CHAPTER IV

THE GRAPH OF THE BINOMIAL COEFFICIENT FUNCTIONS IN THREE DIMENSIONS

4.1 The Graph of f (r,n) Using $m = \infty$ in the Eliminating of the Singularities.

When we eliminate the singularities on the lattice points by taking the limit along the lines with slope to those points, we have the values of lim. f (r,n) as shown in figure 7 (the same values as in Wanida's thesis: figure 4)

Values of r

Values of n	-4	-3	-2	-1	0	1	2	3	4	5	6	7
5	0	0	0	0	1	5	10	10	5	1	0	0
4	0	0	0	0	1	4	6	4	1	0	0	0
3	0	0	0	0	1	3	3	1	0	0	0	0
2	0	0	0	0	1	2	1	0	0	0	0	0
1	0	0	0	0	1	1	0	0	0	0	0	0
0	0	0	0	0	1	0	0	0	0	0	0	0
-1	0	0	0	0	1	-1	1	-1	1	-1	1	-1
-2	0	0	0	0	1	-2	3	-4	5	-6	7	-8
-3	0	0	0	0	1	-3	6	-10	15	-21	28	-36
-4	0	0	0	0	1	-4	10	-20	35	-56	84	-120
-5	0	0	0	0	1	-5	15	-35	70	-126	220	-340

Figure 7: The values of the Binomial coefficient Function on the Lattice Points of the (r,n) Plane

From the above data, we can plot the graph shown in figure 8.

