CHAPTER II.

DESCRIPTION OF THE ANALOGUE COMPUTER

The variable quantities of a physical problem are related in various ways. An analogue computer operates by representing these variables by D.C. voltages, and the circuit is so constructed that the relations between the D.C. voltages are the same as the relations between the physical variables. These relationships are usually in the form of differential equations. In order to understand how a differential equation is solved on an analogue computer, it is necessary study in some detail the basic computing components. These computing components are the building blocks in forming any model and their characteristics are explained in sufficient detail below to enable the reader to follow the discussion in Chapter IV.

(2.1) Operational Amplifiers[1],[2]

An operational amplifier, sometimes called a D.C. amplifier, which performs the basic mathematical operations necessary for the solution of the problems is the heart of the modern analogue computer. Operational amplifiers used in an analogue computer are usually constructed with vacuum-tubes or transistors and are voltage amplifiers, having an output voltage range ±100 volts for vacuum-tube and ±10 volts for transistorized. The D.C. amplifier used in an analogue computer is a high gain direct-coupled amplifier with negative feedback. In practice, the forward gain of the amplifier, denoted by -A, has a absolute value in the range of 10^4-10^8 for all expected computer operations. The gain of an amplifier is given by

$$A = -\frac{e_o}{e_g} \qquad \text{-----------} \qquad (2.1)$$

where e_g represents the grid voltage, and e_o the output voltage of the amplifier. (See Fig.2.1).

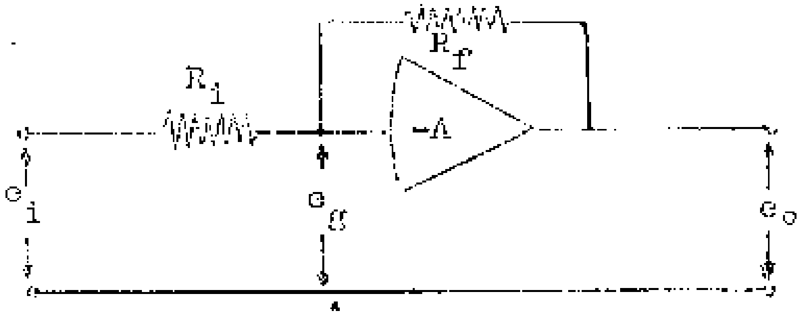


Fig.2.1: The D.C.-operational amplifier

In Fig.2.1, e_i represents the input voltage, R_f the feedback resistor and R_i the input resistor.

From equation (2.1), if -A approaches $-\infty$, e_g approaches zero. In practice this conditon is realized by using high-gain amplifiers. The linear mathematical operations are performed by using a high-gain D.C. amplifier, as shown in equation (2.2)

$$e_o = - \frac{R_f}{R_i} e_i \qquad \text{---------------} \quad (2.2)$$

which can be written $e_o = -K\ e_i$, where $K = \frac{R_f}{R_i}$. This amounts to multiplication by a constant coefficient and this ratio $\frac{R_f}{R_i}$ is comparatively small (less than 100).

From equation (2.2), if $R_f > R_i$ then $e_o > - e_i$ and there is a voltage gain. If $R_f = R_i$, then $e_o = -e_i$ and the amplifier becomes a sign-changer (also called an inverter). Hence any desired gain in equation (2.2) can be produced simply by choosing the correct elements for R_i and R_f. Furthermore amplifiers are capable of performing, in addition, the operations of:[3]

a) Multiplication by -1, or sign-changer

b) Multiplication by a constant

c) Addition or summation

d) Integration. (R_f is replaced by a capacitor)

a) <u>Operational Amplifier as Sign-Changer or Inverter.</u>

From equation (2.2), if $R_f = R_i$ the output voltage will be equal but opposite in sign to the input voltage. We obtained

$$e_o = - e_i \qquad \text{---------------} \quad (2.3)$$

Thus the operational amplifier can be used as a sign-changer or an inverter. The circuit of the sign-changer is represented in Fig.2.2.

