#### CHAPTER III

#### CATCULATION OF THE MODE SHAPE AND FREQUENCIES

We use inverse interpolation and linear interpolation to find the mode shapes and frequencies. Suppose we have a function y = f(x) from which for a given value of  $x_i$  a value  $y_i$  is assigned. To find x when some  $y_j$  is given  $(i \neq j)$  we use inverse interpolation. If  $x_j$  is given  $(i \neq j)$  then we use direct interpolation to find y.

The calculation is done in the following steps: 1. Fix F, and determine the value of  $y_1$  that makes  $y'_{10} = 0$ , by using inverse interpolation.

Repeat the calculation for different frequencies F.
 The results obtained are in Table 15 - 22.

3. From the Table 15 - 22 calculate the corresponding value of  $y_{10}^{''}$  by using direct interpolation. This gives  $y_{10}^{''}$  as a function of F, when  $y_{10}^{\prime} = 0$ . Results to are shown in Table 23 - 30.

4. Determine the frequency F for which  $y_{10}^{''} = 0$ , using inverse interpolation. This is the vibration frequency, and the corresponding values of  $y_0, y_1, y_2, \dots, y_{10}$  give the shape of the mode. The result obtained is F = 1289 H<sub>g</sub> (Hertz), see table 31.

Calculation the value of  $y_1$  that makes  $y_{10}'' = 0$ , where F = 1000

| xio -     | <b>T</b> 1 | 1           | 2          | 3          | 4          | 5         |
|-----------|------------|-------------|------------|------------|------------|-----------|
| -0.3161:0 | -1.0       | All marking | 100        |            |            | Sec. S    |
| -0,13930  | -0.9       | -0.8213439  |            |            |            |           |
| 0,03765   | -0.8       | -0.8212851  | -0.8212976 |            |            |           |
| 0.21470   | ~0.7       | -0.8212766  | -0.8213174 | -0.8212771 |            |           |
| 0.39170   | -0,6       | -0,8212618  | -0.8213224 | -0.8212799 | -0.8212737 |           |
| 0.56870   | -0.5       | -0.8212631  | -0.8213280 | -0.8212810 | -0.8212747 | -0.821271 |

 $T = 1000, DF = 100, T_4 = -1.0, DT_4 = 0.1$ 

Since  $y'_{10} = 0$  the laft-hand column contains the "parts" used in the process. We obtain  $y_1 = -0.0212716$ .

## Thble 16

F = 1100, DF = 100, 74 = -1.0, DF = 0.1

| J'10     | 74   | 1          | 2          | 3          | 4           | 5  |
|----------|------|------------|------------|------------|-------------|--|
| -0.38450 | -1.0 |            | -          |            | sectional 2 |  |
| -0.19110 | -0.9 | -0.8011892 | and of the | and all    |             | the second s |
| 0.002268 | -0.8 | -0.8011728 | -0.8011730 |            |             |  |
| 0.19560  | -0.7 | -0.8011550 | -0.8011723 | -0.8011731 | ALC: A      | N.   |
| 0.3890   | -0.6 | -0.8011635 | -018011807 | -0.8011750 | -0.8011732  |  |
| 0.5825   | -0.5 | -0.8011892 | -0,8011892 | -0.8011729 | -0.8011732  | -0.8011732   |