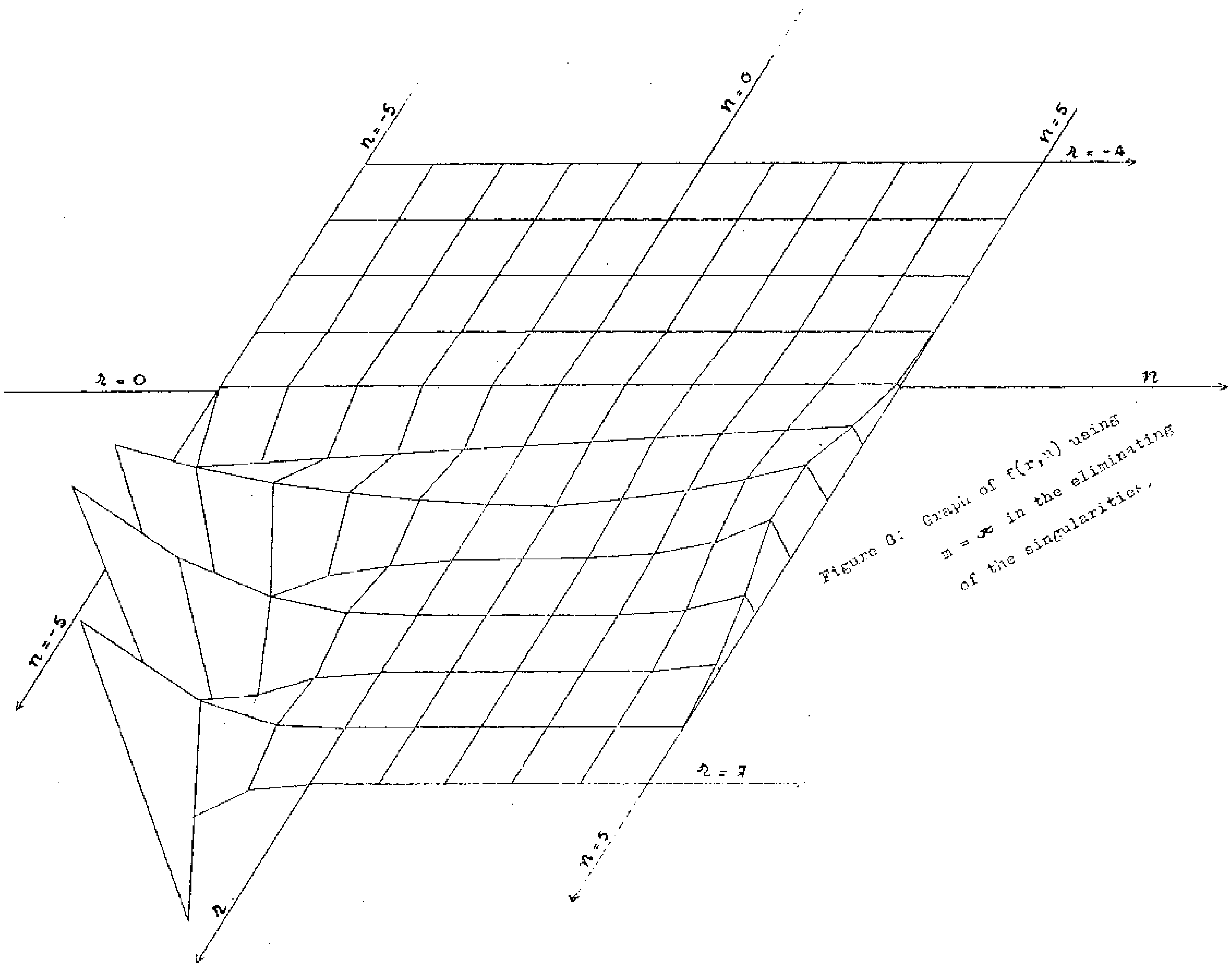


Figure 8: Graph of f(r,n) using $m = \infty$ in the eliminating of the singularities.

4.2 The graph of f(r,n) using $m=1$ in the eliminating of the singularity.

Values of r

Values of n	-5	-4	-3	-2	-1	0	1	2	3	4	5
5	0	0	0	0	0	1	5	10	10	5	1
4	0	0	0	0	0	1	4	6	4	1	0
3	0	0	0	0	0	1	3	3	1	0	0
2	0	0	0	0	0	1	2	1	0	0	0
1	0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	1	0	0	0	0	0
-1	1	-1	1	-1	1	0	0	0	0	0	0
-2	-4	3	-2	1	0	0	0	0	0	0	0
-3	6	-3	1	0	0	0	0	0	0	0	0
-4	-4	1	0	0	0	0	0	0	0	0	0
-5	1	0	0	0	0	0	0	0	0	0	0

Figure 9 : The values of the Binomial Coefficient Function on the Lattice Points of the (r,n) Plane.