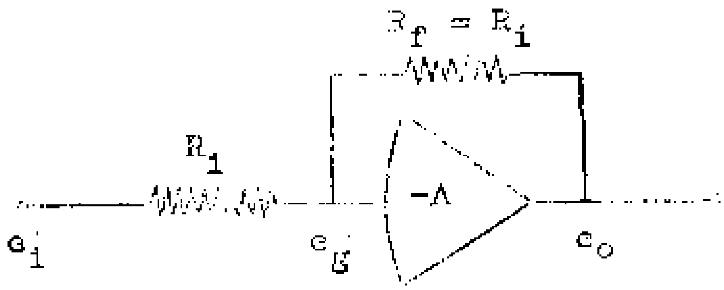


Fig.2.2 Operational amplifier as a sign-changer.

b) Operational Amplifier for multiplying by a constant

If the value of the feedback resistor is β times the value of the input resistor, then the output voltage will be β times the input voltage. From equation (2.2) when $R_f = \beta R_i$, where $\beta > 1$, we have

$$e_o = -\beta e_i , \quad \text{where } \beta > 1 \quad \text{------} \quad (2.4)$$

The sign-changer circuit is shown in Fig.2.3

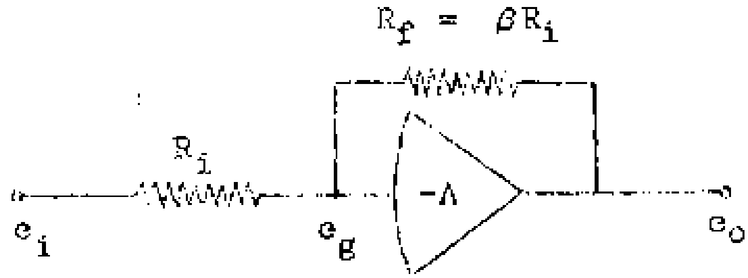


Fig.2.3 Operational Amplifier for Multiplication by β

Division by a constant β can be treated as multiplication using $1/\beta$. The relation between the input voltage and the output voltage is represented in equation (2.5).

$$e_o = -\frac{1}{\beta} e_i \,, \quad \text{where } \beta > 1 \quad \text{------} \quad (2.5)$$

c) Operational Amplifier as Adder or Summer.

More than one input can be applied to the operational amplifier. In this case the operational amplifier becomes an adder or summer.

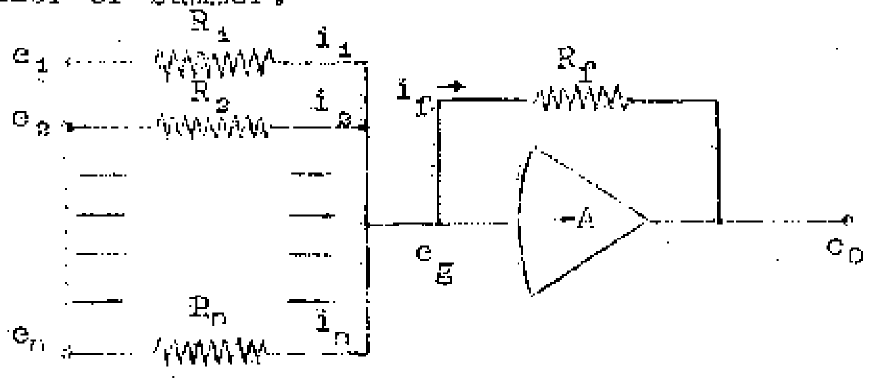


Fig.2.1g Operational Amplifier as Adder.

