# CHAPTER II

# THEORY

## 2.1 The Hall Effect

The Hall effect is one of a large number of galvanomagnetic, thermoelectric and thermomagnetic effects in the problem of transport phenomena in solids. If a rectangular slab of metal is carrying an electric current and a magnetic field is applied perpendicular to one face of the slab and to the current, a voltage is found developed across the faces perpendicular to both the current and the magnetic field.

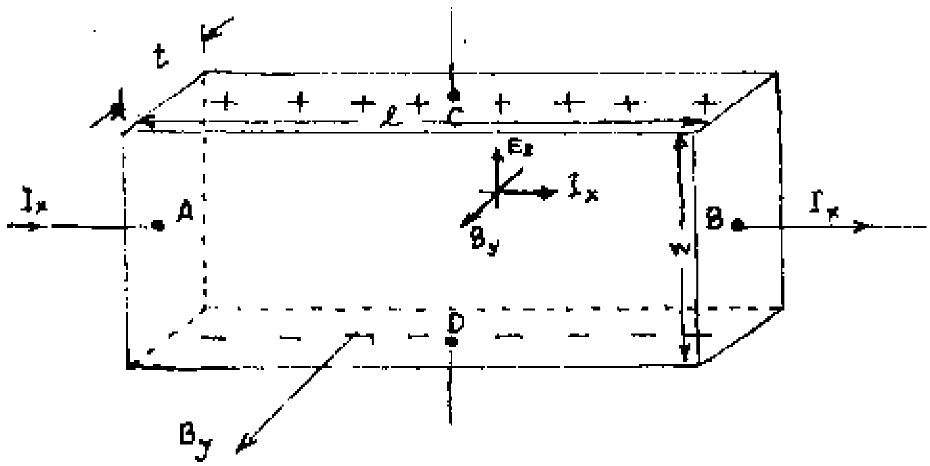


Fig.1 Conditions for Hall Effect for Electron Conduction.

The voltage, called the Hall voltage, is found experimentally to be proportional to the thickness, t, of the specimen;

$$V_H = \frac{R_H I_x B_y}{t} . \qquad (2.1)$$

The constant of proportionality $R_H$ is called the Hall constant or the Hall coefficient. The magnitude and sign of $R_H$ is characteristic of a material, and for the case of thin film it varies with dimensions of the specimen.

## 2.2 Theory of Hall Effect in Electron Gas Model

The theory of Hall effect is formulated in the general study of the problem of electrical and thermal conduction in different materials, metals or semiconductors. A detailed understanding requires knowledge of the band structure and the crystallographic structure of the material. Hall effect provides simple and conclusive evidence of the validity of the postulate of conduction by 'holes' in semiconductors. However, a simple picture of the effect can be given in terms of the very naive electron gas model of Drude (2).

In Drude's model, inside a metal there are free electrons forming a cloud of electron gas. The electrons have an average speed and are moving in random through collision with the immobile metal ions. The electrons travel in straight lines during one relaxation time. When an applied electric field $\vec{E}$ is set up, the electrons acquire an average velocity $\vec{v}$ in the direction of the electric field. Each electron still moves in a more or less random fashion although the electron gas as a whole now has a drift velocity. When a tranverse magnetic field $\vec{B}$ is applied, the electron will experience the Lorentz force and the total force exerted on the electrons is

$$\vec{F} = -e\left[\vec{E} + \vec{v}\times\vec{B}\right] \quad (2.2)$$

In free space the electron will then be deflected in the direction perpendicular to the plane of $\vec{B}$ and $\vec{v}$. But in a solid they soon are stopped at the confining walls of the specimen and gradually set up an electric field that will resist further piling up of the electrons. When this field becomes strong enough to counter the Lorentz force, an equilibrium is attained and the electric current flows on as before. This is the situation in the defining eq.(2.1). To be independent of the dimensions of the specimen, the Hall constant of the material is expressed in terms of the current density, $j_x = \frac{I_x}{wt}$, and the Hall electric field, $E_z = \frac{V_H}{w}$, so that

