A MATHEMATICAL STUDY OF MAXWELL'S EQUATIONS

by

Suchat Chantip

B.Sc., Chulalongkorn University, 1965

Thesis

Submitted in partial fulfillment of the requirements for the

Degree of Master of Science

in

Department of Mathematics

April, 1967

(B.E. 2510)

Accepted by the Graduate School, Chulalongkorn University in partial fulfillment of the requirements for the Degree of master of Science.

....
Dean of the Graduate School

Thesis Committee ....Chairman

....

Bhijayo Panyapin...............

....

Thesis Supervisor ....

Date ..8th..May..1967.....

ACKNOWLEDGEMENT

I wish to express here my gratitude to Dr. Robert H.B. Exell, my thesis supervisor, for his generous help and instruction at all times, for many stimulating ideas, and for lending me some books.

Suchat Chantip.

ABSTRACT

Part one : we find from Maxwell's equations that electromagnetic waves are propagated in ether with a velocity c which is a characteristic constant of the theory (c is the ratio of the electromagnetic unit to the electrostatic unit of charge). This velocity ($c = 3\times 10^{10}$ cm./sec.) agrees with the observed velocity of light in ether. From this and a variety of other reasons, light is identified as consisting of such electro - magnetic waves. The results of the Michelson and Morley experiment indicated that the velocity of light is always constant and equal to c in all systems (at rest or moving.) This requires us to ensure that the form of Maxwell's equations should be the same in all systems of reference and the velocity of electromagnetic waves deduced from them will be the same. In this part we also find that Maxwell's equations are invariant under the Lorentz transformation, but not under the Galilean transformation. This shows the correctness of the Lorentz transformation, and inapplicability of the Galilean transformation.

Part two : the aims of this part are to write Maxwell's equations in compact forms and to introduce the magnetic charge to them . To do this , we procceed as follows :

We combine $\bar{E}$ and $\bar{H}$ into a single complex field $\bar{\Psi}$, ie. $\bar{\Psi} = \bar{E} + i\bar{H}$ where $|\bar{\Psi}|^2$ is proportional to the energy density in the field.

We introduce the vecter $\overline{\Psi}$ into Maxwell 's equations in three dimensional form to reduce them to two equations.

We next discuss the construction of four-vectors, and define the cross product of two vectors in any number of dimensions.

The application of these four-vectors to Maxwell's equations reduces them to two equations in four dimensional form. Then these two equations are reduced to one equation.

Finally magnetic charge is introduced in Maxwell's equations and the new equations are written in the compact forms as before.

TABLE OF CONTENTS

Page

ACKNOWLEDGEMENTS iii

ABSTRACT iv

PART ONE :

CHAPTER I : THE VELOCITY OF ELECTROMAGNETIC WAVES 1

CHAPTER II : TRANSFORMATION OF MAXWELL'S EQUATIONS BY THE GALILEAN TRANSFORMATION 12

CHAPTER III: TRANSFORMATION OF MAXWELL'S EQUATIONS BY THE LORENTZ TRANSFORMATION 18

PART TWO :

CHAPTER I : THE FORMS OF MAXWELL'S EQUATIONS 24

CHAPTER II : VECTORS IN FOUR DIMENSIONS 26

CHAPTER III : MAXWELL'S EQUATIONS IN FOUR DIMENSIONAL FORM 30

CHAPTER IV : MAGNETIC CHARGE 36

BIBLIOGRAPHY 40