The inclusion of more than one input resistor to a high gain d-c amplifier circuit, each resistor having a voltage applied to it, changes the input-output relationship to:

$$\frac{e_o}{R_f} = -\left[\frac{e_1}{R_1} + \frac{e_2}{R_2} + \frac{e_3}{R_3} + \dots\dots + \frac{e_n}{R_n} \right]$$

[That is to say the current i_f flowing through the feedback resistor must be the algebraic sum of currents i_1,

$i_2, \ldots i_n$, flowing through the input resistors since the amplifier input voltage and current are zero. Thus:

$$e_0 = -\left[\frac{R_f}{R_1} e_1 + \frac{R_f}{R_2} e_2 + \ldots\ldots + \frac{R_f}{R_n} e_n \right]$$

$$\therefore e_0 = -\sum_{k=1}^{n} K_k e_k \quad \text{------------} \quad (2.6)$$

where $K_k = \frac{R_f}{R_k}$, $k = 1, 2, \ldots\ldots$, and the e_k are the input voltages.]

Hence the operational amplifier can be used to add. In equation (2.6), by using equal values for all resistors, one obtains a simple algebraic summation with the usual inversion associated with every computing amplifier. If resistors have different values, then each input voltage is multiplied by a factor, given by the ratio of the feedback resistor to the input resistor, before it is added to the sum.

d) Operational Amplifier as Integrator.

Integration is performed by replacing the feedback resistor R_f by a capacitor C_f as shown in Fig.2.5.

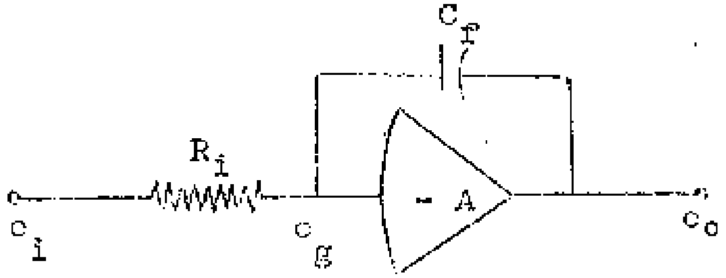


Fig.2.5 Integrator with single-input.

The mathematical relation between the input voltage and the output voltage of the operational amplifier used as an integrator is shown in equation (2.7):

$$e_o(t) = -\frac{1}{R_i C_f} \int_o^t e_i(t)\,dt + e_i(o) \quad \text{---- (2.7)}$$

where $e_i(o)$ is the constant of integration (initial condition) and is the voltage across the feedback capacitor C_f at t = o. Thus the operational amplifier can integrate.

It is necessary to be able to control the operation of integration and also to be able to apply an initial charge to the capacitor to set the initial value $e_i(o)$ of the output voltage e_o. This is done by connecting a relay[7],[8] and an initial condition power supply across the amplifier as shown in Fig.2.6. Initially the relay switch is closed. It is then opened to carry out the computation.

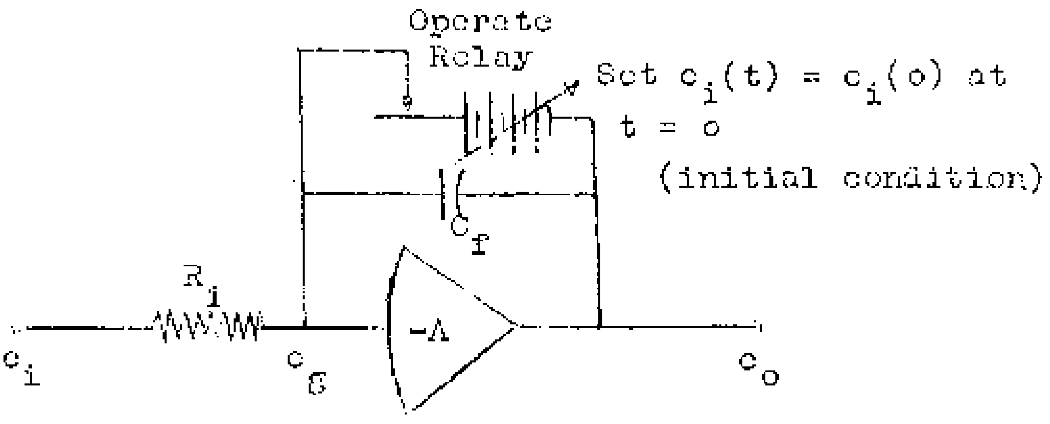


Fig.2.6: Integrator with Operate Relay and Initial Condition.

