CHAPTER II

THEORETICAL CONSIDERATIONS

The Maximum Principal Stress Theory

This is often called the Rankine theory, and it states that inelastic action at any point in a material at which any state of stress exists begin only when the maximum principal stress at the point reaches a value equal to the tensile (or compressive) elastic limit or yield strength of the material as found in a simple tension (or compressive) test, regardless of the normal or shearing stresses that occur on other planes through the point.

i.e. $$\sigma_e = \sigma_1 , \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau^2} \qquad (1)$$

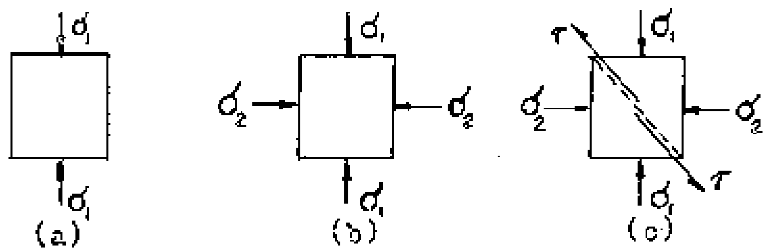

Fig. 1 Stress on a Body.

According to this theory, if block in Fig. 1a reaches its elastic limit when subjected to the stress σ_1, the elastic limit will still be σ_1, even if the block is subjected to the stress σ_2 (Fig. 1b) in addition to σ_1.

The Maximum Shearing Stress Theory

A special case of Coulomb's theory, as proposed by Guest, states that inelastic action at any point in a body at which any state of stress exists begins only when the maximum shearing stress on some plane through the point reaches a value equal to the maximum shearing stress in a tension specimen when yielding starts (see Fig. 1c). This means that the shearing elastic limit must be more than one half the tensile elastic limit, since the maximum shearing stress in a tension specimen (on a 45° oblique plane) is one-half the maximum tensile stress in the specimen.

i.e. Maximum shear stress $= \frac{1}{2}\left(\sigma_1 - \sigma_2\right) = \sigma_e$ (2)

It is seen that, σ_1 will be higher when σ_2 is tensile stress.

The Maximum Strain Theory

This is often called St. Venant's theory, and it states that inelastic action at a point in a body at which any state of stress exists begins only when the maximum strain at the point reaches a value equal to that which occurs when inelastic action begins in the material under a uniaxial state of stress, such as occurs in a specimen in the tension test.

$$\epsilon_c = \frac{\sigma_e}{E} = \frac{1}{E}\left(\sigma_1 - \mu\sigma_2 - \mu\sigma_3\right) \qquad (3a)$$

$$\text{or} \quad \sigma_e = \sigma_1 - \mu\sigma_2 - \mu\sigma_3 \qquad (3b)$$

According to this theory of failure, σ_1 could be increased to a value somewhat higher than σ_e without causing yielding if σ_2 and σ_3 are tensile stress, but for compressive stresses σ_2 and σ_3, the value of σ_1 will be lower than σ_e.

The Maximum Strain Energy Theory

This theory was proposed by Beltrami and Haigh, and states that inelastic action at any point in a body due to any state of stress begins when the energy per unit volume of the material reachs the same value as when the elastic limit is reaches under a uniaxial state of stress, as in a simple test.

i.e. $$\omega = \frac{\sigma_e^2}{2E} = \frac{1}{2E}\left[(\sigma_1^2+\sigma_2^2+\sigma_3^2) - 2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1)\right] \quad (4a)$$

or $$\sigma_e^2 = \sigma_1^2+\sigma_2^2+\sigma_3^2 - 2\mu(\sigma_1\sigma_2+\sigma_2\sigma_3+\sigma_3\sigma_1) \quad (4b)$$

The Energy of Distortion Theory

The total strain energy per unit volume as given by equation 4a may be resolved into two component parts, one part associated with the change in volume of unit volume and the other part associated with the (volume-constant) distortion or change in shape of unit volume. Hence

$$w = \omega_v + \omega_d \quad (5)$$

$$\omega_v = \frac{1}{2}(\sigma_{avg}^2/E_v) \quad (6)$$

then $$\omega_d = \frac{(1+\mu)}{6E}\left[(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2\right] \quad (7)$$

This theory was developed independently by H.Hencky and

R. von Mises, and it states that inelastic action at any point in a body under any combination of stresses begins only when the strain energy of distortion per unit volume at the point is equal to the strain energy of distortion per unit volume at any point in a bar stressed to the elastic limit under a state of uniaxial stress as in a simple tension (or compression) test. Therefore

$$\sigma_e^2 = \frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right] \qquad (8)$$

Since change of shape involves shearing stresses, the energy of distortion theory is sometimes called (somewhat erroneously) The Shear Energy Theory.

