Chapter 4

T_c equations

In this chapter I will show how to obtain T_c from Eq.(3.55).

Since the structure of the matrix in equation (3.55), which determines T_c, depends on the nature of different conducting layers in the unit cell and their surrounding.

The transition temperature T_c for a three-layer type is given by the solution of the determinant,

$$\begin{vmatrix} 1-N_dV_dF+K'\left(1-\frac{V_{er}}{V_d}\right) & -\left[\frac{V_d}{V_c}V_{er}N_dF+K'\left(1-\frac{V_{er}}{V_c}\right)\right] & 0 \\ -\left[\frac{V_c}{V_d}V_{er}N_cF+K\left(1-\frac{V_{er}}{V_d}\right)\right] & 1-N_cV_cF+K\left(1-\frac{V_{er}}{V_c}\right) & -\left[\frac{V_c}{V_d}V_{er}N_cF+K\left(1-\frac{V_{er}}{V_d}\right)\right] \\ 0 & -\left[\frac{V_d}{V_c}V_{er}N_dF+K'\left(1-\frac{V_{er}}{V_c}\right)\right] & 1-N_dV_dF+K'\left(1-\frac{V_{er}}{V_d}\right) \end{vmatrix} = 0$$

One superconducting layer per unit cell

We first consider the one layer per unit cell system,

V_c

Fig. 4.1 One superconducting layers per unit cell here $V_d = 0$, $K = K' = 0$ and hence

$$1 - N_c V_c F = 0$$

where
$$F = 2\pi T_c \sum_{n=0} \frac{1}{\omega_n}$$

$$= \ln(1.13\,\omega_D / T_c) \quad \text{(see appendix)}$$

then

$$T_c = 1.13\omega \exp[-1/N_c V_c] \tag{4.1}$$

here ω is the cutoff frequency.

Two superconducting layers per unit cell

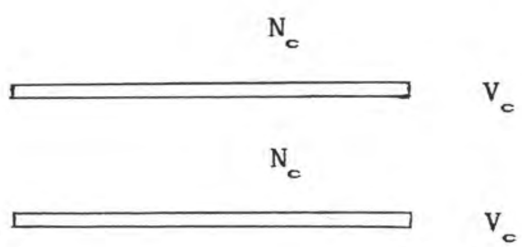

Fig. 4.2 Two superconducting layers per unit cell

Let us now calculate the critical temperature for the sample where there are two superconducting layers per unit cell, our T_c is determined by the condition

$$\begin{vmatrix} 1 - N_d V_d F + K'\left(1 - \dfrac{V_{er}}{V_d}\right) & -\left[\dfrac{V_d}{V_c} V_{er} N_d F + K'\left(1 - \dfrac{V_{er}}{V_c}\right)\right] \\ -\left[\dfrac{V_c}{V_d} V_{er} N_c F + K\left(1 - \dfrac{V_{er}}{V_d}\right)\right] & 1 - N_c V_c F + K\left(1 - \dfrac{V_{er}}{V_c}\right) \end{vmatrix} = 0. \tag{4.2}$$

Solving Eq.(4.2), we obtain

$$T_c = 1.13 \quad \exp\left[- \frac{N_cV_c+N_dV_d+N_dK\left(V_d-\frac{V_{er}^2}{V_c}\right)+N_cK\left(V_c-\frac{V_{er}^2}{V_d}\right)}{2N_cN_d(V_cV_d-V_{er}^2)} \right.$$

$$+ \left\{ \left[N_cV_c(1+K')+N_dV_d(1+K)-V_{er}^2\left(\frac{N_dK}{V_c}+\frac{N_cK'}{V_d}\right)\right]^2 \right.$$

$$\left. \left. - 4N_cN_d(V_cV_d-V_{er}^2)\left[1+K\left(1-\frac{V_{er}}{V_c}\right)+K\left(1-\frac{V_{er}}{V_d}\right)\right] \right\}^{1/2} \Big/ 2N_cN_d(V_cV_d-V_{er}^2) \right] \tag{4.3}$$

In solving Eq.(4.2) for F, we obtain two solutions for T_c. The solution which gives the highest T_c is acceptable, while the other with the lower T_c value is rejected because the system is already superconducting once the system reaches the high T_c value.

Consider limit t tends to zero, $N_c=N_d$, and $V_c=V_d$. From Eq.(4.3) we obtain

$$T_c \quad = \quad 1.13\omega \exp(-1/N_c(V_c+V_{er})) \tag{4.4}$$

which is precisely the result of Ihm and Yu (46).

From equation (4.3), it is concluded that T_c is always raised by the presence of V_{er}. We also see clearly that the effect of small but finite t is detrimental to high-T_c superconductivity in consistent with ref.(55). Recently Cava et. al.(56) synthesized $La_{1.6}Sr_{0.4}CaCuO_2O_6$, the layer-stacking sequence in this compound is $LaO\text{-}CuO_2\text{-}Ca\text{-}CuO_2$. This system is a bi-layer superconductor. The upper and lower CuO_2 layers of this system are equivalent.

Equation (4.3) also gives

$$T_c = 1.13\omega \exp(-1/N_c(V_c+V_{er})) \tag{4.5}$$

for this system, assuming that $t \neq 0$. Eqs.(4.4) and (4.5) are identical this is because of the fact that in a system of equivalent double layers, whether there is a direct electron hopping process or not, does not change the physical situation at all.