Several voltages can be connected to the input of the integrating amplifier, as shown in Fig.2.7.

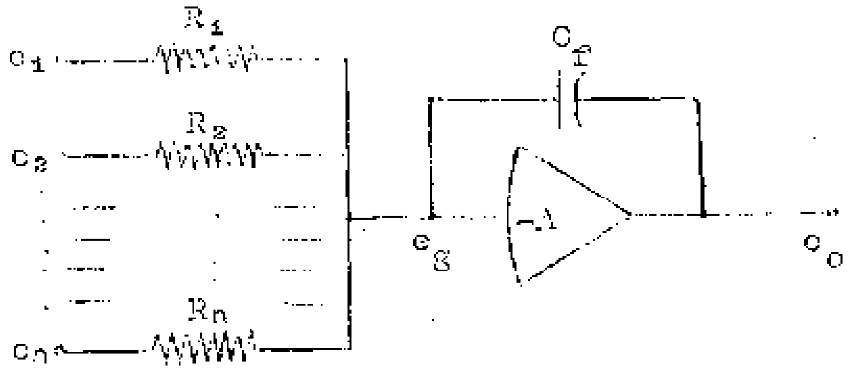


Fig.2.7: Multiple inputs integrator

The output voltage of the integrating amplifier for multiple inputs is:

$$e_o(t) = -\frac{1}{C_f}\int_o^t\left(\frac{e_1(t)}{R_1} + \frac{e_2(t)}{R_2} + \ldots\ldots + \frac{e_n(t)}{R_n}\right)dt + e_o(o).$$

If we let $\frac{1}{C_f R_i} = K_i$, then we obtain

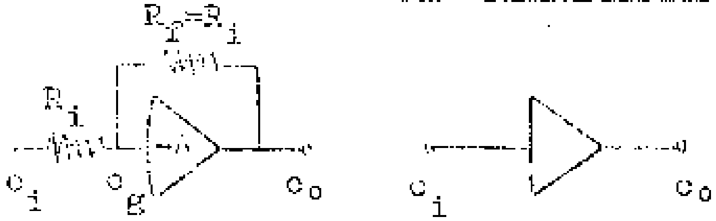
$$e_o(t) = -\int_o^t \sum_{i=1}^{n} K_i e_i(t)dt + e_o(o) \quad \text{-------(2.8)}$$

where $e_o(o)$ is the constant of integration.

Furthermore, an amplifier may be used to differentiate with respect to t, but since this operation magnifies errors and fluctuates in the computer voltage, it is rarely used.

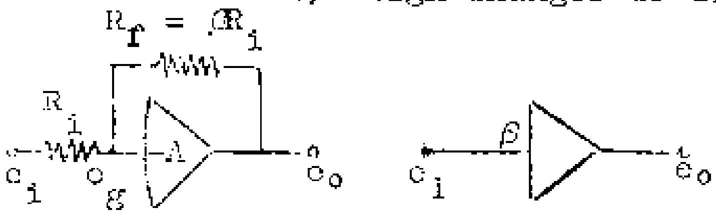
The symbols frequently used to represent the various functions of an operational amplifier necessary to solve differential equations are shown in Fig.2.8, together with the corresponding mathematical functions.