When the yield stress and Poisson's ratio as obtained from a simple tension ~~test~~ test and a torsion test are known, the principal stresses are divided by the yield stress, then plotted on the curve sheet two at a time, as shown in Fig. 2. This represents, in a non-dimensional form, the combination of stresses σ_1 and σ_2 necessary to cause failure for each theory.

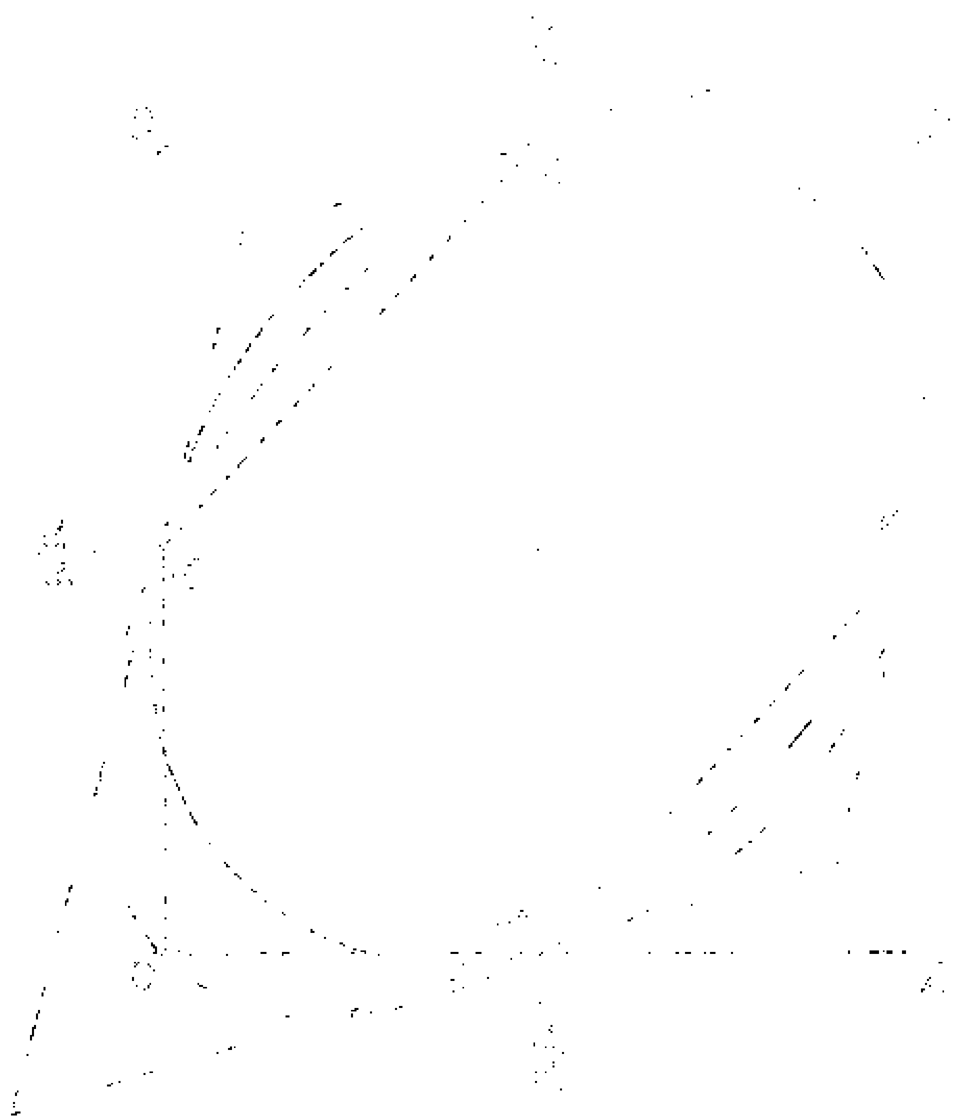

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Fig.2 [illegible]