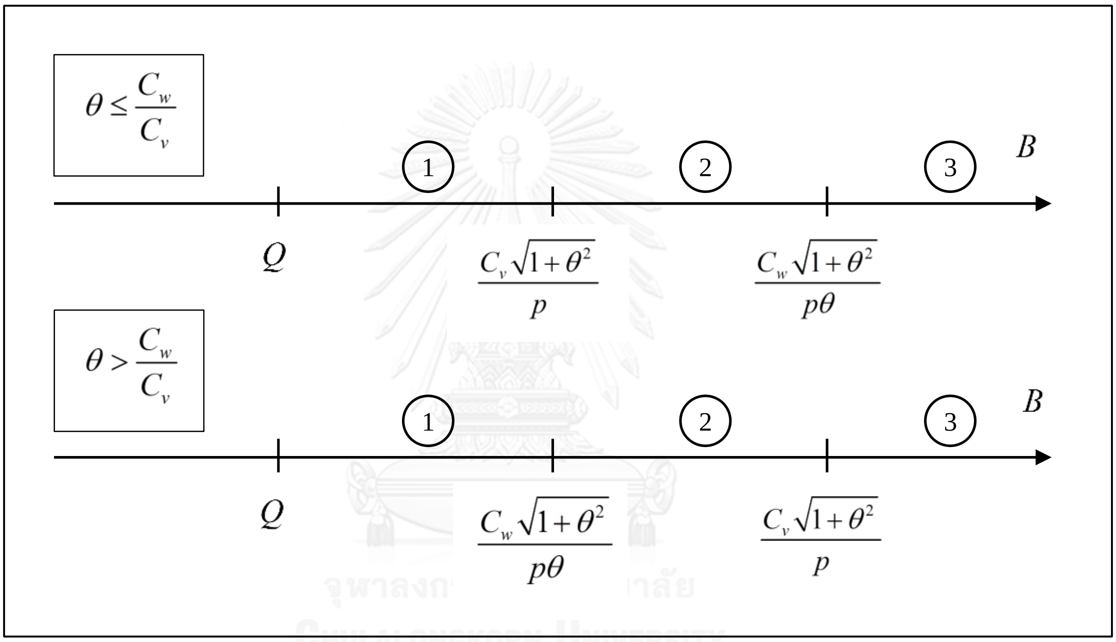

Figure III-10 The integral interval for the booking request level for the *case 2.1*

The cost function for this case is composed of two main functions: $\min()$ and $\max()$, which require more steps of comparing between parameter values. The spoilage cost in this case is

$$CS_{2.1} = a \max \left\{ \begin{array}{l} d \min\left(\frac{(B-Q)p}{\sqrt{1+\theta^2}}, C_v - \frac{Qp}{\sqrt{1+\theta^2}} \right) \\ \min\left(\frac{(B-Q)p\theta}{\sqrt{1+\theta^2}}, C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}} \right) \end{array} \right\}$$

The steps for determining the conditions and integral interval in this case are as follow. First, the values in each $\min()$ function are compared. In this first step, by solving inequality problems, the basic integral intervals are obtained and they are the same as the integral interval for the booking request level shown in Figure III-10. Last, each of the minimum value based on the first condition are compared in the $\max()$ function. As seen in Figure III-10, the integral interval is based on the booking request density, θ; thus, the steps described above have to be done according to the booking request density, θ, and can be divided into two situations. The steps outlined below are for the first step: comparing between values in each $\min()$ function.

- When $\theta \le \frac{C_w}{C_v}$

When the booking request level, B, is in the middle between the overbooking level, Q, and the $\frac{C_v\sqrt{1+\theta^2}}{p}$ term; the minimum values of each $\min()$ function are $\frac{d(B-Q)p}{\sqrt{1+\theta^2}}$ and $\frac{(B-Q)p\theta}{\sqrt{1+\theta^2}}$; therefore, these two terms are compared in the $\max()$ function if the booking request level, B, is in this region. The cost function can be written as:

$$CS_{2.1} = \int_{Q}^{\frac{C_v\sqrt{1+\theta^2}}{p}} a\max\left[\frac{d(B-Q)p}{\sqrt{1+\theta^2}}, \frac{(B-Q)p\theta}{\sqrt{1+\theta^2}}\right] f(B)\,dB \qquad (3.34)$$

When the booking request level, B, is in the middle between $\frac{C_v\sqrt{1+\theta^2}}{p}$ and $\frac{C_w\sqrt{1+\theta^2}}{p\theta}$, the minimum values of each $\min()$ function are $d\left(C_v - \frac{Qp}{\sqrt{1+\theta^2}}\right)$ and $\frac{(B-Q)p\theta}{\sqrt{1+\theta^2}}$; therefore, these two terms are compared in the $\max()$ function if the booking request level, B, is in this region. The cost function can be written as:

$$CS_{2.1} = \int_{\frac{C_v\sqrt{1+\theta^2}}{p}}^{\frac{C_w\sqrt{1+\theta^2}}{p\theta}} a\max\left[d\left(C_v - \frac{Qp}{\sqrt{1+\theta^2}}\right), \frac{(B-Q)\,p\theta}{\sqrt{1+\theta^2}}\right] f(B)\,dB \tag{3.35}$$

When the booking request level, B, is more than $\frac{C_w\sqrt{1+\theta^2}}{p\theta}$, the minimum values of each $\min()$ function are $d\left(C_v - \frac{Qp}{\sqrt{1+\theta^2}}\right)$ and $C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}}$; therefore, these two terms are compared in the $\max()$ function if the booking request level, B, is in this region. The cost function can be written as:

$$CS_{2.1} = \int_{\frac{C_w\sqrt{1+\theta^2}}{p\theta}}^{Q} a\max\left[d\left(C_v - \frac{Qp}{\sqrt{1+\theta^2}}\right), C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}}\right] f(B)\,dB \tag{3.36}$$

- *When* $\theta \le \frac{C_w}{C_v}$

When the booking request level, B, is in the middle between the overbooking level, Q, and the $\frac{C_w\sqrt{1+\theta^2}}{p\theta}$ term; the minimum values of each $\min()$ function are $\frac{d(B-Q)\,p}{\sqrt{1+\theta^2}}$ and $\frac{(B-Q)\,p\theta}{\sqrt{1+\theta^2}}$; therefore, these two terms are compared in the $\max()$ function if the booking request level, B, is in this region. The cost function can be written as:

$$CS_{2.1} = \int_{Q}^{\frac{C_w\sqrt{1+\theta^2}}{p\theta}} a\max\left[\frac{d(B-Q)\,p}{\sqrt{1+\theta^2}}, \frac{(B-Q)\,p\theta}{\sqrt{1+\theta^2}}\right] f(B)\,dB \tag{3.37}$$

When the booking request level, B, is in the middle between $\frac{C_w\sqrt{1+\theta^2}}{p\theta}$ and $\frac{C_v\sqrt{1+\theta^2}}{p}$, the minimum values of each $\min()$ function are $\frac{d(B-Q)\,p}{\sqrt{1+\theta^2}}$ and

$C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}}$; therefore, these two terms are compared in the $\max()$ function if the booking request level, B, is in this region. The cost function can be written as:

$$CS_{2.1} = \int_{\frac{C_w\sqrt{1+\theta^2}}{p\theta}}^{\frac{C_v\sqrt{1+\theta^2}}{p}} a \max\left[\frac{d\left(B-Q\right)p}{\sqrt{1+\theta^2}}, C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}}\right] f\left(B\right) dB \tag{3.38}$$

When the booking request level, B, is more than $\frac{C_v\sqrt{1+\theta^2}}{p}$, the minimum values of each $\min()$ function are $d\left(C_v - \frac{Qp}{\sqrt{1+\theta^2}}\right)$ and $C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}}$; therefore, these two terms are compared in the $\max()$ function if the booking request level, B, is in this region. The cost function can be written as:

$$CS_{2.1} = \int_{\frac{C_v\sqrt{1+\theta^2}}{p}}^{Q} a \max\left[d\left(C_v - \frac{Qp}{\sqrt{1+\theta^2}}\right), C_w - \frac{Qp\theta}{\sqrt{1+\theta^2}}\right] f\left(B\right) dB \tag{3.39}$$

To summarize the six equations above, they are put together in the summations as shown in Table III-11. The six equations above: Eq. (3.34) - (3.39), then, compare their values inside the $\max()$ function.

Table III-11: Cost function after $\min()$ function for *case 2.1*

$Q \le \frac{C_v\sqrt{1+\theta^2}}{p}$ and $Q \le \frac{C_w\sqrt{1+\theta^2}}{p\theta}$	
$\theta \le \frac{C_w}{C_v}$	$\int_{Q}^{\frac{C_v\sqrt{1+\theta^2}}{p}} a\max\left[\frac{d(B-Q)p}{\sqrt{1+\theta^2}}, \frac{(B-Q)p\theta}{\sqrt{1+\theta^2}}\right] f(B)dB + \int_{\frac{C_v\sqrt{1+\theta^2}}{p}}^{\frac{C_w\sqrt{1+\theta^2}}{p\theta}} a\max\left[d\left(C_v-\frac{Qp}{\sqrt{1+\theta^2}}\right), \frac{(B-Q)p\theta}{\sqrt{1+\theta^2}}\right] f(B)dB + \int_{\frac{C_w\sqrt{1+\theta^2}}{p\theta}}^{Q} a\max\left[d\left(C_v-\frac{Qp}{\sqrt{1+\theta^2}}\right), C_w-\frac{Qp\theta}{\sqrt{1+\theta^2}}\right] f(B)dB$
$\theta > \frac{C_w}{C_v}$	$\int_{Q}^{\frac{C_w\sqrt{1+\theta^2}}{p\theta}} a\max\left[\frac{d(B-Q)p}{\sqrt{1+\theta^2}}, \frac{(B-Q)p\theta}{\sqrt{1+\theta^2}}\right] f(B)dB + \int_{\frac{C_w\sqrt{1+\theta^2}}{p\theta}}^{\frac{C_v\sqrt{1+\theta^2}}{p}} a\max\left[\frac{d(B-Q)p}{\sqrt{1+\theta^2}}, C_w-\frac{Qp\theta}{\sqrt{1+\theta^2}}\right] f(B)dB + \int_{\frac{C_v\sqrt{1+\theta^2}}{p}}^{Q} a\max\left[d\left(C_v-\frac{Qp}{\sqrt{1+\theta^2}}\right), C_w-\frac{Qp\theta}{\sqrt{1+\theta^2}}\right] f(B)dB$

All the cost functions based on conditions above are calculated in Matlab using computer language and processing. The next section describes the computer process that is used in order to obtain the optimal overbooking level from the cost functions formulated in this section.

3.10 Computer Processing

From the previous section, cost functions based on conditions are obtained. In order to find the optimal overbooking level, $Q*$, which minimizes the summation of all the cost functions in every conditions, the computer processing must be used because the model is too complex and is composed of many cost functions. The overall model is non-differentiable as a result of the many discontinuous cost functions. The computer process for finding the optimal overbooking level, $Q*$, is illustrated in Figure III-11 below.