Computer Diagram	Computer Symbol	Mathematical Functions



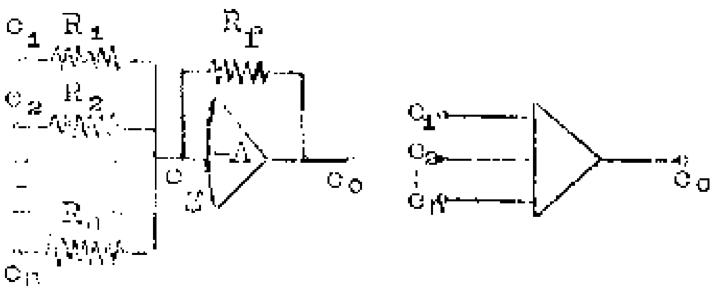
$e_o = -e_i$

a) Sign-Changer or Inverter



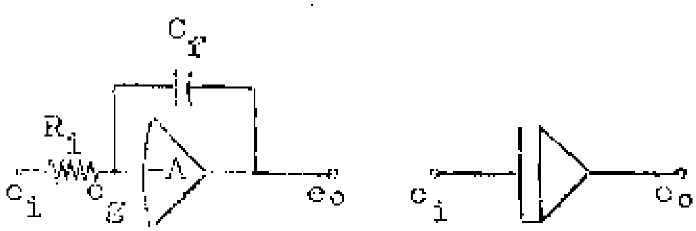
$e_o = -\beta e_i$

b) Multiplication by β



$e_o = -\sum_{i=1}^{n} e_i$

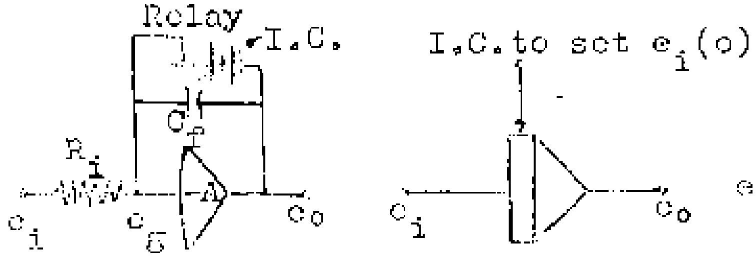
c) Adder or Summer



$e_o(t) = \frac{1}{R_i C_f} \int_o^t e_i(t)dt$

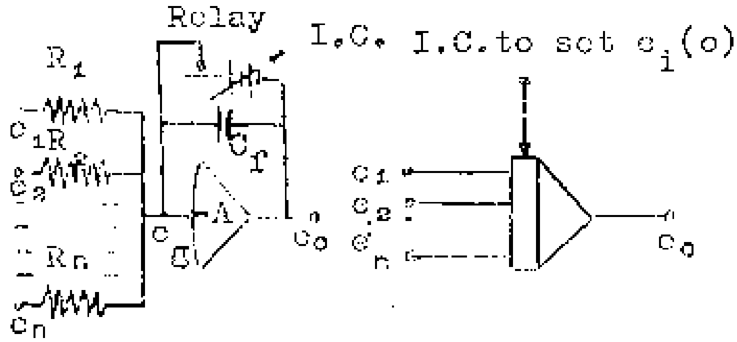
d) Single Input Integrator

Computer Diagram | Computer Symbol | Mathematical Functions



$$e_o(t) = -\frac{1}{R_i C_f} \int_o^t e_i(t)dt + e_i(o)$$

e) Single Input Integrator with I.C.



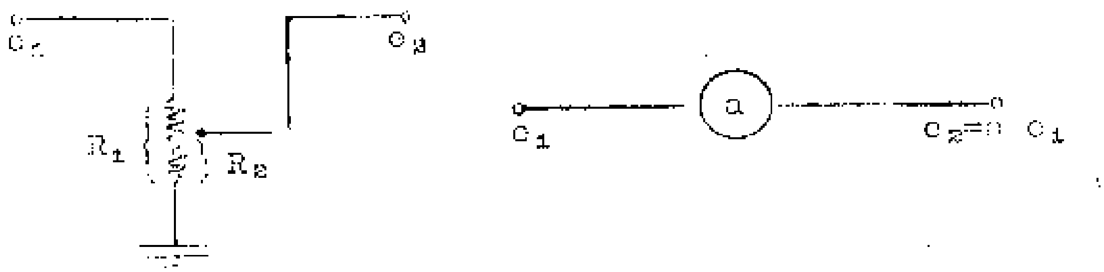
$$e_o(t) = -\int_o^t \sum_{i=1}^{n} K_i e_i(t)dt + e_o(o)$$

f) Multiple Inputs Integrator with I.C.