$$R_H = \frac{E_z}{j_x B_y} \qquad (2.3)$$

At equilibrium there is no force acting on the electrons, $\vec{F}=0$, implying that

$$e\vec{E} = e\vec{B} \times \vec{v} \; . \qquad (2.4)$$

If $\vec{v}$ is the drift velocity of the electron gas, then the current density is

$$\vec{j} = ne\vec{v} \; , \qquad (2.5)$$

where n is the concentration of the electrons taking part in conduction. With $\vec{B}=(0,B_y,0)$; $\vec{v}=(v_x,0,0)$, and $\vec{E}=(0,0,-E_z)$, it is apparent from (2.4) that

$$-E_z = (\frac{1}{ne}) B_y j_x \; .$$

Comparing with (2.3), we have

$$R_H = \frac{1}{ne} \; . \qquad (2.6)$$

In this simple model, the Hall coefficient gives the value of the concentration of conduction electrons. If the current carrier is differently charged, the Hall electric field will change direction and the Hall coefficient will change sign. The sign convention is such that, for electrons, the Hall coefficient is negative.

From the macroscopic point of view, the electrical conductivity of a metal is defined by

$$j_x = \sigma E_x , \tag{2.7}$$

so that, in view of eq. (2.5),

$$\sigma = ne\frac{v_x}{E_x} . \tag{2.8}$$

If Ohm's law is obeyed, as is normally the case, then $\sigma$ must be independent of $E_x$ so that,

$$\sigma = ne\mu_c , \tag{2.9}$$

where $\mu_c$ is the electron drift velocity per unit applied electric field, or the conduction mobility defined macroscopically by

$$v_x = \mu_c E_x . \tag{2.10}$$

From eqs.(2.6) and (2.9), it can be written that

$$|R_H|\sigma = \mu_H , \tag{2.11}$$

where $\mu_H$ is a quantity analogous to $\mu_c$ and is called the Hall mobility.

## 2.3 Hall Effect in Bismuth in Band Theory

The naive Electron Gas Model has many difficulties. A serious one is its inability to explain the positive Hall coefficient in some substances. This and other difficulties are resolved by the band theory. The band theory will not be explained in detail here, but rather, familiarity with the following concepts in modern band theory of solids will be assumed in the following discussions of Hall effect in bismuth;conduction by positive current carriers or holes, effective mass ( a tensorial quantity), Brillouin zone and Fermi surface, and the crystallographic structure of bismuth. Detailed theory of Hall effect can be found in the literature (2, 3, 4).

Pure bismuth has rhombohedral crystal lattice and its electrical properties are sometimes labelled as semi-metal. Its resistivity is $120 \times 10^{-6}$ ohm-cm, that is, in between values for metals and semiconductors. Carrier concentration in bismuth is of the order of $10^{17}/cm^3$ . The Fermi surface is composed of a set of three equivalent eccentric ellipsoids in momentum space for electrons and an ellipsoid of revolution around the trigonal axis for holes. The three ellipsoids can be transformed into one another by $120^\circ$ rotations around the trigonal axis. The Fermi surface of electrons encloses $< 10^{-3}$ of the Brillouin zone.

Three bands contribute to conduction in bismuth. There are a light mass conduction band and a valence band which are separated by an energy gap of about 0.007 ev. and a heavy mass valence band of holes the top of which is about 0.0185 ev. above the bottom of the conduction band.