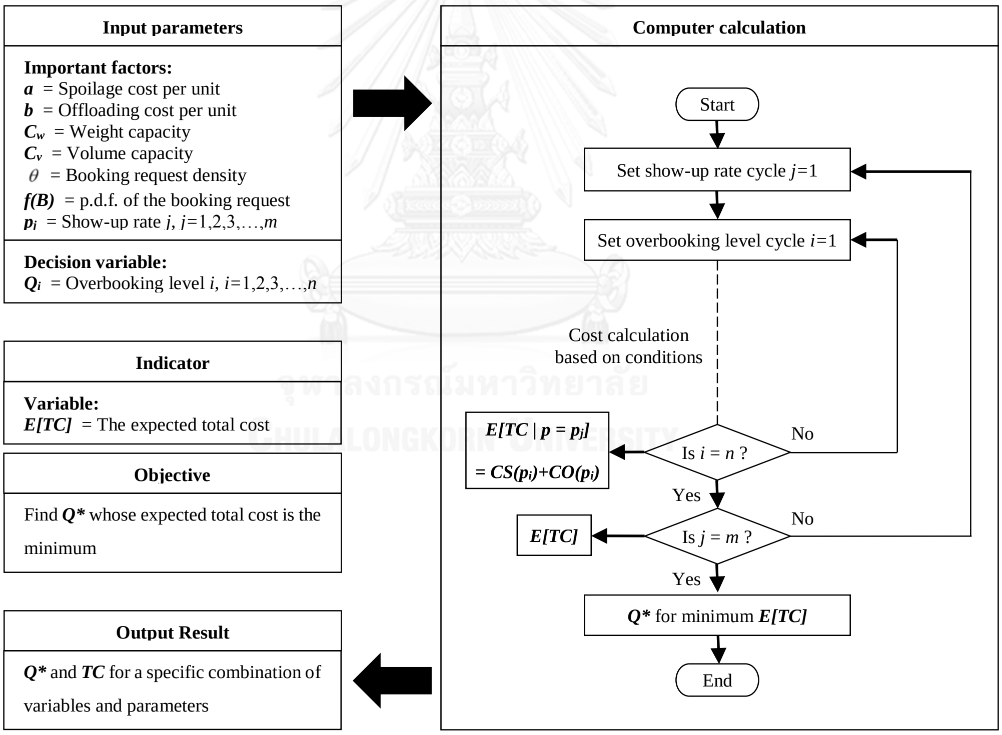

Figure III-11 Procedure for obtaining the optimal overbooking level

As seen in the figure above, there are two main inputs for obtaining the optimal overbooking level, $Q*$, and there are:

1. Important factors

The important factors consist of parameters, random variables, and their probability functions. A set of combination of these important factors is called “scenario.” One scenario can have only one optimal solution as this is a mathematical model, which always gives the same answer for a combination set of the factors, not a simulation, of which the answers may vary.

2. Decision variable

As mentioned above, the model cannot be differentiated; therefore, obtaining the optimal overbooking level can be accomplished by plugging a range of the overbooking level into the model, and letting the computer process select whichever overbooking level that minimizes the total cost. Thus, Q_i is inputted as a range. Although the nature of this decision variable is continuous, this research defines this variable to be integers and increasing by one unit at a time; for example, a range of [100,102] means 100, 101, and 102. This might seem to be an incorrect way of finding an optimal continuous decision variable, but it simplifies the process and allows obtaining an optimal continuous decision variable for a discontinuous model feasible. Not only that, although the optimal solution retrieved from this method might not be the real solution, the solution of this method has an error of less than ± 1, of which the difference in the expected total cost value would be insignificant.

After all the important factors and decision variable are defined and inputted, the computer process begins. It compares parameter values, calculates the costs based on conditions, summates them for every possible values of the given show-up rate to be an expected total cost for a specific value of the overbooking level, and repeats the steps until all the input overbooking level are used. The last step is to select the overbooking level which gives the minimum expected total cost: the optimal overbooking level.

All the steps described above are accomplished in Matlab program in Windows platform. To run this model, it requires a high performance computer with Matlab program installed; moreover, it demands a lot of time to run the model for a specific combination of variables and parameters. In other words, this model is not yet practical for everyday usage. The simplified models are presented in the later chapter in this thesis. The next chapter shows computational experiments on this two-dimensional air cargo overbooking model, and comprises sensitivity analysis, experimental design, and analysis of variance in order to give insight of how each parameter affects the optimal overbooking level obtained from this model.

CHAPTER IV

COMPUTATIONAL EXPERIMENTS

This chapter gives insights about the effects of each variable and parameter on the optimal overbooking level. This chapter is divided into three sections: sensitivity analysis, experimental design, and analysis of variance. First, sensitivity analysis is used in order to give basic idea of how each variable and parameter affects the results. Next, experimental design is applied in order to appropriately design the number of experiments run. Last, in order to know which variables and parameters affect the results significantly and which does not, analysis of variance method is employed as there are many factors that have to be taken into account.

4.1 Sensitivity Analysis

There are many factors involved in the two-dimensional air cargo overbooking model presented in this thesis. The total cost function depends solely on those factors. The optimal overbooking level depends on the total cost value. Therefore, the concept of the optimal overbooking level in terms of the total cost must be understood, and the main effects of the factors must be studied. Before going into more detail of each factors, the basic relationships between the total cost and the overbooking level has to be explained. The first figure displayed below shows that there is only one value of the overbooking level: the optimal overbooking level, which generates the minimum total cost. The insight knowledge are described below.

As stated previously, there are two random variables involved in the two-dimensional air cargo overbooking model in this thesis. One is the booking request level, and another is the booking request show-up rate. The booking request show-up rate is a discrete random variable, and is defined to be uniform as for this study. The booking request show-up rate is easy to defined since, because of the discrete random variable, it does not need to specify any of the parameters of the distribution. On the other hand, for the booking request level, it is a continuous random variable. The parameters for its distribution must be specified in order to make it capable of being integrated. The distribution of the booking request level used in this study is a normal distribution. A normal distribution requires two parameters, i.e., mean, and variance. Therefore, the booking request mean and variance must be specified in order for the cost functions to be able to be integrated.

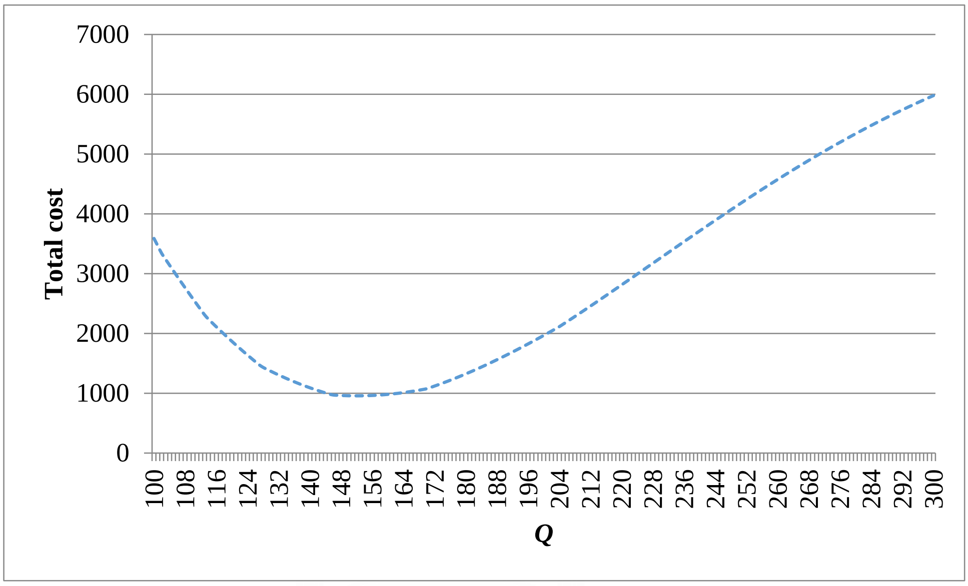

Figure IV-1 The total cost as the overbooking level increases

Figure IV-1 shows the trend of the total cost as the overbooking level, Q, increases. At lower values of Q, the total cost is high and starts dropping as Q increases. The total cost will continuously drop until Q reaches the optimal overbooking level at which the total cost will be at minimum. When Q passes the optimal overbooking level, the total cost will rise again. However, the farther Q passes the optimal overbooking level, the slower the rising rate will be. The reason behind this is when Q is lower than the optimal overbooking level, there is a higher chance that the capacities will be spoiled caused by cancellations and no-shows. On the contrary, when Q is higher than the optimal overbooking level, there is a risk that the show-up booking requests will be offloaded. Therefore the optimal overbooking level minimizes those risks and consequently minimizes the total cost.

Sensitivity analysis considers one factor at a time to observe the main effect on the results of each factor. This section gives a brief idea of how the tendency of the optimal overbooking level goes when a factor is increased or decreased. This section is divided into subsections according to the variables and parameters, and they are outlined as the following paragraph.

4.1.1 Spoilage Cost and Offloading Cost per Chargeable Unit Weight

The first two experiments study about the effects of the ratio between the spoilage cost and offloading cost per chargeable unit weight: a/b and b/a.

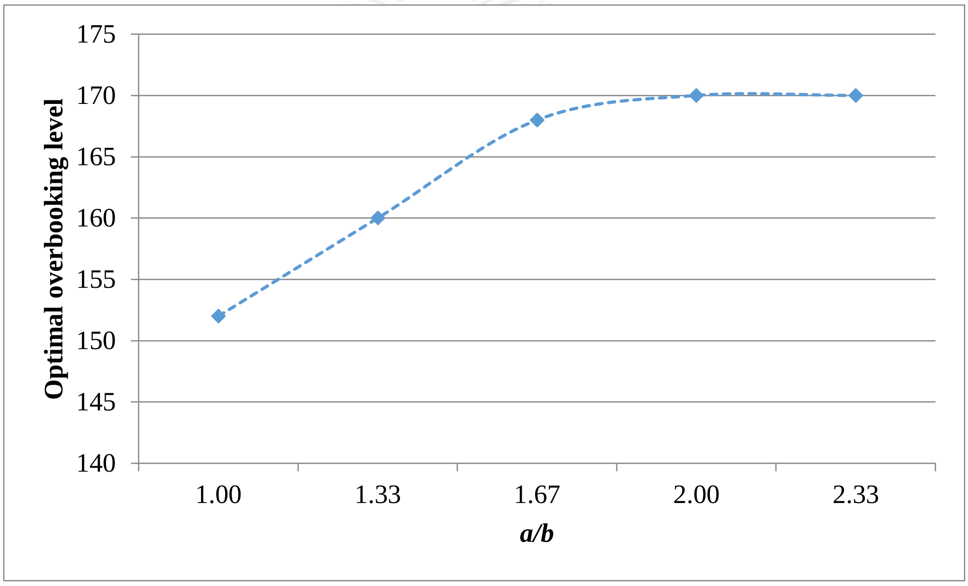

Figure IV-2 The optimal overbooking level as a/b increases

Figure IV-2 shows the optimal overbooking level at different ratios between spoilage cost and offloading cost per chargeable unit weight, a/b. It can be observed that the optimal overbooking level increases as a/b increases. This is because as a/b increases, it costs more if the capacities are spoiled. As a result, the optimal overbooking level increases in order to minimize the total costs.