JA = -0.8011732

 $P = 1300, DP = 100, y_1 = -1.0, Dy_1 = 0.1$ 

50

| J10   | 34  | 1.1.7                               | 2                                   | 3  | 4          | 5                                    |
|---|---|-------------------------------------|-------------------------------------|--|------------|--------------------------------------|
| -0.5haho  | -1.0                                      |                                     | and the state                       | and the second                           |            |                                      |
| -0-31130  | -0.9                                      | -0.7652964                          |                                     |  |            |                                      |
| -0,08031  | -0.8                                      | -0.7652405                          | -0.7652211                          |  | 5          |                                      |
| 0.15070   | -0.7                                      | -0.7652490                          | -0.7652615                          | -0.7652362                               |            |                                      |
| 0.38480   | -0,G                                      | -0.7652156                          | -0.7652736                          | -0.7652202                               | -0.7652401 |                                      |
| 0.61280   | -0.5                                      | -0.7652355                          | -0.7652759                          | -0.7652274                               | -0.7652392 | -0.765241                            |
| Contraction of the second s | Proprietory and the state of the state of | Compression and an average services | protect and a second second real of | and which as an end of the second second |            | Study was an dealer with the barrant |

y10 = 0

NAMES OF STREET, STREE

Ja = -0.7652418

 $T = 1200, DF = 100, y_q = -1.0, Dy_q = 0.1$ 

| 710     | 74   | 1          | 2           | 3           | 4.          | 5          |
|---------|------|------------|-------------|-------------|-------------|------------|
| -0_1598 | -1.0 |            | Sec. 1      |             |             | - Anne     |
| -0.2484 | -0.9 | -0,7824976 |             |             |             |            |
| -0.0370 | -0,8 | -0.7821976 | -0.7824976  | 1 Courses   |             |            |
| 0*174   | -0.7 | -0.7821976 | -0.7821976  | -07821:976  |             |            |
| 0.3858  | -0,6 | -0.7824976 | -0.7821976  | -0.7821976  | -0.782:976  |            |
| 0.5972  | -0.5 | -0.7824976 | -0.7821:976 | -0.7821,976 | -0.7821:976 | -0.7824976 |

310 = 0

74 = -0.7824976

alle

21

 $P = 4000, DF = 100, y_1 = 1.0, Dy_1 = 0.1$ 

| J40     | y y  | Anna         | 2           | 3          | 4                   | 5          |
|---------|------|--------------|-------------|------------|---------------------|------------|
| -0.6326 | -1.0 | in manageria |             |            | Conne de            |            |
| -0,3202 | -0.9 | -0.7493664   |             |            | e contra los terrar | manin      |
| -0.1278 | +0.8 | -0,7493661   | -0.74:93661 |            | 1                   |            |
| 0.1215  | -0.7 | -0.7493330   | -0.7493661  | -0.7493664 |                     |            |
| 0.3769  | -0,6 | -0.7493413   | -0.7493536  | -0.7493629 | -0.7493677          |            |
| 0.6293  | -0.5 | -0.7493462   | -0.7493586  |            | -0.7493664          | -0.7493690 |

J4 = -0.7493696

Table 20

 $\ddot{P} = 1500, DF = 100, y_1 = 1.0, Dy_1 = 0.1$ 

| Vio      | V1   | 1           | 2          | 3           | 4           | 5              |
|----------|------|-------------|------------|-------------|-------------|----------------|
| -0.7305  | -1.0 |             |            |             | 14          |                |
| -0.1:550 | -0.9 | -0.7318457  |            | -Toti - All |             |                |
| -0.1796  | -0.8 | -0.7317976  | -0.7347662 | (interest   |             |                |
| 0.09576  | -0.7 | -0.7347687  | -0.7347821 | -0.7347766  |             | 1.0            |
| 0.3711   | -0.G | -0.73474.95 | -0.7347927 | -0.731:7748 | -0.732:7772 | order the sup- |
| 0.6466   | -0.5 | -0.7347687  | -0.7348139 | -0.7317766  | -0.7307766  | -0.734777      |