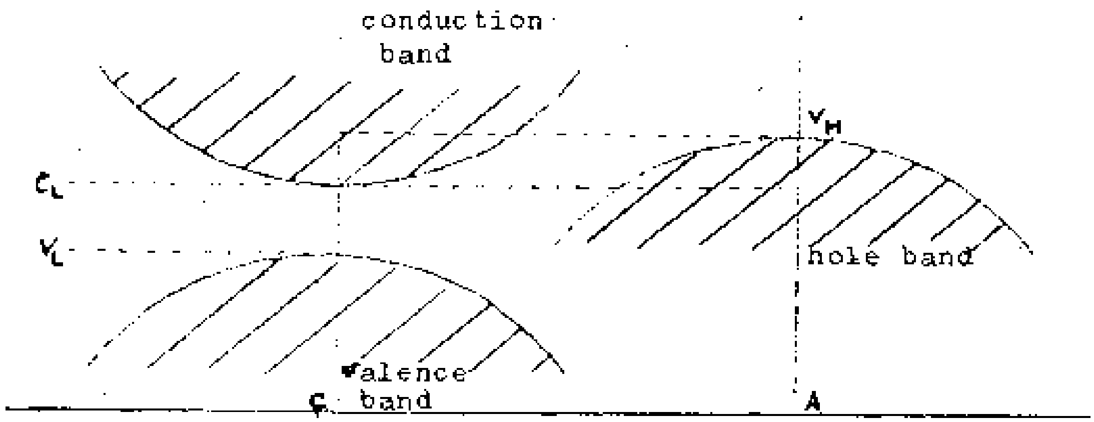


Fig.2 E vs k Diagram for Bi.

$c_L$ and $v_L$ represent the light mass bands.

$v_H$ is the heavy mass band.

Because of the overlap of the bands, bismuth does not behave as a semiconductor. Its electrical conductivity is due to a small number of light electrons at the bottom of the conduction band and an equal number of heavy holes at the top of the valence band. The galvanomagnetic effects are thus determined by the detailed structure of the bottom of the conduction band.

In a rhombohedral crystal like bismuth, there is anisotropy in the Hall effect. This anisotropy is expressed by two Hall coefficients, $R_{H_{\parallel}}$ and $R_{H_{\perp}}$, defined as Hall coefficients measured when the magnetic field is parallel and perpendicular, respectively, to the trigonal axis. $R_{H_{\parallel}}$ is positive and $R_{H_{\perp}}$ is negative.

## 2.4 Measurement of the Hall Effect.

### 2.4a Galvanomagnetic and Thermomagnetic Effects.

The Hall voltage can be measured directly with a sensitive galvanometer calibrated as a microvoltmeter, but this is not perfectly straightforward. As mentioned before, the Hall effect is but one of a family of galvanomagnetic and thermomagnetic effects. Referring to Fig.1, the same situation that produces the Hall effect also produces the Ettingshausen effect. This is the appearance of a temperature difference $\Delta T$ between C and D.

Since a heat flow in metal is also a motion of electrons, it is plausible to suggest that if, for $I_x = 0$, there is a heat flow along the specimen in Fig.1, a potential difference and a temperature difference $\Delta T$ might again appear between C and D. This actually happens, and these two phenomena are, respectively, the Nernst effect and the Righi-Leduc effect. They are called transverse thermomagnetic effects.

It will now be shown that the three effects may interfere with the measurement of $R_H$. Assume now that the leads to C,D,A,B are of a metal different from the specimen. Assume also that when $B_y = 0$, and the current is flowing, C,D are not quite on an equipotential line so that a voltage $V_I$ exists between them. This will usaully be the case because it is difficult to attach the leads exactly on an equipotential. The measured voltage across CD, for $B_y \neq 0$ can then be written

$$V_1 = V_H + V_E + V_N + V_{RL} + V_I$$

The contributions, apart from $V_I$, are from the four galvano- and thermomagnetic effects. The thermomagnetic effects are in action because there will certainly be a heat flow as well as a current along the specimen - if only because there will be Peltier heating at A and cooling at B, or vice versa. There may also be an accidental temperature gradient. The Ettingshausen and Righi-Leduc effects both produce a $\Delta T$ between C and D and, since C and D are junctions, there will be a thermoelectric voltage in the galvanometer circuit. This voltage is indicated by $V_E + V_{RL}$ .