Conversely, the ratio between spoilage cost and offloading cost per chargeable unit weight, a/b, can be switched between the numerator and the denominator, and the ratio becomes b/a. The impact of the alternative ratio, b/a, on the optimal overbooking level is shown in the following figure with the explanations below.

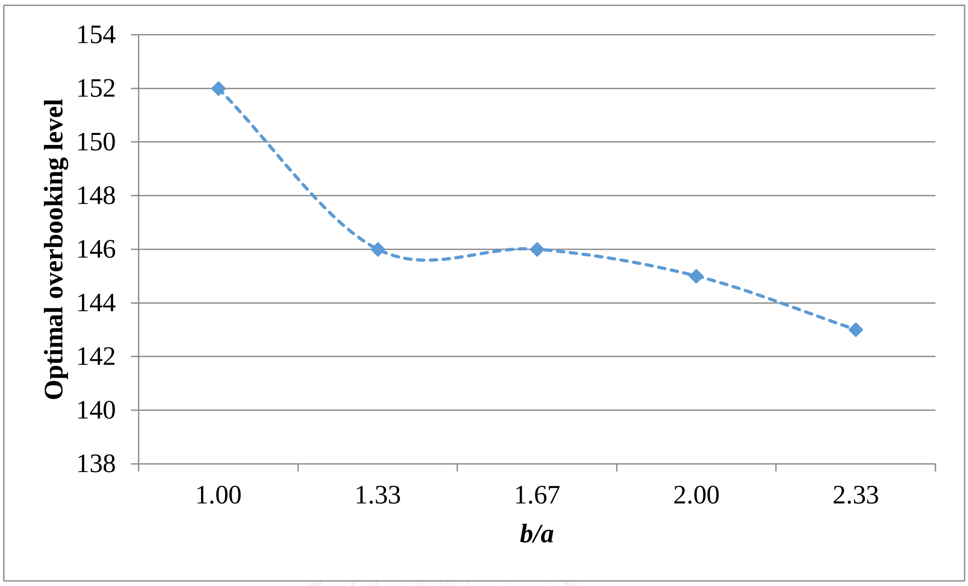

Figure IV-3 The optimal overbooking level as b/a increases

Figure IV-3 shows the optimal overbooking level as the ratio between offloading cost and spoilage cost per chargeable unit weight, b/a, increases. As seen in Figure IV-3, the optimal overbooking level decreases as b/a increases. This is because as b/a increases, it costs more if the show-up booking requests are offloaded. As a consequence, the optimal overbooking level decreases in order to reduce the risk of the booking requests are being offloaded.

Another observation from the two experiments about the spoilage cost and offloading cost per chargeable unit weight above is that the optimal overbooking level is the same as long as the ratio between spoilage cost and offloading cost per chargeable unit weight, which can be a/b or b/a, does not change regardless of the value of a and b. For example, in Figure IV-3, as long as the ratio between a and b is 1, the optimal overbooking level is always 152. The actual values of a and b in this example are 1500 and 1500, respectively, but they can be any values as long as the ratio between those values is still the same, e.g., 1000 and 1000. This is why the ratio between a and b are used as an indicator instead of their actual values.

4.1.2 Show-up Rate

Another important factor that affects the optimal overbooking level is the show-up rate of the booking requests, p. The optimal overbooking levels at different values of the show-up rate while other factors remain the same are illustrated in Figure IV-4.

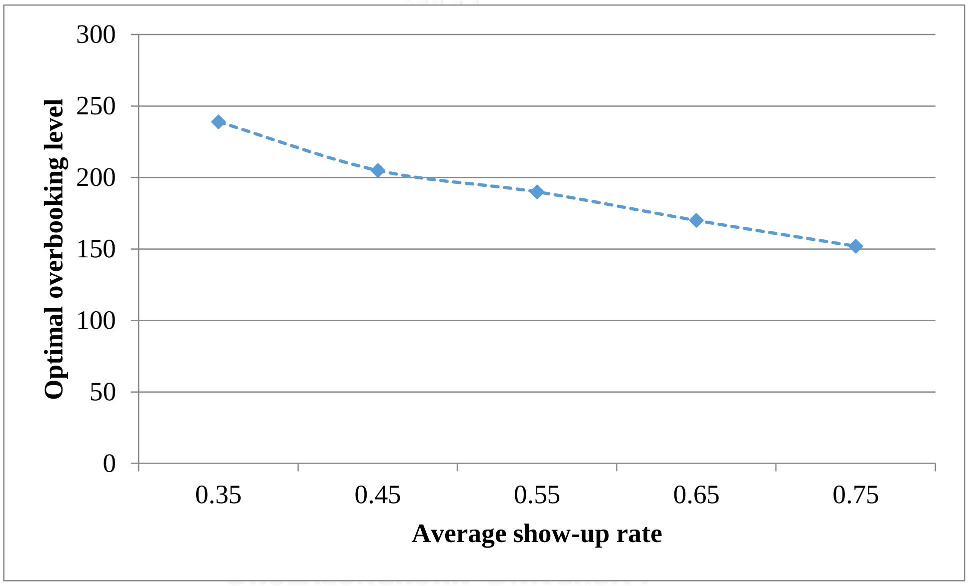

Figure IV-4 The optimal overbooking level as the average show-up rate increases

Figure IV-4 above demonstrates the optimal overbooking level as the average show-up rate increases. It can be observed that the optimal overbooking level gradually declines as the average show-up rate goes up. Generally, the lower the average show-up rate is, the higher the risk which the capacities are spoiled will be. In order to minimize that risk, the result suggests higher optimal overbooking level. In the same way, the higher the average show-up rate is, the better the chance that the show-up booking requests are offloaded will be. Therefore, the model suggests that the optimal overbooking level should be lower in order to minimize that risk. As a result, the trend for optimal overbooking level is downward while the show-up rate increases as seen in Figure IV-4.

4.1.3 Booking Request Mean and Variance

Booking request mean and variance also play a vital role in determining the optimal overbooking level; therefore, the trend of the optimal overbooking level depends on the booking request mean and variance and has to be studied.

Generally, when the booking request and show-up rate are deterministic, the optimal overbooking level is determined easily. By defining the booking request variance to be close to 0, the booking request mean can be assumed to be deterministic. In the first example, the optimal overbooking level when the booking request and show-up rate are deterministic is shown in Figure IV-5. In this example, the weight and volume capacities are 30 and 40, respectively; thus, the magnitude of the capacity is 50 by using Pythagoras's theorem. In order to keep things simple, the booking request density is 0.75 as it equals to the ratio between the weight and volume capacities. The show-up rate is 0.75.

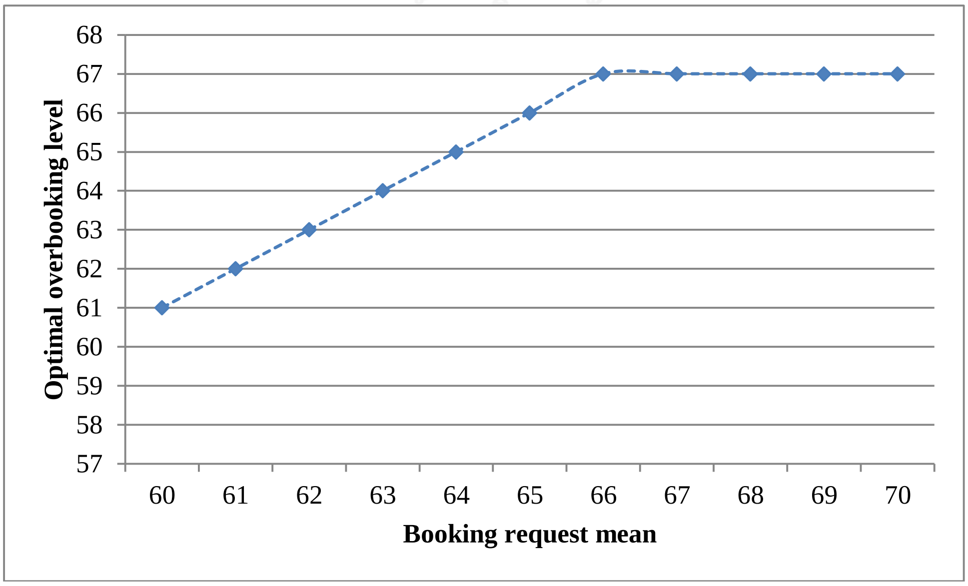

Figure IV-5 The optimal overbooking level as the booking request mean increases when the booking request variance is close to 0 and the show-up rate is 0.75

As expected, it can be observed that the booking request means that are over 67 are no longer affect the optimal overbooking level. This is because when the booking request mean equals to 67, the show-up booking request is about exactly 50, which is the magnitude of the capacity. In conclusion, when the booking request mean and show-up rate are deterministic, the optimal overbooking level can be determined easily by using the booking request value if the show-up booking request is not higher than the capacity. If the show-up booking request is higher than the capacity, the optimal overbooking level does not increase, and is limited at the capacity divided by the show-up rate.

The two figures below are the trend of the optimal overbooking level at different values of the booking request mean and variance when the booking request and show-up rate are stochastic. The explanations are as follow.

The first figure shows the trend of the optimal overbooking level when there is a growth in the booking request mean.