74 = -0.7347778

 $F = 1600, DF = 100, y_1 = -1.0, Dy_1 = 0.1$ 

| J10      | 21           |                | 2           | 3          | 4          | 5         |
|----------|--------------|----------------|-------------|------------|------------|-----------|
| -0.8364  | -1.0         | Sala September |             |            | a. arrent  |           |
| -0.5362  | -0.9         | -0.7213857     |             | THE REAL   | artister ( |           |
| -0.2360  | -0.8         | -0+7213857     | -0.7213852  |            |            |           |
| -0.06b14 | -0.7         | -0.7213671     | -0.7213.707 | -0.7213737 | and the    |           |
| 0.393    | <b>₩0.</b> 6 | -0.7213625     | -0.7213719  | -0.7213730 | -0.7213727 |           |
| 0.6645   | -0.5         | -0.7213674     | -0.7213774  | -0.7213832 | -0.7213726 | -0.721372 |

J4 = -0.7213728

F = 1700, DF = 100, Ja = -1.0, Dy = 0.1

| 710     | <b>V</b> 4 | 1          |            | . 3        | 4          | 5          |
|---------|------------|------------|------------|------------|------------|------------|
| +0.9505 | -1.0       |            |            |            | Sec. 2     |            |
| -0.6238 | -0.9       | -0.7090629 |            |            | 14         |            |
| -0.2974 | -0.3       | -0.7090603 | -0.7090579 |            |            | Section 1  |
| 0.02954 | -0.7       | -0.7090424 | -0.7090757 | -017090740 | C. Same    |            |
| 0.3562  | -0.6       | -0.7090380 | -0.7090171 | -0.7090530 | -0.7090758 |            |
| 0.6829  | -0.5       | -0.7090424 | -0.7090531 | -0.7090564 | -0.7090747 | -0.7090770 |

J10 = 0

JA = -0.7090770



Calculation the value of  $y_{10}^{\prime\prime}$  corresponding to

y4 = -0.8212716

 $P = 1000, y_1 = -1.0, Dy_1 = 0.1, 1 = 0, 1, 2, ---5$ 

| 11.1.1 | 34   | Ja - Ja    | 740       | 1            | 2            | 3          | 4            | 5         |
|--------|------|------------|-----------|--------------|--------------|------------|--------------|-----------|
| 111111 | -    | 0.1787284  |           | a raine      |              |            |              |           |
|        | -0.9 | 0.0787284  | -0.01166  | -0.0040233   |              |            |              | 1 1 2     |
|        | +0.8 | -0.0212716 | -0.001961 | -0,001,0233  | -0.0000233   |            | 12.4.2.1     |           |
|        | -0.7 | -0.1212716 | 0.007744  | -0.001:021:8 | -0.0010230   | -0.0040234 |              |           |
|        | -0.6 | -0,2212716 | 0.01744   | -0.0040211   | -0.0010295   | -0.0010226 | -0.001:021:4 |           |
|        | -0.5 | -0.3212716 | 0.02744   | -0.0040233   | -0.001,011,2 | 0.0040239  | -0.001;0234  | -0.004027 |

0

 $y_{0} = y_{1} = -0.8212716$  $y_{10}'' = -0.0010272$ 

F = 1100, yq = -1.0, Dyq = 0.1, 1 = 0,1,2---,5

| 71   | y <sup>a</sup> - 2 <sup>7</sup> | J''10     | 1           | 2          | 3          | ALL BARRE      | 5          |
|------|---------------------------------|-----------|-------------|------------|------------|----------------|------------|
| -1.0 | 0.1988267                       | -0.02658  |             |            |            | S. And         |            |
| -0.9 | 0.0988267                       | -0.011:73 | -0.0030150  | inisia i   |            |                | 1.1.1.1.3  |
| -0.8 | -0.0011733                      | -0.002875 | -0.0030131  | -0.0030131 |            | an in a fair a |            |
| -0.7 | -0.1011733                      | 0.00898   | -0.0030404  | -0,0030131 | -0,0030131 | - Alberta      | unante h   |
| -0.6 | -0,2011733                      | 0.02083   | -0.003011:7 | -0,0030193 | -0,0030131 | -0,0030131     |            |
| -0.5 | -0.3011733                      | 0.052690  | -0.0030292  | -0.0030104 | -0.0030131 | -0.0030131     | -0.0030131 |