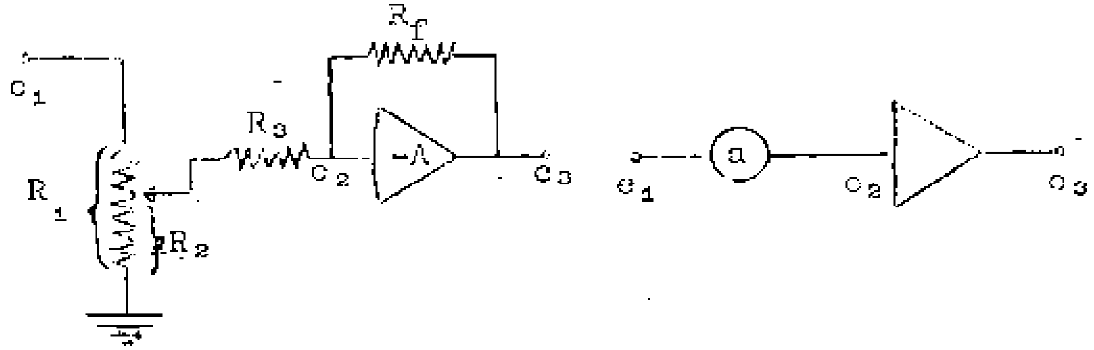
Fig.2.8: Computer Diagrams, Symbols and Mathematical Functions of the Operational Amplifier.

(2.2) Coefficient Setting Potentiometers.(6)

Coefficient Setting Potentiometers are used in an analogue computer setups to perform multiplication by a constant less than unity. These are necessary for setting the coefficients of equations, and the potentiometer circuit is shown in Fig.2.9.



a) Schematic diagram b) Symbol of Potentiometer.



c) Loading of a Potentiometer and Symbol.

Fig.2.9: The Coefficient Setting Potentiometer.

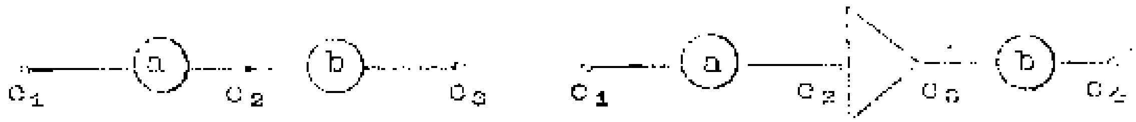
In Fig.2.9a, we have

$$e_2 = \frac{R_2}{R_1} e_1 .$$

Since $R_2 \leq R_1$, the ratio $\frac{R_2}{R_1}$, denoted by a, is less than 1.

Hence $e_2 = a\, e_1$, where $a \leq 1$. ------- (2.9)

In practice, a potentiometer is usually connected as shown in Fig.2.9c. Two coefficient setting potentiometers of equal resistance cannot be used directly in series without an intervening operational amplifier (see Fig.2.10).



a) Not Possible. b) Possible.

Fig.2.10: The Method of Using Two Potentiometers.

(2.3) Multiplier and Function Generator.[7]

(a) Quarter-Square Multiplier.

Multiplication of two variable voltages is a non-linear operation which is necessary on a general purpose computer. A "quarter-square" technique is used to effect this operation, use being made of the identity:

$$xy = \frac{1}{4}[(x+y)^2 - (x-y)^2] \qquad \text{---------} \ (2.10)$$

The block diagram representation of a quarter-square multiplier is shown in Fig.2.11.