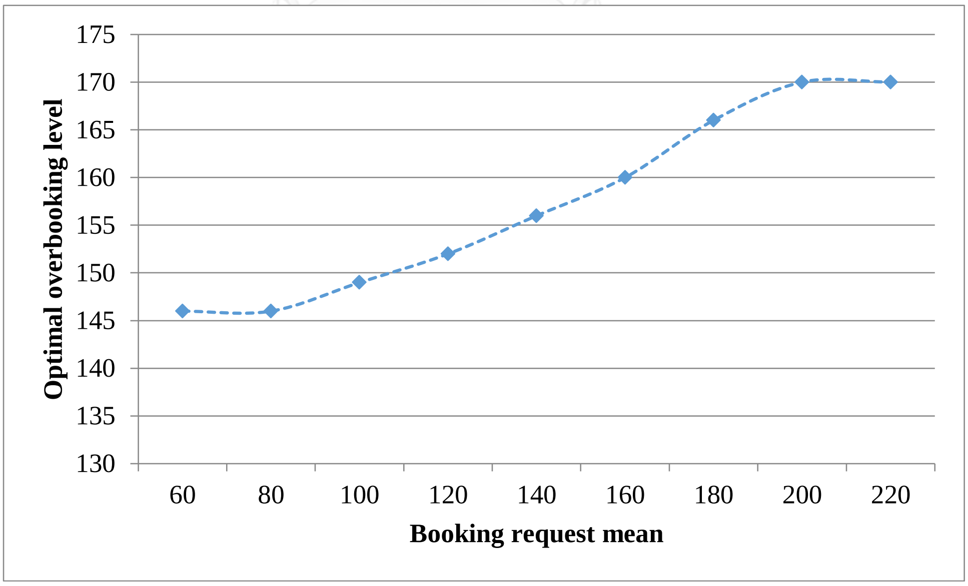

Figure IV-6 The optimal overbooking level as the booking request mean increases

Figure IV-6 presents the optimal overbooking level at different booking request means. As expected, the optimal overbooking level increases if there are more booking requests. As shown in Figure IV-6, at lower booking request mean levels, i.e., 60 to 80, the change in booking request mean has no impact on the optimal overbooking level because the booking request level is too low. Likewise, at higher booking request mean levels, i.e., 200 to 220, increasing booking request mean also has no impact on the optimal overbooking level anymore since the booking request mean is too high. When the booking request mean is high, it is possible to say that the booking request level is approaching infinity. It is likely that the booking requests will be offloaded more if the overbooking level is increased. The model, then, suggests that the optimal overbooking level be high enough to minimize the risk that the capacities are being spoiled. As well as minimizing the risk that the booking requests are being offloaded. However, the optimal overbooking does not continue to increase anymore because there will be only more offloaded booking requests occurred, and not reducing any spoiled capacities. In between those values of booking request mean, i.e., 80-200, there is a continuous increase in the optimal overbooking level. It is increased because the model tries to reduce the opportunity lost, which is spoilage cost, caused by the rejection and spoiled capacities.

The second figure shows the trend of the optimal overbooking level when the booking request variance increases.

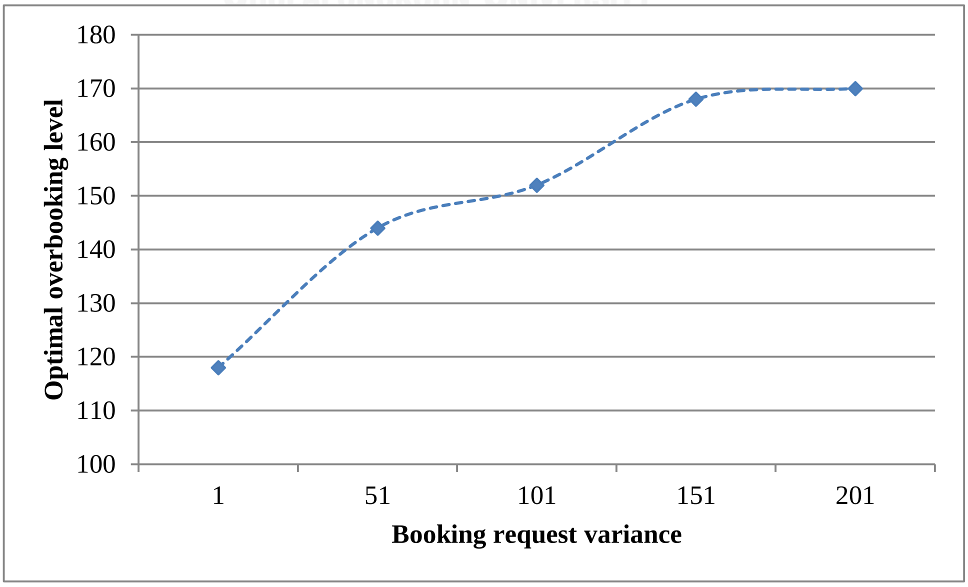

Figure IV-7 The optimal overbooking level as the booking request variance increases

Figure IV-7 indicates the trend of the optimal overbooking level at different values of the booking request variance. The graph above suggests that the optimal overbooking level be increased as the booking request variance increases. The reason behind this is as the booking request variance grows, the booking request level is likely to be either higher or lower than its mean. As mentioned before, when there are more booking requests, the optimal booking level is increased in order to minimize the risk that the capacities are spoiled which leads to spoilage cost. On the other hand, when the booking request variance increases, there is a chance that the booking request level may be lower; however, the lower booking request level does not cause more spoilage and offloading costs. Thus, the model suggests higher optimal overbooking level as the booking request variance increases.

4.1.4 Volume and Weight Capacity

Another two important factors for determining the optimal overbooking level are the volume and weight capacities. Both factors have significant impact on the optimal overbooking level. The effects of the two capacities are intuitive; that is, both capacities should cause the optimal overbooking level to increase if they are increased. However, in some cases, increasing the two capacities might not be able to increase the optimal overbooking level. Below, the figures and explanations are provided.

First, the impact of the volume capacity on the optimal overbooking level is studied. The first figure shows the trend of the optimal overbooking level when the volume capacity is increased.

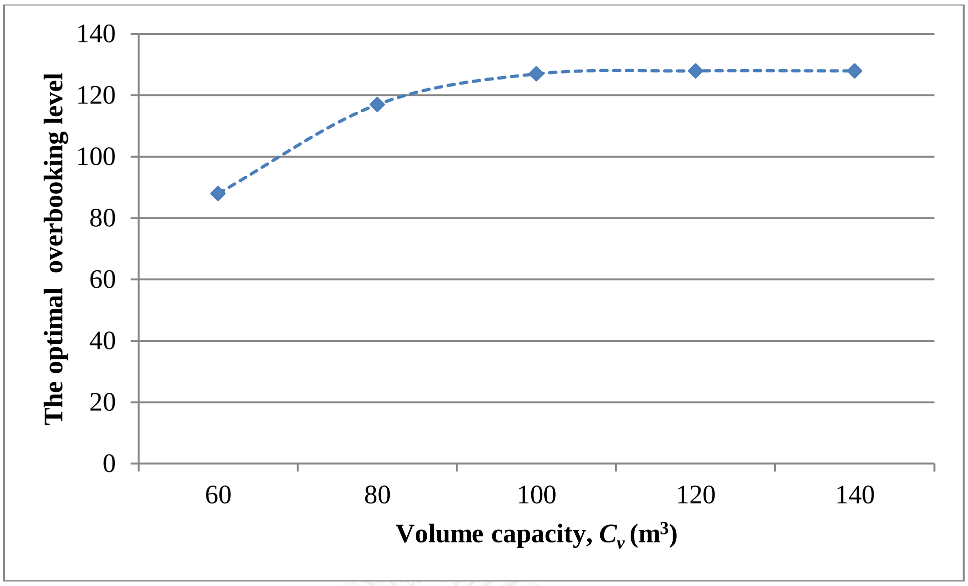

Figure IV-8 The optimal overbooking level as the volume capacity increases

Figure IV-8 shows the optimal overbooking level at different values of the volume capacity. It can be observed that the optimal overbooking level increases as the volume capacity increases as expected. However, when the volume capacity reaches and passes the point where C_w / C_v is equal to the booking request density, θ, the optimal overbooking level no longer increases; the optimal overbooking level stays the same no matter how high the volume capacity is. This is because when the volume capacity is low, the booking request density, θ, is more than the C_w / C_v and the volume capacity is an important factor to determine the optimal overbooking level. Therefore, increasing the volume capacity in this range causes the optimal overbooking level to increase. On the other hand, when the booking request density, θ, is less than C_w / C_v, the optimal overbooking level is determined by the weight capacity, and not the volume capacity anymore. Therefore, increasing the volume capacity when the booking request density, θ, is less than C_w / C_v does not increase the optimal overbooking level.

In the same way as the first figure, the trend of the optimal overbooking level when the weight capacity is increased is shown in the second figure.

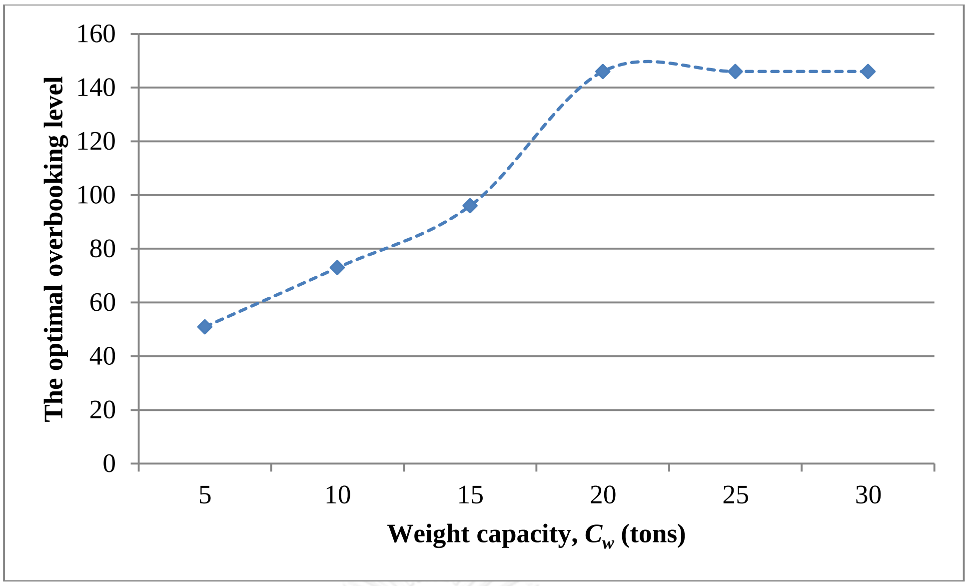

Figure IV-9 The optimal overbooking level as the weight capacity increases

Figure IV-9 shows the optimal overbooking level at different values of the weight capacity. As expected, it can be observed that the optimal overbooking level increases as the weight capacity increases. However, when the weight capacity reaches and passes the point where C_w / C_v is equal to the booking request density, θ, the optimal overbooking level no longer increases; the optimal overbooking level stays the same no matter how high the weight capacity is. This is because when the weight capacity is low, the booking request density, θ, is more than the C_w / C_v and the weight capacity is an important factor to determine the optimal overbooking level. Therefore, increasing the weight capacity in this range causes the optimal overbooking level to increase. On the other hand, when the booking request density, θ, is less than C_w / C_v, the optimal overbooking level is determined by the volume capacity, and not the weight capacity anymore. Therefore, increasing the weight capacity when the booking request density, θ, is less than C_w / C_v does not increase the optimal overbooking level.

From the observations above about the capacities, it can be conclude that the main effects for each capacity are positive. This means when the capacities increase, the optimal overbooking level should increase.

4.1.5 Booking Request Density

Booking request density, θ, is one of very important parameters that impacts on the optimal overbooking level; however, the effect of the booking request density depends on the ratio between the weight and volume capacities, C_w / C_v.