- y'10 = 0
- Jo = J = -0.8011733
  - y'10 = -0.0030131

 $y = 1200, y_1 = -1.0, Dy_1 = 0.1, 1 = 0, 1, 2, ---5$ 

| 71   | Jo - Ji    | J'10       | 1          | 2          | 3          | 4           | 5          |
|------|------------|------------|------------|------------|------------|-------------|------------|
| -1.0 | 0.2175024  | -0.02259   |            |            | 1          |             |            |
| -0.9 | 0.1175024  | -0.01833   | -0.001574  | 1.1        |            | erena i da  | 1          |
| -0.8 | 0.0175024  | -0.001:078 | -0_0015035 | -0.0015851 |            |             |            |
| -0.7 | -0.0824976 | 0.01018    | -0.00158.5 | -0.0015822 | -0.0015846 |             |            |
| -0.6 | -0.1824976 | 0.021111   | -0.0015842 | -0.0015811 | -0.0015833 | -0_0015857  |            |
| -0.5 | -0.2824976 | 0.03870    | -0.0015842 | -0.0015820 | -0.0015814 | -0.00\$5901 | -0.0015771 |

$$y'_{10} = 0$$
  
 $y_{10} = y_{1} = -0.7821976$   
 $y''_{10} = -0.0015771$ 

 $P = 1300, y_{q} = -1.0, Dy_{q} = 0.1, 1 = 0, 1, 2-5$ 

| 7 <b>1</b> | Ve - VA    | J'10      | 1.<br>Secondaria | 2          | 3           | 4             | 5               |
|------------|------------|-----------|------------------|------------|-------------|---------------|-----------------|
| -1.0       | 9.2347130  | -0.03948  | the mark         | 1999-11-1  |             | ar en en esta | 10 N 1 1 1      |
| -0.9       | 0.1347130  | -0.02255  | 0.0002569        |            |             | departuro     | togeneration in |
| -0.8       | 0-0347130  | -0.005626 | 0.0002188        | 0.0002380  | a constants | an al surray  | a que trances   |
| -0.7       | -0.0652870 | 0.01130   | 0.00021;95       | 0.0002190  | 0.0002348   |               |                 |
| -0.6       | -0.1652870 | 0.02823   | 0.0002169        | 0.0002512  | 0.00021:76  | 0.0002215     |                 |
| -0.5       | -0.2652870 | 0.04516   | 0.0002469        | 0,00021/69 | 0.00002583  | 0.0002299     | 0.0002176       |

$$y'_{10} = 0$$
  
 $y_0 = y_1 = -0.7652870$   
 $y'_{10} = 0.0002176$ 

F=1400, y = -1.0, Dy = 0.1,1=0, 1,2,....5

|      |             |  |  |   |  |                              | and the second sec |
|------|-------------|--|--|---|--|------------------------------|--|
| 71   | 70- 71      | <b>F</b> <sup>et</sup><br><b>F1</b> 0  | 1  | 2   | 3  | 4                            | 5  |
| -1.0 | 0.2506304   | -0.04733   | 196  |   |  |                              | -  |
| -0.9 | 0.1506304   | -0.02745   | 0.0024853  |   |  |                              |  |
| -0.8 | 0.0506304   | -0,00758   | 0.0021785  | 0.0024751   |  | 1.00                         |  |
| -0.7 | -0.0493696  | 0.01229  | 0.0034825  | 0.0024802   | 0.0024778  |                              |  |
| -0.6 | -0.11:93696 | 0.03216  | 0,0024803  | 0.0024836   | 0.0094790  | 0.0024772                    |  |
| -0.5 | +0.2493696  | 0.05204  | 0.0024653  | 0.0025027   | 0.0024804  | 0.00254157                   | 0.0024772  |
|      |             | and the second sec | Contraction of the second statement of the second sec | And the second se | and the second state of th | COLUMN TWO IS NOT THE OWNER. | Street, Square, and statistics.  |