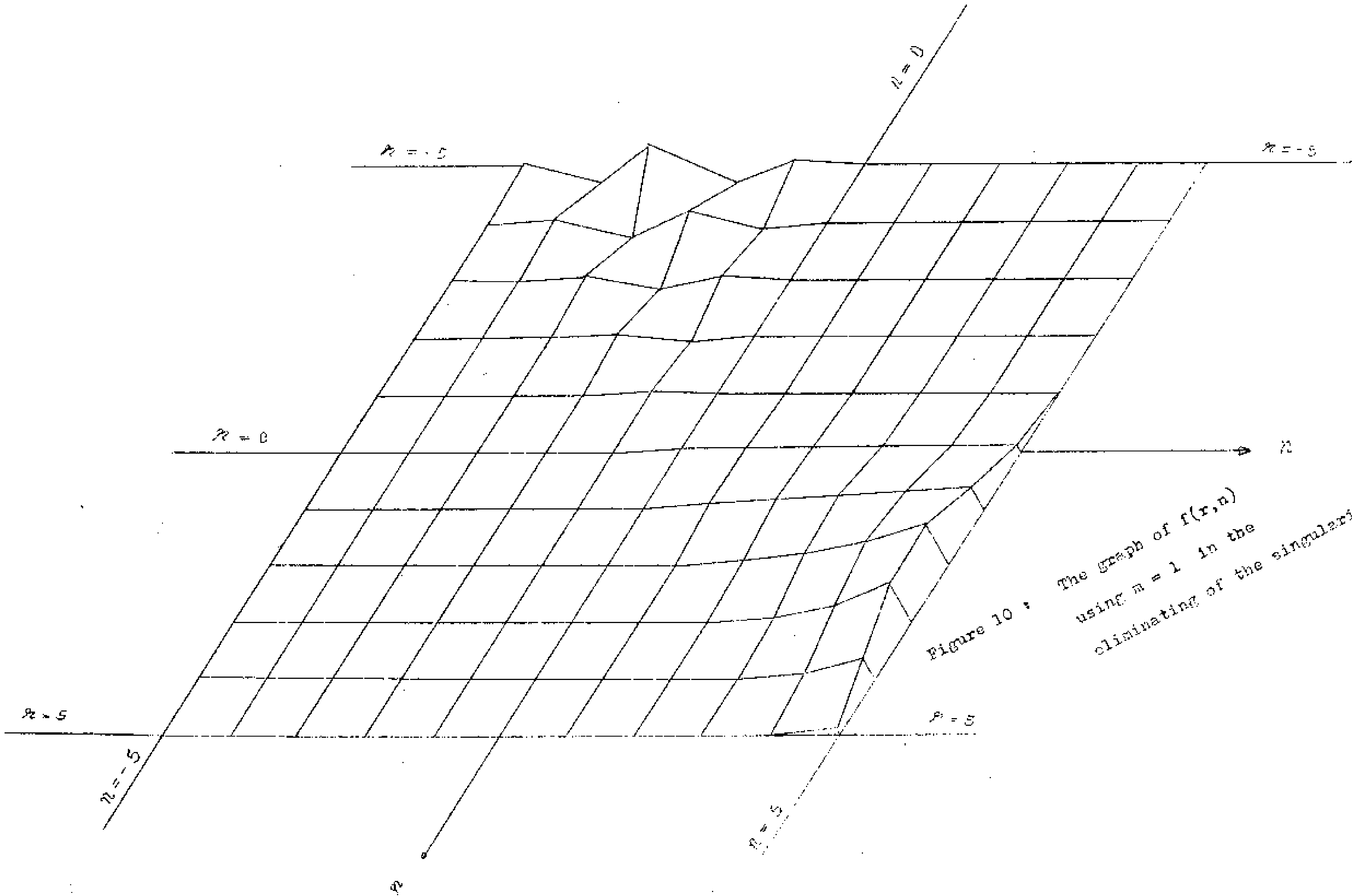
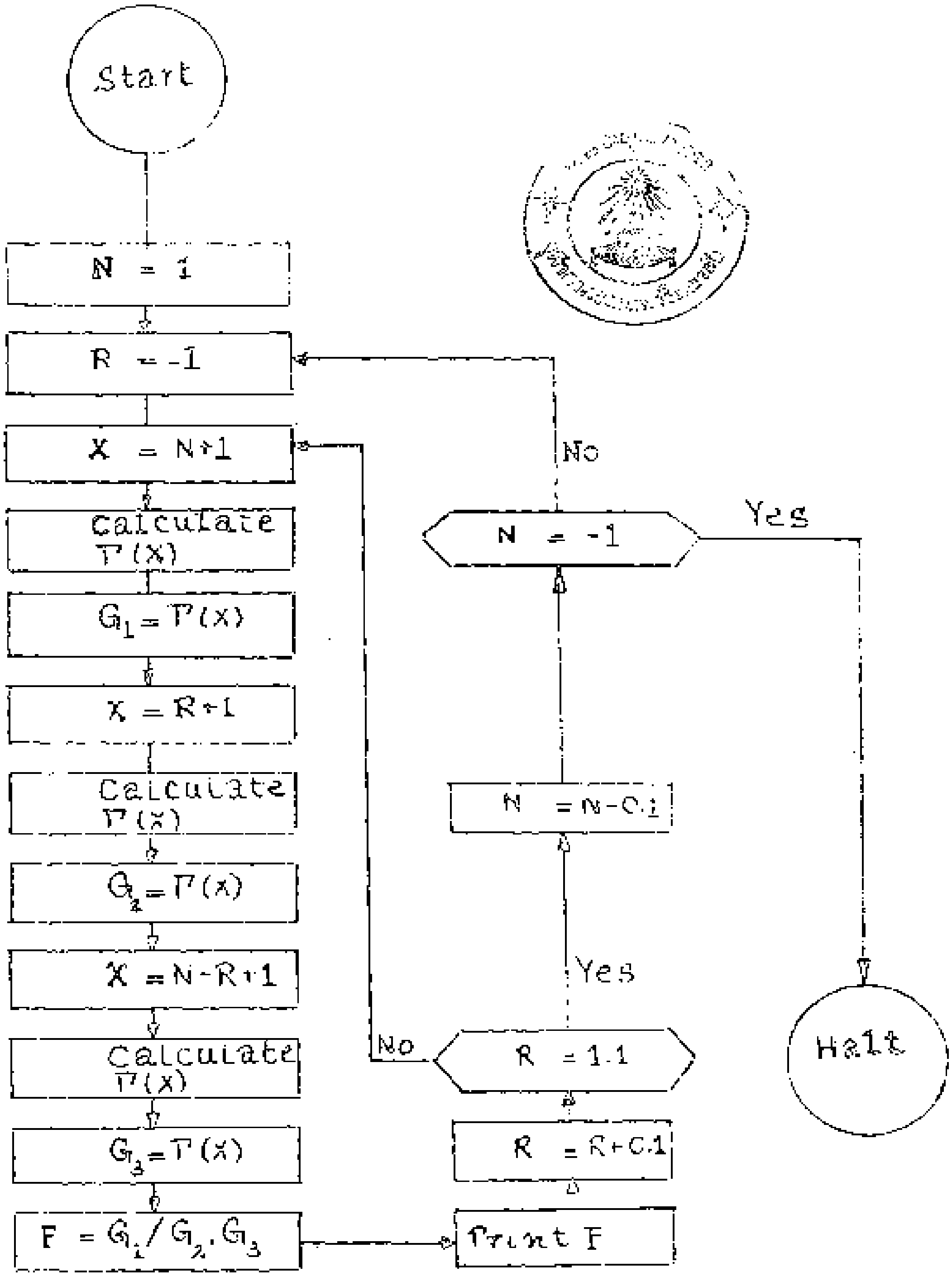