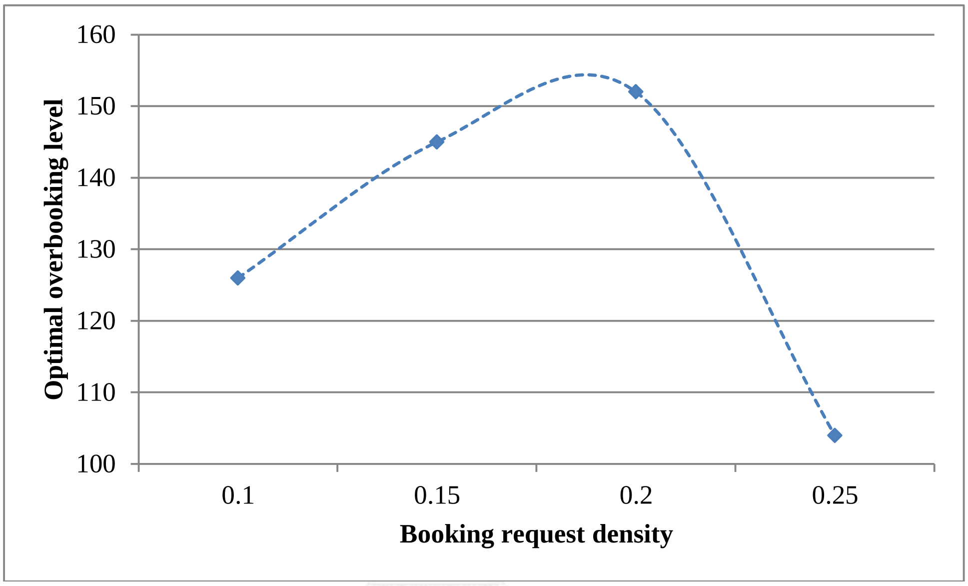

Figure IV-10 The optimal overbooking level as the booking request density increases when $C_v = 100$ and $C_w = 20$

Figure IV-10 demonstrates the optimal overbooking level trend when θ changes its value. It can be observed that the optimal overbooking level increases as θ increases until when θ reaches the ratio between weight and volume capacity, i.e., C_w and C_v , respectively; the optimal overbooking level, then, drops down as θ passes C_w / C_v and continues to increase. This is caused by the normal behavior of the two-dimensional air cargo overbooking model. When θ is lower than C_w / C_v , the volume capacity, C_v, can be utilized efficiently; that is, ideally, it can be filled one hundred percent with show-up booking request volume, B_{sv}, whereas the weight capacity, C_w, cannot be utilized efficiently. The volume capacity, C_v, becomes an important factor for calculating the total cost in order to find the optimal overbooking level. When θ is low, the optimal overbooking level is close to the volume capacity, C_v, in this case, it is 100. The optimal booking level reaches its highest point when θ

is equal to C_w / C_v because both volume and weight capacities, C_v and C_w, can be utilized efficiently. As θ passes C_w / C_v, there is a sudden drop in the optimal overbooking level.

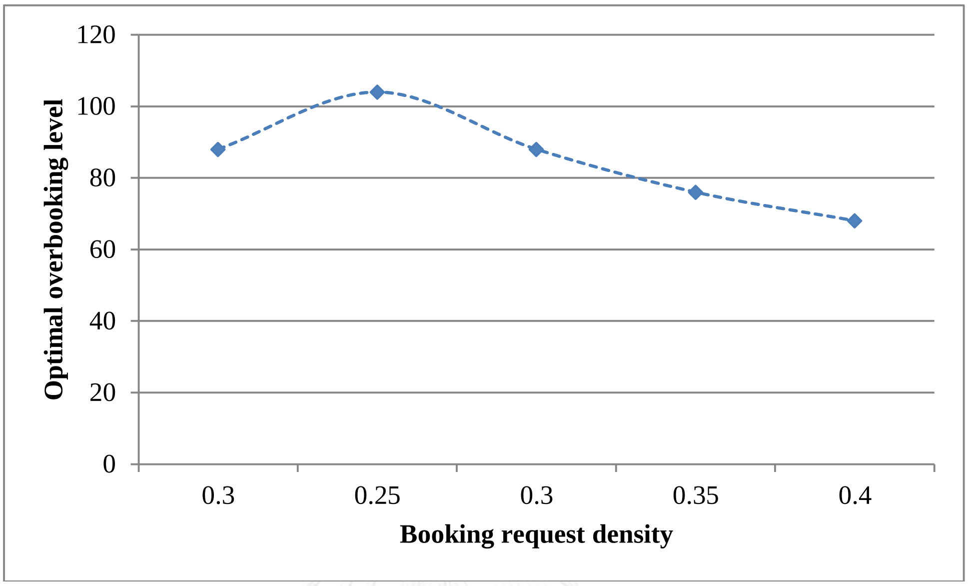

Figure IV-11 The optimal overbooking level as the booking request density increases when $\theta > C_w / C_v$

In the same way, when θ is higher than C_w / C_v, only the weight capacity, C_w, can be utilized efficiently; that is, ideally, it can be filled one hundred percent with show-up booking request weight, B_{sw}, whereas the volume capacity, C_v, cannot be utilized efficiently. The weight capacity, C_w, becomes an important factor for calculating the total cost in order to obtain the optimal overbooking level. Figure IV-11 shows the optimal overbooking level decreases as θ increases when θ is more than C_w / C_v. As θ passes and gets higher than C_w / C_v, the optimal overbooking level decreases and gets closer to C_w.

From the explanations for two cases above, it is possible to conclude that the capacities are utilized the most efficient when θ is equal to C_w / C_v, and the optimal overbooking level will be highest in that very specific case.

4.2 Design of Experiments

Due to the large number of factors, the design of experiments is used in order to design the appropriate number of the combination sets of the factors to be experimented with. Before the effects of every factor, including the interactions between them, can be correctly analyzed; the optimal overbooking for appropriate combination sets of factors must be known. This is why this step is necessary and has to be done before the analysis of variance. From the previous section, the main effect of each factor can be summarized in Table IV-1.

Table IV-1: Main effect of each factor on the optimal overbooking level

Factor	Main effect
b/a ratio	Negative
Average show-up rate	Negative
Booking request mean	Positive
Booking request variance	Positive
Volume capacity, C_v	Positive
Weight capacity, C_w	Positive
Booking request density, θ	Depends on C_w / C_v

As seen in Table IV-1, the booking request density, θ, does not have any main effects. Its effect depends on the ratio between the weight and volume capacities, C_w / C_v. Therefore, this factor is omitted at first when analyzing the effects of each factor. The booking request density, θ, is defined to be equal to the ratio between the weight and volume capacities, C_w / C_v, for all experiments. There are six factors left to be considered after eliminating the booking request density, θ. 2^n factorial experiment is used in order to properly screen the effects of each factor with interactions. Adopting this experiment makes the number of trials be sixty-four trials. Each factor is given with two levels; one level is for the minimum value for that factor, and the other is the maximum value. The minimum and maximum values for each factor are summarized in Table IV-2.

Table IV-2: The minimum and maximum values for each factor

Factor	Minimum value	Maximum value
b/a ratio	1	5
Average show-up rate	0.35	0.75
Booking request mean	80	200
Booking request variance	20	100
Volume capacity, C_v	100	200
Weight capacity, C_w	20	100

After the minimum and maximum values of each factor have been defined, the experiments are run in order to find the optimal overbooking level for all the sixty-four combinations of the factors. The optimal overbooking level obtained from all the combinations of the factors are, then, analyzed using analysis of variance method as shown in the next section.

4.3 Analysis of Variance

The results are analyzed using analysis of variance in Matlab program. The p-values for each factor are shown in Table IV-4.

From this section onwards, there are more variable notations that needs to be defined because these variables have to be used in analysis of variance and regression analysis. The variable notations used from this section onwards are summarized in Table IV-3.

Table IV-3: The variable notations used in analysis of variance and regression analysis

Notation	Meaning
b/a	Ratio between the offloading and spoilage costs per chargeable weight
$\overline{p}$	Average show-up rate
$\overline{B}$	Booking request mean
$Var(B)$	Booking request variance
C_v	Volume capacity
C_w	Weight capacity

Table IV-4: Multi-factor analysis of variance with interactions results

Source	p-value	Interpretation
b/a	0	Significant
$\overline{p}$	0	Significant
$\overline{B}$	0	Significant
$Var(B)$	0	Significant
C_v	0	Significant
C_w	0	Significant
$b/a*\overline{p}$	0.1855	Insignificant
$b/a*\overline{B}$	0.3122	Insignificant
$b/a*Var(B)$	0	Significant
$b/a*C_v$	0.3653	Insignificant
$b/a*C_w$	0.8725	Insignificant
$\overline{p}*\overline{B}$	0.005	Significant
$\overline{p}*Var(B)$	0	Significant
$\overline{p}*C_v$	0	Significant
$\overline{p}*C_w$	0	Significant
$\overline{B}*Var(B)$	0.3122	Insignificant
$\overline{B}*C_v$	0.0008	Significant
$\overline{B}*C_w$	0.1099	Insignificant
$Var(B)*C_v$	0.0019	Significant
$Var(B)*C_w$	0.1099	Insignificant
C_v*C_w	0	Significant

The interpretations above are based on 95% confidence level. It is a standard confidence level for many research. To make sure that the selected factors and their interactions to be put into regression analysis in the next step actually affect the optimal overbooking level notably. The greyed-out cells in Table IV-4 are the factors that does not affect the optimal overbooking level significantly enough to be worth selecting them. The other white cells in Table IV-4 are the selected factors and their interactions as they affect the optimal overbooking level significantly enough.

In this section, the base factors and interactions for predicting the optimal overbooking level have been identified. The next chapter, *MODEL SIMPLIFICATION*, uses these factors and interactions as a foundation for performing stepwise regression method in order to find the best simplified model in terms of R-sq(adj) value.

CHAPTER V
MODEL SIMPLIFICATION

As stated previously, the full overbooking model is very complicated. It requires coding, the necessary program, and takes so much time for obtaining the optimal overbooking level. For practical use, the full overbooking model has to be simplified. In reality, the optimal overbooking level has to be identified easily, shortly, and not too complicated. One way to get a practical overbooking level is to forecast it using the relationships between known parameters. Therefore, the full overbooking model is simplified by adopting the idea of forecasting. This chapter presents two simplified models: 1) regression model with interactions and 2) naïve method. The regression model is presented in *Regression Analysis* section whereas the naïve method is presented in *Naïve Method* section.

5.1 Regression Analysis

Due to the complexity of the two-dimensional air cargo overbooking model, the model cannot be simplified by reducing the variables, parameters, or terms easily. Therefore, regression method is used in order to predict the appropriate optimal overbooking level for a specific combination set of the factors. Regression is used when there is a response that needs to be predicted. In this case, the optimal overbooking level is predicted given that the values of each factor are known. Regression method is effective for many situations. For simplification of this overbooking model, it is very useful and effective because, as already stated, the formulated overbooking model cannot be simplified easily, and requires some time to obtain the optimal overbooking level. Regression method can give the appropriate optimal overbooking level and does not require the same amount of time that running the full overbooking model would.