y<sub>10</sub>= y<sub>1</sub> = -0.7493696

340 = 0.6024772

Tablo 28

F=1500, y1= -1.0, Dy1= 0.1, 1=0, 1,2,...5

| y1   | ys- yi    | J''10    | 1         | 2         | 3         | 4         | 5         |
|------|-----------|----------|-----------|-----------|-----------|-----------|-----------|
| -1.0 | -0.265222 | -0.05623 |           |           |           | 1         |           |
| -0.9 | -0.165222 | -0.03312 | 0.0050628 |           |           | 1.        |           |
| -0.8 | -0.065222 | -0.01000 | 0.0050761 | 0.0050842 |           |           | 1.1       |
| -0.7 | 0.034778  | 0.01310  | 0.0050618 | 0.0050619 | 0.0050697 |           |           |
| -0.6 | 0.134778  | 0.03621  | 0.0050628 | 0.0050628 | 0.0050772 | 0.0050671 | 1. 19     |
| -0.5 | 0.234778  | 0.05933  | 0.0050681 | 0.0050650 | 0.0050800 | 0.0050679 | 0.0050660 |

 $y'_{10} = 0$  $y_{0} = y_{1} = -0.734778$  $y''_{10} = 0.0050660$ 

F=1600, ya= -1.0, Dya= 0.1, 1=0, 1, 2, ....5

| 71   | Jor Jg     | 340      | 1         | 2         | 3         | 4         | 5         |
|------|------------|----------|-----------|-----------|-----------|-----------|-----------|
| -1.0 | 0,2786272  | -0.06630 | 1         |           |           |           |           |
| -0.9 | 0.1786272  | -0.03964 | 0.0079820 |           |           |           |           |
| -0.8 | 0.0786272  | -0.01298 | 0.0079820 | 0.0079820 |           |           |           |
| -0.7 | -0.0213728 | 0.01367  | 0.0079760 | 0.0079770 | 0.0079780 |           |           |
| -0.6 | -0.1213728 | 0.04033  | 0.0079720 | 0.0079770 | 0.0079770 | 0.0079780 |           |
| -0.5 | -0,2213728 | 0,06700  | 0.0079600 | 0.0079870 | 0.0079830 | 0.0079820 | 0.0079732 |

540ª 0

Jo= Ja = -0.7213288

340= 0.00797316

F=1700, yq = -1.0, Dyq = 0.1, 1 = 0, 4, 2,...5

| ALC: NOT THE OWNER.                             | to the first group by a property of           | adder vormanitärik aterenigente Farminette   | Distantial Including the rest of the local distance of the local d | the same in the same of the | states and the second states of the second states o | A DESCRIPTION OF TAXABLE PARTY.    | A Design of the local division of the local |
|---|---|--|--|---|--|------------------------------------|---|
| ¥1  | Je- 91  | y40  | 1  | 2   | 3  | 4                                  | 5   |
| -1.0  | 0,2909230                                     | -0+07765   |  |   | 1.1.1  |                                    |   |
| -0.9  | 0.1909230                                     | -0.04711   | 0.0111979  |   | 12010  | 1.1.1.1.1.1                        | 1.11  |
| -0.8  | 0.0909230                                     | -0.01658   | 0+0111788  | 0.0111614   |  |                                    |   |
| -0.7  | -0.0090770                                    | 0.01395  | 0.0111788  | 0.0111788   | 0.0111772  |                                    |   |
| -0.6  | -0.1090770                                    | 0.04448  | 0.0111788  | 0.0111788   | 0.0111788  | 0.0111771                          | 1.000   |
| -0.5  | -0.2090770                                    | 0.07502  | 0.0111804  | 0.0111895   | 0.0111699  | 0.0111775                          | 0.0111770   |
| And Man Property lies, Name of Street, or other | a planter or an experiment, the second second | And International Contraction of the International Contractional Contr | THE REPORT OF AN ADDRESS OF THE PARTY OF   | The state line was a special provided by which  | procession and the second s  | processing the second data product | the second s  |

