CHAPTER V

COMPARISION OF THE LORENTZ TRANSFORMATION USING COORDINATE VELOCITIES WITH THE NEW FORM.

IN the preceeding chapters we have found, using the assumptions:

1. Transformation is linear.
2. The composition of two transformations is a transformation of the same form.
3. The transformation is symmetric. ,

and working with proper velocities, that the Lorentz transformation is

$$\left.\begin{aligned} x' &= (1 + v^2/k^2)^{\frac{1}{2}}x - vt, \\ t' &= - vx/k^2 + (1 + v^2/k^2)^{\frac{1}{2}}t \ , \end{aligned}\right\} \qquad (1)$$

by equations (11) and (12), Chapter II, and equation (5) , Chapter IV.

As stated in Chapter I the equation of the Lorentz transformation may be written

$$\left.\begin{aligned} x' &= \frac{x}{(1 - \bar{v}^2/c^2)^{\frac{1}{2}}} - \frac{\bar{v}}{(1 - \bar{v}^2/c^2)^{\frac{1}{2}}}t \ , \\ t' &= \frac{t}{(1 - \bar{v}^2/c^2)^{\frac{1}{2}}} - \frac{\bar{v}\,x}{c^2(1 - \bar{v}^2/c^2)^{\frac{1}{2}}} \ , \end{aligned}\right\} \qquad (2)$$

where $\bar{v}$ is the coordinate velocity of S' in S and c is the coordinate velocity of light.

The relation between the proper velocity v and the coordinate velocity $\bar{v}$ is given by the equation

$$v = \frac{\bar{v}}{(1 - \bar{v}^2/c^2)^{\frac{1}{2}}},$$

according to the definition in Chapter I.

We shall now verify that when we substitute $\frac{\bar{v}}{(1 - \bar{v}^2/c^2)^{\frac{1}{2}}}$ for v in equations (1) we obtain equations (2).

Substituting $v = \frac{\bar{v}}{(1 - \bar{v}^2/k^2)^{\frac{1}{2}}}$ into equations (1) we get, after some manipulations,

$$\left.\begin{aligned} x' &= \frac{x}{(1 - \bar{v}^2/k^2)^{\frac{1}{2}}} - \frac{\bar{v}}{(1 - \bar{v}^2/k^2)^{\frac{1}{2}}}\,t \\ \text{and} \quad t' &= -\frac{1}{k^2}\,\frac{\bar{v}}{(1 - \bar{v}^2/k^2)^{\frac{1}{2}}}\,x + \frac{t}{(1 - \bar{v}^2/k^2)^{\frac{1}{2}}} \end{aligned}\right\} \quad (3)$$

which is the same as equations (2) if $k = c$.