There are many types of regression method, e.g., simple regression, multiple regression, linear regression, nonlinear regression such as polynomial regression, regression with or without interactions, and other combinations of the above, etc. That said, this study adopts only multiple regression with interactions, and it is able to predict the optimal overbooking level well enough. Most of the factors are based on the results of analysis of variance from the last chapter. However, there are some parameters which are modified to be inserted into the regression model, and the term derived from those parameters is neither a linear term nor an interaction of parameters. From the last chapter, there is one factor that has been omitted when performing the analysis of variance as it has no main effect on the optimal overbooking level, but, instead, its effect depends on the volume and weight capacities. The booking request density, θ, is omitted because it needs to be compared with on the ratio between the weight and volume capacities, C_w / C_v, in order to know its effect. As seen in Figure IV-10 and Figure IV-11, the optimal overbooking level is at its highest point when the booking request density, θ, is equal to the ratio between the weight and volume capacities, C_w / C_v. When they are not equal, two of the capacities are not utilized efficiently and the optimal overbooking level decreases. Thus, another effect of this factor is added in terms of the booking request density, θ; the volume capacity, C_v; and the weight capacity, C_w. As mentioned before, the optimal overbooking level decreases when the booking request density, θ, and the ratio between the weight and volume capacities, C_w / C_v are not equal; therefore, the absolute value function is used in order to give the same direction of effect when they are not equal; and the added factor can be written in a mathematical form as:

$$\left| \theta - \frac{C_w}{C_v} \right|$$

This function always returns a positive value whether the booking request density, θ, is more or less than the ratio between the weight and volume capacities, C_w / C_v. This way, it can be expected that the main effect of this function is recognized as negative in the regression model.

All the significant factors and the added factor are formed together, and taken as base variables in regression analysis. Stepwise method is, then, applied in order to find the most appropriate group of factors and their interactions, which gives the highest R-sq(adj) value, that should be put in the model. When the highest R-sq(adj) value is achieved, it means that the most appropriate group of factors and their interactions has been identified. Putting more factors into the model may increase the prediction accuracy, but it is not worth it as the accuracy gained is negligible. Therefore, the stepwise method is said to be a standard method in regression analysis to find the most appropriate group of factors that is included in the model by monitoring the R-sq(adj) value.

By applying the stepwise method to the multiple regression with interactions model at an alpha value of 0.1 using Minitab program, the summary of the results can be shown in Table V-1.

Table V-1: Stepwise regression analysis results summary

Factor	Coefficient	p-value
constant	91.71	0.005
$\lvert \theta - C_w / C_v \rvert$	-405.4	0.000
C_v	0.6091	0.078
C_w	1.4058	0.000
$b / a * Var(B)$	-0.11954	0.000
$C_v * \overline{B}$	0.0038731	0.000
$C_v * Var(B)$	0.003998	0.056
$C_w * \overline{p}$	-1.5969	0.000
$\overline{p} * \overline{B}$	-1.0373	0.000

The regression results always come with the R-sq and R-sq(adj) values. Those values obtained from performing this regression model are shown in Table V-2.

Table V-2: Measurement values corresponding to the regression model

Measurement	S	R-sq	R-sq(adj)
Value	7.33779	98.6%	98.3%

From the regression analysis above, the multiple regression model can be written in a mathematical expression as shown below:

$$Q = 91.71 - 405.4\left|\theta - \frac{C_w}{C_v}\right| + 0.6091C_v + 1.4058C_w - 0.11954\left(\frac{b}{a}Var\left(B\right)\right) + 0.0038731C_v\bar{B} + 0.003998C_vVar\left(B\right) - 1.5969C_w\bar{p} - 1.0373\bar{p}\bar{B} \quad (5.1)$$

As for general, majority of the people who apply regression analysis to their work tend to give priority to R-sq(adj) more than R-sq. Since R-sq value can always be increased if the number of factors is increased, R-sq value is biased and partly depends on the number of the factors. On the other hand, R-sq(adj) is unbiased; meaning that it calculates the R-sq based on the importance of that factor if it has to be put into the model. If that factor is not important enough, putting that factor into the model decreases the value of R-sq(adj). Therefore, R-sq(adj) is a standard, and unbiased value that should be taken into account when comparing the performance of the regression model.

The multiple regression results above show the R-sq(adj) value of 98.3%. This means the optimal overbooking level can be explained by this model approximately 98.3%, and is accurately predicted by this multiple regression model. With this multiple regression model, the appropriate optimal overbooking level can be determined. The time required for the results is far less than that of running the full overbooking model, and the results are accurate enough, and satisfied.

However, in addition to the R-sq(adj) value that has to be considered, regression validation must be performed in order for the regression model to be able to apply. There are four residual plots that needs to be validated: 1) normal probability plot 2) histogram 3) fitted plot and 4) observation order plot.

One of the regression assumptions is that the residuals' distribution must be normal; thus, the first two plots give the information of the residuals' distribution. If the red dots are formed in a straight line in the normal probability plot, and the histogram graph looks symmetrical and normal, the normal validation is qualified.

Another assumption is that the residuals have to be random and does not depend on the fitted value, or the order that the values are observed. The other two plots give the information whether the residuals are appeared in random or not. The red dots in both plots have to be arbitrary with approximate the same amount of the positive and negative residual values. If the red dots in those two plots appear to have no trend, the regression model is valid.

From the explanations above, the four plots of the multiple regression results are illustrated in Figure V-1.

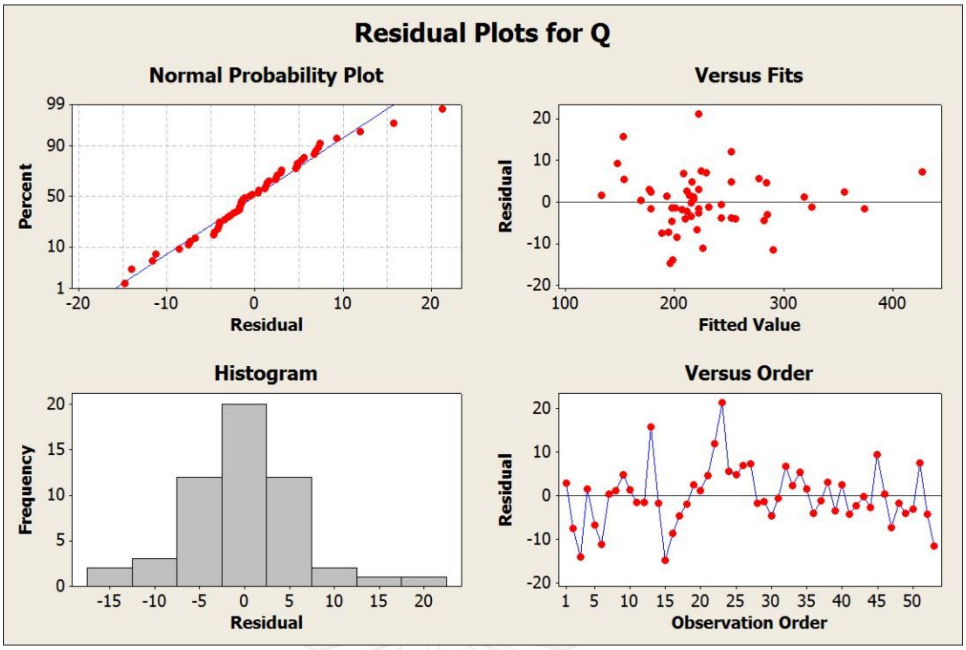

Figure V-1 Residual plots for the predicted optimal overbooking level

As seen in Figure V-1, it can be observed that the red dots in the normal probability plot appear to be a straight line, and the histogram seems symmetrical and normal. Moreover, the residuals look random and does not depend on the fitted value or observation order. The residual values have no trend in fitted value and observation order plots. Therefore, the necessary assumptions for this multiple regression model are valid and this regression model is able to be applied in order to predict the optimal overbooking level.

5.2 Naïve Method

Naïve method is an alternative method which requires the least factors in order to predict the optimal overbooking level. This method uses the common sense of guessing the appropriate value of the overbooking level when most of the variables and parameters are unavailable or unknown.

Generally, the overbooking level is estimated from two factors: 1) capacity and 2) show-up rate. For the two-dimensional air cargo overbooking problem, the capacity is also two-dimensional; therefore, the level of the two-dimensional capacity, which can be calculated from the volume and weight capacities, is used. This method assumes that the booking request level is more than enough, and the capacities are always fully filled with the booking requests.

Basic idea of estimating the optimal overbooking level is to reserve more capacity that is equal to the booking request level that would not show-up. When the booking request level is at full capacity, show-up booking request level is calculated from the multiplication of the show-up rate and the capacity. The show-up booking request level can be expressed in terms of notations as:

$$Cp$$

Therefore, the booking requests that would not show-up can be calculated by the multiplication of the not-show-up rate, which can be obtained from one minus the show-up rate, and the capacity; and it can be written as:

$$(1-p)C$$

To reserve more capacity that is equal to the booking request level which would not show-up, the overbooking level is calculated by adding the not-show-up booking request level to the capacity. The reserved capacity, which is the predicted overbooking level, becomes:

$$Q = C + (1-p)C$$

By reforming and applying the two-dimensional characteristic of the capacities, and using the average show-up rate as a representative value of the overall show-up rates; the predicted overbooking level for this naïve model becomes:

$$Q = 2\sqrt{C_v^2 + C_w^2} - \bar{p}\sqrt{C_v^2 + C_w^2} \qquad (5.2)$$

The performance of this model is explored in the next section.

5.3 Model Comparison

In this section, the two simplified models are compared in terms of the total cost. Intuitively, the performance of the multiple regression model should outperform the naïve method model as it takes more variables and parameters to predict the optimal overbooking level. In addition, the R-sq(adj) value of the multiple regression model is at 98.3% which is considered as very high. Therefore, in most cases, the multiple regression model is able to predict the optimal overbooking level of which the total cost is almost identical to that of the real optimal overbooking level. However, there are situations that the naïve method model is capable of predicting the proper optimal overbooking level.

There are many factors that can be considered when performing comparison between the overbooking models. This section selected two changeable factors which are not included the naïve method model in order to observe the performance of each model.

The first factor to be observed is the offloading and spoilage costs per chargeable unit weight, b/a, as, most of the time, they are not equal to each other; and they are inconstant. Figure V-2 shows the total costs generated from different overbooking models.