J10 = 0

y.= y.= -0.7090770

340= 0.0111770

Calculation the value of F that makes  $y_{40}^{\prime\prime} = 0$ 

| 340  | P                     | 1                                  | 2                     | 3    | 4                      | 5  | 6                        | 7               |
|--|-----------------------|------------------------------------|-----------------------|------|------------------------|--|--------------------------|-----------------|
| 0.0111770  | 1700                  | 1351                               |                       |      |                        |  |                          |                 |
| 0.0079732  | 1600                  | 1351                               |                       |      |                        |  |                          |                 |
| 0,0050660  | 1500                  | 1334                               | 1304                  |      |                        |  |                          |                 |
| 0.0024772  | 1400                  | 1315                               | 1299                  | 1294 |                        |  |                          |                 |
| 0.0002176  | 1300                  | 1292                               | 1290                  | 1289 | 1289                   |  |                          |                 |
| -0.0015774   | 1200                  | 1262                               | 4277                  | 1283 | 1287                   | 1289   |                          | 1.00            |
| -0.0030131   | 1100                  | 1227                               | 1261                  | 1277 | 1286                   | 1289   | 1289                     |                 |
| -0.0000272   | 1000                  | 1185                               | 1211                  | 1269 | 1284                   | 1289   | 1289                     | 1289            |
| integration of integration on the later star fails and | COLUMN TWO IS NOT THE | AND AND PROVIDE AND THE OWNER WHEN | CONTRACTOR OF TAXABLE |      | the rest of the second | And the lot of the lot | the second second second | - laure and and |

We obtain F = 1289

5. Then we find  $y_4 = -0.76710bl_1$ , by using inverse interpolation, for F = 1289 H<sub>2</sub> which is shown in the Table 32. 6. This frequency F = 1289 H<sub>2</sub> is the vibration frequency, for which  $y_{10}^{\prime\prime} = 0$ , and the corresponding values of  $y_0, y_1, y_2 \cdots y_{10}$  can be obtained as follows:

From the equations (8) and (9)

$$y_{4} = -y_{-4} = -0.7674044$$

$$z_{4} = \frac{CE^{2}a_{0}y_{0}}{2}, \text{ for the from } C = 0.185 \times 10^{-9}$$

$$\frac{C}{2} = 0.925 \times 10^{-10}$$

$$CE^{2} = 0.0003078$$

$$\frac{CE^{2}}{2} = 0.0004537$$
Ethem  $z_{4} = 0.0004537$  (1). (-1.0)

= -0,0001537

By using the relations(6) and (7) we obtain

$$y_{2} = \frac{z_{4}}{(a_{4})^{3}} + 2y_{4} - y_{0}$$
  
=  $\frac{-0.0004537}{(1.0)^{3}} + 2(-0.7671044) - (-4)$   
=  $-0.5343625$   
 $z_{2} = 05^{2} a_{4}y_{4} + 2z_{4} - z_{0}$   
=  $-0.0005435$ 

and so on. The table of these results is shown below, table 33.

Calculation the value of  $y_1$  corresponds to F = 1239

| Fi   | F - H1 | y4          | 1          | 2          | 3                | 4            | 5         |
|------|--------|-------------|------------|------------|------------------|--------------|-----------|
| 1000 | 289    | -0.8212715  |            |            |                  |              | 1         |
| 1100 | 189    | -0.8011732  | -0.7631874 |            |                  | Presentation | North La  |
| 1200 | 89     | -0.7824976  | -0.7652132 | -0.7670728 | en resigning for | n miner      |           |
| 1300 | -11    | -0.76524:18 | -0.7673296 | -0.7671018 | -0.7670986       |              | 121       |
| 1400 | -111   | -0.7493696  | -0,7693224 | -0.7670525 | -0.7670618       | -0.7671026   |           |
| 1500 | -211   | 40.7317778  | -0.7712781 | -0.7670103 | -0.7670543       | -0.7671010   | -0.767104 |