Figure 10 : The graph of f(r,n) using m = 1 in the eliminating of the singularities

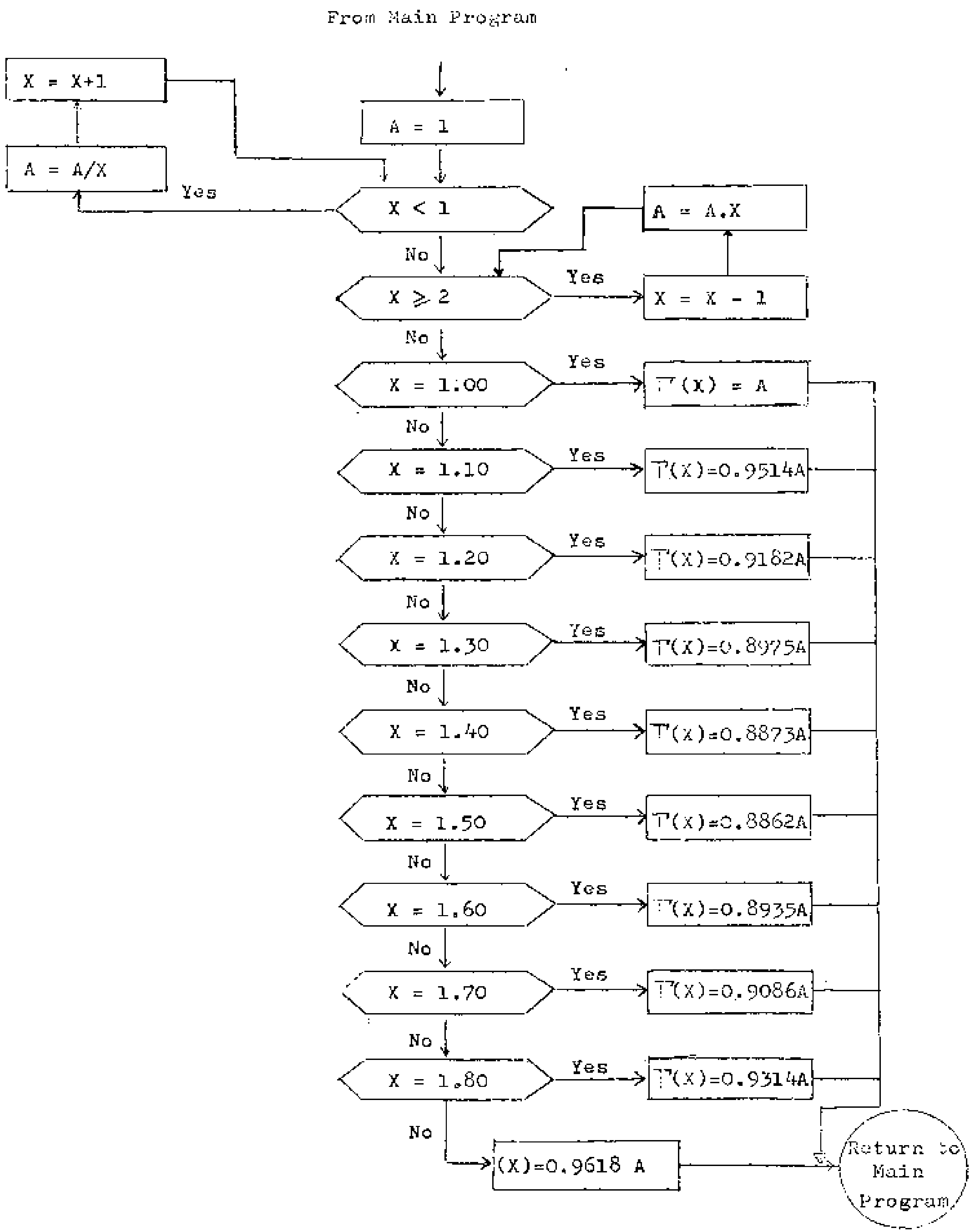
4.3 Graph of f(r,n) in the neighbourhood of (0,0).