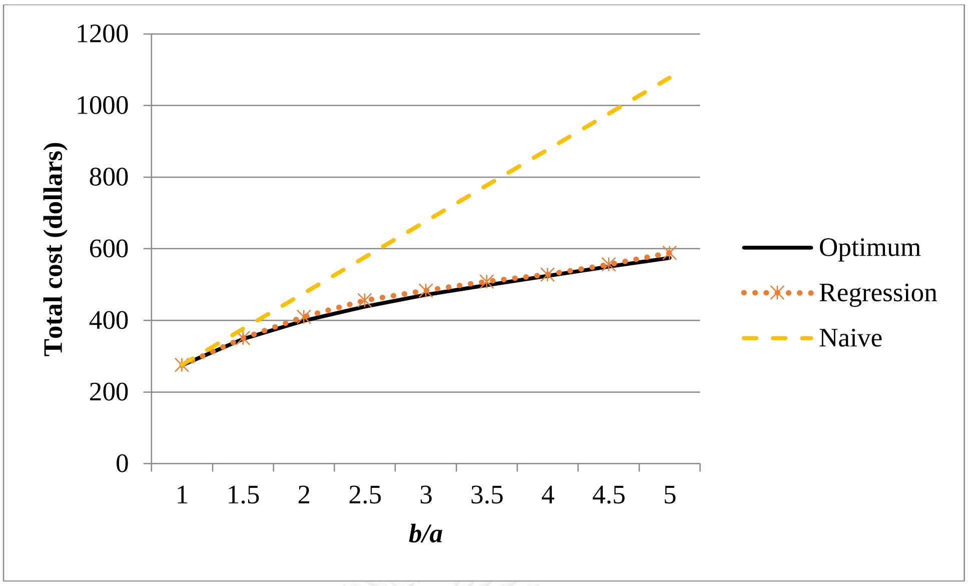

Figure V-2 The total cost comparison as b/a increases

Figure V-2 shows the total cost for each model at different values of the ratio between offloading and spoilage costs per chargeable unit weight, b/a. It can be observed that the total costs of the multiple regression model, which is the orange line, are very close to that of real overbooking model, which is the blue line. This means the multiple regression model predicts the optimal overbooking level closely enough that the total costs are very close to the minimum total cost. In this case, naïve method model works well when the ratio between offloading and spoilage costs per chargeable unit weight, b/a, is low. As the naïve method model does not consider the offloading and spoilage costs, when the ratio between the two costs, b/a, changes; the naïve method model can no longer predicts the optimal overbooking level accurately.

Another important factor which is omitted in many overbooking models is the booking request variance. Often enough, the assumption of the booking requests are more than the capacities is used, which, consequently, ignores the booking request variance as it does not make any difference anymore when the booking request level is large enough. This includes the naïve method model; this model omitted the booking request level and variance because it assumes that the booking requests are large enough to always fill the capacities.

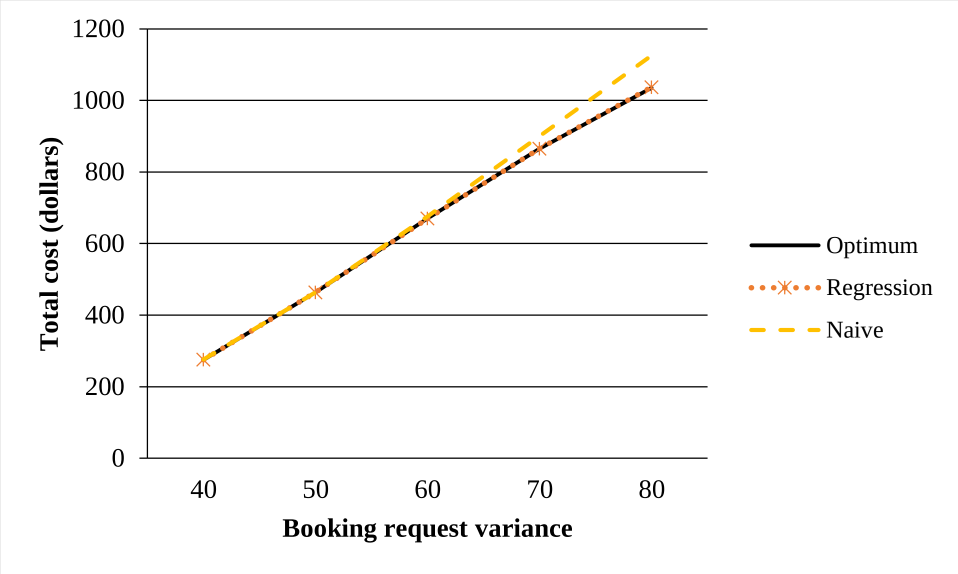

Figure V-3 The total cost comparison as the booking request variance increases

Figure V-3 shows the total costs of each model at different values of the booking request variance. It can be seen that the optimal total costs, the straight line, and the multiple regression total costs, the dotted line, are almost identical. The total costs generated from the multiple regression model are almost completely superimposed the optimal total costs. This means, again, the multiple regression model is capable of predicting the optimal overbooking level which generates the total cost nearly at the minimum. Naïve method model, in this case, performs better than expected as the total costs generated from the naïve method model follow the trend of the optimal total costs. Although the total costs may start to separate when the booking request variance is higher, the results are acceptable in this situation.

The difference in the booking request mean also play a vital role in predicting the optimal overbooking level. As the naïve method does not take the booking request mean into account when predicting the optimal overbooking level, whereas the regression method does; the difference should be presented between these two methods when predicting the optimal overbooking level at different booking request means. Figure V-4 shows the total costs for each model at different booking requests. It should be noted that the capacity level used in Figure V-4 is at about 140.

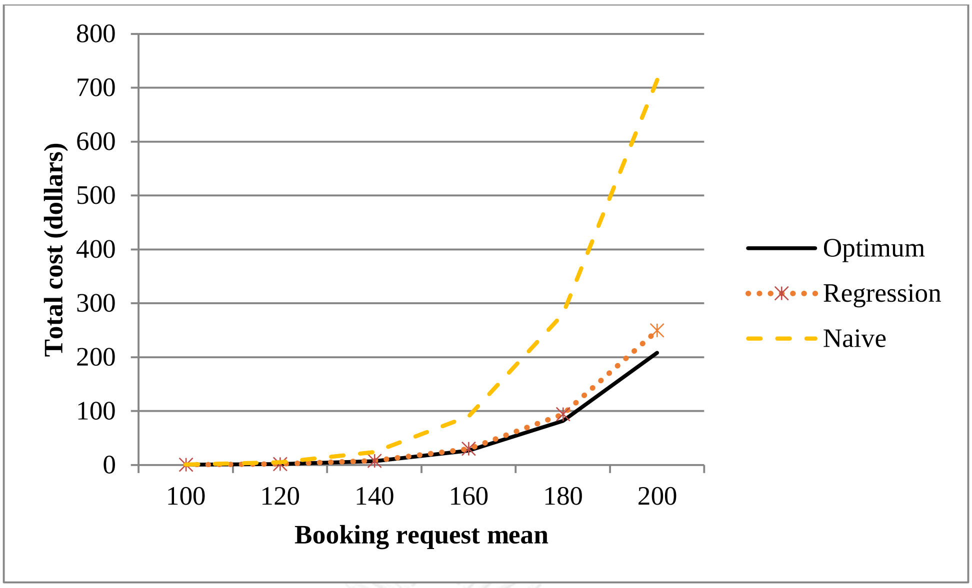

Figure V-4 The total cost comparison as the booking request mean increases

It can be obviously seen that as the booking request mean increases, the naïve method is worse in terms of the total cost. By taking a closer look, when the booking request mean is at the capacity level or lower, the naïve method model can predict the optimal overbooking level well enough. This is because the naïve method model does not take the booking request mean into account, and assume the booking request mean to be at the capacity level. Moreover, when the booking request mean is lower than the capacity level, the error in overbooking level does not seem to affect the total cost much. Therefore, the naïve method model can predict the optimal overbooking level well when the booking request mean is at the capacity level or lower.

Another important factor that is not included in the naïve method model is the booking request density. The next figure shows the performance of each model at different booking request densities. It should be noted, again, that the ratio between the weight and volume capacities used in this illustration is 0.2. Figure V-5 illustrates the total costs generated by each prediction method model at different booking request densities.

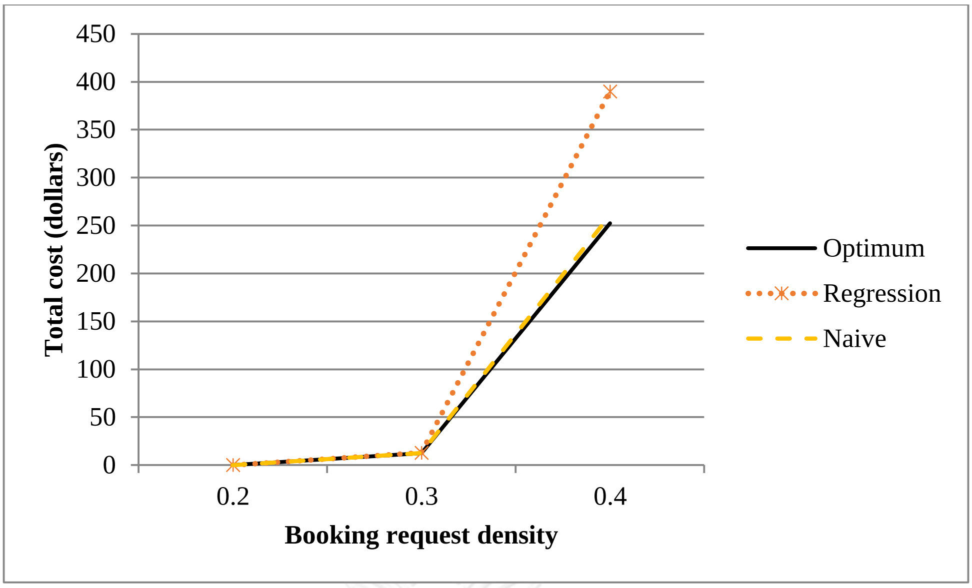

Figure V-5 The total cost comparison as the booking request density increases

Strangely, in this example, it can be observed that as the booking request density goes farther away from the ratio between the weight and volume capacities, the regression model generates higher total cost than the naïve method model. However, this does not mean that the regression model predicts less accurate than the naïve method model. This occurrence has a logical explanation.