J = -0.7671044

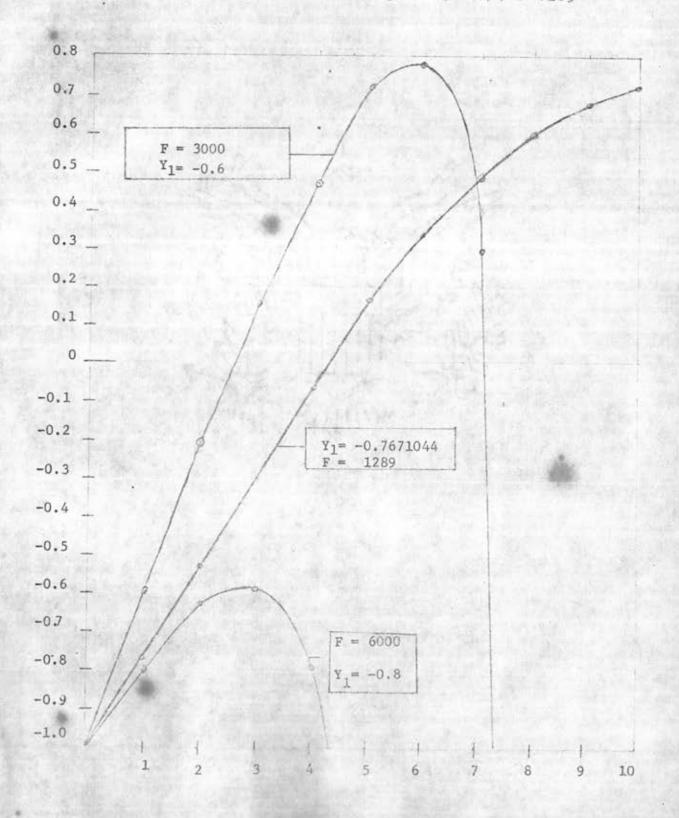
Values of y1 and Z1, 1=0, 1, 2, 3 ..... 10, corresponding to F=1289, y1 = -0.7671044

| 1  | Zi+1=CF2e171+ 221- 2 | 1-1 JIH " (a) 3 + 241- JI-1 |
|----|----------------------|-----------------------------|
| 0  | Z0# 0.0000000        | y₀= -1.0000000              |
| 1  | 24= -0.0001537       | Ja= -0. 7674044             |
| 2  | Z2= -0.0005435       | ¥2 <sup>==</sup> -0+5343625 |
| 3  | 23= -0.0010978       | 73= -0.3021641              |
| 4  | Z24= -0.0017451      | ¥1,= +0.0710635             |
| 5  | Z5= -0.0024143       | y5 0.1582920                |
| 6  | Z6= -0,0031200       | V6m 0.3304196               |
| 7  | 87= -0.0037748       | yy= 0.4775872               |
| 8  | Z8= -0.0043561       | 78th 0,5945564              |
| 9  | 29= -0.0048459       | yg= 0.6766768               |
| 10 | Z40-0.0091521        | y40= 0.7200300              |

They are represented in graphical form in the Figure (1.)



Node Shape of the actually frequency, F=1289



CONCLUSION



For the non-uniform bar case we can find the made shope by using the difference equation and the given parameters varying along the bar length. From the derived table one can obtain the made shape by using the interpolation and inverse interpolation method.

Thus by using the numerical method with the aid of a computer and a deak calculator we can obtain the mode shape and its natural frequencies. These results are useful in finding the evertence of musical instruments for which the cross-section is non-uniform.