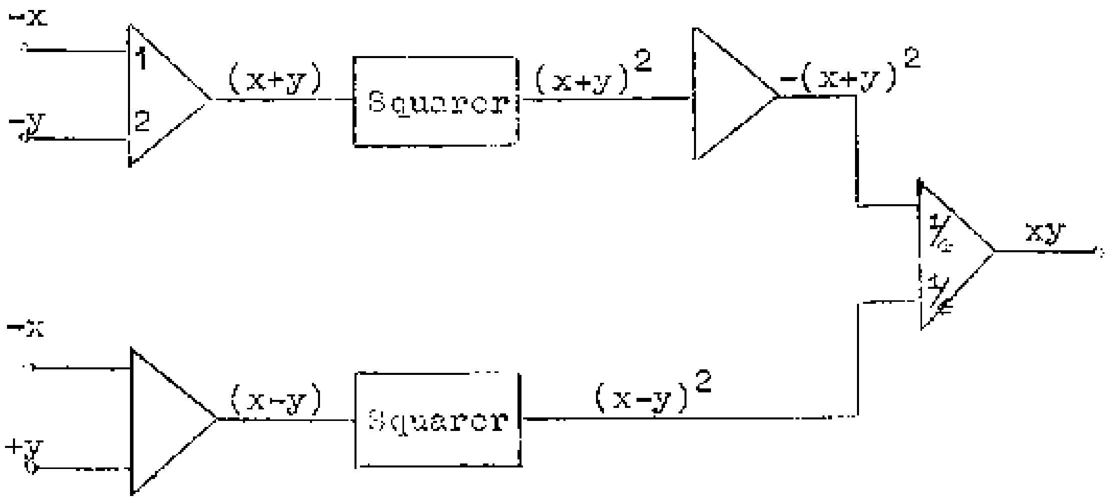


Fig.2.11: Block Diagram of a Quarter-square Multiplier

(b) Function Generator

The function generators available for an analogue computer use are of various types. In general, function generator in an analogue computer is done by the use of straight-line segments which are combined to approximate arbitrary curves. The functions are produced by the use of diode-tubes and each diode-tube makes a line segment. The number of straight-line segments usable in the representation normally varies from 5 to 22, depending upon the manufacturer of the equipment. An example for the generation of an arbitrary function as shown in Fig.2.12.