To investigate the values of f(r,n) in the neighbourhood of the origin the region $-1 \leqslant r \leqslant 1$ and $-1 \leqslant n \leqslant 1$ was divided into 400 equal squares of side 0.1 . At each corner the value of f(r,n) was computed and the results are shown in Fig. 11. The calculations were made by R.H.B Exell using the I B M 1620 computer in the Computer Science Labaratory, Chulalongkorn University. The block diagram of the method, omitting programming details, is shown below.

CALCULATION OF $F = \dfrac{\Gamma(N+1)}{\Gamma(R+1)\,\Gamma(N-R+1)} = \dfrac{G_1}{G_2 G_3}$



CALCULATION OF GAMMA X



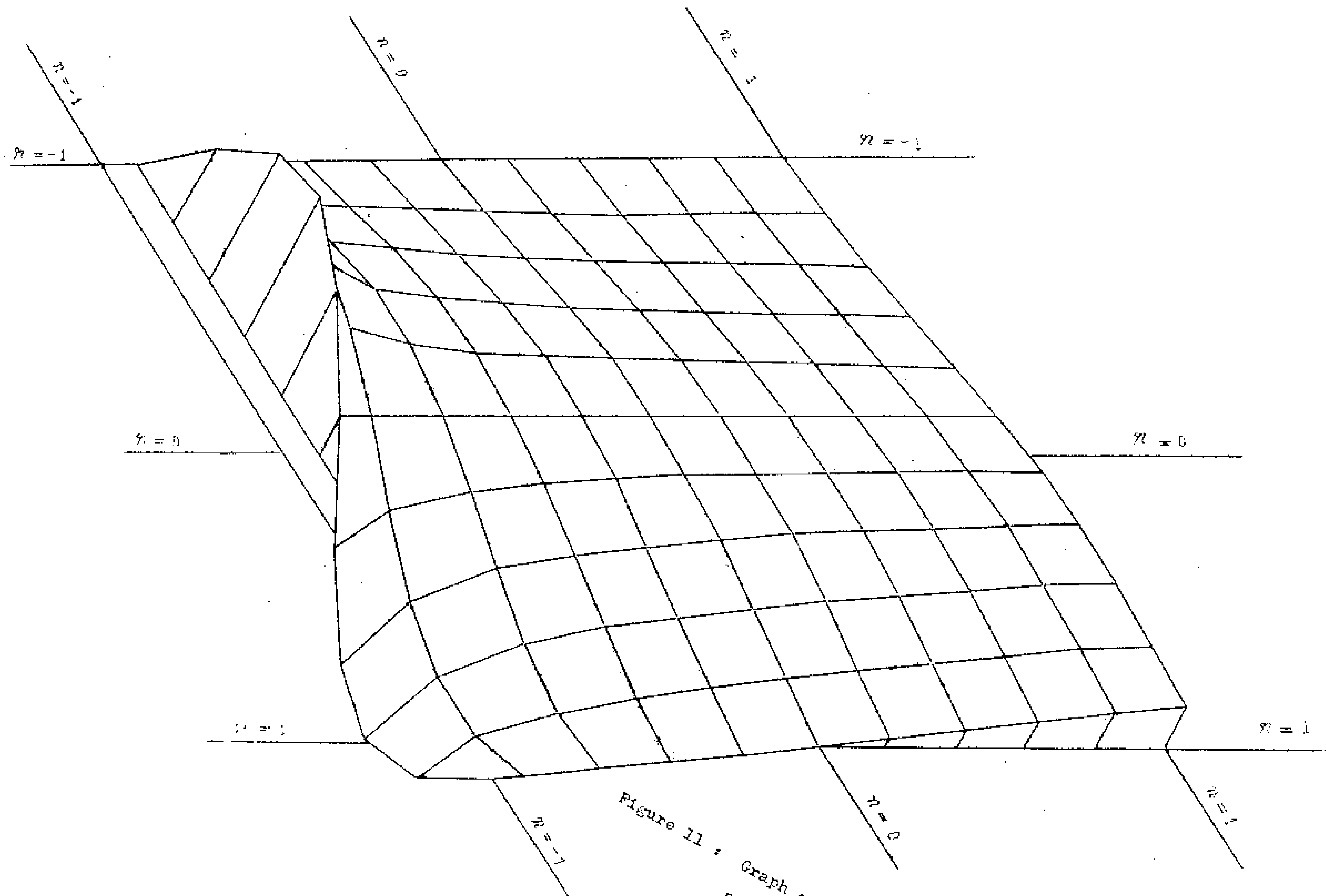


Figure 11 : Graph of f(r,n) in the neighbourhood of the origin.