For two inequivalent double layer, Eq.(4.3) yields

$$T_c = 1.13\omega \exp\left[-\frac{N_cV_c+N_dV_d+N_dK\left(V_d-\frac{V_{er}^2}{V_c}\right)+N_cK\left(V_c-\frac{V_{er}^2}{V_d}\right)}{2N_cN_d(V_cV_d-V_{er}^2)}\right] \tag{4.6}$$

where $t=0$.

<u>Three identical layers per unit cell</u>

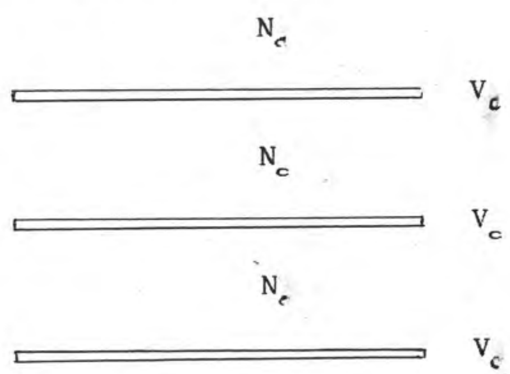

Fig. 4.3 Three identical layers per unit cell

For three identical layers, the interactions $V_c=V_d$, if $t=0$ and $N_c=N_d$, the 3X3 determinant of equation (3.55) gives

$$T_c = 1.13\omega \exp(-1/N_c(V_c+\sqrt{2}\,V_{er})) \tag{4.7}$$

We recover the formula of Ihm and Yu (46).

We next consider, for example the compound $Tl_2Ba_2Ca_2Cu_3O_{10}$, here the TlO-TlO-CuO_2 layers bounding the middle CuO_2 sheets are mediating the interaction and maintaining its 2 dimensional property. These layers have the density of states, at the Fermi level, of the same magnitude with the carrier from the middle CuO_2 layer. It is plausible that the excitations from the TlO-TlO layers have a strong influence on the interactions in the bounding CuO_2 layer, the two outer CuO_2 layers are therefore different from the middle CuO_2 layer. We have for this system

$$\begin{aligned}
T_c = 1.13\omega \exp\Bigg[- \Bigg[& N_cV_c\left(1+K'\left(1-\frac{V_{er}}{V_d}\right)\right)+N_dV_d\left(1+K\left(1-\frac{V_{er}}{V_c}\right)\right) \\
& + 2KV_{er}\left[\left(1-\frac{V_{er}}{V_d}\right)\frac{V_dN_d}{V_c}+\left(1-\frac{V_{er}}{V_c}\right)\frac{V_cV_{er}}{V_d}\right]\Bigg) / 2N_cN_d(V_cV_d-2V_{er}^2) \\
& - \Bigg[\Bigg(N_cV_c\left(1+K'\left(1-\frac{V_{er}}{V_d}\right)\right)+N_dV_d\left(1+K\left(1-\frac{V_{er}}{V_c}\right)\right)+2KV_{er}\Bigg(\left(1-\frac{V_{er}}{V_d}\right)\frac{V_dN_d}{V_c} \\
& + \left(1-\frac{V_{er}}{V_c}\right)\frac{V_cV_{er}}{V_d}\Bigg)\Bigg)^2 - 4N_cN_d[V_cV_d-2V_{er}^2]\left[1+K\left(1-\frac{V_{er}}{V_c}\right)+K'\left(1-\frac{V_{er}}{V_d}\right)\right. \\
& \left. - KK'\left(1-\frac{V_{er}}{V_c}\right)\left(1-\frac{V_{er}}{V_d}\right)\right]\Bigg]^{1/2} \Bigg/ 2N_cN_d(V_cV_d-2V_{er}) \Bigg]
\end{aligned} \tag{4.8}$$

For the bounding layers of insulating nature $V_d=0=V_{er}$, non-zero hopping becomes important and we have a system of a superconducting plane interposed between two insulating sheets.

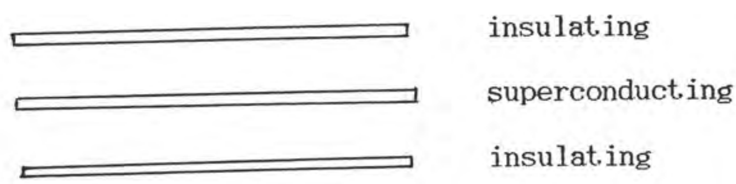

Fig. 4.4 A superconducting layer between two insulating layers.

Equation (4.8) yields

$$T_c = 1.13\omega\exp\{ -1/N_cV_c - \pi^2t^2N_d/4T_{co}\}. \tag{4.9}$$

The critical temperature decreases with the increased direct hopping between layers.

In order to further understand the physics of high-T_c superconductor it is important to study the influence of the decoupling of the CuO_2 bilayer by the insulating layer.

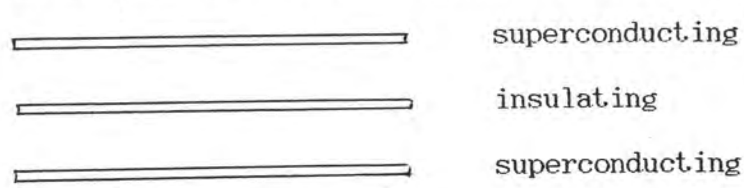

Fig. 4.5 An insulating layer between two superconducting layers.

Here $V_c = 0 = V_{er}$, and Eq.(4.8) yields

$$T_c = 1.13\omega\exp\{ -1/N_dV_d - \pi^2t^2N_c/4T_{co}\}. \tag{4.10}$$

This last equation indicates that the coupling between the CuO_2 layers across the insulating layer is reduced.