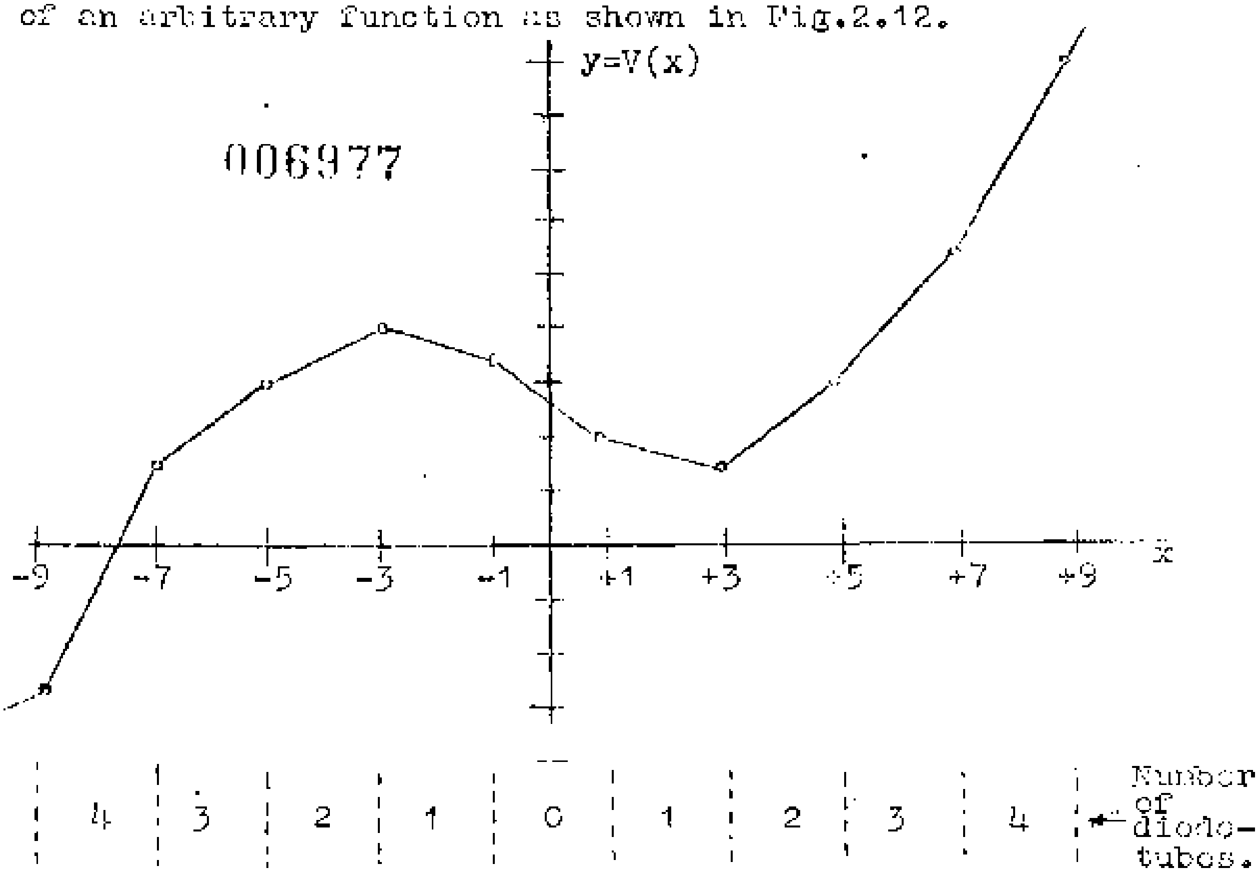


Fig.2.12: The generation of an arbitrary function.

Sometimes the computer does not have function generators, or the function generator of the computer cannot construct the required function. Other techniques may be used to construct functions. For example a linear function of t may be constructed by supplying a constant potential to the input of an integrator. The output of the integrator is represented by a straight line, as shown in Fig.2.13.

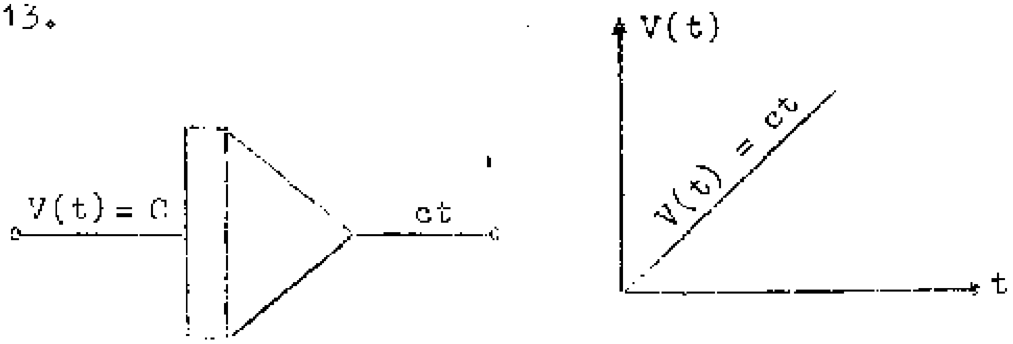


Fig.2.13: The method for constructing a linear function of t.

Furthermore, we may use a signal generator[a] for the function generator of the computer. But the method of using the signal generator will not be discussed in this thesis.

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