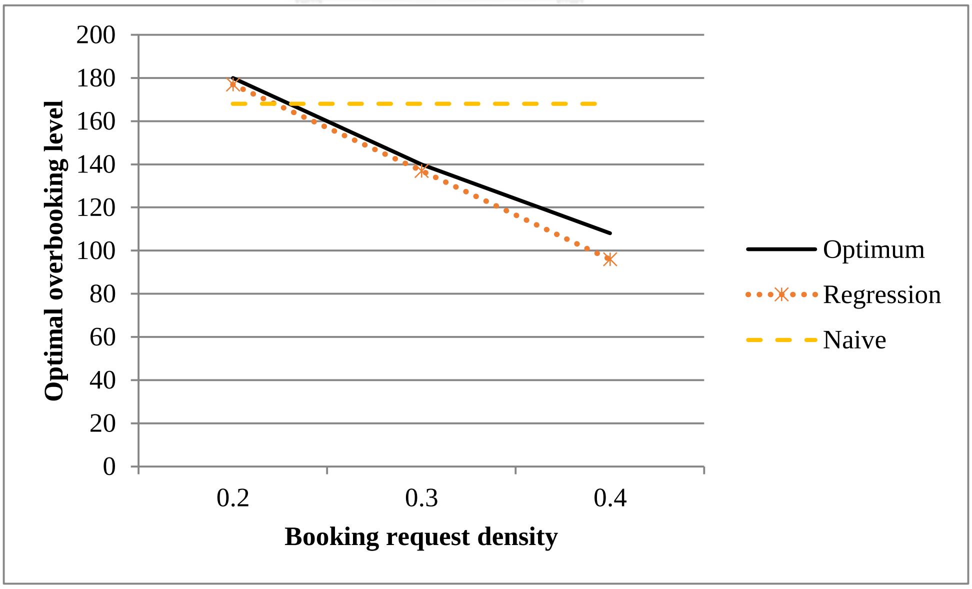

Figure V-6 The optimal overbooking level comparison as the booking request density increases

Figure V-6 shows the optimal overbooking level predicted by each model as the booking request density increases farther away from the ratio between the weight and volume capacities of 0.2. As expected, the regression model can predict the optimal overbooking level closer to the real optimal overbooking level than the naïve method model However, in this example, the regression model predicts the optimal overbooking level lower than the real optimal overbooking level as the booking request density increases which causes more spoilage cost. On the other hand, the naïve method model predicts the optimal overbooking level higher than the real optimal overbooking level which should cause more offloading cost. However, the booking request used in this illustration is lower than the capacity; therefore, by setting the overbooking level too high does not cause much offloading cost compared to spoilage cost generated when the overbooking level is set too low. Therefore, in this illustration, the spoilage cost generated by the regression model is higher than the offloading cost obtained by the naïve method model even though, in fact, the regression model predicts the optimal overbooking level more accurate.

In conclusion, this chapter presents optimal overbooking level prediction methods as the full model is too complicated and takes much time to obtain the optimal overbooking level. The models presented in this chapter are multiple regression with interactions model and naïve method model. Most of the time, the multiple regression model can predict the optimal overbooking level accurately enough to the real optimal overbooking level obtained by running the full overbooking model with an R-sq(ad) of 98.3%. On the other hand, the naïve method model is not always able to predict the optimal overbooking level closely to the real optimal overbooking level obtained by running the full overbooking model, although it is easier to use as it requires less factors. However, there are situations that the naïve method works just fine; for example, the important factors for running the full overbooking model and multiple regression model may be unavailable, or the company knows that the booking request level is about capacity and may not need the real optimal overbooking level; the company may just need a rough estimation, then the naïve method model can give just a right answer.

CHAPTER VI
CONCLUSION

This thesis presents the two-dimensional air cargo overbooking model and proposed methods to simplify it. There are three air cargo overbooking models presented in this thesis. The first overbooking model is the full two-dimensional air cargo overbooking model, and requires computer processing within the Matlab program due to its complexity. This overbooking model formulates the spoilage and offloading costs in every condition possible at the date of departure. The spoilage cost in this overbooking model is calculated differently than others' research have ever studied. The other two models are simplified models. One of them uses multiple regression with interactions method; while another method (called Naïve method) uses common sense of guessing the appropriate overbooking level. From this study, this chapter can be divided into three main sections: *1) Summary*, *2) Problems and Obstacles*, and *3) Suggestions*.

6.1 Summary

By studying this research, the two-dimensional air cargo overbooking models presented in this thesis, and, also, the results from each model can be summarized in many aspects

6.1.1 The Two-Dimensional Air Cargo Overbooking Model

There are some main points that should be summarized about the two-dimensional air cargo overbooking model. The main points are outlined below.

1) Spoilage Cost Calculation

The two-dimensional air cargo overbooking model presented in this thesis has an objective of minimizing the total cost. The total cost is composed of spoilage and offloading costs. The offloading cost calculation concepts are the same as it is an actual cost that occurs when there are offloaded booking requests whereas the spoilage cost calculation in this overbooking model is different. In this study, spoilage cost occurs only when there is rejection, and both show-up booking request volume and weight are lower than the capacities.

2) Random Variables and Parameters

The random variables are the booking request level and the booking request show-up rate. The booking request level is a continuous random variable whereas the booking request show-up rate is a discrete random variable. In fact, the booking request show-up rate should be a continuous random variable, but due to the complexity of the model, it cannot be integrated if the booking request show-up rate is continuous. By altering the booking request show-up rate to a discrete random variable, the model is still practical and is easier to find the optimal overbooking level.

Most of the parameters are well-known and accepted, including the volume and weight capacities, and the spoilage and offloading costs per chargeable unit weight. The booking request density, however, should be a random variable. Again, due to the complexity of the two-dimensional air cargo overbooking model, the model is unable to be integrated if the booking request density is a continuous random variable. Therefore, the booking request density is altered to be one of the parameters; that is, it is a constant value for each run. It still makes sense as this research mainly concerns with the overall booking requests at the date of departure; thus, the booking request density can be just a value that represents the overall booking requests.

3) **Usage and Results**

This two-dimensional air cargo overbooking model requires a program called Matlab in order to find the optimal overbooking level. In addition, the optimal overbooking level finding takes a lot of time. Therefore, it is not practical to be applied in real life. However, the results from this model always at the optimal value: the total cost is always at minimum. Thus, this model can be used as a benchmark for other models.

6.1.2 The Multiple Regression Model

The multiple regression model is used in order to predict the optimal overbooking level as the full model takes much more time and a specific program to obtain the optimal overbooking level. Thus, the regression model simplifies the procedure of obtaining the optimal overbooking level by putting all important factors in the regression model and obtaining the predicted answer in no time. There are some points that should be summarized in the multiple regression model. The points are outlined below.

1) **Important Factors**

Obtaining the important factors for the multiple regression model involves two main procedure: 1) analysis of variance and 2) stepwise regression method. First, the design of experiments is used in order to define the proper combinations of factors to be performed in analysis of variance. As the model includes interactions, the least number of experiments that is able to calculate the interactions between factors is 2^n, where n is the total number of the factors. Then, analysis of variance is performed with the results according to the design of experiments.

The $\left|\theta - \frac{C_w}{C_v}\right|$ factor is first omitted in the design of experiments and analysis of variance procedures. This factor is separately put back into the starting factors in the stepwise procedure. Then, the stepwise method is performed in order to find the combination of factors, their interactions, and their coefficients that gives the highest R-sq(adj) value. By performing the steps above, the multiple regression model for the optimal overbooking level prediction can be expressed as:

$$
\begin{aligned}
Q \quad = \quad & 91.71 - 405.4\left|\theta - \frac{C_w}{C_v}\right| + 0.6091C_v + 1.4058C_w - 0.11954\left(\frac{b}{a}Var\left(B\right)\right) \\
+ \quad & 0.0038731C_v\bar{B} + 0.003998C_vVar\left(B\right) - 1.5969C_w\bar{p} - 1.0373\bar{p}\bar{B}
\end{aligned}
$$

2) Usage and Results

The multiple regression model does not require a fancy program, any calculators will work. In this study, the optimal overbooking level prediction is done in Microsoft Excel program. It predicts the optimal overbooking level in no time. This multiple regression model has an R-sq(adj) of 98.3%. Thus, it can be said that the optimal overbooking level can be predicted at the accuracy of 98.3%. Therefore, this model is capable of being implemented extensively.

6.1.3 The Naïve Method Model

In the same way as the regression model, the naïve model approximately predicts the optimal overbooking level, but requires much less factors. There are some points regarding to the naïve method model to be summarized. The points are outlined below.

1) Assumption

The naïve method model assumes that the booking requests are always more than the capacities. The factors that are included in the naïve method model are the capacities and the average show-up rate. The optimal overbooking level prediction equation of the naïve method can be expressed as:

$$Q = 2\sqrt{C_v^2 + C_w^2} - \bar{p}\sqrt{C_v^2 + C_w^2}$$

2) Usage and Results

This model is the easiest model to implement as it requires the least factors to predict the optimal overbooking level. However, with the easy implementation of the model, this model does not always give accurate predicted optimal overbooking level. Unlike the multiple regression model, there is no accuracy measurement value in the naïve method model; thus, it has less reliability on the results.

In conclusion, the multiple regression model can be implemented extensively to determine the appropriate overbooking level without having to run the full model and takes far less time. On the other hand, the naïve method model is the easiest to implement and requires least factors. The naïve method model is appropriate to use when there are too few variables and parameters available although it does not always accurately predict the optimal overbooking level.

6.2 Problems and Obstacles

There were problems and obstacles encountered along the way while studying this research. The problems and obstacles are outlined below.

1. Defining variables and parameters was among the first problems and obstacles encountered in this thesis. In the full model, the booking request level, show-up rate, and density should all be continuous random variables. At first, all the factors described were defined as continuous random variables, but the model could not be integrated. Therefore, the booking request show-up rate was redefined to be a discrete random variable, and the booking request density was redefined to be a parameter.

2. Due to the complexity of the two-dimensional air cargo overbooking model, the optimal overbooking level could not be identified easily; therefore, the solution finding of this model was one of the challenges that had to be accomplished.

3. Challenges due to the program was another problem in completing this thesis. The program used almost solely in the third and fourth chapter was Matlab. This program requires computer coding knowledge. Without proper coding techniques and the program, this thesis could not be completed, or could be much harder to be completed.

6.3 Future Research Suggestions

The overbooking technique has been extensively applied to many companies and industries; however, there are few research on air cargo overbooking problem. Moreover, most of the air cargo overbooking models are still one-dimensional. Therefore, there are plenty of rooms that the air cargo overbooking model can be improved.

This research already improved some of the gaps in the field of air cargo overbooking. However, in this research, there are still imperfections in the two-dimensional air cargo overbooking model. The points at which the future research might be worth looking into are outlined below.

1. Booking request level

The booking request level in this thesis is considered as a continuous random variable with a distribution of normal. In reality, the distribution of customers is often exponential. The distribution should be altered to other distributions as well; and the results corresponding to the distributions should be monitored.

2. Booking request show-up rate

The booking request show-up rate in this thesis is considered as a discrete random variable with a distribution of uniform. This variable should be a continuous random variable with some other distributions.

3. Booking request density

The booking request density in this thesis is considered as a parameter which means, in an experiment, the booking request density does not change and remains constant. This variable, too, should be a continuous random variable with various distributions.

4. Data

Future research may use real data to test the overbooking model in order to reliably verify the model. This research focuses on the theory and uses fake data, which is estimated from the real ones, to run the model.

5. Assumption

This thesis makes a lot of assumptions which may be inappropriate to be assumed in real-life situations. Future research might be able to improve some of the assumptions made in this thesis.

The suggestions above are the only basic ones of many points that can be improved in this thesis. There are many more gaps in the field of air cargo overbooking problem that is waiting for future research to look into and improve them.

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