If now three more measurements, with $I_x$ or $B_y$ reversed, are made in rapid succession so that the longitudinal temperature gradient due to Peltier heating does not have time to reverse, then

$$V_2 = - V_H - V_E + V_N + V_{RL} - V_I \qquad (I_x \text{ only reversed}),$$

$$V_3 = V_H + V_E - V_N - V_{RL} - V_I \qquad (I_x \text{ and } B_y \text{ reversed}),$$

$$V_4 = - V_H - V_E - V_N - V_{RL} + V_I \qquad (B_y \text{ only reversed}).$$

It follows that

$$V_H + V_E = \frac{V_1 - V_2 + V_3 - V_4}{4} .$$

This reversing procedure is often used in Hall measurements with direct currents. $V_E$ cannot be eliminated but may, in fact, be much smaller than $V_H$. In principle, $V_E$ could be avoided by making leads and specimen of the same material.

2.4b Methods of measurement of Electrical Conductivity and Hall Coefficient.

These properties can be measured using the circuit arrangement shown in Fig. 11. The specimen is provided with two pairs of contacts, A,B near the ends to enable a current to be passed through it, and a transverse pair of potential probes C,D near the center. If a direct current is now passed through the specimen, then the resistance, and hence the conductivity may be found by measuring the current and voltage between A and B. Provided this voltage is measured with a potentiometer the result obtained will be independent of contact resistance at A or B. Since the contact resistance may often be comparable with or greater than the bulk resistance of the sample, a large error may be introduced by thermo-electric effects if the temperature of the specimen is not uniform. This error can be eliminated by reversing the current, taking a second reading and averaging the pair.

If now a magnitic field perpendicular to the flat face of the specimen is applied the Hall voltage is obtained by measuring the potential between C and D. It will usually be found that in the absence of a magnetic field there is a voltage between C and D owing to their alignment being imperfect. This is eliminated by reversing the magnetic field and measuring the potential between C and D again. Errors may also be introduced if a temperature gradient is present, but these are eliminated by reversing the current and taking another pair of readings with the magnetic field normal and reversed. Averaging these four readings elimitates all errors except that produced by the Ettingshausen effect.

A potentiometer which can measure down to 1μv is required for these measurements. A good mirror galvanometer is the most convenient instrument for balancing the potentiometer.

The simplest way of measuring magnetic fields is by use of a calibrated search coil and a ballistic galvanometer.

The effect of shape of the specimen on the results must also be considered. The current terminals across the ends tend to short out the Hall voltage. This shorting, however, has been shown to be negligible for specimens whose length-to-width ratio is greater than 3 or 4 (4).

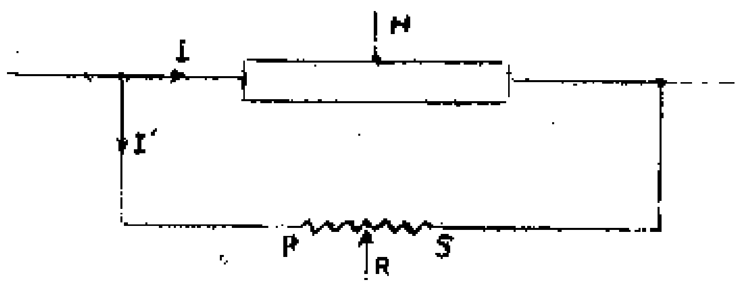


Fig.3 Three-probe Circuit for Measuring the Hall Effect.

Fig.3 shows an arrangement of the specimen circuit which gives a high degree of precision for Hall measurements. Only the Hall contact H, is applied to the specimen, which is shunted by the potential divider PRS. With the magnet off, R is adjusted to give zero between R and H. When the magnet is switched on a Hall voltage appears between R and H, but it is only half the voltage that would be seen if R were attached to the specimen. This arrangement has the advantages that the resistance of PRS can be any convenient value and that only three contacts need be made to the specimen. This is a real advantage when dealing with specimens in the form of thin films.